Elements of Machine Design

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ELEMENTS OF MACHINE DESIGN

[IN MKS SYSTEM]

by

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PREFACE

Although at present several books by foreign authors exist in the subject of Machine Design, many students and teachers alike have felt the need for a book on the subject particularly suited to the syllabi in machine design for the degree course in mechanical engineering of Indian Universities. The present book is an attempt to fill the gap.

The authors have tried to develop the subject from the fundamentals and have given various practical applications of the important formulas. Instead of working out designs of complete machines such as steam engines, internal combustion engines, boilers, etc., the authors have preferred to concentrate upon the design of basic elements which constitute these machines. A sufficient number of problems has been solved to illustrate clearly the applications of the different theoretical results.

A book of this kind naturally draws from several sources of information and knowledge. We acknowledge our deep gratitude to many teachers, who have contributed to our knowledge of the subject directly or through their writings.

Our thanks are due to the Senates of the Universities of Manchester, Gujarat, Bombay, Poona and Roorkee, and also of the M. S. University of Baroda and Sardar Vallabhbhai Vidyapeeth and to the Council of the Institution of Civil Engineers (England) for permission to include questions taken from the papers set at their examinations.

We have great pleasure in expressing our gratitude to Dr. M. A. Date for his help and encouragement. Finally the authors wish to acknowledge their indebtedness to Shri R. C. Patel of Charotar Book Stall for his very painstaking work in reading through the proofs, to Shri L. D. Bhatt for typing the manuscript, to Prabhat Process Studio, Ahmedabad for excellent block making and to the management and the conscientious staff of the Anand Press, Anand for the beautiful printing and get up of the work.

The authors wish their readers a profitable and enjoyable journey through the text book. All comments and opinions suggesting any improvement in the book will be much appreciated.

Vallabh Vidyanagar
1st January, 1962

N. C. Pandya
C. S. Shah

PREFACE TO FOURTH EDITION

We have availed of the opportunity of printing this edition to enlarge the text so as to increase its utility to the student world. New topics such as brakes, clutches, press, etc. have been added. Students are also introduced to the S.I. system, which is now adopted internationally. Some illustrative examples solved by the M.K.S. system have again been solved by the S.I. approach. This, it is believed, would familiarise the student with S.I. units. Special mention must be made of the second chapter, in which fatigue considerations in design have been dealt with. In the majority of chapters the principle of fatigue design is taken into account in proportioning the parts. It is hoped that these additions to the text materials of the previous edition would be found helpful especially to the final year degree students.

Our thanks are due to the Senates of Benaras Hindu University and the University of Rajasthan for permission to include questions taken from the papers set at their examinations.

Suggestions to improve the usefulness of this book to students will be gratefully welcomed.

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CHAPTER 1

MATERIALS OF CONSTRUCTION AND THEIR PROPERTIES

1-1. Introduction:

A machine is a combination of resistant bodies, with successfully constrained relative motions, which is used for transforming other forms of energy into mechanical energy or transmitting and modifying available energy to do some particular kind of work. The machine is known as a heat engine when it receives heat energy and transforms it into mechanical energy. The majority of machines receive mechanical energy, and modify it so that the energy can be used for doing some specific task for which it is designed; common examples of such machines being hoist, lathe, screw jack, etc.

The transmission and modification of energy within the machine require the inclusion of a number of elements, which are so designed that they carry with safety the forces to which they are subjected; in addition the desired motion is produced so that the machine can perform its task successfully. The analysis of forces involved and the design of machine parts, so that they can perform their duties *without failure or undue distortion,* lie within the province of machine design. In study of this subject we are required to apply constantly our knowledge of mathematics, classical mechanics, strength of materials, mechanics of machines, metallography and technical drawing.

1-2. General considerations:

One of the first point to be decided when designing a certain machine part is the material of which the part is to be made. The choice of the material is governed by the following important considerations:

(i) Suitability of the material of the component for working conditions during service

(ii) Amenability of the material to the processes required in making the component

(iii) Cost of material in relation to selling price of the component.

The quantity required, delivery date, material availability and scrap utilisation are the other factors which determine the choice of material.

Materials of construction are classified as metallic or non-metallic. The metallic materials are further classified as ferrous and non-ferrous. Ferrous materials consist chiefly of iron with comparatively small addition of other materials. Non-ferrous materials contain little or no iron.

Ferrous materials are iron and its alloys such as cast iron, malleable cast iron, wrought iron and steel. Non-ferrous materials include copper, zinc, tin, lead and aluminium and alloys produced by combining these elements such as brass, bronze, duralumin etc. Non-metallic materials include rubber, leather, plastics etc.

1-3. Mechanical properties of materials of construction:

The proper and efficient use of the materials of construction requires considerable knowledge of their mechanical properties. The mechanical properties of the materials are those properties which describe the behaviour of the material under mechanical usage. The most important mechanical properties are strength, elasticity, stiffness, ductility, hardness, malleability, resilience, toughness, creep and machineability.

Strength is the ability of the material to resist stress without failure. Several materials such as structural steel, copper, aluminium, etc. have equal strength in tension or compression, but their strength in shear is about two-thirds of the strength in tension, while in grey cast iron the strength in tension and shear is a fraction of the strength in compression. The measure of the strength is the ultimate stress. The values of strength usually fall when the metal is heated.

Elasticity is the property of regaining original shape after deformation. All the materials of construction are elastic but the degree of elasticity varies with different materials. This property is exceedingly important in precision tools and machines. Steel is highly elastic material.

Plasticity is the property that enables the formation of a permanent deformation in a material. Stiffness is the property by virtue of which a material can resist deformation. The measure of the stiffness is the modulus of elasticity. This property is desirable in materials used in machines, columns, beams and machine tools.

Ductility is the property of material that enables it to be drawn out or elongated to an appreciable extent before rupture occurs. The percentage

elongation and the percentage of reduction of area before rupture of a test specimen are measures of the ductility of a material.

✓ Brittleness is opposite to ductility. Brittle materials show little deformation before rupturing.

Materials with more than 15% elongation are usually considered ductile. Those with less than 5% elongation are considered brittle. Those between 5 and 15% are of intermediate ductility. Mild steel, wrought iron, copper and aluminium are ductile materials Cast iron is the most brittle material. The property of ductility is desirable in machine parts which may be subjected tc sudden and severe loads.

Malleability is the property of the material that enables it to undergo grea change in shape under compressive stress without rupture. Malleable material may be hammered or rolled into any desired shape without rupture. Soft steel wrought iron, copper and aluminium are malleable metals.

Hardness of a material enables it to resist abrasion or indentation. It i usually stated relative to the hardness of other materials. This property i decreased by heating.

Resilience is that property of the material which enables it to store energ and resist shock and impact. The measure of resilience is the amount of energy that can be stored per unit volume after being stressed to elastic limit Thi property is desirable in materials for springs.

Toughness is the property which enables a material to be twisted, bent or stretched under a high stress before rupture The measure of toughness is the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is decreased by heating

Creep is that property of the material which causes the material under constant stress to deform slowly but progressively over a period of time. Creep occurs in steel at high temperatures. This property assumes importance in design of turbines, boilers and internal combustion engines.

Machineability is the readiness with which a material may be worked with cutting tools. Creep is the prop of mat by war which mat can deform over a period of const. stress

1-4. Determination of mechanical properties:

In order to determine the mechanical properties of the material, certain tests are carried out in mechanical testing laboratories. These tests are carried out according to various standard procedures laid down for the purpose. The simplest test that can be made on most materials is the *static tensile test.* The procedure to carry out this test is suggested by **Indian Standards Institution.** The following informations are obtained from this test:

✓(i) Ultimate tensile strength ✓(ii) Elastic limit

✓(iii) Yield point ✓(iv) Percentage elongation

✓(v) Percentage reduction in area

Other tests commonly employed are compression, torsion, flexure, cold bending, hardness, impact and fatigue. The data of these various tests are usually shown graphically by the stress strain diagrams.

1-5. Processing methods — Hot working processes:

The working of a metal may be divided into two types of processes: (i) Hot working and (ii) Cold working.

By hot working is meant processes such as rolling, forging, extrusion and hot pressing. In this working the metal is heated sufficiently to make it plastic and easily worked. The temperature of the heated metal or alloy should be above the re-crystallisation temperature. This temperature is different for different metals.

Hot rolling is used to create a bar of material of particular shape and dimensions. The principal rolled steel sections are plates, angles, tees, channels and joists; round, hexagonal and square bars for forging and machining operations; sheets, rails, etc. All of them are available in many different sizes and in different materials. The materials most available in the hot rolled bar sizes are steel, aluminium and copper alloys. Tubes may be manufactured by hot rolling of strip or plate; the product may be butt welded or lap welded.

Forging is the hot working of metals by hammers, presses or forging machines. For small work the forging is carried out with hand hammers but for large work hammers and forging machines are used. Forging alters the internal structure of metal which results in increased strength and ductility. Compared with castings, forgings have greater strength for the same weight. Forging should be carried out between proper temperature range. If the temperature is too high the metal will be weak and brittle. If the temperature is too low, there will be internal stresses which may lead to distortion or cracking.

Many small parts are drop forged. In drop forging, solid lump with little or no previous treatment by hand is squeezed between dies to the shape required with one or more blows from a drop hammer. The component can be made to dimensions and with a good surface so that machining may be unnecessary. The limitations of this process are that the number of parts should be great and complicated shapes cannot be produced as they could not be removed from dies.

Extrusion is a process where a heated blank is caused to flow through a restricted orifice under great pressure. Very complicated shapes may be produced by the extrusion process. The process is restricted to materials of low melting points such as brass, aluminium and certain alloys of tin, lead and other soft metals.

Hot pressing consists of forming metal to shape in a very rigid type of power press. A hot piece of metal is pressed and extruded in suitable dies into a smoothly finished piece to accurate dimensions. Automobile valves are formed by this process.

1-6. Cold working processes:

By cold working is meant the forming of a metal usually at room temperature. Though this temperature is higher sometimes but always lower than re-crystallisation temperature. Cold working may vary from a simple bend to great deformation produced by deep pressing and tube drawing. The result of cold work is to increase the hardness and the tensile strength but to decrease the ductility and shock resistance. Cold worked parts have a bright new finish, are more accurate and require less machining. Where cold work is considerable, the part may be annealed at some intermediate stage or stages of work. In cold working the surface of a material is very important as scale may be worked into the finished article with serious results. Some of cold working processes are drawing, heading, spinning, stamping, etc.

Drawing is a process by which the cross section of a metal is diminished by pulling it through an accurately formed hole in a drawing die. The operation is performed cold and only simpler forms can be produced without excessive resistance and tearing

Heading is a cold working process in which the metal is gathered or upset The operation is commonly used to make screw and rivet heads The blank is usually a piece of wire of suitable length and cross section, one end is cold forged in dies to form the desired shape of the head Annealing may be required after cold heading

Spinning is the operation of working sheet material around a rotating form into a circular shape. Pressure is applied to the sheet by means of a blunt nosed tool which presses it against the former. This is an economical method of forming parts if the quantities are small.

Stamping is the term used to describe punch press operations such as blanking, coining, forming and shallow drawing.

1-7. Casting:

Casting is the oldest form of metal shaping and is still the basic engineering process since most metals are melted and cast from ores. Castings are made of iron, steel, various brasses and bronzes, aluminium and its alloys and the various white metal alloys.

Patterns may be made of wood or metal and with its help the sand mould is formed in which molten metal is poured. The mould is dried before the metal is poured. Metal in cooling solidifies to the form outlined in the mould.

In die casting process the mould is usually made of steel and molten metal is poured or forced under pressure into the mould. This method is used for mass production only.

Non-ferrous alloys are sometimes cast centrifugally. Molten metal is poured into a rapidly rotating cylindrical mould and is held against the mould by centrifugal force so that core is not required. On cooling the casting is complete. Such castings are generally denser and more homogeneous than ordinary sand castings. This process is limited to simple shapes and to fairly large quantities.

The following precautions should be observed in design of castings:

(i) All sections should be designed as far as possible with a uniform thickness.
(ii) All walls should be sufficiently thick to allow the molten metal to flow freely into all corners.
(iii) Adjoining sections should be designed with generous fillets or radii.
(iv) Parts should be designed so that patterns may be drawn readily from the moulds.
(v) A complicated part should be designed in two or more castings. These castings are assembled by fasteners.
(vi) Where the section uniformity is not possible, light sections should be blended into heavy sections.

The thickness of casting determined by calculations is often too small to permit the production of good casting. The following are the minimum values of the thickness for various castings:

Material	Minimum thickness in mm
Grey cast iron	6
Malleable cast iron	6
Steel casting	6
Brass	3
Bronze	3
Aluminium	3

1-8. Powder metallurgy:

It is the art of making small components by heat treatment of compressed metallic powders sometimes with inclusion of non-metallic material.

The powdered metals in desired proportions are compressed in moulds under a very high pressure varying from 700 to 14,000 kg/sq cm depending on the metal. The compacted part is heated at a temperature which is less than melting point of the major ingradient. The disadvantages of this method are (i) low strength of the component (ii) higher cost of material and (iii) the limited range of materials which can be used.

Filaments of refractory metals such as tungsten, self lubricating bearings, tungsten carbide tips for cutting tools and iron alloys for permanent magnets are examples of articles made from powdered metal. By this process small components can be made out of some metals whose melting point is too high to allow the use of die casting.

1·9. Finishing processes:

In many processes such as die casting, rolling, extruding, etc. accuracy and smoothness may be obtained so that components manufactured by these methods require no further finishing operations in order to use the component. However, castings, forgings and welded parts do not have the accuracy of dimensions or necessary smoothness. Therefore such parts may be subjected to further finishing operations. Finishing operations are necessary from many considerations such as lightness, attractive appearance, etc.

Finishing may be accomplished by abrasives or by cutting tools, the former are finding more extended use. The finishing processes comprise turning, milling, shaping, slotting, planing, drilling, reaming, boring, broaching, grinding, honing and lapping. Manytimes hand scraping is used to finish the product.

1-10. Accuracy:

When two parts have to fit together, one within the other, a definite difference in size is usually necessary depending upon the nature of the fit required. This difference in dimension between two parts is called the *allowance*. As it is difficult to manufacture any part true to size, certain maximum permissible deviation from the given dimension is allowed. This variation which covers imperfection of workmanship is termed *tolerance*.

This variation of dimension depends upon the kind of component. Tolerances and allowances have been standardized in each country by standards association. In our country, the recommended values for tolerances and allowances are given in specification IS: 919 of 1959. IS is an abbreviation for Indian Standards. These standards are laid down by Indian Standards Institution.

If close tolerances are necessary for interchangeability of parts, jigs and fixtures may be carefully prepared so that hole locations and other dimensions may be duplicated on any number of parts. Jigs guide the tool as well as hold the work while fixtures only hold the work and simplify and regulate the set up.

Except for drilling machines, all machine tools of cutting type give about the same degree of accuracy under ordinary conditions.

1-11. Ferrous metals — Cast Iron and Wrought Iron:

All ferrous metals are made by refining pig iron and adding to it other elements to produce a desired combination of mechanical properties. Cast iron is an alloy of iron, carbon and silicon and is hard and brittle metal. The carbon content is always more than 1·7% and often around 3%. The carbon may be present in two forms: as free carbon or graphite and as combined carbon or iron carbide (Fe_3C). Cast irons in which the carbon is mainly graphite are grey in colour and are called grey cast irons. They are extensively used for machine parts because they are inexpensive, can be given almost any desired form and have high compressive strength. Graphite is an excellent lubricant, and grey cast iron is easily machined as the tool is lubricated and chips break off readily. The freedom with which articles slide over a smooth surface of cast iron is largely due to graphite in the surface. However, *brittleness and lack of ductility and shock resistance prohibit their use in parts subject to high tensile stress or suddenly applied loads.* Its use above a temperature of 300°C is avoided.

Alloys in which carbon is in form of iron carbide are referred to as white cast iron because they have a whitish appearance. Iron carbide is a hard, brittle substance and its presence increases the hardness of cast iron. *White cast iron is almost unmachineable and is used somewhat in parts which require abrasion resistance.*

Silicon is used as a softener in cast iron. Increasing the silicon content of cast iron increases the free carbon and decreases

the combined carbon. Manganese tends to harden cast iron as it promotes combined carbon. In a foundry balance has to be struck between silicon and manganese contents so as to obtain a machineable but strong casting.

The speed of cooling has a considerable influence on the final hardness of cast iron. Castings of light section cool more rapidly than heavy castings thus results in formation of more combined carbon and less free carbon with a consequent increase in hardness. For these reasons for light castings more silicon is required to encourage formation of graphite.

Malleable cast iron is white cast iron which is rendered malleable by proper annealing. Malleable iron is an inexpensive material tougher than grey cast iron and more resistant to bending and twisting. Malleable cast iron is useful for many purposes *such as gear housing, brake pedals, plough, tractor and various* automobile parts.

In some castings, in order to have a hard durable surface, the casting is chilled. Such castings are produced by burying iron plates in the mould; as a result, the metal coming in contact with these plates will be cooled rapidly and will be harder than the rest of the casting. Chilled castings are used to some extent as wheels of rail road cars.

Wrought iron is a mechanical mixture of pig iron and uni- [illegible] slag. It possesses the important [illegible] toughness. It is suitable [illegible] It has also got excellent welding properties. With the introduction of steel the use of wrought iron has decreased although it is still used extensively for chains and crane hooks, for bolts subjected to shock loads, for pipes, pipe fittings and culvert plates. The ultimate strength is about three quarters of that of structural steel while the price is approximately three times that of mild steel. Several processes are used in the production of wrought iron of which the puddling process is most commonly used.

Chromium, nickel and molybdenum are the most common alloying elements used in cast iron. Chromium increases the hardness and wear resistance. Nickel increases strength and density and improves wearing qualities and raises the machine-

ability. Molybdenum increases stiffness, hardness, tensile strength and impact resistance.

1-12. Steel:

It is an alloy of iron and carbon in which the carbon content is less than 1·7%. It is produced by oxidizing the impurities in molten pig iron and then adding the amount of necessary carbon which will give required combination of strength, ductility and hardness. Since carbon is the controlling element, the steel is known as plain carbon steel.

The processes commonly used for manufacture of steel are (i) the open hearth process, (ii) the Bessemer process and (iii) the electric furnace process. The particular process used depends on the chemical analysis of pig iron to be refined and upon the desired quality of the steel to be produced. The finished molten steel is to be poured into ingots in sizes suitable for use by rolling mills.

Dead mild steel contains carbon upto 0·1%. It is softest and most ductile metal and possesses excellent machineability and weldability. It is rolled into sheets and is available in bar forms. It is used for rivets and solid drawn steel tubes. Mild steel contains carbon ranging from 0·1 to 0·35%. The welding properties and the machineability are good. The steel is strong and is harder than dead mild steel but less ductile. It is used for plate work, bars for general work and rolled steel sections for structural work.

Medium carbon steel contains carbon ranging from 0·35 to 0·65%. With increasing carbon content the strength and hardness increases but ductility decreases. The welding properties and machineability become poorer with increasing carbon percentage.

High carbon steel contains carbon ranging from 0·65 to 1·5%. This steel is harder and less tough. Welding is very difficult and machining except grinding can be done only in the annealed state. Steel is used in the heat treated condition and the properties of the steel varies with the heat treatment given. The table on page 11 gives a list of typical applications of plain carbon steels.

Carbon Ranges %	Uses of plain carbon steel
0·05 — 0·10	Wire, rivets, sheets, stampings, welding stock, cold drawn parts
0·10 — 0·20	Screws, machine parts, carburised parts structural shapes
0·20 — 0·30	Gears, levers, shafting, welded tubing, carburised parts
0·30 — 0·40	Seamless tubings, connecting rod, crane hooks, axles, shafts
0·40 — 0·50	Gears, studs, heavily loaded shafts, forgings
0·50 — 0·60	Railway rails
0·60 — 0·70	Setscrews, lock washers, hard drawn spring wire, locomotive tyres, drop hammer dies
0·70 — 0·80	Hammers, wrenches, anvil faces, band saws, spring, cultivator disc, plough beams
0·80 — 0·90	Cold chisels, hand tools, punches, leaf spring, music wire, shovels, rock drill, harrow blades, plough shears
0·90 — 1·00	Knives, axes, dies, springs, harrow blades
1·00 — 1·10	Knives, taps, drills, milling cutters
1·10 — 1·20	Drills, lathe tools
1·20 — 1·30	Knives, files, metal cutting tools
1·30 — 1·40	Razors, metal cutting saws, wire drawing dies

1-13. Alloy steels:

Carbon steel is an alloy of iron and carbon with small amounts of manganese, silicon, sulphur and phosphorus while alloy steels are steels to which elements other than carbon, are added to produce special effects such as greater resistance to corrosion, higher tensile strength, increased toughness and hardness. The most common alloying metals are nickel, chromium, manganese, silicon, molybdenum, tungsten and vanadium.

Alloy steels are standardized. The design engineer must be familiar with the effects of the various alloying elements on the properties of steel. A short guide is given below:

Carbon: By increasing the carbon content the tensile strength rises, the elongation drops, the hardness increases and weldability declines.

Silicon: The addition of silicon gives dense castings. The elastic limit is raised by increasing silicon content. However, weldability and forgeability are reduced.

Manganese: The primary effect of adding manganese is to increase wear resistance.

Nickel: The addition of nickel improves the tensile strength and elongation. Nickel steels are highly heat resistant and rust resistant.

Chromium: The addition of chromium increases the tensile strength, elongation and hardness. Chrome steels are highly heat resistant and rust resistant. They are frequently used in conjunction with nickel to give chrome nickel steels.

Phosphorus: It improves strength, but reduces elongation and impact strength.

Molybdenum: It increases strength at elevated temperatures and eliminates temper brittleness.

Tungsten: It increases the hardness.

Aluminium: Aluminium steels are suitable for nitriding.

Copper: It improves rust resistance.

For further informations and data on this topic, the engineering handbooks should be referred to.

Many varieties of carbon steels and alloy steels are used for the construction of machinery. The Indian Standards Institution has adopted a standard IS 1762-1961 for uniform system of designation of steel.

A steel may be designated by a group of symbols, indicating the important characteristics such as tensile strength, carbon content, alloy content, sulphur and phosphorous limits, weldability, surface finish, surface condition, steel quality and treatment. The following prefixes may precede the designation of steel to avoid confusion with designation of other materials:

S for wrought steel
CS for cast steel

If the steel is to be designated on the basis of its tensile strength without detailed chemical composition, the symbol '*St*' is to be followed by the value of minimum ultimate tensile strength in kg/mm². '*St* 50' designates steel whose minimum ultimate tensile strength is 50 kg/mm². For plain carbon steels, the letter *C* is followed by the average carbon content in hundredths of a percent. Plain carbon steel containing carbon from 0·10 to 0·18% will be designated by '*C* 14'. For alloy steels the carbon content in hundredths of a percent shall be used without the prefix '*C*'. For carbon and alloy tool steels the letter '*T*' is to be followed by the average carbon content in hundredths of a percent. The alloy index shall consist of chemical symbols of the significant elements arranged in descending order of percentage contents. The nominal or average percentage of each alloying element shall be indicated by an index number following its chemical symbol.

A nickel chromium molybdenum alloy steel with grain size controlled and case carburised with composition

Carbon	0·12 to 0·18
Silicon	0·10 to 0·35
Manganese	0·6 to 1·00
Nickel	1·00 to 1·50

Chromium	0·75 to 1·25
Molybdenum	0 08 to 0 15

is designated as 15 *Ni* 13 *Cr* 1 *Mo* 12 *Ge*.

Here the first numbers indicate the average carbon content in hundredths of a percent. The rest of the numbers indicate the average percentage of the alloying elements designated by its chemical symbols which precedes the numbers. The underlined number is number after the decimal points. Last letter '*G*' indicates steel quality while '*e*' indicates treatment given to the steel. In this manner the chemical composition of any alloy can be given. By writing letters at the end, weldability, surface finish, surface condition, steel quality and treatment can be designated, detail of which is given in IS: 1762-1961.

1-14. Non-ferrous metals and alloys:

Copper: It ranks next to iron and carbon in importance to industry. Pure copper is soft, ductile and tough and can withstand severe bending and forging without failure. It can be drawn into wires and hammered into sheets. It is a good conductor of electricity. Electrolytic copper is used for wires and conductors. Pure copper is used for castings, condenser tubes, water pipes and in chemical works, food and brewing plants, etc. due to its anticorrosive properties. Arsenical copper containing upto 0·5% arsenic and smaller traces of other elements is used for heater tubes, rivets, etc. Copper is rolled into sheets from which tanks and low pressure boilers are made for special uses.

Zinc: On account of its resistance to corrosion, it is used as a protective covering forming what is known as galvanised iron or steel. Zinc is electro-positive to iron and is used to prevent the galvanic action. Galvanized iron pipes are used for water supply pipes. Tanks are generally galvanized after being riveted.

Copper-Zinc alloys - Brasses:

The alloys of copper and zinc are known as brasses. These alloys are highly resistant to corrosion, machine easily and make good bearing material. The properties of a brass vary with its zinc percentage. Zinc percentage varies from 5 to 45. By increasing the percentage of zinc the ductility of the alloy increases but after 37%, there is a fall in ductility. When the zinc percentage is less than 20%, the alloy is known as red brass, which, is used for plumbing of pipe and connections, rivets, hard ware etc. When the percentage of zinc lies between 20 and 35, the alloy is known as cartridge brass which is the most ductile of all the brasses. It is used for stamping and deep drawing, cartridge

to corrosion in air and sea-water, making it a valuable metal for naval use.

Superalloys :

The rotating blades and discs of turbomachineries are subjected to large tensile stresses and the strongest of stainless steels will not be adequate as a suitable material. To meet the demands of these machines *Superalloys* are developed. These alloys have exceptionally high tensile strength and oxidation resistance at very high temperatures.

There are three classes of superalloys:

(i) Class A — wrought alloys strengthened by work hardening
(ii) Class B — wrought alloys strengthened by heat treatment
(iii) Class C — casting alloys

The following table gives the composition and uses of various kinds of non-ferrous alloys:

Kind of alloy	Composition of alloy	Uses
Standard brass	70% copper; 30% zinc	Rolling into sheets or drawing into tubes for locomotives, cartridge, pump liners
Admiralty brass	70% copper; 29% zinc; 1% tin	Steam condenser tubes
Cast brass	—	Bearings, hydraulic fittings
Muntz metal	60% copper; 40% zinc	Suitable for hot working by rolling, stamping or extruding
Ordinary bronze	95% copper; 5% tin	Worms, gears, pump bodies, bushes
Phosphor bronze	90% copper; 9·7% tin; 0·3% phosphorus	Bearings, worm wheels, rods, sheets
Manganese bronze	60% copper; 35% zinc; 5% manganese	Under water shafts and fittings
Delta metal	55% copper; 41% zinc; 2% lead; 2% iron	Parts of marine engine, screw propellers, ordnance, chemical, hydraulic, mining plants, sanitary fittings
Gun metal	90% copper; 10% tin	Small valves, fittings for water services
Engineer's bronze	88% copper; 10% tin; 2% zinc	Engine parts, steam fittings, hydraulic machineries
Cupro nickel	75% copper; 25% nickel	Coinage, casing of rifle bullets, condenser tubes
Monel metal	67% nickel; 28% copper and the rest iron, manganese and carbon	Valve parts for superheated steam, turbine blades, pumps and condenser tubes, in chemical industries for vats and coils exposed to corrosive influences, dyeing plants, artificial silk processes

Kind of alloy	Composition of alloy	Uses
Constantan	50% copper; 50% nickel	Standard resistance; thermo couple junction metal
Manganin	84% copper; 12% manganese; 4% nickel	
Silveroid	55% copper; 45% nickel	Decorative work in connection with shop fronts, hotel entrances, etc
German Silver	50% copper, 20% nickel 30% zinc	Ornamental work of motor cars, shop and house fittings
Nichrome	80% nickel, 20% chromium	Heating elements for domestic and industrial electrical appliances; annealing pots, pyrometer sheaths, glass making, Diesel engine valves; pulverised fuel and oil burners
Coronite	65% nickel; 15% chromium, 20% iron	
Perminvar	45% nickel, 30% copper; 25% cobalt	Transformer core, magnetic shield
Invar	36% nickel; 64% iron	Surveying tapes, piston struts, compensation collars
Elinvar	36% nickel; 50% iron, 12% chromium, 1% Manganese, 1% carbon and tungsten	Standard tuning forks, chronometer balance springs
Pewter	80% tin; 20% lead	Drinking vessels, domestic appliances
Plumber's solder	34% tin; 66% lead	Soldering and tinning purposes
Type metal	79.5% lead; 20.25% antimony; 0.25% bismuth	Types for printing
Fusible metal	50% bismuth; 25% lead, 25% tin	Fusible plug of steam boilers
Babbitt metal	87.75% tin; 4% copper, 8% antimony, 0.25% bismuth	Bearing linings
White metal	80% lead, 20% antimony	Bearing linings
Duralumin	95% aluminium; 4% copper; 0.5% manganese; 0.5% magnesium	Light structures, extruded sections, sheets
T. Alloy	92.5% aluminium; 4% copper; 2% nickel; 1.5% magnesium	I.C. engine pistons

1-15. Non-metallic materials:

The commonly adopted non-metallic materials are leather, rubber, asbestos and plastics.

Leather is used for belt drives and as a packing or as washers. It is very flexible and will stand considerable wear under suitable conditions. The modulus of elasticity varies according to load.

Rubber is used as a packing, belt drive and as an electric insulator. It has a high bulk modulus and must have lateral freedom if used as a packing ring.

Asbestos is used for lagging round steam pipes and steam boilers.

1-16. Plastics:

These materials have come into extensive use now-a-days. The name plastic materials has been derived from the state of plasticity existing at a certain stage in their manufacture. This makes it possible to give plastic products any desired shape.

They are classified into two main categories: *Thermoplastics,* which soften under the application of heat and can be repeatedly moulded. *Thermosetting* plastics which, under pressure and heat are cured and polymerised so that the plastic assumes a different chemical combination, becomes hard and will not deform when again subjected to heat.

The basic compounds in both categories are available mainly in powder, tablet, liquid and sheet form, and are converted into the finished product by moulding, die casting under pressure and conventional casting, by pneumatic vacuum moulding, by machining and by extrusion using screw presses. With thermosetting plastics the finished shape is usually obtained from powder by compression moulding in a die under heat and pressure. Thermoplastic compounds may be formed by extrusion, compression or injection moulding. Sheets of thermoplastic materials may be reshaped by heating. With some shapes additional machining operations, cutting, drilling, etc are necessary.

Plastics are produced on a synthetic or less frequently on a natural resin base. Apart from resins, most plastics contain what is known as a filler, to provide particular properties such as colour, strength and impact and wear resistance. Fillers include

paper, fabric, chipped-wood moulding compound, graphite, wood veneer, textile, glass fibers, asbestos and more recently molybdenum disulphide, which provides excellent lubricating and wear properties — particularly when introduced into nylon.

The good features of plastic materials are

(i) Low cost
(ii) Light weight
(iii) Good resistance to shock and vibration
(iv) Self lubrication, which means low friction and high wear resistance
(v) Heat and electric insulating properties
(vi) Resistance to corrosion, and
(vii) Ease of fabrication.

The unfavourable features of plastics are

(i) Low strength
(ii) High thermal expansion
(iii) Low heat resistance
(iv) High creep and deformation under load, and
(v) Embrittlement at low temperature

The following table gives the list of some of the plastics most commonly used in Mechanical Engineering

Name of plastic	Uses
(Textolite Laminated fabric)	Gear wheels, machine tool slide ways, pulleys, and bearing liners
Wood laminate	Shells of large sized bearings, pulleys and gears and as a substitute for non-ferrous metals
Compressed-wood plastic	Bearing material, and substitute for non-ferrous metals and for making pipes, hand rails, etc
Fibreglass	Hulls of small ships, boats and yachts, and automobile bodies
Fluorinated plastics (Ethylene polymers)	Lining of friction surfaces, packings, electric and radio parts, pipes and pipe valves
Polyamide resins (Capron and nylon)	High speed gears, compressor discs and blades, and parts with high impact strength and abrasion resistance
Faolite	Pipes to convey chemically aggressive fluids
Polythylene	Pipes

EXAMPLES I

1. What are the important considerations that govern the choice of a material?
2. Classify the materials of construction.
3. Define the meaning of the term base metal as applied to engineering materials.
4. What is meant by the term "mechanical properties of material"?
5. Define in general the properties of strength and elasticity.
6. What is meant by ductility, malleability and plasticity?
7. Explain the term resilience.
8. What is meant by toughness and how is it measured?
9. Why is brittleness an undesirable property, especially for materials to be used as machine parts?
10. Explain the terms creep and machineability.
11. What useful information is obtained from a static tensile test?
12. Name hot working processes.
13. What are the advantages of forged components?
14. What are the advantages of using extruded parts?
15. Name two articles which are shaped by cold working.
16. Explain the terms: drawing, heading, spinning and stamping.
17. Enumerate the precautions to be taken while designing castings.
18. Define powder metallurgy. Outline the general process used.
19. Discuss the advantages and the limitations of powder metallurgy.
20. Name several examples of parts that can be made of powdered metal.
21. Name finishing processes.
22. Define tolerance and allowance.
23. What is the main difference between a jig and a fixture?
24. How does the carbon content affect cast iron, wrought iron and steel with reference to hardness and toughness?
25. What advantages has cast iron as construction material?
26. For what particular parts is malleable iron used?
27. What advantages has malleable iron over white or grey cast iron?
28. What advantages are there in using alloy cast iron?
29. Name the processes by which steel is commonly produced.
30. Classify carbon steel.
31. Explain the difference between carbon steel and alloy steel.
32. How are alloying elements effective in changing the properties of steel?
33. What alloy steel is suitable for springs?
34. What is Thackeron?
35. For what types of service are brasses and bronzes used?

36. What are the constituents and physical properties of monel metal?

37. What is the advantage of aluminium bronze over tin bronze?

38. *Why is duralumin said to be a successful steel substitute?*

39. Where do we use muntz metal, invar, nichrome, phosphor bronze and constantan?

40. Name the important non-metallic materials of construction.

41. State what materials are commonly used for the following parts of an I. C. engine:

(a) Cylinder block (b) Piston (c) Piston rings (d) Big-end bearing
(e) Crankshaft (f) Crankcase.

42. For what purpose is each of the following alloys widely used:

(a) mild steel (b) high carbon steel (c) grey cast iron (d) malleable cast iron (e) 60/40 brass (f) 67/33 lead-tin alloy?

43. Give one typical application of each of the following alloys:

(a) 60/40 brass
(b) tin base bearing metal
(c) 70/30 lead-tin alloy
(d) monel metal as cast
(e) muntz metal

44. Explain what the following terms mean

(a) hot rolling (b) drop forging (c) cold drawing

What effects would these operations have on properties of alloys?

45. Enumerate the properties of materials required to be considered by the designer when selecting the same for a particular machine part. Discuss with particular reference to the following cases

(a) Connecting rod of an aero engine
(b) Frame of a punch press
(c) Frame of a power hammer
(d) Crane hook
(e) Propeller shaft of a ship
(f) Surface plate
(g) Piston of a scooter engine
(h) Manhole cover for a drain.

46. (a) What are the chief physical characteristics of materials that are important in deciding on their choice as material for manufacture of different types of machine elements?

(b) Discuss in detail the material used and the special property which makes it most suitable for use in manufacturing the following:

(i) Cylinder block of an aero engine
(ii) Boiler shell
(iii) Pipes
(iv) Pulleys
(v) Gears

(vi) Rim of locomotive wheels
(vii) Pump bodies
(viii) Worm wheels.

47. Select suitable materials for the manufacture of the following:
(i) Drop hammer dies
(ii) Metal cutting saws
(iii) Die castings
(iv) Electrical switch boxes
(v) Condenser tubes
(vi) I. C. engine pistons
(vii) Bearings
(viii) Heating coils for furnaces

Clearly give reasons for your choice of the material, give the composition and the properties of the same.

CHAPTER II

DESIGN CONSIDERATIONS IN MACHINE PARTS

2-1. Loads:

The object of the machine is to transmit motion through its various links to some particular part, where useful work is to be done. In carrying out their functions various parts of the machine are subjected to forces, whose thorough analysis is essential in order to design these parts. These forces may be classified as follows:

(a) Useful forces due to energy to be transmitted by the part

(b) Dead weight forces due to weights of individual parts in a machine

(c) Frictional forces

(d) Inertia forces due to changes in velocity

(e) Forces due to changes of temperature

In addition, other forces may exist due to poor workmanship, due to non-homogenity of material and due to reduction of area caused by deterioration of the material.

Another way of classifying the loads is as follows.

(a) Static load which does not change in magnitude, direction or point of application

(b) Live load which varies in magnitude and/or direction. Live loads are of two kinds: first, those which are of the same kind but change in magnitude, as does the varying weight of traffic passing over a bridge; in second kind of live loads the magnitude change from a maximum of one kind to a maximum of the opposite kind, [redacted] is the case with the piston-rod of a double acting engine. Here the stress set up is said to change sign.

(c) Shock loads are applied with velocity, they are also known as impact loads. Examples of impact loads are blows of hammer, rough road reactions to the wheels and axles of motor cars, etc.

For the same magnitude of load a machine part must be made progressively heavier in design as the load varies in type from a static to a shock load.

2-2. Stress:

When any body is subjected to an external force, there are set up within the body resisting forces called stresses, which resist the change of form of the body and are measured in terms of force exerted per unit area. In engineering practice, the force is generally given in kg and the resisting area in sq cm and so the stress is often abbreviated to kg/sq cm.

In practice the word stress is used for two purposes; to indicate force per unit area generally referred to as unit stress and to indicate total internal force within a member generally called total stress. The unit stress is given by the equation

$$f = \frac{\Delta F}{\Delta A} \quad \text{where}$$

f = unit stress

ΔF = total force either parallel or perpendicular to area ΔA.

Two kinds of basic stresses exist in a machine part: (a) normal stresses, which always act normal to the stressed surface under consideration and (b) shearing stresses, which are parallel to the stressed surface.

Other stresses are either similar to the basic stresses or are combination of these stresses. For example, the stresses in the frame of a punching machine are normal stresses as well as the combination of normal and shear stresses across certain sections.

2-3. Strain:

A strain is deformation. All material bodies which are subjected to external forces are necessarily deformed. For example, a long rod subjected to an axial tensile load would be elongated, while a column supporting an axial compressive load would be shortened. The total deformation produced in a member is designated by Greek letter δ. If the length of the member be l, the deformation per unit length is given by

$$\text{unit strain} = \varepsilon = \frac{\text{total deformation}}{\text{original length}} = \frac{\delta}{l}.$$

The unit strain has got no mathematical unit.

2-4. Stress-Strain Diagram — Modulus of Elasticity:

When a stress is applied to a body, a corresponding strain is produced. On removal of the stress, the strain will disappear provided the unit stress developed by the applied force has not exceeded a certain limit called the elastic limit. If the stress has exceeded the elastic limit, after the removal of the applied force, the body will retain a permanent deformation. An elastic limit is the maximum unit stress which can be imposed on a body without causing the permanent strain. Within the elastic limit, the strain produced is directly proportional to the stress applied. This law was first demonstrated by Robert Hooke, the English scientist in 1678.

The following table gives the elastic limit for certain metallic materials of construction:

ELASTIC LIMIT

Material	Tension kg/sq cm	Compression kg/sq cm	Shear kg/sq cm
Mild steel	2,450	2,450	1,470
Wrought iron	2,100	2,100	1,260
Malleable cast iron	2,100	2,100	1,400
Cast iron	420	1,750	420
Nickel steel	3,500	3,500	2,660
Aluminium alloy	2,240	2 240	1,470
Copper (hard drawn)	2,800	2 800	1,610
Brass 70/30	1,750	1,750	1,050
Bronze	1,750	1,750	1,050

The Young's modulus or modulus of elasticity, which is usually denoted by the letter E, is the ratio of the unit stress to the corresponding unit strain, provided the unit stress does not exceed the elastic limit. The modulus of elasticity is constant within elastic limit and is a measure of body's stiffness or rigidity. It may be defined crudely as the stress which would double the length of a piece of the material if elastic extension could go on to such an extent. The modulus of elasticity is practically *same* for all the steels. This means that some steels have a much higher elastic limits than others but all the steels stretch the same amount for the same stress providing the elastic limit is not exceeded.

The following table lists the modulii of elasticity under tension and compression for some of the metallic materials of construction:

MODULUS OF ELASTICITY

Material	E, modulus of elasticity kg/sq cm
Aluminium	700,000
Brass	980,000
Bronze	840,000
Cast iron	1,050,000
Copper	1,050,000
Mild steel	2,100,000
Wrought iron	1,750,000

When any member of a machine is subjected to a unit stress in excess of its elastic limit, the member is unsafe. As the stress is increased, the deformation increases rapidly. Finally the stage will be reached when the member will rupture. The maximum unit stress that a material can bear before rupture is known as ultimate strength of the material which is two to four times its elastic limit.

The relation between the stress and strain is shown graphically by a curve known as stress-strain diagram for a material. Fig. 2-1 shows such a diagram for a medium steel bar in tension. Tensile tests under static loading are most commonly used as they are simplest to carry out and at the same time permit a fair prediction of the behaviour of a material under other types of loading.

Tensile tests are carried out in testing machines designed to apply truly axial loads. The test specimens are of uniform cross section and whenever possible they confirm to the sizes and shapes suggested by Indian Standards Institution. The strain is determined over a measured length of the specimen called the *gauge length* which is taken some distance from the end fixings. During the test the relation between the force stretching the specimen and the elongation of the specimen is recorded automatically on a diagram.

The following are some of the recommendations of IS: 1608-1960 for the test piece of steel products other than sheet, strip, wire and tube:

(i) The cross section of the test piece may be circular, square, rectangular or in special cases of other form. For test pieces of rectangular section the ratio of 4:1 for sides should not be exceeded.

(ii) There should be a transition curve between the gripped heads and parallel length. The gripped heads may be of any shape to suit the

holders of the testing machine. Section, bars, etc. may be tested without being machined.

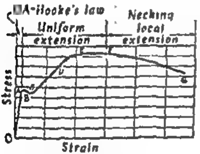
(iii) As a rule the diameter of machined cylindrical test pieces should not be less than 4 mm.

(iv) As a rule only proportional test pieces complying with the requirement l_0 = *gauge length* = 5.65 $\sqrt{\text{original cross section of the gauge length}}$ should be used for the tensile test

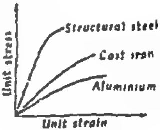
(v) The parallel length shall be between $l_0 + \frac{d}{2}$ and $l_0 + 2d$ shall always be used for arbitration purpose.

(vi) If prismatic test pieces with rectangular cross sections are to be cut from a parcel of rolled sections, a uniform parallel length shall be adopted which may be obtained from the formula $l_0 + 2d$ where l_0 and d refer to the test piece with the largest cross section.

If the yield stress is to be determined, the rate of increase of stress on the test piece is not more than 1 kg/mm² per second from a stress of 5 kg/mm² until the yield point is reached.



Stress-strain diagram for a medium steel bar
Fig. 2-1



Comparison of Stress-strain diagrams
Fig. 2-2

Let us study the stress-strain diagram of fig 2-1 very carefully. As can be seen at the beginning strains grow directly as stresses in the portion *OA upto certain stress called the proportional limit* Consequently Hooke's law holds upto the proportional limit With a further increase in load the diagram becomes curved

Proportional limit is the stress at which the curve just begins to deviate from a straight line, i.e., deviate from direct proportionality between stress and strain.

However if stresses do not exceed a certain magnitude, the *elastic limit*, the material retains its elastic properties, i.e., after

unloading the specimen recovers its original form and dimensions. Elastic limit is the stress at which the material just begins to retain a permanent set. The difference between the proportional limit and the elastic limit is not great and in practice no distinction is usually made between the two.

If the load is increased still further, the moment is reached such as a point B when deformations in fact begin to grow without the load being increased. The stress at which deformations grow with no increase in load is called the *yield point*. Having elongated to a certain extent under a constant value of the force, the material regains its ability to resist extension, i.e., it hardens and beyond the point C, the diagram rises though far more gradually than before. At the point F the stress reaches its maximum value which is called the *ultimate strength*. The ultimate strength is the maximum stress that can be developed in the material.

Upon reaching the magnitude of the ultimate strength a sharp local reduction of area, so called *necking* occurs. As the test progresses the cross sectional area of the specimen at the neck reduces rapidly. The rupture of the specimen takes place at the narrowest section of the neck. The stress at the time of final fracture is called the *breaking strength*.

Elastic range of the diagram is the portion of the diagram in which the material reacts elastically. The portion of the diagram upto the elastic limit is the elastic range. The remainder of the diagram is the *plastic range*.

In addition to the mechanical characteristics of the material, the *percentage elongation* at rupture and the *percentage reduction in area* of cross section at ruputre, which are the important properties, can be obtained.

$$\text{Percentage elongation} = \frac{l_1 - l_0}{l_0} \times 100$$

where l_0 = original specimen length between gauge points
l_1 = specimen length after rupture between gauge points.

$$\text{Percentage reduction in area} = \frac{A_0 - A_1}{A_0} \times 100$$

where A_0 = the original cross sectional area
A_1 = area of the neck after rupture.

Percentage reduction in area characterises ductile properties more exactly than the percentage elongation.

The total elongation of the tensile test specimen upto the point of fracture is made up of (i) uniform extension and (ii) large local extension.

The uniform extension is proportional to the guage length while the large local extension is due to waisting Prof Unwin suggested the following formula for total extension.

$$x = b \times l + C\sqrt{A}$$

where x = total extension
b = constant
l = gauge length
C = another constant
A = cross sectional area

By means of an extensive experiments Barba proved that geometrically similar test specimens of the same material deform similarly if they are so proportioned that $\frac{l}{\sqrt{A}}$ = constant

As a rule only proportional test pieces complying with the requirement $l = 5{\cdot}65\ \sqrt{A}$ should be used

The characteristic shapes of the tensile test diagram for structural steel, cast iron and aluminium are shown in fig. 2-2. In stretching specimens of brittle materials a number of peculiarities can be observed. The tensile test diagram of cast iron is typical of all brittle materials. It is seen from the diagram that the deviation from Hooke's law begins very early, and fracture occurs suddenly with very small deformations and without necking.

Example:

1. In a tensile test on a specimen of mild steel 2 cm in diameter and gauge length 10 cm, the following readings were recorded.

Load kg	*500*	*1,000*	*1,500*	*2,000*	*2,500*	*3,000*	*3,500*
Extension 10^{-3} cm	*0·757*	*1·51*	*2·26*	*3·01*	*3·80*	*4·67*	*5·31*

Deduce the value of Young's modulus.

When the specimen was afterwards tested to destruction, the maximum load recorded was 14,990 kg, the diameter of the neck was 1·16 cm and the length between the gauge marks was 13·6 cm. Calculate the ultimate tensile strength, percentage reduction in area and the percentage elongation.

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\text{load}}{\text{area}} \times \frac{\text{length}}{\text{extension}} = \frac{\text{length}}{\text{area}} \times \frac{\text{load}}{\text{extension}}$$

$$= \frac{\text{length}}{\text{area}} \times \text{slope of the load extension graph.}$$

$$\text{Average slope of the load extension graph} = \frac{3500}{5 \cdot 31} \times 10^3$$

$$E = \frac{10}{\frac{\pi}{4} \times 2^2} \times \frac{3500}{5 \cdot 31} \times 10^3 = 2 \cdot 1 \times 10^6 \text{ kg/cm}^2.$$

$$\text{Ultimate strength} = \frac{\text{ultimate maximum load}}{\text{original cross sectional area}}$$

$$= \frac{14990}{\frac{\pi}{4} \times 2^2} = 4{,}780 \text{ kg/cm}^2.$$

$$\text{Percentage elongation} = \frac{\text{increase in gauge length}}{\text{original gauge length}} \times 100$$

$$= \frac{13 \cdot 6 - 10}{10} \times 100 = 36\%.$$

Percentage reduction in area

$$= \frac{\text{reduction in cross sectional area}}{\text{original cross sectional area}}$$

$$= \frac{\frac{\pi}{4}[2^2 - 1 \cdot 16^2]}{\frac{\pi}{4} \times 2^2} \times 100 = 66 \cdot 3\%.$$

Exercises:

1. *Describe briefly and sketch on the same axes of load and extension the type of curve you would expect to obtain from tensile tests to destruction on specimens of the same size of (a) mild steel (b) steel containing 1% of carbon (c) aluminium.*

Write down with reference to curve (a) how you would evaluate the stress at the yield point and (ii) the ultimate strength marking the corresponding load points on the curve.

2. *Define (a) proportional limit, (b) elastic limit, (c) yield point, (d) ultimate strength and (e) breaking strength.*

3. *Define (a) gauge length, (b) percentage elongation and (c) percentage reduction in area.*

4. Explain the following terms:

(a) Elastic range
(b) Plastic range
(c) Permanent set.

5. A tensile specimen of non-ferrous alloys of 2 sq cm and gauge length 8 cm gave the following results·

Load kg	*1,000*	*2,000*	*3,000*	*4,000*	*5,000*	*6,000*	*7,000*	*8,000*	*9,300*
Extension $\times 10^{-3}$ *cm*	*3·15*	*6·30*	*9·46*	*12·62*	*15·78*	*18·95*	*22·03*	*26·00*	*Fracture*

At fracture the length between the gauge points was 8·88 cm. Draw a graph of these results and determine the modulus of elasticity for the material and the percentage elongation.

Ans. $1{\cdot}23\ 10^6$ *kg/sq cm; 11%.*

2·5. Poisson's ratio:

It is found by experiments that a longitudinal strain is accompanied by a transverse strain. Both these strains are opposite in nature. The ratio of the transverse unit strain to the longitudinal unit strain is constant for a particular material This ratio is called Poisson's ratio. The following table gives the average values of this ratio for some materials·

POISSON'S RATIO

Material	Poisson's ratio
Cast iron	0 270
Mild steel	0 303
Hard steel	0 295
Wrought iron	0 278
Brass	0 350
Copper	0·340

2·6. Modulus of rigidity:

The modulus of elasticity in shear or torsion, which is denoted by the letter G, is the ratio of the unit shear stress to the unit shear strain. It is also known as the modulus of rigidity.

Theoretically E and G are related by the equation

$$G = \frac{E}{2\,(1+\mu)} \quad \text{where } \mu \text{ is Poisson's ratio.}$$

If E and G are known the above relation can be used to determine the value of the Poisson's ratio by the relation

$$\mu = \frac{E}{2G} - 1.$$

2·7. Bulk modulus:

A material under the action of equal compressive stresses p in three mutually perpendicular directions is subjected to a hydrostatic pressure p. The term hydrostatic is used because the material is subjected to the same stresses as would occur if it were immersed in a fluid at a considerable depth. Because of this pressure, the body of volume v will be reduced in volume by the amount say δv; then the ratio $\frac{\delta v}{v}$ is called the volume strain. The ratio of the hydrostatic pressure to the volumetric strain is called the bulk modulus of the material and is denoted by K. Then

$$K = \frac{p}{\frac{\delta v}{v}} \qquad \text{(i)}$$

The relation between E, the modulus of elasticity and K is given by the equation

$$K = \frac{E}{3\,(1-2\mu)} \qquad \text{(ii)}$$

where μ = Poisson's ratio.

We should expect the volume of a material to diminish under a hydrostatic pressure. In general, if K is always positive, we must have

$$1 - 2\mu > 0$$

or $$\mu < \tfrac{1}{2} \qquad \text{(iii)}$$

Thus the Poisson's ratio for metals in elastic strain is always less than $\frac{1}{2}$. For plastic strains of a metallic material there is a negligible change in volume, and Poisson's ratio is approximately $\frac{1}{2}$.

2-8. Basic requirements of the machine elements:

The following are the basic requirements of the elements of any machines:

(i) Strength
(ii) Stiffness
(iii) Wear resistance
(iv) Light weight and minimum dimensions
(v) Employment of easily available materials
(vi) Processability
(vii) Safety
(viii) Life and reliability
(ix) Compliance with I.S.

From the above requirements we shall consider in this text book mainly the fundamental requirements which determine the efficiency of the machine elements viz. strength, stiffness and wear resistance.

Strength: The machine element must not collapse or be permanently deformed by the forces applied within a specified service life. Breaking and deterioration of the working surfaces of the machine element is unallowable.

Stiffness: Elastic deformation of the machine element under the effect of the applied forces should not exceed certain pre-set values.

Wear resistance: Within the specified service life, the wear of machine element should not impair its mating with other elements, nor should it result in undue reduction of strength.

As the strength and stiffness are the main criteria in determining the serviceability of the part, they should be calculated first at the designing stage. The wear resistance of the machine elements can be increased by increasing the strength and hardness of the working surfaces (e.g. by case hardening or nitriding) or by increasing the contact surface. Wear of friction parts can be reduced by normal lubrication. The service life of the whole machine or an individual part of it differs for machines of different application.

The machine element should be sufficiently strong, stiff and wear resistant while having the minimum possible dimensions and weight. These requirements can be achieved by:

(i) Employing light weight rolled sections
(ii) Resorting to up-to-date methods of surface hardening
(iii) Using high-strength grades of cast iron and light alloys
(iv) Introduction of non-metallic materials to replace ferrous and non-ferrous metals and
(v) Improving the design of machine elements.

2-9. Factor of safety: Selection of allowable stresses:

The necessary and sufficient strength of a machine element is ensured by imparting to it dimensions and shape which exclude breakage and permanent deformations. In order to utilise completely the mechanical properties of a material and to reduce the weight of the parts and the machine as a whole, *the highest stresses obtained when proportioning machine parts by formulas of strength of materials should not exceed the allowable values but should approach them closely.*

$$\text{Allowable stress} = \frac{\text{critical stress}}{\text{factor of safety}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

The critical stress induced by static loading of ductile material is expressed by the yield point or the yield strength; in case of brittle materials the critical stress is expressed by ultimate strength; under variable load the critical stress is denoted by the endurance limit.

The appropriate value of the factor of safety for any design depends upon a number of considerations, some of which are as follows:

(i) Degree of economy desired
(ii) Selected values of strength characteristics (yield point, ultimate strength or endurance limit)
(iii) Load conditions such as static, varying or shock loads
(iv) Degree of accuracy of force analysis of members
(v) Permanency of design and vital importance of the machine part
(vi) Dependability of the material
(vii) Accessibility of parts for inspection and maintenance
(viii) Difficulty in preventing deterioration of the material
(ix) Degree of safety to human life and property
(x) Possible imperfections of workmanship.

Machine designers generally work with predetermined allowable stresses, which are based upon extensive laboratory tests made to study the mechanical properties of materials used and upon vast amount of accumulated experience. The selection of the proper factor of safety depends upon and reflects the judgement and experience of the designer. If the factor of safety is too high, there will be undue increase in weight and dimensions of the machine. Conversely if the factor of safety is too low, there will be a danger of unallowable deformation as well as ultimate failure of the part in the course of operation.

Let us consider the common method of determining safety factors and allowable stresses.

The common method of determining safety factors is known as *separate factor method* in which the required safety factor is determined as the product of a number of coefficients which take into account the influence of a number of factors on it.

Factor of safety $n = n_1 \times n_2 \times n_3$ .... (ii)

In the above equation n_1 takes into account the accuracy of determination of the magnitude and nature of application of load and the degree to which the assumptions made in carrying out force analysis are valid for the actual working condition. The value of n_1 varies from 1·1 to 2

The coefficient n_2 accounts for the degree of homogenity of the material. For cast iron the value of this coefficient varies from 1·5 to 2·5 while for steel forgings and rolled sections its value varies from 1·2 to 1·5.

The coefficient n_3 is introduced only when extra dependability of a part in service must be ensured; the coefficient n_3 then varies from 1 to 1·5.

The method of determining allowable stresses by the separate factor method depends on the nature of stress change. There are three basic types of stress change in time:

(i) Static stress created by static load
(ii) Repeated stress created by a repeated loading cycle
(iii) Reversed stress created by a reversed loading cycle.

Under a constant load, the allowable stress for a ductile material (annealed, normalized, hardened and high tempered steel) is

$$f_t = \frac{f_y}{n_1 \times n_2 \times n_3} \quad \text{.....................} \quad \text{(iii)}$$

where $n_2 = 2{\cdot}5 \left[\frac{f_y}{f_u}\right]$

which means that n_2 depends on the ductility of the material.

$$f_s = (0{\cdot}5 \text{ to } 0{\cdot}6)\, f_t \quad \text{.....................} \quad \text{(iv)}$$

Any discontinuity or change of section such as holes, notches, bends, grooves, scratches is a *stress raiser.* It will result in a concentration of stress or a localised stress that is greater than the average or nominal stress obtained by using the formulas of strength of materials. Stress concentration factors are computed mathematically by theory of elasticity or determined by various experimental techniques, the most common being the photo elastic method, which uses transparent models of various plastics.

Stress concentrations are insignificant for ductile materials only when the loads are static. Since a ductile material under a steady load yields at points of high stress concentration if the stress exceeds the yield strength. The local plastic deformations result in the redistribution and equalisation of stresss throughout the entire section of the part. More over the material becomes even stronger at the point of stress concentration. Therefore the stress concentration factor is not introduced into the calculations of the allowable stresses for parts made of ductile material and subjected to a constant load.

For brittle materials

$$\text{allowable stress} = \frac{f_u \times n_s}{n_1 \times n_2 \times n_3 \times k_s} \quad \text{.....................} \quad \text{(v)}$$

where n_s = scale factor which takes into account a reduction of the ultimate strength of the brittle material with an increase in the absolute dimensions of a part

k_s = effective stress concentration factor under a static load.

The following are some of the stress raisers:

Holes, grooves, tool marks, keyways, blow holes, surface roughness, small fillets, sudden changes of section, welds, clamps, point where a thread ends, etc.

The values of scale factors and stress concentration factors may be obtained from relevant reference books.

Stress concentrations are significant for ductile materials only when the loads are repeated. The stress at the concentration points may exceed the endurance strength and if so, the part eventually fails by fatigue. Therefore while calculating allowable stresses, the effective stress concentration factor should be considered. For a reversed stress cycle

$$\text{allowable stress} = \frac{f_e \times n_s \times \beta}{n_1 \times n_2 \times n_3 \times k_f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

where f_e = endurance limit for reversed stress cycle

β = coefficient which takes into account the influence the surface finish of the part has on its endurance

n_s = scale factor

and k_f = stress concentration factor for completely reversed stress cycle.

The value of β varies from 0 6 to 1.

The another method consists in the selection of allowable stresses or factors of safety in accordance with the kind of material, deformation and nature of loading. Such tables have been compiled for various branches of mechanical engineering and diverse types of machine elements. The tabular method of determining allowable stresses is widely used in bolts, riveted and welded joints, bearings, belt drives and friction drives and gears. The separate factor methods are widely used in highly specialised branches of engineering such as automotive and air craft industries.

The table on page 38 gives specific informations on the factors of safety to be used under different load conditions:

Sometimes the factor of safety is defined as the ratio of limit stress to allowable stress. The limit stress is defined as the highest value of the induced stress which the part can develop without either permanent deformation or failure. If we define the factor of safety in this manner, the factor of safety chosen for any given material may vary from 1 5 to 6 depending upon the load conditions.

$$\text{Factor of safety} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Material	Kinds of load			
	Steady load or dead load	Varying load or live load		Shock load
		Stress always of the same kind	Stress alternating	
Cast iron	4	6	10	15
Wrought iron	4	6	8	13
Mild steel	4	6	8	13
Cast steel	5	6	8	15
Brittle alloys	5	6	10	15
Soft metal and alloys	5	6	8	12
Leather	8	10	14	16

2-10. Procedure for designing a machine element:

The following procedure will be adopted for designing a machine element:

(i) The design diagram is drawn in which the shape of the element and the nature of its mating with other elements are presented in simplified form. The forces applied to this element are assumed to be either concentrated or distributed according to a fundamental law.

(ii) The forces acting upon the element during machine operation are determined and the machine element is recognised i.e. it is to be designed as a link, beam, shaft or column.

(iii) Suitable material is selected for the component and either factor of safety or allowable stresses are determined.

(iv) By using the relevant formulas of strength of materials, the dimensions of the machine element are determined considering the strength, stiffness and wear resistance.

(v) Lastly the drawing of the component is prepared showing its dimensions, manufacturing accuracy, surface finish and other data pertinent to its manufacture.

For many machine elements there are standard sizes as shafts, bolts, keys, *I* beams, which means that such sizes are more readily available in the market and are also cheaper. The designer always uses standard items and standard proportions unless he

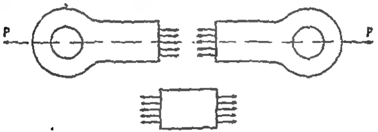
feels strongly that some custom design is desired. We shall give informations on standard sizes as we proceed.

Sometimes the designer sets the preliminary optional dimensions and shape of the part on the basis of its purpose, nature of jointing with other parts and a general lay out of the unit. Under such circumstances check calculation of the element is performed to determine the actual stresses and safety factors for the given design diagram and the forces.

For static loads the actual stresses are compared with the allowable stresses while for variable stresses the actual safety factor is determined and compared with that required.

2-11. Tensile stress:

Fig. 2-3 shows an eye bar which is subjected to a pair of *axial* forces P. The tendency of the load is to elongate or stretch the eye bar. In such a case the eye bar is said to be in tension and the



Tensile stress

Fig. 2-3

load is said to be the tensile load This load produces axial tensile stresses internally in a member on a plane lying perpendicular to its axis. If a section is cut out of the bar, the internal tensile stresses are disclosed, and being tensile stresses, they are of course, pulling on each of the remaining ends of the bar as well as on the section removed. The stressed area considered is a plane surface lying normal to the direction of stress. In general it is the cross-sectional area of the member and it is mostly assumed that the stress is uniformly distributed over that area.

Let P = external force

A = cross sectional area of the member

l = length of the member

E = modulus of elasticity

f_t = unit tensile stress

δ = total elongation.

$$\therefore \quad f_t = \frac{P}{A} \quad \cdots\cdots (i)$$

and
$$\delta = \frac{Pl}{AE} \quad \cdots\cdots (ii)$$

By means of formula (ii) the total elongation can be calculated which is a desirable thing to do for very long tensile members as elongation of members is limited in many machine parts. By means of formula (i) the cross sectional dimensions of the member can be obtained for a given load when permissible tensile stress is known.

Examples:

1. The side links of a compressed air riveter have to sustain a load of 8,200 kg each. Assuming that the width of the cross section is to be three times the thickness and that the maximum allowable stress is not to exceed 700 kg/sq cm, determine the dimensions for the link.

The side link of a compressed air riveter is subjected to a load of 8,200 kg and is of rectangular section. As the permissible tensile stress intensity is 700 kg/sq cm, the minimum cross sectional area required will be $\frac{8200}{700} = 11{\cdot}70$ sq cm.

If t be the thickness of the link, its width will be $3t$. The area of the link will be $3t^2$.

$$\therefore \quad 3t^2 = 11{\cdot}7$$

or
$$t = \sqrt{\frac{11{\cdot}7}{3}} = 1{\cdot}98 \text{ cm; we adopt } t = 2 \text{ cm.}$$

$$W = 2 \times 3 = 6 \text{ cm.}$$

Note: It should be noted that the theoretical result should be changed to a number that is slightly larger than the theoretical; thus the actual answer is more practicable value that will suit the scale of a designer. The designer must have knowledge of sizes which are available in the market. The designer should refer various Indian Standards for preferred sizes as recommended by Indian Standards Institution.

2. Fig. 2-4 shows a coil chain of a crane which is required to carry a maximum load of 4 tonne. Determine the diameter d of the link stock if the permissible tensile stress in the link material is not to exceed 650 kg/sq cm.

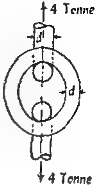


FIG. 2-4

Maximum tensile load on the chain

$$= 4 \times 1000 = 4{,}000 \text{ kg}.$$

Minimum cross sectional area of the link stock required $= \dfrac{4000}{650} = 6{\cdot}16$ sq cm

If d be the diameter of the link stock, then

$$2 \times \frac{\pi}{4} d^2 = 6{\cdot}16$$

or $$d = \sqrt{\frac{6{\cdot}16 \times 4}{2\pi}} = 1{\cdot}98 \text{ cm}; \text{ we adopt 2 cm.}$$

Exercises:

1. The connecting rod of a reciprocating steam engine is subjected to a maximum live load of 6.500 kg. Determine the diameter of the rod at the thinnest part if the permissible tensile stress is limited to 325 kg/sq cm.
Ans. 5 5 cm.

2. A belt is required to transmit 25 h p from a pulley running at a mean belt speed of 20 metre/sec. The angle of contact between the belt and the pulley is 160°, and the coefficient of friction is 0 25. The safe working stress for the belt is 30 kg/sq cm and the thickness of the belt is 6 mm. Determine the width of the belt required. *Ans 12 cm*

3. A boiler of 4,500 kg weight is suspended from the hook of a crane by means of a coil chain as shown in fig. 2-5. Determine the diameter of the link stock of the chain if the permissible stress in the chain material is limited to 650 kg/sq cm. *Ans. 2 5 cm.*

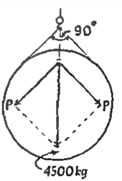


FIG. 2-5

4. An empty elevator weighs 4,000 kg and its balance-weights weigh 5,000 kg. The four wheels over which the ropes run weigh each 200 kg; their diameter is 1·5 metre and radius of gyration 0·6 metre. The maximum load to be lifted is

1,500 kg with an acceleration of 1·5 metre/sec². Suggest the suitable size of the rope. The permissible load on the rope is given by the equation $P = 300\ d^2$ where d is the diameter of the wire rope in cm.

Ans. $d = 2$ *cm.*

5. *A tie bar has to carry a load of 12 tonne. What must be the width of the bar 13 mm thick, if there is a rivet hole 22 mm diameter on its centre line? Working stress for the tie bar is 0·75 tonne/sq cm.*

Ans. *15 cm.*

6. *A hydraulic press exerts a total push of 350 tonne. This load is carried by two turned steel rods, supporting the upper head of the press. Calculate the diameter, and find the extension in each rod in a length of 250 cm. Safe stress 850 kg/sq cm;* $E = 2{\cdot}1 \times 10^6$ *kg/sq cm.*

Ans. *17 cm; 0·099 cm.*

7. *The maximum tension in the lower link of a Porter governor is 40 kg. If the link is of circular cross section, suggest the suitable size of the link if the permissible stress in the link is 300 kg/sq cm.*

Ans. *0·5 cm.*

2-12. Compressive stress:

When a pair of axial forces push on a member and shorten it, the forces are called compressive forces and they produce axial compressive stresses and the body is said to be in compression. Such a condition is shown in fig. 2-6 where a short post is supporting at its top an axial load P which is counteracted by the reacting force P at its base. When a section is cut out of this short post, the internal compressive stresses are disclosed as shown in fig. 2-6. The compressive stresses push on the section removed as well as on the two remaining ends of the post. The stressed area is a plane surface normal to the axis of the member and it is assumed that the stress is uniformly distributed over the cross sectional area of the member.

Let P = the external force (compressive)

A = cross sectional area of the member

l = length of the member

E = modulus of elasticity

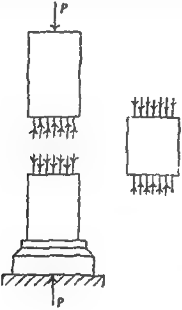
f_c = unit compressive stress

δ = total shortening of the member.

$$\therefore \quad f_c = \frac{P}{A} \quad \ldots\ldots \text{(i)}$$

and
$$\delta = \frac{Pl}{AE} \quad \ldots\ldots \text{(ii)}$$

Both the above formulas apply only when the length of the member is short. *If the length of the member is more than twenty times the least radius of gyration of its normal cross section it should be treated as a column and the stresses should be determined by one of the column formulas, and not by short compression member formula.*



Compressive stress
Fig. 2-6

Example:

1. Calculate the diameter of the wrought iron rod which is to be under a compressive load of 35,000 kg in order that the allowable unit stress may not exceed one-fourth of the elastic limit which is 2,100 kg/sq cm.

Permissible stress $= \frac{2100}{4} = 525$ kg/sq cm.

External compressive load $= 35{,}000$ kg.

$\therefore$ Minimum cross sectional area of the rod $= \frac{35000}{525}$

$= 66{\cdot}7$ sq cm.

If d be the diameter of the rod, then

$$\frac{\pi}{4} d^2 = 66{\cdot}7$$

or $$d = \sqrt{\frac{66{\cdot}7 \times 4}{\pi}} = 9{\cdot}2 \text{ cm; we adopt } 9{\cdot}5 \text{ cm.}$$

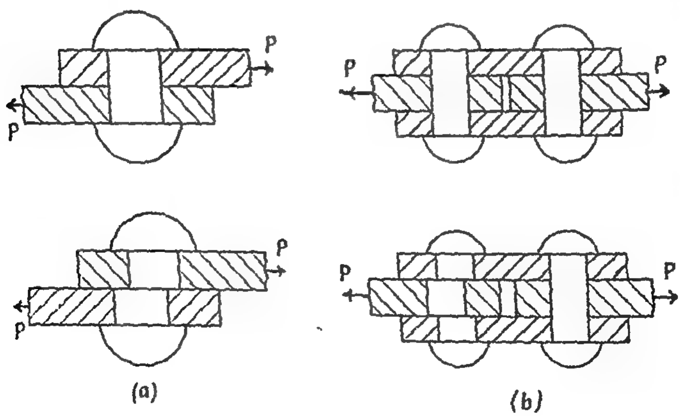
Exercise:

1. The piston rod of a steam engine is 5 cm in diameter and 60 cm long. The diameter of the piston is 40 cm and the maximum steam pressure is 9 kg/sq cm. Determine the compression and extension of the piston rod if the modulus of elasticity is $2{\cdot}1 \times 10^6$ kg/sq cm.

Ans. 0·165 mm.

2-13. Shearing stress:

Two forces, which are equal in magnitude and pull opposite each other along parallel lines of action will cause relative sliding or slipping of adjacent portions of the body in parallel lines. This condition is called shear, the illustration of which is shown in fig. 2-7. Fig. 2-7(*a*) shows the rivet in single shear, while in



Single shear

Double shear

Shear failure of rivets

Fig. 2-7

fig. 2-7(*b*) the rivet to the left has failed in double shear. Rivets, cotter and knuckle pin are examples of bodies under a shearing stress. This type of stress differs from tensile and compressive stress in that the stressed plane (the shear plane) lies parallel

with the direction of the stress rather than perpendicular to it, as in the cases of tensile and compressive stresses.

In the case of bolts, rivets and beams forces producing shear stresses generally act transversely to the longitudinal axis of the member. The stressed area is lying parallel to the direction of stress. The relation existing between the external force and the area A resisting shear is given by the equation

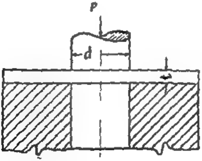
$$f_s = \frac{P}{A} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (i)$$

In fig. 2-7(a), the rivet is in single shear. If d be the diameter of the rivet, the area that resists the shear of the rivet is $\frac{\pi}{4}d^2$.

$$\therefore \quad f_s = \frac{P}{\frac{\pi}{4}d^2} = 1{\cdot}3\,\frac{P}{d^2} \quad \ldots \ldots \ldots \ldots \ldots \quad (ii)$$

When the rivet is in double shear as in fig 2-7(b), the area that resists the shear of the rivet is $2 \times \frac{\pi}{4}d^2$.

$$\therefore \quad f_s = \frac{P}{2 \times \frac{\pi}{4}d^2} = 0{\cdot}65\,\frac{P}{d^2} \quad \ldots \ldots \ldots \quad (iii)$$



Ultimate shearing strength
FIG. 2-8

When metal plates are to be cut to specified size and shape or the holes are to be punched or drilled, the tools used to perform the operations must overcome the ultimate shearing resistance of the material to be cut. For fig. 2-8, the area to be sheared is πdt. If U_s be the ultimate shear strength of the material, the maximum shear resistance to the tool will be $\pi dt\ U_s$.

Examples:

1. *What must be the diameter of the rivet in fig. 2-7(a) if the value of the shearing force P is 4,500 kg and the permissible stress is not to exceed 1,000 kg/sq cm?*

The minimum shear area to be provided by the rivet, which is in single shear, will be $\frac{4500}{1000} = 4{\cdot}5$ sq cm.

If d be the diameter of the rivet, then

$$\frac{\pi}{4} d^2 = 4{\cdot}5$$

or $$d = \sqrt{\frac{4{\cdot}5 \times 4}{\pi}} = 2{\cdot}4 \text{ cm.}$$

2. *Determine the smallest size of a hole that can be punched in a 15 mm thick mild steel plate, having an ultimate shear strength of 33 kg/sq mm. The allowable crushing stress in the punch is 120 kg/sq mm.*

Note: Shear strain is taken advantage of in cutting metals by means of dies and steel blades. Unlike machines and other engineering structures where strain must not be allowed to exceed the elastic limit, strain in cutting is carried to the failure stage of the material along the plane of shear.

Let d mm be the minimum diameter of the hole that can be punched in the plate. The crushing resistance of the punch should be greater than or at the most just equal to the maximum punching force.

The maximum punching force $= \pi \times d \times 15 \times 33$ kg.

The crushing resistance of the punch $= \frac{\pi}{4} d^2 \times 120$ *kg.*

$$\therefore \quad \frac{\pi}{4} d^2 \times 120 \geq \pi d \times 15 \times 33.$$

From the above inequality, we get

$d \geq \frac{33}{2}$ mm; we adopt $d = 17$ mm.

Exercises:

1. *The cover plate of a punch is subjected to an upward force of 3,800 kg while the hole is being punched. Sliding of the cover plate relative to the punch frame is prevented by means of two dowel pins made of steel. Allowing an allowable shear stress of 1,000 kg/sq cm, determine the diameter for the dowel pin.* *Ans.* 5 *cm.*

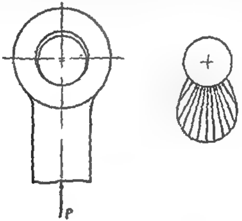
2. *A cast iron motor support is to have four holes in its base for mounting purposes. If the base is 2 cm thick and the holes are clearance*

hole for 25 mm bolts, what is the shearing force to be overcome if the ultimate shear strength for cast iron be 1,750 kg/sq cm? *Ans.* 34,400 *kg.*

3. *A centrifuge has a small bucket which weighs 0·2 kg with contents and which is suspended on a manganese bronze pin at the end of a horizontal arm. The pin is 1·8 cm in diameter and is in double shear under the action of a centrifugal force. If the arm turns at 10,000 r.p.m. and the centre of gravity of the bucket is 30 cm from the axis of rotation, determine the shear stress induced in the pin.* *Ans.* 1,320 *kg/sq cm.*

2-14. Bearing stress:

Compressive stresses exerted on an external surface of a body, when one object presses against another, is referred to as bearing stresses, or contact stresses. Examples of bearing stresses are shown in figs. 2-6 and 2-9. In fig. 2-6, the post bears on a plate,



Distribution of bearing pressure for pin connection
FIG. 2-9

the plate bears on the footing and the footing bears on the soil. The resulting stresses between post and plate, between plate and footing and between footing and the soil are the bearing stresses. The stressed areas are the plane surfaces normal to the direction of the load. Bearing, on a curved surface such as a pin, is shown in fig. 2-9, where the load is applied to the member by pins. The distribution of pin pressure will not be uniform but will be in accordance with the shape of the surfaces in contact and deformation characteristics of the two materials. So we use the average

bearing stress formula by using the projected area which is the product of diameter of the pin and the length of pin in contact.

$$f_b = \frac{P}{ld} \quad \text{.......................................} \quad \text{(i)}$$

where P = load on the pin

l = length of the pin in contact

d = diameter of the pin

f_b = average bearing stress.

The most frequent cause of failure of many machine parts is the wear of the sliding surfaces when these surfaces have relative motion under load. To ensure normal service life of the components the bearing pressure intensity should not exceed the given limit, which depends upon number of factors. Wear resistance is one of the basic requirement of the elements of any machine.

We quote some examples where bearing pressure intensity is to be considered while designing the components.

(1) Eccentric sheave and straps (2) Cross head and guide (3) Brake block and rotating drum (4) Pin in a knuckle joint (5) Threads in a nut (6) Crank pin and its bearing (7) Thrust block (8) Clutch lining (9) Fulcrum pin in a lever (10) Cotter and socket (11) Power screws (12) Guide ways of machine tools e.g. planing machine (13) Journal bearings.

Examples:

1. The crank pin of a high speed stationary engine sustains a maximum load of 3,500 kg due to steam pressure. Assuming an allowable bearing pressure of 70 kg/sq cm, determine dimensions for the pin. Assume that the bearing length of the pin is 1·2 times the diameter of the pin.

Minimum bearing area required $= \frac{3500}{70} = 50$ sq cm.

Let d be diameter of the pin, then length of the pin in the bearing will be $1{\cdot}2d$ and the projected bearing area will be $d \times 1{\cdot}2d = 1{\cdot}2d^2$

$\therefore \quad 1{\cdot}2d^2 = 50$

or $\quad d = \sqrt{\frac{50}{1{\cdot}2}} = 6{\cdot}45$ cm.

$\therefore$ We adopt $d = 6{\cdot}5$ cm and $l = 8{\cdot}0$ cm.

2. Fig. 2-10 shows a 30 mm diameter round headed bolt passing through a heavy supporting steel plate. Under the tensile pull P = 9,000 kg, the bolt might fail in three ways: (i) in tension across 30 mm diameter shank, (ii) in shear on the cylindrical surface formed if the shank of the bolt should pull straight out of its head, and

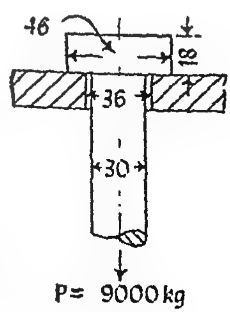


Fig. 2-10

(iii) in bearing on contacting surfaces between the bolt head and supporting plate. Calculate average tensile, shearing and bearing stresses resulting from the load P.

$$\text{Tensile stress} = \frac{9000}{\frac{\pi}{4} \times 3^2}$$

$$= 1.275 \text{ kg/sq cm.}$$

Area that resists shear of the bolt head will be $\pi \times 1{\cdot}8 \times 3{\cdot}6 = 20{\cdot}35$ sq cm.

$$\text{Shear stress} = \frac{9000}{20{\cdot}35} = 442 \text{ kg/sq cm.}$$

$$\text{Bearing area} = \frac{\pi}{4}[4{\cdot}6^2 - 3{\cdot}6^2] = 6{\cdot}44 \text{ sq cm.}$$

$$\text{Bearing stress} = \frac{9000}{6{\cdot}44} = 1{,}397 \text{ kg/sq cm}$$

Exercises:

1. Find area of the crosshead guide bar for a horizontal engine cylinder diameter 35 cm, 60 cm stroke and connecting rod 150 cm long, the boiler pressure being 7 kg/sq cm by gauge. Safe bearing pressure on the guide is limited to 3·5 kg/sq cm. *Ans. 410 sq cm.*

2. A rim clutch is to consist of four pairs of wood lined jaws engaging a cast iron rim of 75 cm diameter 50 h.p. is to be transmitted at a speed of 180 r.p.m. Assuming allowable bearing pressure to be 2 kg/sq cm and coefficient of friction 0·2, determine the minimum area of each block in contact with the rim. *Ans. 135 sq cm.*

3. Each bearing of an electric motor sustains a load of 400 kg Assuming $\frac{l}{d}$ *ratio to be 1, determine length of the bearing if permissible bearing pressure intensity is limited to 10 kg/sq cm.* *Ans. 6·5 cm*

4. A cone clutch with a cone semi angle of 12 degree is to transmit 15 h p. at 750 r.p.m. The width of the face is to be $\frac{1}{5}$ *the mean diameter and normal pressure between the contact faces is limited to 0·7 kg/sq cm. Determine main dimensions of the clutch. Assume the coefficient of friction to be 0·2.* *Ans. Width 3 cm, inside diameter 15 cm.*

5. In design of eccentric, the thickness and diameter of the sheave are calculated from bearing considerations. The net area of the valve face on which the steam pressure acts may be taken as a rectangle 30 cm by 20 cm;

steam pressure 9 kg/sq cm by gauge; coefficient of friction 0·2; safe bearing pressure between sheave and strap 5 kg/sq cm. Determine diameter and thickness of the sheave if their ratio be 10.

Ans. Diameter 50 cm, thickness 5 cm.

6. A bolt of diameter 'd' is enlarged near the head to a diameter D as shown in fig. 2-11. The head is cylindrical having diameter 1·5D and thickness t. If the bolt carries a pull of 35 tonnes find the dimensions d, D and t. Assume the following safe stresses for the material:

$f_t = 10$ *kg/sq mm*, $f_s = 5$ *kg/sq mm and*

$f_c = 6$ *kg/sq mm.*

Ans. $d = 7$ *cm;* $D = 8$ *cm;* $t = 2{\cdot}5$ *cm.*

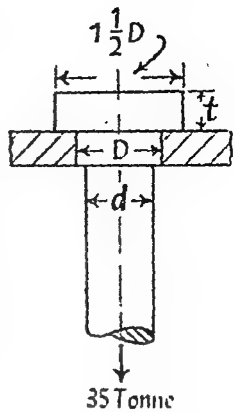


FIG. 2-11

2-15. Bending:

Bending produces a simple combination of the normal tensile and the compressive stresses. The fundamental assumptions underlying standard bending formulas are *that the plane sections at right angles to the neutral axis remain plane in the bent condition, that the modulus of elasticity is the same in tension and compression and that the material obeys Hooke's law.*

When a machine part is bent, the outer fibres are under tension and inner under compression. There is an intermediate layer where there is no stress. This is known as the neutral surface. The magnitude of the stress induced in a fibre due to bending moment is proportional to its distance from the neutral axis. The bending stress at any distance y from the neutral axis is given by

$$f = \frac{My}{I} \qquad \text{(i)}$$

where f = bending stress

M = bending moment

I = second moment of area about neutral axis.

The neutral axis passes through the centroid of the section. Equation (i) does not apply for stresses above the proportional limit and for bending stresses developed when the beam is bent about some axis of cross section other than a principal axis. In design, M should be the maximum bending moment because

maximum stress will be produced by maximum bending moment. The maximum value of stresses will occur for the maximum values of y, which we denote by c for symmetrical beam sections. The ratio $\frac{I}{c}$ is called the rectangular section modulus of the section and is denoted by the letter Z and has dimensions of cm³. The section modulus is equal to its moment of inertia divided by the distance from the neutral axis to the outermost fibre. The design formula is written as

$$M = f.Z \quad \ldots\ldots\ldots\ldots\ldots \quad (ii)$$

where M = maximum bending moment

f = safe bending stress

Z = rectangular section modulus of the section used.

The values of Z for typical symmetrical sections used in design of machine elements have been given in Appendix I.

If the material of the machine part subjected to bending has different resistance in tension and compression (for instance cast iron) section which is symmetrical with respect to neutral axis becomes non-economical. In case of symmetrical sections such as circular, rectangular, square, etc the maximum stresses on the extreme fibres are equal The dimensions of the section will be determined by allowable tensile stress which is lower than allowable compressive stress The stress in the compressed zone will be less than the allowable stress The material will not be used completely In unsymmetrical section such as T section, the neutral axis will be displaced towards the flange The stresses on the extreme fibres in the flange are less than those in the web When bending moment diagram does not change the sign, it is advisable to place the flanges in the stretched zone and the web in the compressed zone In this case it is possible to choose dimensions of the web and flanges in such a manner that permissible tensile and compressive stress intensities are reached simultaneously. Thus the material will be used to the full. Unsymmetrical I sections can also be used.

The following four rules will result in the most effective use of steel for bending loads:

1. Place flange material as far as possible from the neutral axis Connect flanges with web section
2. Avoid reductions in sectional area below requirements for horizontal stiffness.
3. Connect ends of beams rigidly to supporting members for maximum strength and stiffness.
4. Place joints in low stress areas.

2-16. Shear stresses in a beam:

When a beam is subjected to a transverse load, in addition to bending stresses, it is also subjected to a transverse shear stress. The shear stress varies from zero at the outer fibres to a maxi-

mum at the neutral surface, while the bending stresses vary from zero at the neutral surface to the maximum at the outer fibre.

The total vertical shear divided by cross sectional area of the beam is the average shear stress at the section but not the maximum shearing stress, which is 50% more than average stress for rectangular sections and 33% more than the average stress for circular sections. For detailed information, one should consult books on structural mechanics.

Examples:

1. The spindle shown in the sketch is used to connect the brake shoe of an industrial spring set brake to the brake arm. The loads P are each equal to 500 kg. Assuming an allowable stress of 1,250 kg/sq cm, determine the diameter of the spindle.

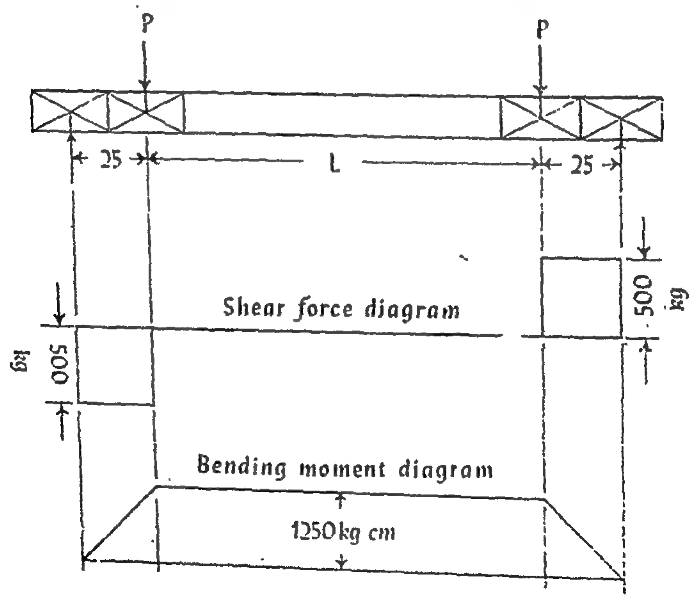


Fig. 2-12

This is an example in which the major part of the spindle is subjected to pure bending. Shear force and bending moment diagrams are shown in fig. 2-12.

Maximum bending moment $= 500 \times 2{\cdot}5 = 1{,}250$ kg cm.

If d cm be the diameter of the spindle, then

$$1250 = \frac{\pi}{32} d^3 \times 1250$$

or $$d = \sqrt[3]{\frac{1250}{1250} \times \frac{32}{\pi}} = 2{\cdot}2 \text{ cm.}$$

2. *A mild steel pump lever rocking shaft is shown in fig. 2-13. For the loading indicated, suggest the suitable diameter for the centre portion if the permissible stress is limited to 650 kg/sq cm.*

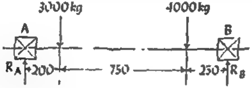


FIG. 2-13

To find reactions at A and B, we take moments about supports.

Taking moment about the support A, we get

$$R_B \times 120 = 3000 \times 20 + 4000 \times 95$$

$\therefore$ or $$R_B = \frac{3000 \times 20 + 4000 \times 95}{120} = 3{,}670 \text{ kg.}$$

Similarly R_A can be obtained by taking moment about support B.

$\therefore$ $$R_A = \frac{4000 \times 25 + 3000 \times 100}{120} = 3{,}330 \text{ kg.}$$

B.M. at 3,000 kg load $= 3330 \times 20 = 66{,}600$ kg cm.

B.M. at 4,000 kg load $= 3670 \times 25 = 91{,}750$ kg cm.

If d cm be the diameter of the solid shaft, then

$$91750 = \frac{\pi}{32} \times d^3 \times 650$$

or $$d = \sqrt[3]{\frac{91750 \times 32}{650 \times \pi}} = 11{\cdot}2 \text{ cm; we adopt 12 cm.}$$

Exercises:

1. *A cast iron pulley transmits 20 h.p at 180 r.p.m., diameter of the pulley being 70 cm. The pulley has 4 straight arms of elliptical section, the major axis being twice the minor axis. Determine dimensions*

of the elliptical section at the boss if the permissible stress be limited to 105 kg/sq cm.

Ans. Major axis 7·5 cm; minor axis 3·8 cm.

2. *Determine the depth of a guide bar of an engine, assuming that the greatest thrust on it occurs at the middle of its span of 90 cm when the crank is at right angles to the line of its stroke. The ratio of the length of the connecting rod to crank is 5. Diameter of the cylinder is 40 cm and the steam pressure is 6 kg/sq cm gauge. The width of the guide bar is 15 cm. The permissible stress intensity is limited to 250 kg/sq cm.*

Ans. 8 cm.

3. *The roof of a combustion chamber of a locomotive boiler is strengthened by a number of girder stays 70 cm span, spaced 20 cm apart. Three bolts 17·5 cm apart attach the plates to each stay. The section of each stay is rectangular, the thickness being 3/8 of the depth. Suggest suitable thickness for the stay if the pressure in the boiler is 12 kg/sq cm gauge. The permissible stress intensity is 700 kg/sq cm.*

Ans. Depth 16 cm; width 6 cm.

4. *In a certain trailer the axle load is 3,000 kg. The wheel track is 160 cm and the distance between the centres of the springs on which the chasis is supported is 110 cm. The axle is made of mild steel square sections with the ends turned to receive bearings. Design the axle if the permissible stress in the material is limited to 700 kg/sq cm.*

Draw the axle showing the spring pads welded to it.

Ans. 7 cm square.

2-17. Torsion:

A machine member subjected to the action of two equal and opposite couples acting in parallel planes is said to be in torsion and the stresses induced are known as the *torsional shear stresses,* which vary in magnitude from zero at the centroidal axis to the maximum at the outer fibre. The torsional shear stress in a member of circular cross section is obtained by the equation

$$f_s = \frac{T \times y}{I_p} \qquad \text{(i)}$$

where f_s = shearing stress at a distance y from the centroidal axis

T = applied torque

I_p = polar second moment of area of the section about the centroidal axis.

In design, we are concerned with the maximum stress which will occur at the outer fibre. The formula (i) for a solid shaft of diameter d will be reduced to

$$f_s = \frac{16T}{\pi d^3} \quad \text{......(ii)}$$

The deformation or angular twist of the shaft is given by the equation

$$\theta = \frac{Tl}{GI_p} \quad \text{......(iii)}$$

where θ = angular twist in a length l, in radians

T = applied torque

G = modulus of rigidity.

The angle of twist in degrees for a circular shaft of diameter d is given by the equation

$$\theta = \frac{584\ Tl}{Gd^4} \quad \text{......(iv)}$$

The formula (ii) is to be used when the member is to be designed for strength, while formula (iv) is to be used when it is to be proportioned for stiffness.

For a square shaft of side a, the equation (ii) will be

$$f_s = \frac{4.8T}{a^3} \quad \text{......(v)}$$

Note: While designing the shaft transmitting certain horse power at a given speed, the following equation becomes very useful

If we consider the horse power, the relation will be as under.

Let T be the torque transmitted by the shaft in kg cm at a speed N r.p.m.

$$\therefore \quad \text{H.P.} = \frac{2\pi \frac{T}{100} \times N}{4500} = \frac{2 \cdot \frac{22}{7} \cdot \frac{T}{100} \times N}{4500} = \frac{TN}{71620}.$$

$$\therefore \quad \text{Torque (kg cm)} = \frac{71620 \times \text{H.P.}}{\text{Speed in R.P.M.}} \quad \text{......(vi)}$$

Steel in rolled structural shapes or built up sections is very efficient in resisting torsion. With steel, torsionally rigid sections are easily developed by the use of stiffners. Castings on the other hand are restricted because of difficulties in coring, need for draft, etc.

The following three basic rules should be observed while designing machinery members to resist torsional loading.

(i) Use closed sections where possible.

(ii) Use diagonal bracing.

(iii) Make rigid end connections.

The *solid* or *tubular round closed section* is the best for torsional loading since the shear stresses are uniform around the circumference of the member. Next to a tubular section, the best section for resisting torsion is a closed square or rectangular tubular section.

The poorest sections for torsional loading are open sections, flat plates, angle sections, channel sections, Z bar sections, T bar sections, I beam sections and tubular sections which have a longitudinal slit.

In case of a solid rectangular shaft having b as the larger size, the maximum torsional shear stress is given by

$$f_s = \frac{T}{\alpha b d^2} \qquad \ldots\ldots(vii)$$

where α is a constant whose value depends upon the ratio of $\frac{b}{d}$.

The value of α is given below for various values of the ratio $\frac{b}{d}$.

The value of α equals

$$\frac{1}{3 + 1{\cdot}8 \times \frac{d}{b}}.$$

$\frac{b}{d}$	1·00	1·5	2	2·5	3	4	6	8	10	∞
α	0·208	0·231	0·246	0·258	0·267	0·282	0·299	0·307	0·313	0·333

The maximum shear stress of a rectangular section in torsion lies on the surface *at the middle of the long side.* Some what lower maximum shear stress is at the middle of the short side. The ratio of two maximum shear stresses is given by

$$\frac{\text{Maximum shear stress at the middle of the long side}}{\text{Maximum shear stress at the middle of the short side}} = \frac{b}{d} \qquad \ldots(viii)$$

The shear stress at the corners of a solid rectangular or square section is zero.

Actual tests show that the torsional resistance of an open section made up of rectangular areas nearly equals the sum of the torsional resistances of all the individual rectangular areas. For example *the torsional resistance of an I beam is approximately equal to the sum of the torsional resistances of the two flanges and a web.*

The torsional resistance R of a rectangular section having b as the larger side and t as the thickness is given by

$$R = \beta\ bt^3 \qquad \text{(ix)}$$

where β depends on the ratio of $\frac{b}{t}$.

For all practical purposes, the value of β may be taken as $\frac{1}{3}$.

When the value of torsional resistance of a section is known, the angle of twist is obtained by the relation

$$\theta = \frac{T \times l}{R \times G} \qquad \text{(x)}$$

Let us consider the following three arrangements of two channel sections as shown in fig 2-14.

Case 1:

In the arrangement of fig. 2-14(a), two channel sections are separated but fastened together by end plates. In this arrangement the section has not much torsional resistance.

Case 2:

When these two channels are securely fastened back to back, there is suitable torsional resistance. This arrangement is shown in fig. 2-14(*b*). The torsional resistance can be obtained as a sum of torsional resistances of three rectangular areas. Two webs are considered as one solid web and top and bottom flanges are considered solid.

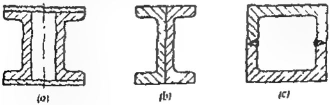


Fig. 2-14

Case 3:

In arrangement shown in fig. 2-14(*c*), two channels are welded toe to toe to form a box section, the torsional resistance would be

greatly increased. The torsional resistance of such a section can be obtained from the values given in table on page 56.

Examples:

1. *A steel shaft transmits 150 horse power at 115 r.p.m. The maximum twisting moment during each revolution exceeds the mean by 30%. Suggest the suitable diameter for the solid shaft if the shear stress is not to exceed 650 kg/sq cm.*

$$\text{Mean torque in kg cm} = \frac{71620 \times \text{h.p.}}{\text{r.p.m.}}$$

$$= \frac{71620 \times 150}{115} = 93{,}500 \text{ kg cm.}$$

Maximum torque $= 1{\cdot}3 \times 93500 = 122{,}000$ kg cm.

If d cm be the diameter of the solid shaft, then

$$122000 = \frac{\pi d^3}{16} \times 650$$

or $$d = \sqrt[3]{\frac{122000}{650} \times \frac{16}{\pi}} = 10 \text{ cm.}$$

2. *The cross section of an aluminium alloy lever suitable for general engineering purposes is a rectangle of depth 5 cm and thickness 2 cm. It is subjected to a torque of 1,500 kg cm. Determine the maximum value of the torsional shear stress if the polar modulus for a rectangular section is*

$$Z = \frac{b^2 d}{3 + 1{\cdot}8\dfrac{b}{d}}$$

where b = *thickness and*

d = *depth of the section.*

$$Z = \frac{2 \times 2 \times 5}{3 + 1{\cdot}8 \times \frac{2}{5}} = 5{\cdot}37 \text{ cm}^3.$$

Maximum shear stress will be at the middle of the longer side and will be equal to

$$f_s = \frac{\text{torque}}{\text{modulus of section}} = \frac{1500}{5{\cdot}37} = 280 \text{ kg/sq cm.}$$

Lower maximum at the middle of the short side

$$= 280 \times \tfrac{2}{5}$$

$$= 112 \text{ kg/sq cm.}$$

Shear stresses at the corners of the section will be zero.

3. Compare the torque carrying capacity of a thin walled circular tube of mean diameter D and thickness t if

(i) tube is closed

(ii) tube is cut longitudinally.

Also compare the torsional rigidity if the same torque is applied to both these tubes.

Let f_s be the permissible shear stress intensity for the material of the tube.

Torque T of the hollow tube can be approximately written as

$$T = \pi\, Dt\, f_s . \frac{D}{2} = \frac{\pi}{2}\, D^2 t\, f_s.$$

We consider that the cut circular section is equivalent to a rectangular section having one side πD and the other side t.

Torque of a slit tube can be written as

$$T' = f_s \times \tfrac{1}{3}\, \pi\, Dt^2.$$

Thus the ratio of two torques will be

$$\frac{T}{T'} = \frac{\frac{\pi}{2} D^2 t\, f_s}{f_s \times \frac{1}{3} \pi D t^2} = \frac{3}{2}\left(\frac{D}{t}\right).$$

Thus torque transmitting capacity of the closed tube is considerably greater.

The angle of twist of a circular tube is given by

$$\theta = \frac{Tl}{GI_p} = \frac{Tl}{G \times \pi\, Dt . \frac{D^2}{4}} = \frac{4\, Tl}{\pi D^3 Gt}$$

where T = torque applied

l = length of the tube

G = modulus of rigidity

I_p = polar second moment of area of the section. For the same torque applied to the longitudinally cut hollow circular section

$$\theta' = \frac{Tl}{GR} = \frac{Tl}{G \times \frac{1}{3} \pi D t^3} = \frac{3Tl}{\pi D G t^3}$$

The ratio of the two twists will be

$$\frac{\theta}{\theta'} = \frac{4\, Tl}{\pi D^3 Gt} \times \frac{\pi D G t^3}{3 Tl} = \frac{4}{3}\left(\frac{t}{D}\right)^2$$

From the above we see that for the same torque applied to both these tubes, the ratio of the induced shear stresses has a value

of the order of $\frac{D}{t}$ and the ratio of the angles of rotation has a value of the order $\left(\frac{D}{t}\right)^2$. *The quantity D is considerably greater than t. Consequently a closed section is appreciably stronger and to an even greater degree more rigid than a similar open section.*

Exercises:

1. *Calculate the diameter of the solid shaft to transmit 75 h.p. at 180 r.p.m. if the angle of twist on a length of 4 metre is not to exceed 0·4°. Assume $G = 0{\cdot}84 \times 10^6$ kg/sq cm.*

Ans. 12 *cm.*

2. *The propeller shaft of a car is in the form of a hollow tube, 5 cm outside diameter and 3 mm thick. Determine the maximum shear stress in the tube when the shaft transmits 50 h.p. at 2,800 r.p.m.*

Ans. 108·2 *kg/sq cm.*

3. *A motor car shaft consists of a steel tube 30 mm internal diameter and 3 mm thick. The engine develops 12 h.p. at 2,000 r.p.m. What will be the maximum stress in the tube when the power is transmitted through a 4:1 gearing?*

Ans. 340 *kg/sq cm.*

4. *Two steel shafts A and B are to be joined together by means of a flanged coupling. Both shafts are firmly fixed at the ends remote from the coupling. On assembly it is found that the holes in the coupling halves have an angular misalignment of 3° so that it is necessary to twist the shafts to enable the coupling to be assembled. If shaft A is 4 cm diameter by 150 cm long and shaft B 2·5 cm diameter by 90 cm long, determine the shear stress present at the surface of each shaft after assembly has taken place. Assume that the coupling is rigid.*

Ans. Shaft A; 58·6 *kg/sq cm;*
Shaft B; 61·7 *kg/sq cm.*

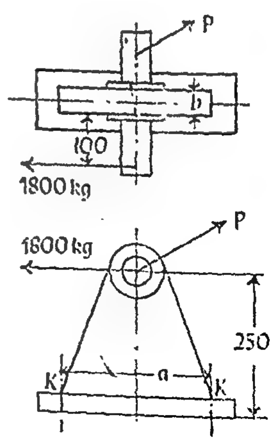


Fig. 2-15

5. *Pin P carries a load of 1,800 kg and is supported by a bracket as shown in fig. 2-15. Determine the section KK, assuming only the*

bending action of the load. Take an allowable bending stress of 400 kg/sq cm. Also determine the maximum shear stress produced in the cross section KK, if the polar modulus of section is given by $\dfrac{ab^2}{3+1\cdot 8\dfrac{b}{a}}$

2-18. Eccentric loading:

In structures as there is no relative motion at various joints it is generally assumed that they are centrally loaded; this may not be the case for machine parts when friction at the pin joints is taken into consideration, as the line of action of the force is tangential to the friction circle. As a result, the link is subjected to bending in addition to direct stress. Manytimes in the design of machine parts circumstances are such, that it is not practicable to make the link straight due to interference from the adjoining links. To avoid such interference an offset connecting link is provided. In some cases due to defective workmanship, i.e., due to drilling error, etc., the load axis may not coincide with the geometrical axis of the member, as a result the member is subjected to eccentric loading.

Frames of machines, such as a punching machine, portable hydraulic riveter, clamp, etc are subjected to eccentric loading

Now we consider the stress analysis of such members.

When the line of action of load on a short prismatic bar is parallel to, but does not coincide with, the centroidal axis of the supporting member as shown in fig. 2-16 (b), the load is referred to as an *eccentric load* and distance e is called the eccentricity of the load. The moment Pe will bend the member about the axis XX, thus adding tensile and compressive bending stresses to the already existing compressive direct stresses.

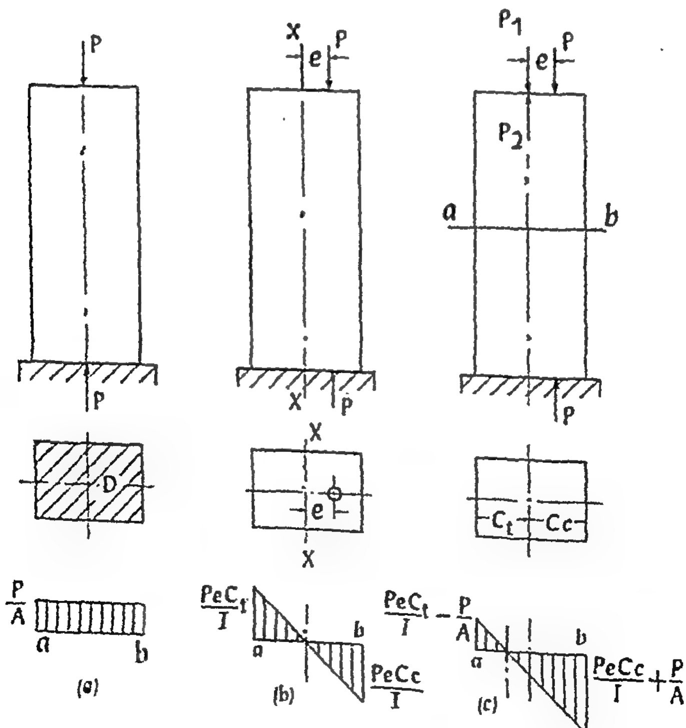
In order to determine the stresses, two forces P_1 and P_2 equal in magnitude to the applied force P may be introduced along the longitudinal axis without altering the equilibrium of the body as shown in fig. 2-16(c).

Stress due to load P_1 on the cross sections of the bar will be uniform and is equal to $\dfrac{P_1}{A}=\dfrac{P}{A}$. as shown in fig. 2-16(a).

The forces P_2 and P form a couple of the magnitude Pe which will induce compressive stress at b and tensile stress at a. The resulting stresses are obtained by the principle of superposition.

$$\left.\begin{aligned}\text{Stress at } a &= \frac{PeC_t}{I} - \frac{P}{A}\\ \text{Stress at } b &= \frac{PeC_c}{I} + \frac{P}{A}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where I is the second moment of area about the neutral axis and C_t and C_c are the distances of the extreme fibres from the neutral axis on the tension and compressive sides respectively. When the maximum bending stress is less than the direct stress there will be compressive stresses over the entire cross section. If the maximum bending stresses are more than the direct compressive stress, there will be a line of zero stress parallel to the neutral axis, which divides the cross section into two zones as shown in



Eccentric loading

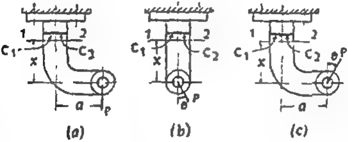
FIG. 2-16

fig. 2-16(*c*). Tensile stresses are on the left of the line of zero stress and compressive stresses are on the right.

When loading on the member is tensile, the equations of this section may be used by interchanging the subscripts *c* and *t*.

2-19. Combined stresses. Bending combined with direct stress:

In many instances we come across members which are subjected to both direct and bending loads as shown in fig. 2-17. All the cases of fig. 2-17 are loaded in such a manner that across any section 1-2 the stresses induced are the sum of direct and bending stresses. In general, the maximum combined stress will be the sum of the direct and bending stresses.



Combined straining action

Fig. 2-17

Let A be the area of the cross section, f_1 the stress at side 1 of the section, f_2 the stress at the side 2 of the section, C_1 distance of centre of gravity of section from side 1, C_2 the distance of centre of gravity of section from side 2, a the perpendicular distance of point of application of load P from the neutral axis and I second moment of area of section 1-2 about an axis perpendicular to the plane of the paper and passing through the centre of gravity of the section.

$$\left.\begin{aligned} f_1 &= \frac{P}{A} - \frac{PaC_1}{I} \\ f_2 &= \frac{P}{A} + \frac{PaC_2}{I} \end{aligned}\right\} \text{fig. 2-17}(a) \qquad \left.\begin{aligned} f_1 &= \frac{P\cos\theta}{A} + \frac{Px\sin\theta C_1}{I} \\ f_2 &= \frac{P\cos\theta}{A} - \frac{Px\sin\theta C_2}{I} \end{aligned}\right\} \text{fig. 2-17}(b)$$

$$\left.\begin{aligned} f_1 &= \frac{P\cos\theta}{A} - \frac{(Pa\cos\theta + Px\sin\theta)\,C_1}{I} \\ f_2 &= \frac{P\cos\theta}{A} + \frac{(Pa\cos\theta + Px\sin\theta)\,C_2}{I} \end{aligned}\right\} \text{fig. 2-17}(c)$$

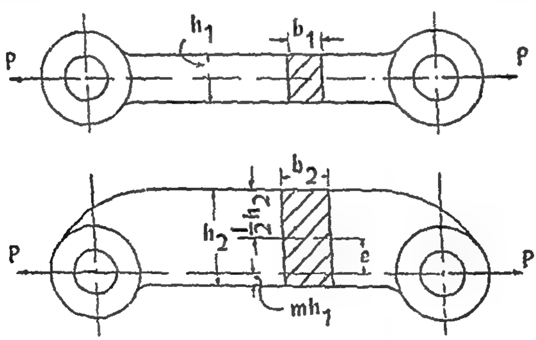
Similar expressions hold good when the load is compressive instead of tensile.

For symmetrical sections, the maximum stress will be tensile if the direct stress is tensile and it will be compressive if the direct stress be compressive.

Since three basic stresses exist — direct, bending and torsional shear stress as in the shaft, four combinations of stresses are possible (i) direct and bending, (ii) bending and torsional, (iii) torsional and direct and (iv) direct, bending and torsional. In this section we have considered the first, and other cases will be considered at the appropriate places in the book.

2-20 Offset connecting links and C shaped frames:

In the design of machine parts we come across offset connecting links as shown in fig. 2-18. For such links, in order to bring



Offset connecting link

FIG. 2-18

economy of the material, the following rule, which can be derived from first principles, is observed. When the line of action of the load cuts the section, the width h_2 is kept as small as possible and area of the section is increased by increasing the thickness b_2. When the line of action of the load does not cut the section, as in C shaped frames of punches, shears, presses and riveters, the thick-

ness b_2 is kept small and area of the section is increased by increasing the width h_2.

Examples:

1. The line of pull in a tension specimen 1 cm in diameter is parallel to the axis of the specimen but is displaced from it. Determine the eccentricity of the pull when the maximum stress is 20% greater than the mean stress on a section normal to the axis.

Let P be the load acting on the specimen and A be the area of cross section. Let e be the eccentricity of the applied pull.

$$\text{Direct stress} = \frac{P}{A}.$$

$$\text{Bending stress} = \frac{PeC_t}{I} \text{ with the notations of sec. 2-18.}$$

$$\therefore \quad \frac{P}{A} + \frac{PeC_t}{I} = \frac{120\,P}{100\,A}.$$

$$\therefore \quad \frac{e\,C_t}{I} = \frac{1}{5} \times \frac{1}{A}.$$

$$\therefore \quad e = \frac{1}{5} \cdot \frac{I}{AC_t} = \frac{1}{5} \times \frac{\frac{\pi}{32}d^3}{\frac{\pi}{4}d^2} = \frac{d}{40} = \frac{1.0}{40}$$

$$= 0.025 \text{ cm}$$

2. The crank arm of a steam engine has a rectangular cross section of 21 cm by 10 cm. In dead centre position of the crank the maximum compressive force acting is 7,000 kg. The line of action of the force is parallel to and at a distance of 12 cm from the principal axis of the section. Determine the maximum tensile stress induced in the crank arm.

The cross sectional area is $21 \times 10 = 210$ sq cm.

$$\text{Direct compressive stress} = \frac{7000}{210} = 33.3 \text{ kg/sq cm}$$

Due to eccentricity of 12 cm, a bending moment of magnitude $7000 \times 12 = 84{,}000$ kg cm is acting on the section.

$$\text{Maximum value of bending stress} = \frac{84000}{\frac{1}{6} \times 21 \times 10^2}$$

$$= 240 \text{ kg/sq cm.}$$

$\therefore$ Maximum compressive stress $= 240 + 33.3 = 273.3$ kg/sq cm.

Maximum tensile stress $= 240 - 33.3 = 206.7$ kg/sq cm.

3. A mild steel bracket shown in fig. 2-19 is subjected to a pull of 500 kg acting at an angle of 30° to the vertical. The bracket has a rectangular section whose thickness is $\frac{3}{8}$ of its depth. Determine the section of the bracket if the permissible stress in the material is limited to 550 kg/sq cm.

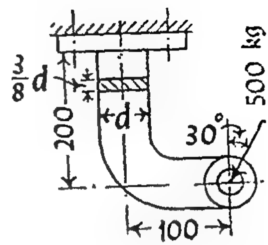


FIG. 2-19

Let d be the depth of the section.
Width of the section will be $\frac{3}{8}d$.

Area of cross section will be $d \times \frac{3}{8}d = \frac{3}{8}d^2 = 0{\cdot}375d^2$.
Modulus of section will be

$$Z = \frac{1}{6} \times \frac{3}{8} d \, . \, d^2 = \frac{d^3}{16} = 0{\cdot}0625d^3.$$

The force of 500 kg can be resolved into two components.
Horizontal component will be
500 sin 30° = 0·5 × 500 = 250 kg.
Vertical component will be
500 cos 30° = 0·866 × 500 = 433 kg.
Direct tensile stress due to vertical component will be

$$\frac{433}{0{\cdot}375d^2} = \frac{1150}{d^2} \text{ kg/sq cm.}$$

Bending moment acting on the section due to vertical component = 433 × 10 = 4,330 kg cm. Due to this bending moment, tensile stresses are induced on the inner fibres of the bracket, while compressive stresses are induced on the outer fibres. Maximum value of tensile stress on inner fibre will be

$$\frac{4330}{0{\cdot}0625d^3} = \frac{69400}{d^3} \text{ kg/sq cm.}$$

Horizontal component will give rise to a bending moment of the magnitude 250 × 20 = 5,000 kg cm which will induce tensile stresses on the inner fibre and compressive on the outer.

Maximum value of tensile stress on inner fibre will be

$$\frac{5000}{0{\cdot}0625d^3} = \frac{80000}{d^3} \text{ kg/sq cm.}$$

Total tensile stress in innermost fibres

$$= \frac{1150}{d^2} + \frac{69400}{d^3} + \frac{80000}{d^3}$$

$$= \frac{1150}{d^2} + \frac{149400}{d^3} \text{ kg/sq cm.}$$

Maximum value of the stress is limited to 550 kg/sq cm.

$$\therefore \quad \frac{1150}{d^2} + \frac{149400}{d^3} = 550.$$

The equation for the determination of d will be $d^3 - 2{\cdot}10d - 272 = 0$ which is a cubic equation of the form $x^3 + ax + b = 0$ which may be solved by Cardan's solution. The real root of the equation is given by

$$x = \left\{ -\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right\}^{1/3} + \left\{ -\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}} \right\}^{1/3}.$$

The value of d will be 6·5 cm.

Thickness will be $\frac{3}{8} \times 6{\cdot}5 = 2{\cdot}44$ cm; we adopt $t = 2{\cdot}5$ cm.

4. *A load of 2 tonnes acts on the frame shown in fig. 2-20. The frame is of equal width at all sections namely 150 mm and is 25 mm thick. Determine the stress at section SS, XX and YY.*

Stresses in section SS:

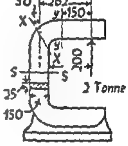


FIG. 2-20

Area of section $= 2{\cdot}5 \times 15$
$= 37{\cdot}5$ sq cm.

Modulus of section $= \frac{1}{6} \times 2{\cdot}5 \times 15^2$
$= 93{\cdot}7 \text{ cm}^3$.

Bending moment $= 2\,[20 + 7{\cdot}5]$
$= 55$ tonne cm.

Direct tensile stress $= \dfrac{2}{37{\cdot}5}$
$= 0{\cdot}0534$ tonne/sq cm.

Bending stress (maximum) $= \dfrac{55}{93{\cdot}7} = 0{\cdot}586$ tonne/sq cm.

Tensile stress at inner edge $= 0{\cdot}586 + 0{\cdot}0534 = 0{\cdot}6394$ tonne/sq cm.

Compressive stress at outer edge
$= 0{\cdot}586 - 0{\cdot}0534 = 0{\cdot}5326$ tonne/sq cm.

Stresses in section XX:

The stress distribution in an inclined section XX can be investigated as follows: Let $\theta = 30°$ be the inclination of this section to the plane parallel to the direction of the force.

The following stresses will occur in section XX:

(a) Couple $2 \times 26{\cdot}2 = 52{\cdot}4$ tonne cm produces bending giving a maximum tensile stress at the inner corner and a maximum compressive stress at the outer corner.

(b) Component of 2 tonnes at right angle to section produces uniform tensile stress over the section.

(c) Component of 2 tonnes parallel to the section produces uniform shear stress across section.

As the frame is of equal width and equal thickness at all sections, area and modulus of section of section XX will be $37{\cdot}5 \sec\theta = 43{\cdot}3$ sq cm and $93{\cdot}7 \sec^2\theta = 125$ cm³ respectively.

Bending stress $= \dfrac{52{\cdot}4}{125} = 0{\cdot}42$ tonne/sq cm, tensile at the inner corner and compressive at the outer.

Component of 2 tonnes perpendicular to section will be $2 \sin 30° = 0{\cdot}5 \times 2 = 1$ tonne.

Component of 2 tonnes parallel to section $= 2 \cos 30° = 0{\cdot}866 \times 2 = 1{\cdot}732$ tonnes.

Tensile stress (uniform) over section $= \dfrac{1}{43{\cdot}3} = 0{\cdot}0231$ tonne/sq cm.

Shear stress (uniform) over section $= \dfrac{1{\cdot}732}{43{\cdot}3} = 0{\cdot}04$ tonne/sq cm.

Maximum tensile stress at the inner corner will be $0{\cdot}42 + 0{\cdot}0231 = 0{\cdot}4431$ tonne/sq cm and the maximum compressive stress at the outer corner will be $0{\cdot}42 - 0{\cdot}0231 = 0{\cdot}3969$ tonne/sq cm. Since the shear stress acts at right angles to the stresses, maximum stresses in the section can only be found by the formula

$$p_n = \tfrac{1}{2}\left[p + \sqrt{p^2 + 4q^2}\right].$$

In the region of the inner corner, the major principal stress will be

$$\tfrac{1}{2}\left[0{\cdot}4431 + \sqrt{0{\cdot}4431^2 + 4 \times 0{\cdot}04^2}\right] = 0{\cdot}447 \text{ tonne/sq cm}$$

and the maximum shear stress $\frac{1}{2}\sqrt{p^2 + 4q^2} = 0{\cdot}226$ tonne/sq cm.

Similarly the major principal stress (compressive) and maximum shear stress in the region of outer corner will be 0·402 and 0·203 tonne/sq cm respectively.

Stresses in section YY:

The stresses in a section through one limb of the frame parallel to the line of action of the load will be bending stresses

due to bending moment $2 \times 15 = 30$ tonne cm. The bending stresses will be tensile at the inner edge and compressive at the outer edge.

Maximum bending stress at outer fibres will be

$$\frac{30}{93 \cdot 7} = 0 \cdot 32 \text{ tonne/sq cm.}$$

Due to transverse shear the maximum shear stress of the magnitude $\frac{1 \cdot 5 \times 2}{37 \cdot 5} = 0.08$ tonne/sq cm will be induced at the neutral axis while the shear stress at the outer fibres will be zero.

Exercises:

1. It is necessary to bend a certain link of rectangular section as shown in fig. 2-21 in order to prevent interference with another part of the machine. The link is to support a tensile load $F = 1{,}800$ kg with a permissible stress of 1,150 kg/sq cm. The inner edge of the mid section is displaced from the centre of the line of pins by distance $a = 7 \cdot 5$ cm. Neglecting the effect of curvature, determine the dimensions b and h if $h = 3b$.

Ans. $b = 2.5$ *cm,* $h = 7.5$ *cm*

FIG. 2-21

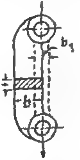


FIG. 2-22

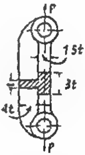


FIG 2-23

2. The symmetrical link shown by dotted lines in fig. 2-22 transmits a force of 4,000 kg. (a) Determine dimensions t and b_1 of the body of the link, assuming that the permissible tensile stress is 700 kg/sq cm and that $b_1 = 2.5\ t$. (b) Due to certain changes made in the design of the machine it was found necessary to replace the original link by an unsymmetrical one shown in figure by full lines, having the same thickness as the original link, determine the depth b of the new link.

Ans. $t = 1 \cdot 6$ *cm;* $b_1 = 4$ *cm,* $b = 15$ *cm.*

3 Determine dimensions of the principal cross section of the cast iron link shown in fig. 2-23 The maximum tensile load is $P = 3{,}700$ kg.

The following stress values should not be exceeded. $f_t = 210$ *kg/sq cm;* $f_c = 840$ *kg/sq cm.* *Ans.* $t = 23$ *mm.*

4. *The load P on C clamp shown in fig. 2-24 is 3,750 kg. Assuming that the clamp is made of cast steel and* $b = 3h$ *and* $e = 20$ *cm and that there is an allowable stress of 1,000 kg/sq cm, determine the dimensions b and h.* *Ans.* $h = 3{\cdot}8$ *cm;* $b = 11{\cdot}5$ *cm.*

5. *A C-clamp frame has a rectangular section 2 cm × 6 cm. The central line of the screw clamp is 10 cm from the neutral axis of the frame. What load must be applied by the screws if the tensile stress is limited to 850 kg/sq cm? Calculate the maximum value of the compressive stress.* *Ans.* *925 kg; 656 kg/sq cm.*

6. *The cross section of the frame of a punch press, where the bending is maximum, is the shape of a T. The top of the T is a rectangle 30 cm × 5 cm and the stem a rectangle 5 cm × 10 cm with 5 cm side adjacent to the top. The distance is 20 cm from the nearest edge of the T to the punching force. Calculate the stress developed when the punch is punching 18 mm diameter hole from a brass plate 5 mm thick. The ultimate shear strength of the brass is 2,100 kg/sq cm.* *Ans. 555·4 kg/sq cm.*

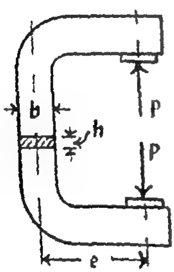


FIG. 2-24

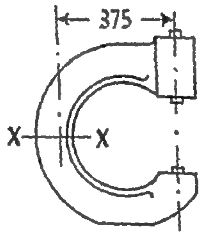


FIG. 2-25

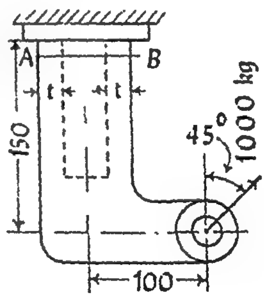


FIG. 2-26

7. *The punch press shown in fig. 2-25 has a tensile section modulus at a section X-X of 820 cm³, a compressive section modulus of 492 cm³ and an area of 162 sq cm. If this punch is used to punch holes out of a mild steel plate 3·2 mm thick, suggest the diameter of the largest hole that can be punched, if the ultimate strength of the plate in shear be 3,360 kg/sq cm. Assume a design stress of 420 kg/sq cm for the frame of the punch.*

Ans. *17 mm.*

8. *A cast iron bracket carries a load of 1,000 kg at 45° to its centre line. The section AB (fig. 2-26) is a hollow square 10 cm outside side.*

Determine the thickness of the bracket wall if the permissible tensile stress intensity in cast iron is limited to 125 kg/sq cm. *Ans.* 13 mm.

9. *The spindle of a drilling machine is subjected to a maximum axial load of 950 kg during operation. Determine the diameter of the solid cast iron column if the tensile stress is limited to 400 kg/sq cm. The distance between the axis of the spindle and the axis of the column is 40 cm.* *Ans.* 10 cm.

2-21. Shearing combined with tensile and compressive stresses:

Many machine parts such as propeller shafts, *C* frames, etc. are acted upon by external forces that produce direct tensile or compressive stresses in addition to the shear stresses at right angles to direct stresses. Similarly many machine parts such as crank-shaft, etc. are subjected to torsion combined with bending or direct compression. In such cases, the value of the maximum tensile or compressive or shear stress is obtained by the following formulas:

Maximum tensile stress $= \frac{1}{2}\left[f_t + \sqrt{f_t^2 + 4f_s^2}\right]$ . . . . (i)

Maximum compressive stress $= \frac{1}{2}\left[f_c + \sqrt{f_c^2 + 4f_s^2}\right]$ . . . . (ii)

Maximum shear stress $= \frac{1}{2}\sqrt{f_t^2 + 4f_s^2}$ or $\frac{1}{2}\sqrt{f_c^2 + 4f_s^2}$. (iii)

In order to use the above formulas it should be remembered that values of f_t and f_s should be taken at the same fibre.

Example:

1. *A propeller shaft transmits a twisting moment of 900,000 kg cm and simultaneous bending moment of 300,000 kg cm and an axial thrust of 12,500 kg. The shaft is 35 cm external and 25 cm internal diameter. Determine the maximum value of the compressive and shear stresses.*

Modulus of section for torsion $= \frac{\pi}{16}\left[\frac{35^4 - 25^4}{35}\right] = 6{,}230$ cm³.

Maximum torsional shear stress $= \dfrac{\text{twisting moment}}{\text{modulus of section}}$

$= \dfrac{900000}{6230} = 144$ kg/sq cm.

Modulus of section for bending $= \frac{1}{2} \times$ modulus of section for torsion.

$= \frac{1}{2} \times 6230 = 3{,}115$ cm³.

Maximum bending stress $= \dfrac{300000}{3115} = 96{\cdot}5$ kg/sq cm.

$$\text{Compression due to thrust} = \frac{12500}{\frac{\pi}{4}(35^2 - 25^2)} = 26{\cdot}6 \text{ kg/sq cm.}$$

$$\text{Hence axial compression} = 26{\cdot}6 + 96{\cdot}5 = 123{\cdot}1 \text{ kg/sq cm.}$$

$$\therefore \text{ Maximum principal stress} = \tfrac{1}{2}\left[123{\cdot}1 + \sqrt{123{\cdot}1^2 + 4 \times 144^2}\right]$$
$$= 218{\cdot}6 \text{ kg/sq cm.}$$

$$\text{Maximum shear stress} = \tfrac{1}{2}\left[\sqrt{123{\cdot}1^2 + 4 \times 144^2}\right]$$
$$= 157 \text{ kg/sq cm.}$$

Exercises:

1. *A steel pin is subjected to a shearing force 2,500 kg and the direct compression of 4,100 kg. Determine the suitable diameter of the pin if the permissible compressive stress is limited to 500 kg/sq cm.*

Ans. *3·8 cm.*

2. *At a point in a propeller shaft there are external twisting and bending moment of 16,000 kg cm and 12,000 kg cm respectively. In addition the shaft is subjected to an axial thrust of 2,000 kg. If the maximum permissible stresses are 750 kg/sq cm in compression and 450 kg/sq cm in shear, what must be the minimum diameter at the point considered?*

Ans. *6·5 cm.*

2-22. Theories of Elastic Failure:

Whenever a machine part is subjected to a system of combined stresses, e.g. as in crankshafts, propeller shafts, turbine rotors, etc. it is important to know what caused failure. The particular action causing failure is known as the *criterion of failure.*

Any state of stress can be specified completely by value of three principal stresses, each of which may be *tensile* or *compressive.* We can express the normal stress, strain, shear stress and strain energy in terms of these principal stresses. *The criterion of failure can be represented as an equation involving principal stresses together with certain constants and an experimentally determined stress, usually tensile.*

Several theories have been proposed each assuming a different criterion for failure. The standard theories are as follows:

(i) Maximum principal stress (Rankine's theory)
(ii) Maximum shear stress (Guest's theory)
(iii) Maximum strain (St. Venant's theory)
(iv) Maximum total strain energy (Haigh' theory)
(v) Distortion energy (Von Mises and Hencky)

We consider two dimensional or bi-axial stress system. Let f_1 and f_2 be the principal stresses produced by the combined action of forces and let f be the elastic limit in simple tension.

Maximum principal stress theory:

This theory assumes that failure will occur when the maximum principal stress f_1 is equal to the elastic limit in tension.

According to this theory, failure will occur when

$f_1 = f$ ....(i)

This is the theory most commonly assumed for *brittle materials*. However when the principal stress have opposite signs, the results do not agree with those obtained by experiment.

Maximum shear stress theory:

This theory assumes that elastic failure occurs when the maximum shear stress for a complex system is equal to the shear stress at the elastic limit in simple tension.

According to this theory failure will occur when

$$\frac{f}{2} = \frac{1}{2}[f_1 - f_2]$$

or $f = (f_1 - f_2)$ . . . .(ii)

It is usual to apply this theory to ductile materials, good approximation being obtained between theory and experimental results. The results are on the safe side independent of whether the principal stresses have like or unlike signs.

Maximum strain theory:

This theory assumes that elastic failure is deemed to occur when the maximum principal strain of the complex system is equal to the maximum strain at the elastic limit in simple tension.

According to this theory the failure will occur when

$$\frac{f}{E} = \frac{1}{E}[f_1 - \mu f_2]$$

or $f = (f_1 - \mu f_2)$

where μ = Poisson's ratio. ....(iii)

Results obtained by this theory are not always valid, and it is not now in general use.

Maximum total strain energy theory.

This theory assumes that failure occurs when the strain energy per unit volume for the complex system is equal to the

strain energy per unit volume at the elastic limit in simple tension. According to this theory failure will occur when

$$f^2 = f_1^2 + f_2^2 - 2\,\mu f_1 f_2 \quad \text{(iv)}$$

Results obtained by using this theory give good approximation with experimental results for ductile materials.

Distortion energy theory: (Shear strain energy theory):

The total strain energy is made of two parts:

(i) Energy due to volume expansion or contraction

(ii) Energy of distortion

This theory assumes that failure occurs when the strain energy of distortion per unit volume for the complex system is equal to the strain energy of distortion at the elastic limit in simple tension.

According to this theory failure will occur when

$$f^2 = f_1^2 + f_2^2 - f_1 f_2 \quad \text{(v)}$$

This theory gives satisfactory result for ductile materials.

If n be the factor of safety, then

$$n = \frac{\text{stress at failure}}{\text{working stress}} = \frac{f\ \text{(yield stress)}}{\text{working stress}}$$

According to various theories of failure, the factors of safety can be written as under:

Maximum principal stress theory:

$$n = \frac{f}{f_1} \quad \text{(vi)}$$

Maximum shear stress theory:

$$n = \frac{f}{f_1 - f_2} \quad \text{(vii)}$$

Maximum principal strain theory:

$$n = \frac{f}{f_1 - \mu f_2} \quad \text{(viii)}$$

Maximum total strain energy theory:

$$n = \frac{f}{\sqrt{f_1^2 + f_2^2 - 2\mu f_1 f_2}} \quad \text{(ix)}$$

Maximum distortion energy theory:

$$n = \frac{f}{\sqrt{f_1^2 + f_2^2 - f_1 f_2}} \quad \text{(x)}$$

We shall consider these theories at a later stage in the text when designing machine elements. At this place we consider a design

of a simple bolt subjected to an axial load accompanied by a shear load.

Example:

1. A bolt is subjected to a direct tensile load of 2,000 kg and a shear load of 1,500 kg. Suggest the suitable size of the bolt according to various theories of elastic failure, if the yield stress in simple tension is 3,600 kg/sq cm. A factor of safety of 3 should be used. Take Poisson's ratio as 0·25.

Let d cm be the minimum diameter of the bolt.

$$\text{Direct tensile stress on the bolt} = \frac{2000}{\frac{\pi}{4}d^2}$$

$$= \frac{2540}{d^2} \text{ kg/sq cm.}$$

$$\text{Direct shear stress on the bolt} = \frac{1500}{\frac{\pi}{4}d^2} = \frac{1910}{d^2} \text{ kg/sq cm.}$$

If f_1 and f_2 are the maximum and minimum principal stresses, then

$$f_1 = \frac{1}{2d^2}\left[2540 + \sqrt{2540^2 + 4 \times 1910^2}\right]$$

$$= \frac{2650}{d^2} \text{ kg/sq cm.}$$

$$f_2 = \frac{1}{2d^2}\left[2540 - \sqrt{2540^2 + 4 \times 1910^2}\right]$$

$$= -\frac{55}{d^2} \text{ kg/sq cm.}$$

As the factor of safety is 3, the working stress is $\frac{3600}{3} = 1{,}200$ kg/sq cm.

Maximum principal stress theory.

$$f_1 = 1200$$

$$\therefore \quad \frac{2650}{d^2} = 1200 \text{ or } d = \sqrt{\frac{2650}{1200}} = 1{\cdot}49 \text{ cm}$$

Maximum shear stress theory

$$f = (f_1 - f_2)$$

$$\therefore \quad 1200 = \frac{2650}{d^2} - \left(-\frac{55}{d^2}\right) = \frac{2705}{d^2}$$

$$\therefore \quad d = \sqrt{\frac{2705}{1200}} = 1{\cdot}51 \text{ cm.}$$

Maximum principal strain theory

$$f = f_1 - \mu f_2$$

$$1200 = \frac{2650}{d^2} - 0{\cdot}25\left(-\frac{55}{d^2}\right)$$

$$= \frac{2664}{d^2}$$

or $$d = \sqrt{\frac{2664}{1200}} = 1{\cdot}49 \text{ cm.}$$

Maximum total strain energy:

$$f^2 = f_1^2 + f_2^2 - 2\mu\, f_1 f_2$$

$$1200^2 = \frac{1}{d^4}\left[2650^2 + 55^2 + 2 \times 0{\cdot}25 \times 2650 \times 55\right]$$

$$= \frac{7101500}{d^4}$$

or $$d = \sqrt[4]{\frac{7101500}{1440000}} = 1{\cdot}49 \text{ cm.}$$

Distortion energy theory:

$$f^2 = f_1^2 + f_2^2 - f_1 f_2$$

$$\therefore\ 1200^2 = \frac{1}{d^4}\left[2650^2 + 55^2 + 55 \times 2650\right]$$

$$= \frac{7174000}{d^4}$$

or $$d = \sqrt[4]{\frac{7174000}{1440000}} = 1{\cdot}5 \text{ cm.}$$

This theory suggests that the diameter of the bolt is dictated by the maximum shear stress theory. Thus we adopt M 18 bolt, whose stress area is equal to 1·92 sq cm which corresponds to a core diameter of 1·56 cm.

Exercises:

1. *A critical section in a shaft is subjected to bending and twisting simultaneously. The bending moment causes a maximum shear stress of 5·5 kg/sq mm and the twisting moment a shear stress of 3·15 kg/sq mm. Determine the factors of safety according to the above theories of failure, if a tensile test gives a proportional limit of 28·4 kg/sq mm. Take Poisson's ratio as 0·28.*

Ans. 4·08, 3·38, 3·86, 3·78 and 3·65.

2. A spherical shell 360 cm diameter and 3 mm thick is subjected to an internal pressure p. Suggest the suitable value of the internal pressure if the failure of the shell is to be prevented according to the following theories of elastic failure:

(i) Maximum shearing stress
(ii) Total strain energy
(iii) Distortion energy.

The elastic limit of the material of the shell in simple tension is 2,400 kg/sq cm and the Poisson's ratio is 0.3

Assume the factor of safety to be 3..

Ans. 2.25 kg/sq cm.

2-23. Designing for impact loads

Impact loading results not only from actual impact or blow of a moving body against the machine members, but is any sudden application of the load. Impact load may occur in any one of the following methods:

(i) A direct impact, usually by another member or an external body moving with considerable velocity as in a pile driver or a punch press.

(ii) Sudden application of forces, without a blow being involved:

(a) The sudden creation of a force on a member as during the explosive stroke in internal combustion engines

(b) The sudden moving of a force onto a member as when a heavily loaded train or truck wheel moves rapidly over the floor of a bridge.

(iii) The inertia of a member resisting high acceleration and retardation such as rapidly reciprocating levers.

In many cases it is extremely difficult to evaluate impact forces quantitatively. The analysis of the problem is generally more of a qualitative nature and requires recognisation of all the factors involved and their inter relationship.

There are two general methods to select from in designing members to withstand impact loads.

(i) The maximum force exerted by the moving body on the resisting member is estimated by applying an impact factor.

Afterwards this force is considered as a static force and standard design formulas are used.

(ii) Energy to be absorbed by the resisting member is to be estimated; and from this value stresses or deformations by formulas for impact load on members are determined.

The dimensions of the resisting member and the properties of the material in the member that give it maximum resistance to an impact load (energy load) are quite different from those that give the member maximum resistance to a static load.

A metal may have good tensile strength and good ductility under static loading and yet fracture if subjected to a high velocity blow.

The two most important properties of a material that indicates its resistance to impact loading are

(i) Modulus of resilience and

(ii) Ultimate energy resistance.

Both the above properties are obtained from the stress strain diagram.

The *modulus of resilience* represents the capacity of the material to absorb energy *within its elastic range* i.e. without permanent deformation. It represents the energy stored in kg-cm/cu cm.

The *ultimate energy resistance* of a material indicates its toughness or *ability to resist fracture under impact loading.* This is a measure how well the material absorbs energy without fracture.

The following table gives the impact properties of common design materials:

Impact properties of common design materials

Material	Tensile modulus of resilience kg cm/cm³	Ultimate energy resistance kg cm/cm³
Grey cast iron	0·08	5
Malleable cast iron	1·2	270
Mild steel	1·5	1,120
Low alloy steel	2·5 to 2·8	—
Medium carbon steel	2·5	1,120
High carbon steel	6·5	350
Alloy steel	45	1,500

The following are the some of the rules which may be adopted while designing the machine element for impact loading:

(i) Design the member such that maximum volume of the material is stressed to the highest working stress.

(ii) Stress the entire length of the member to the maximum stress.

(iii) Reduce stress concentrations to a minimum and avoid abrupt changes in section.

(iv) Steels with higher yield strengths have higher values of modulus of resilience and are better for impact.

(v) Materials should have sufficient ductility to relieve the stress in any area of high stress concentration.

(vi) Materials should be placed so that the direction of hot rolling is in line with impact force, because the impact strength in this direction is higher than if impact occurs at right angles with the direction of rolling.

(vii) It is important to restrict the weight of the member and yet maintain proper rigidity for its particular use or service. This means light-weight, well stiffened members having sufficient second moment of area (I) should be used

(viii) Where required to build in protection against inertia forces caused by rapid movement of the member, it is important to decrease the possible acceleration through some form of flexible support.

The use of energy absorbing devices such as springs, rubber pads or hydraulic cushions will absorb some of the kinetic energy and thereby reduce the energy absorbed by the member.

The table on page 80 gives the *modulus of resilience* for various types of loading and for circular sections.

The common example of impact load that the designer meets in practice is a connecting rod bolts if the nuts on the connecting rod bolts are allowed to slacken slightly This is, in effect equivalent to a load falling through a short distance before striking the machine part.

If a body of weight W falls through a height h on a machine part of length l and cross sectional area A and of modulus of elasticity E, the maximum stress induced in the machine part will be

Type of loading	Modulus of resilience
Tension or compression	$\frac{f^2}{2E}$
Shear, simple transverse	$\frac{f^2}{2G}$
Torsion round bar (solid)	$\frac{f^2}{4G}$
Bending of beam with simply supported ends and loaded at the centre	$\frac{f^2}{24E}$
Helical spring with axial load	$\frac{f^2}{4G}$
Helical spring with axial twist	$\frac{f^2}{8E}$

$$f = \frac{W}{A}\left[1+\sqrt{1+\frac{2AEh}{Wl}}\right] \quad \text{.......................... (i)}$$

If $h=0$ i.e. the load is applied suddenly without impact, we have

$$f = \frac{2W}{A} \quad \text{.................................... (ii)}$$

Example:

1. The brasses of an automobile engine connecting rod have worn so as to allow play which gives, shock loading equivalent to a weight of 600 kg falling through a height of 0·020 cm. The connecting rod is 25 cm long and has a cross sectional area 3 sq cm. Determine the stress induced in the connecting rod. Compare the maximum stress induced with that by a static load of 600 kg.

If f be the maximum stress induced, then

$$f = \frac{W}{A}\left[1+\sqrt{1+\frac{2AEh}{Wl}}\right].$$

On substitution of values, we get

$$f = \frac{600}{3}\left[1+\sqrt{1+\frac{2\times 3\times 2{\cdot}1\times 10^6\times 0{\cdot}02}{600\times 25}}\right]$$

$= 1{,}044$ kg/sq cm.

Static stress $= \frac{600}{3} = 200$ kg/sq cm.

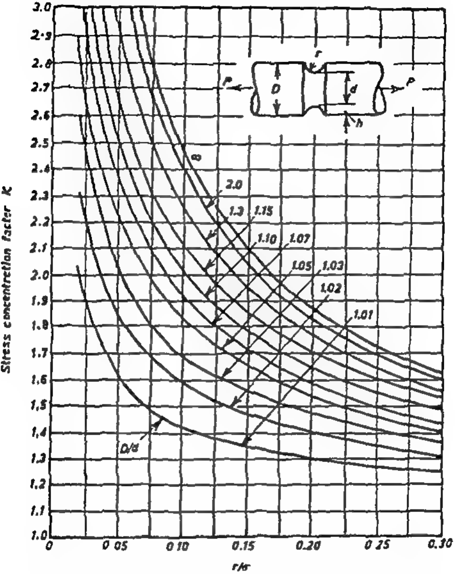


Fig. 2-27

Exercise:

1. *The piston rod of an engine is 5 cm in diameter and 100 cm long. The diameter of the piston is 37·5 cm and the maximum steam pressure is 9 kg/sq cm. Assuming that the steam acts as a suddenly applied load but without shock, determine the maximum stress induced in the rod.*

Ans. *1,000 kg/sq cm.*

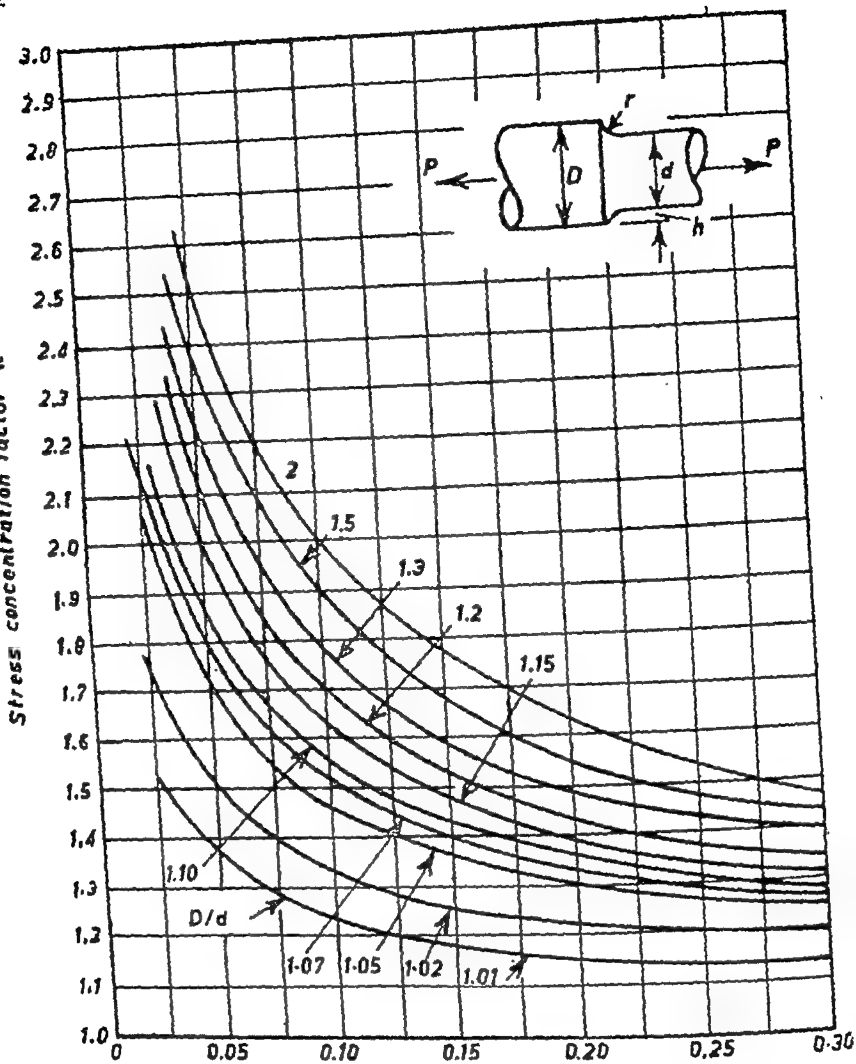


FIG. 2-28

2-24. Stress concentration:

In design procedure considered in earlier sections, we have not considered the effect of abrupt change in cross sections or holes in the machine parts. The stresses produced at these discontinuities are different in magnitude from those calculated by the elementary formulas seen in earlier sections. These stresses are

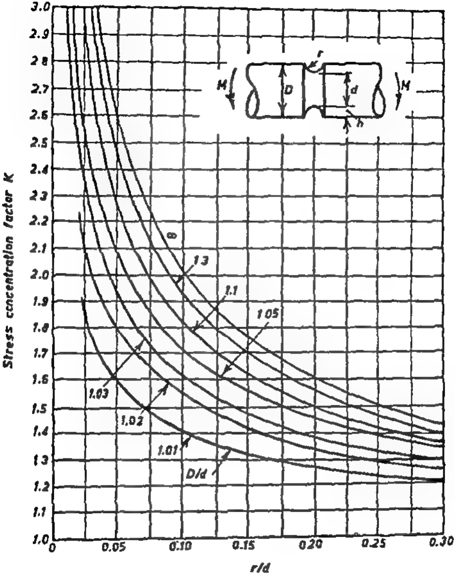


FIG. 2-29

known as the localised stresses which may give rise to a crack, during service conditions, which may lead to a failure of the machine part.

The effect of the localised increase in stress depends on the type of loading, the geometry of the part and the material of the

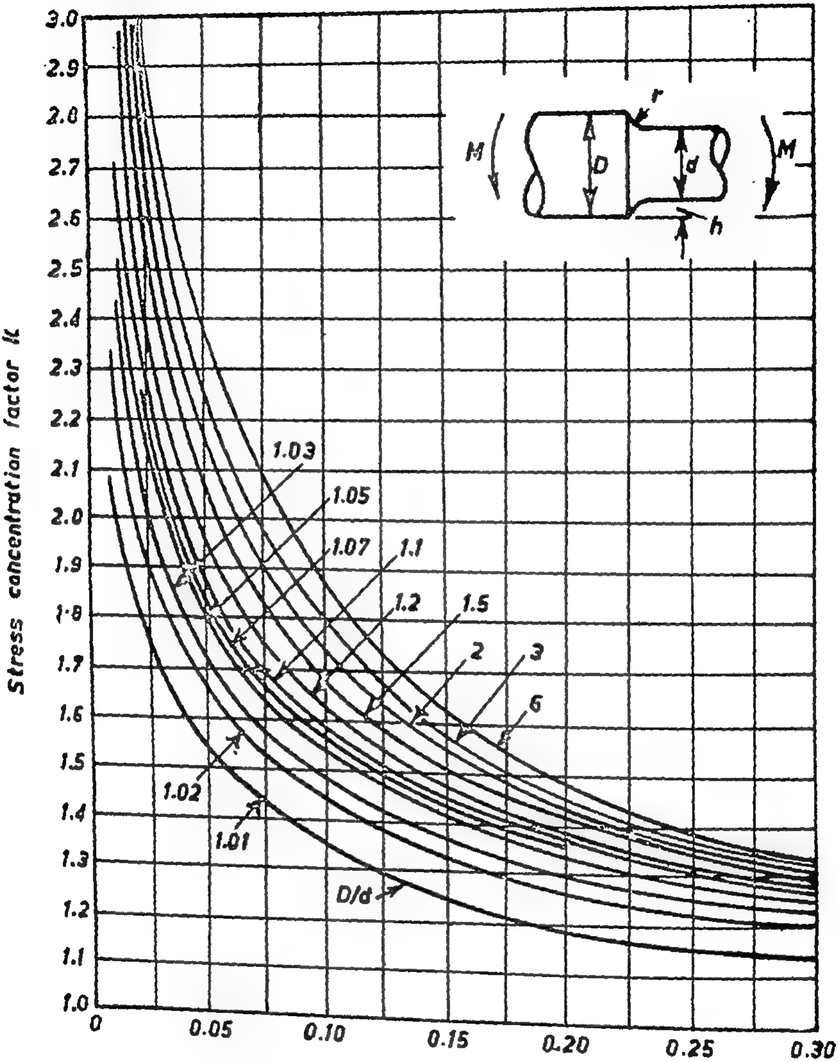


FIG. 2-30

part. The effect of the stress concentration is considered by a factor K where

$$K = \frac{\text{maximum localised stress}}{\text{nominal stress}} \quad \text{.......................(i)}$$

where K is known as the theoretical stress concentration factor or a geometric factor.

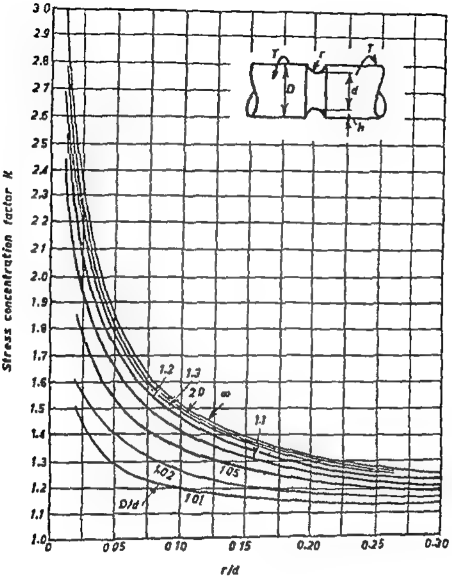


FIG. 2-31

Maximum localised stress is obtained either by theory of elasticity or by experimental stress analysis, while nominal stress is obtained by usual formulas of strength of materials.

Stress concentration, in static loading, is very serious in brittle materials such as cast iron and it is less serious in ductile material owing to the

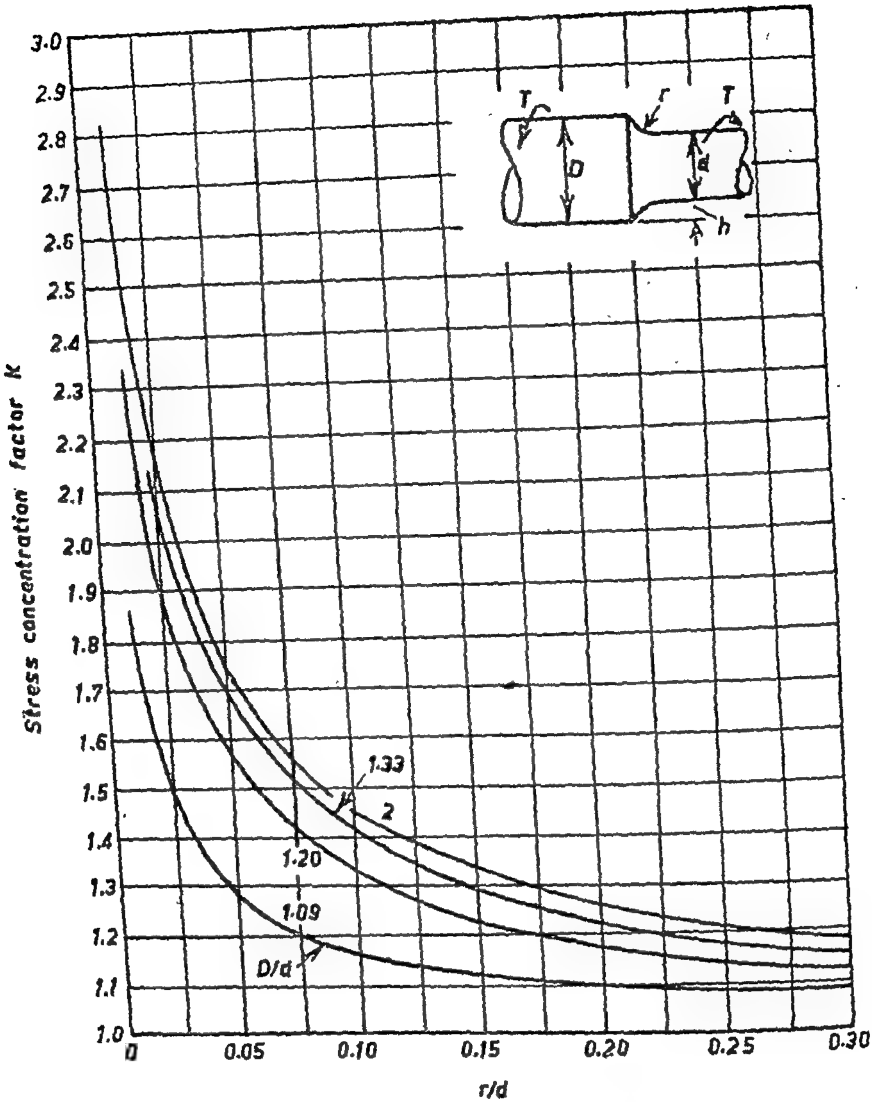


FIG. 2-32

relief of stress concentration by plastic flow. In cyclic loading, stress concentration is very serious for ductile materials also.

Considerable work has been carried out to determine the appropriate stress concentration factors to be applied in design. While it is possible in simple cases to calculate theoretically the values

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In order to study behaviour of materials under cyclic loading, fatigue testing machine is used which imparts a sinusoidal load to the specimen and gives a stress-time diagram as shown in fig. 2-33(a). This is done by placing a unidirectional bending

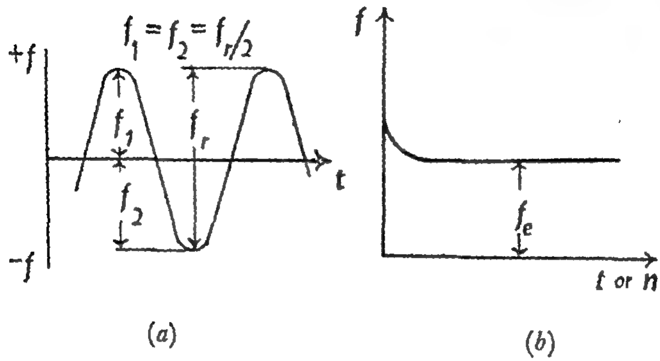


Fig. 2-33

load on a rotating shaft. The stress in the shaft fluctuates between equal values of tension and compression. A record is kept of the number of cycles required to produce a given stress and the results are plotted as shown by stress-cycle curves of fig. 2-33(b). This diagram indicates that if the stress is kept below a certain value called the endurance limit f_e the number of cycles can be increased indefinitely without causing failure. It was originally supposed that the stress-cycle curve did not actually flatten out but that it simply became flat for all practical purposes. Continued research work reveals that the endurance limit exists and if a stress is kept below the endurance limit, the failure due to fatigue will not occur.

The results of various tests suggest that the same factors which influence the ultimate tensile strength of steel also influence the endurance limit. Therefore, there seems to be a relation between the ultimate strength and the endurance limit.

For steel: $f_e = 0{\cdot}5\ U_t$

For cast iron: $f_e = 0{\cdot}4\ U_t$

For non-ferrous metals and alloys: $f_e = 0{\cdot}3\ U_t$

where f_e is the endurance limit and U_t is the ultimate tensile strength.

for stress concentration, it becomes necessary to resort to experimental methods for the more complex cases. Fig. 2-27 to Fig. 2-32 show the graphical results, from which stress concentration factor can be read directly for various values of $\frac{r}{d}$ and $\frac{D}{d}$. The work of R.E. Peterson is outstanding in the field of stress concentration.

2-25. Notch Sensitivity:

It may be defined as *the degree to which the theoretical* effect of stress concentration is actually reached. Notch sensitivity q of the material may be used to determine the fatigue strength reduction factor or actual stress concentration factor K_f as a function of the theoretical stress concentration factor for the case of *repeated loads*. Values for q and K may be found in "Stress Concentration Factors' by R.E. Peterson, and in other literature pertaining to this subject

$$q = \frac{K_f - 1}{K - 1}$$

or $K_f = 1 + q\,(K - 1)$ . . . . (1)

where

q = an experimental notch sensitivity value due to stress concentration, values range from 0 to 1·0.

K = theoretical value of the stress concentration factor. Average values range from 1·0 to 3·0 but could be greater than 3·0.

K_f = the actual stress concentration factor for determining the reduction in the fatigue-strength of the material.

It should be noted that K depends on the geometry of the component, while *fatigue stress concentration* factor, K_f, is dependent on the material and its method of production as well as the geometry.

2-26. Effect of repeated application of load:

Component parts such as crankshafts, connecting rods, gears, springs and frames are subjected to combined stresses as well as repeated stresses in cycles. Experience has shown that the load capacity of a machine part depends not only on the magnitude of the stress and the kind of stress cycle but also on the shape and the size of the part, and even on its surface finish.

Mr. J. M. Lessels states that a steady shear stress produced by a constant torque will not affect the fatigue strength of steel if the shear stress is kept below the value of completely reversed normal stress.

2-27. Fluctuating stress for ductile materials:

In the above article we have discussed the ways of finding the magnitude of completely reversed stress which a material can take indefinitely. The average stress of the cycle was zero. Manytimes the stress situation may be as shown by fig. 2-34 in which the

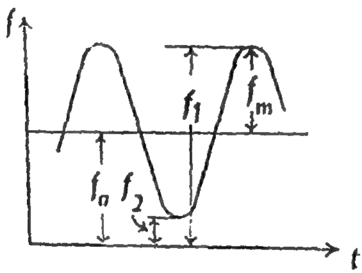


FIG. 2-34

average stress is not zero. Therefore, we should be able to find the maximum stress which can be applied an infinite number of times if endurance limit in reversed bending, f_e, and the stress range f_r are known.

There are three methods proposed to calculate the safe values of fluctuating stress:

(i) Gerber method
(ii) Goodman method
(iii) Soderberg method.

In 1874 Gerber proposed a parabolic relation between the stress range and average stress. The relation proposed by Gerber can be stated as under:

$$f_r = 2f_e\left[1-\left(\frac{f_a}{U_t}\right)^2\right]$$

where f_r = range of stress $= f_1 - f_2$
f_a = average stress
f_e = endurance strength in reversed bending and
U_t = ultimate tensile strength of the material.

Material, subjected to direct stress alternating between tension and compression, have endurance limits of about 85% of the reversed bending endurance limit f_e. For ductile materials subjected to cyclic torsion the endurance limit in shear is about half the reversed bending endurance limit f_e.

For steel:

Direct stress endurance limit $= 0{\cdot}85\ f_e = 0{\cdot}425\ U_t$

Cyclic torsion endurance limit $= 0{\cdot}5\ f_e = 0{\cdot}25\ U_t$

For cast iron:

Direct stress endurance limit $= 0{\cdot}34\ U_t$

Cyclic torsion endurance limit $= 0{\cdot}8\ U_t$

For non-ferrous metals and alloys

Direct stress endurance limit $= 0{\cdot}255\ U_t$

Cyclic torsion endurance limit $= 0{\cdot}2\ U_t$

Cyclic bending combined with cyclic torsion is a situation which does not occur very often. However, the following method is suggested by J. M. Lessels.

Endurance limit $f_e = 2\sqrt{\left(\frac{f_t}{2}\right)^2 + f_s^2}$,

where f_t is the tensile stress produced by completely reversed bending and f_s is the shear stress produced by completely reversed torsion. These two stresses f_t and f_s are in phase.

Let us explain the above statement by a numerical example.

Suppose a shaft is subjected to completely reversed bending stress of 4,000 kg/sq cm and in phase with this, shaft is subjected to a completely reversed torsional stress of 2,500 kg/sq cm.

According to above, we have

$$f_e = 2\sqrt{\left(\frac{4000}{2}\right)^2 + 2500^2} = 6{,}400 \text{ kg/sq cm}$$

Thus, we want a material for this shaft which must have an endurance limit of at least 6,400 kg/sq cm. As the endurance limit of steel is practically half the ultimate tensile strength, we require a steel with an ultimate tensile strength of at least $2 \times 6400 = 12{,}800$ kg/sq cm.

This example shows how the endurance limit of a test sample in reversed bending can be related to the performance of a machine part subjected to combined cyclic stress.

f_a = average stress
f_y = yield point in tension or compression and must be given the same sign as the average stress
K_f = actual stress concentration factor based on the notch sensitivity of the material
f_m = variable stress component
f_e = endurance limit of the material in reversed bending
A = correction factor for type of loading other than reversed bending
B = size correction factor, its value being 0·85 for parts ranging in size from 10 mm to 50 mm
C = surface correction factor.

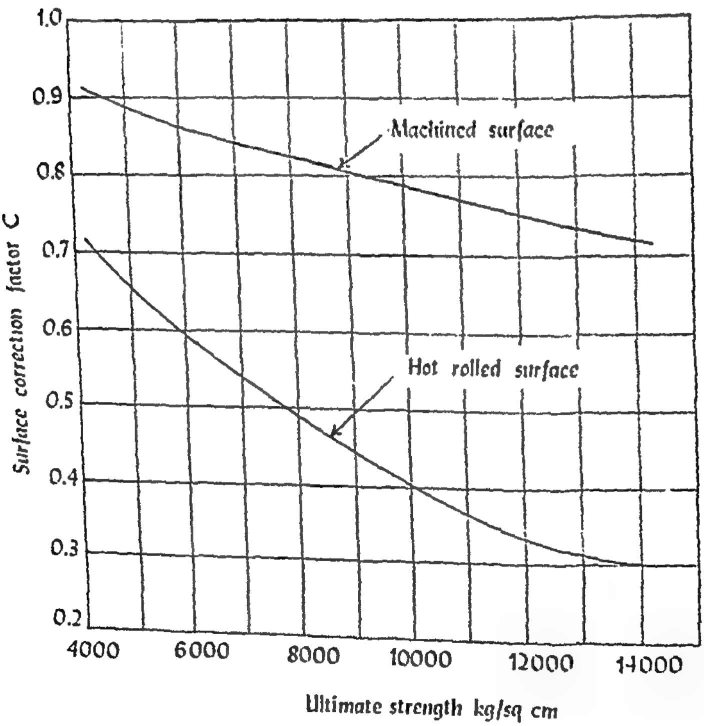


FIG. 2-35

The value of the surface correction factor depends on whether the surface is a machined surface or a hot rolled surface and on the ultimate strength of the part. The values of the surface correction factor can be read from fig. 2-35.

If $f_m = \frac{f_1 - f_2}{2} = \frac{f_r}{2}$

then $f_m = f_e\left[1 - \left(\frac{f_a}{U_t}\right)^2\right]$ .... (i)

f_m is called the variable stress component.

The maximum stress $f_1 = f_a + f_m$ and the minimum stress $f_2 = f_a - f_m$.

As the relation proposed by Gerber is little cumbersome to use, the simple relation has been proposed by Goodman. This relation is on the safer side.

$$f_r = 2f_e\left(\frac{1}{N} - \frac{f_a}{U_t}\right) \qquad \text{Goodman formula}$$

or $$f_m = f_e\left(\frac{1}{N} - \frac{f_a}{U_t}\right). \qquad \text{(ii)}$$

where N = factor of safety.

Soderberg has proposed a relation in which instead of ultimate strength U_t, a yield stress f_y is taken.

$$f_r = 2f_e\left(\frac{1}{N} - \frac{f_a}{f_y}\right) \qquad \text{Soderberg formula}$$

or $$f_m = f_e\left(\frac{1}{N} - \frac{f_a}{f_y}\right) \qquad \text{(iii)}$$

Soderberg formula can be written as

$$\frac{1}{N} = \frac{f_m}{f_e} + \frac{f_a}{f_y} \qquad \text{(iv)}$$

In order to make Soderberg formula a design equation, the experimental value of the endurance limit f_e under reversed bending should be reduced for *size effect, surface effect and type of variable loading if torsional or axial instead of bending.* The calculated variable stress, f_m, should be increased by the actual stress concentration factor K_f for ductile material. For brittle materials the theoretical or geometrical stress concentraction factor K should be applied to, f_a, average stress and K_f to the variable stress.

For ductile materials in tension or compression, Soderberg formula can be written as

$$\frac{1}{N} = \frac{f_a}{f_y} + \frac{K_f\, f_m}{f_e\, ABC} \qquad \text{(v)}$$

where N = suitable factor of safety

Design to avoid fatigue failure:

Since the shape and finish have much influence on a component's behaviour under fatigue conditions, the designer should pay more attention to it. *Functional components* can not be made without some geometrical discontinuities and machining costs, complexity of shape and methods of manufacturing frequently preclude the application of high surface finishes.

But by making unavoidable changes of section occur gradually instead of abruptly, relieving stress concentration by removal of metal at adjacent sections and stipulating suitable surface treatment during manufacture much can be done to avoid fatigue failure. This means particular attention should be paid to bolt heads and shanks, collars, recesses, webs, flanges, holes, keyways, splines, etc, the elimination of punch and stamp marks and poor quality machining and insisting, where necessary, on corrosion protection and suitable quality control in the form of adequate inspection and testing.

Examples:

1. A steel connecting rod is subjected to a completely reversed axial load of 10,000 kg. Suggest the suitable size of the rod using a factor of safety 2. The ultimate strength of the material is 11,000 kg/sq cm and yield strength 9,300 kg/sq cm.

Neglect the column action and the effect of stress concentration.

For a steel the endurance limit for reversed bending is half the ultimate tensile strength.

$\therefore$ f_e = endurance limit $= \frac{1}{2} \times 11000 = 5{,}500$ kg/sq cm.

As the load varies from 10,000 kg to $-$ 10,000 kg, the average or mean load $\dfrac{10000 - 10000}{2}$ will be zero and hence the average stress or mean stress will be zero.

Let d cm be the diameter of the connecting rod.

f_n = variable normal stress component $= \dfrac{4 \times 10000}{\pi d^2}$

The endurance limit correction factor for axial loading is $A = 0{\cdot}85$.

Size factor $= 0{\cdot}85$.

For brittle materials

$$\frac{1}{N} = \frac{f_a}{f_y} \times K + \frac{K_f \cdot f_m}{f_e\ ABC} \quad \text{(vi)}$$

For ductile materials in shear

$$\frac{1}{N} = \frac{f_a.s.}{f_y.s.} + \frac{K_f \cdot f_m.s.}{f_e\ ABC} \quad \text{(vii)}$$

where

$f_a.s.$ = average shear stress
$f_y.s.$ = yield point in shear
$f_m.s.$ = variable shear stress component.

In the above three design equations the factor of safety, N, accounts for variation in material properties, uncertainty of loading, workmanship, lack of test data, accuracy of assumptions, etc. The value of N may be taken from 1·5 to 3 for ordinary design. If the uncertainty is very high and the consequences of failure are serious, the higher value of N may be taken.

Ellipse quadrant relationship

In many cases components will be subjected to combined bending and torsion so that both tensile — compressive and shear stresses are variable.

Let $\pm f$ = the nominal design bending stress
$\pm f_s$ = the nominal design torsion stress
f_e = endurance limit due to bending alone
f_{es} = endurance limit due to torsion alone.

The above quantities can be related as follows:

$$\left(\frac{f}{f_e}\right)^2 + \left(\frac{f_s}{f_{es}}\right)^2 = 1 \quad \text{(viii)}$$

The above equation gives a useful relationship which can be applied to any materials where the theory of maximum shear stress is applicable, *such as all ductile steels, wrought iron, copper and some aluminium and magnesium alloys.*

For the application of the ellipse quadrant relationship, the endurance limit for reversed bending and for reversed torsion must be known. If then the bending stress to be set up by the component is known, the permissible torsional stress can be found out. Conversely, if the torsional design stress is known, the permissible accompanying bending stress can be determined.

On substitution of numerical values in the above equation, we have

$$4200 = \frac{U_t}{1 \cdot 75} - 700.$$

$$\therefore \qquad U_t = 8{,}575 \text{ kg/sq cm.}$$

According to Soderberg, we have

$$f_r = 2f_e \left[\frac{1}{N} - \frac{f_a}{f_y} \right].$$

We assume that $f_y = 0 \cdot 55 U_t$.

On substitution, we have

$$4200 = \frac{U_t}{1 \cdot 75} - \frac{700}{0 \cdot 55}.$$

$$\therefore \qquad U_t = 9{,}550 \text{ kg/sq cm.}$$

From the above calculations, we conclude that a steel with an ultimate tensile strength of 9,550 kg/sq cm will withstand an infinite number of applications of stress fluctuating between 2,800 kg/sq cm and $-$ 1,400 kg/sq cm without failure.

3. A 5 cm diameter shaft made from carbon steel hardened to 180 Brinell is subjected to a torque which fluctuates between 20,000 kg cm to $-$ 8,000 kg cm. Calculate the factor of safety by Soderberg method.

The range of torque $= 20000 - (-8000) = 28{,}000$ kg cm.

$$\therefore \text{ Range of shear stress} = \frac{28000 \times 16}{\pi \times 5^3} = 1{,}140 \text{ kg/sq cm.}$$

$$\text{Average torque} = \frac{20000 - 8000}{2} = 6{,}000 \text{ kg cm.}$$

$$\text{Average shear stress} = \frac{6000 \times 16}{\pi \times 5^3} = 244 \text{ kg/sq cm.}$$

Ultimate tensile strength $= 35 \times$ Brinell number (in kg/cm²)

$= 35 \times 180 = 6{,}300$ kg/sq cm.

Torsional endurance limit $= 0 \cdot 25\ U_t = 0 \cdot 25 \times 6300$

$= 1{,}575$ kg/sq cm.

We further assume that the shear yield point is half the tensile yield point. We further assume that the yield point for steel is 0·60 of the tensile strength.

$\therefore$ Shear yield strength $= 0 \cdot 60 \times 6300 \times \frac{1}{2} = 1{,}890$ kg/sq cm.

According to Soderberg formula,

For a machined surface, the value of surface correction factor can be read from graph, its value being 0·76.

According to Soderberg formula, we get

$$\frac{1}{N} = \frac{f_a}{f_y} + \frac{K_f f_m}{f_e ABC}$$

$$\frac{1}{2} = \frac{0}{9300} + \frac{\dfrac{4 \times 10000}{\pi d^2}}{5500 \times 0{\cdot}85 \times 0{\cdot}85 \times 0{\cdot}76} = \frac{40000}{3000\,\pi d^2}$$

or $\therefore\ d = \sqrt{\dfrac{40000 \times 2}{3000 \times \pi}} = 2{\cdot}95$ cm; we suggest 3 cm as the diameter of the connecting rod.

2. *Bending stress in a machine part fluctuates between a tensile stress of 2,800 kg/sq cm and compressive stress of 1,400 kg/sq cm. What should be the minimum ultimate tensile strength to carry this fluctuation indefinitely according to*

(a) *Gerber's formula* (b) *Goodman's formula* (c) *Soderberg's formula.*

The factor of safety may be assumed to be 1·75. Assume that the yield point is never likely to be less than 55% of the tensile strength or greater than 93% of it.

$$f_r = 2800 - (-1400) = 4{,}200 \text{ kg/sq cm.}$$

$$f_a = \frac{2800 - 1400}{2} = 700 \text{ kg/sq cm.}$$

Endurance limit in reversed bending is equal to $0{\cdot}5U_t$.

According to Gerber's formula, we have

$$f_r = 2\,f_e\left[1 - \left(\frac{f_a}{U_t}\right)^2\right].$$

On substitution, we have

$$4200 = U_t\left[1 - \frac{700^2}{U_t^2}\right]$$

or $U_t^2 - 4200\ U_t - 700^2 = 0$

$\therefore$ $U_t = 4{,}312$ kg/sq cm.

According to Goodman

$$f_r = 2\,f_e\left[\frac{1}{N} - \frac{f_a}{U_t}\right].$$

$$f_{s.a.} = \frac{16}{\pi d^3}\left[\frac{3000-1000}{2}\right] = \frac{5200}{d^3} \text{ kg/sq cm}$$

$$f_{s\cdot m\cdot} = \frac{16}{\pi d^3}\left[\frac{3000-(-1000)}{2}\right] = \frac{10200}{d^3} \text{ kg/sq cm.}$$

The value of the yield point in shear, f_{ys}, for use in the equivalent shear stress equation may be taken as 0·6 times the yield point in tension. This is in close agreement with experimental torsional shear stress. Hence in Soderberg's equation we take $A = 0{\cdot}6$.

We take $A = 0{\cdot}6$, $B = 0{\cdot}85$ and $C = 0{\cdot}62$.

$f_{ys} = 0{\cdot}6\ f_y = 0{\cdot}6 \times 4200 = 2{,}520$ kg/sq cm.

$$f_{e\cdot s} = \frac{5100}{d^3} + \left[\frac{2520}{2800}\right]\left[\frac{10200}{d^3 \times 0{\cdot}6 \times 0{\cdot}85 \times 0{\cdot}62}\right]$$

$$= \frac{34100}{d^3} \text{ kg/sq cm.}$$

Equivalent maximum shear stress

$$= \frac{1}{d^3}\sqrt{\left(\frac{97300}{2}\right)^2 + 34100^2} = \frac{60000}{d^3} \text{ kg/sq cm.}$$

For use in the maximum shear design equation, the value of the yield point in shear, f_{ys}, should be taken as 0·5 times the yield point in tension. *This equation is based on the maximum shear theory of failure which considers a member in simple tension.*

By equating the equivalent maximum shear stress to the permissible shear stress we get

$$\frac{60000}{d^3} = \frac{0{\cdot}5 \times 4200}{2} = 1050$$

or $$d = \sqrt[3]{\frac{60000}{1050}} = 3{\cdot}86 \text{ cm.; we adopt 4 cm.}$$

Design Note:

When a machine element is subjected to both a variable normal stress and a variable shear stress, the equivalent maximum shear stress may be determined by using the theory of combined stresses. For this purpose we use the Soderberg design equation.

The equivalent normal stress

$$f = f_a + \frac{f_y}{f_e}\cdot\frac{K_f\cdot f_m}{ABC}$$

The equivalent shear stress

$$f_s = f_{a.s} + \frac{f_{y.s}}{f_e}\cdot\frac{K_f\cdot f_{m\cdot s}}{ABC}$$

$$\frac{1}{N}=\frac{f_m \text{ shear}}{f_e \text{ shear}}+\frac{f_a \text{ shear}}{f_y \text{ shear}}$$

$$=\frac{\frac{1140}{2}}{1575}+\frac{244}{1890}=\frac{1}{2{\cdot}035}$$

$\therefore \quad N = 2{\cdot}035.$

4. A hot rolled steel shaft is subjected to a torsional load that varies from 3,000 kg cm clockwise to 1,000 kg cm anticlockwise as an applied bending moment at a critical section varies from 4,000 kg cm to −2,000 kg cm. Suggest the suitable size for the shaft if the material has an ultimate strength of 5,600 kg/sq cm and a yield strength of 4,200 kg/sq cm. Take the factor of safety as 2. The shaft is of uniform diameter and no key way is present at the critical section.

For steel the endurance limit for reversed bending is half the ultimate tensile strength. Therefore f_e = endurance limit $= \frac{1}{2} \times 5600 = 2{,}800$ kg/sq cm.

Equivalent normal stress due to bending

Let d cm be the diameter of the shaft at the critical section

$$f_{max}=\frac{32\times 4000}{\pi d^3} \text{ and } f_{min}=-\frac{32\times 2000}{\pi d^3}$$

$$f_a=\frac{32}{\pi d^3}\left[\frac{4000-2000}{2}\right]=\frac{10200}{d^3} \text{ kg/sq cm}$$

$$f_m=\frac{f_{max}-f_{min}}{2}=\frac{32}{2}\left[\frac{4000}{\pi d^3}+\frac{2000}{\pi d^3}\right]$$

$$=\frac{30600}{d^3} \text{ kg/sq cm}$$

We take the value of $B = 0{\cdot}85$ and $C = 0{\cdot}62$ and neglecting the effect of stress concentration if any, the equivalent normal stress

$$=\frac{10200}{d^3}+\left[\frac{4200}{2800}\right]\left[\frac{30600}{d^3\times 1\times 0{\cdot}85\times 0{\cdot}62}\right]$$

$$=\frac{97300}{d^3} \text{ kg/sq cm.}$$

Equivalent shear stress due to torsion:

$$f_s\,max=\frac{16\times 3000}{\pi d^3}=\frac{48000}{\pi d^3} \text{ kg/sq cm}$$

$$f_s\,min=-\frac{16\times 1000}{\pi d^3}=-\frac{16000}{\pi d^3} \text{ kg/sq cm.}$$

$$\frac{1}{2} = \frac{1020}{3800\ d^2} + \frac{1020}{d^2 \times 3500 \times 0{\cdot}7 \times 1 \times 0{\cdot}88}$$

$$= \frac{0{\cdot}743}{d^2}$$

$$\therefore \quad d = \sqrt{0{\cdot}743 \times 2} = 1{\cdot}22 \text{ cm.}$$

For an infinite life,

$$\frac{1}{2} = \frac{1020}{3800\ d^2} + \frac{1020}{d^2 \times 2600 \times 0{\cdot}7 \times 1 \times 0{\cdot}88}$$

$$= \frac{0{\cdot}91}{d^2}$$

$$\therefore \quad d = \sqrt{2 \times 0{\cdot}91} = 1{\cdot}35 \text{ cm.}$$

Thus economy can be effected by designing the member for a finite life.

Exercises:

√ 1. *The maximum pressure of air in the cylinder of a double acting air compressor of 50 cm bore is 9 kg/sq cm. What should be the diameter of the piston rod if there are no stress raisers and no column action? Take the factor of safety as 1·75. Indefinite life is desired. Ultimate tensile strength is 17,500 kg/sq cm. Take A = 0·85, size factor of 0·85 and machine surface factor of 0·78.*

Ans. 30 *mm.*

√ 2. *A steel connecting rod is to be subjected to a reversed axial load of 15,000 kg. Determine the diameter of the rod, using a factor of safety 1·8. The ultimate tensile strength of the material is 10,000 kg/sq cm. Neglect the effect of size factor and surface correction factor.*

Ans. 3 *cm.*

3. *A shaft supported as a simple beam, 45 cm long, is made of AISI 3120 steel. With the shaft rotating a steady load of 800 kg is applied midway between the bearings. The surfaces are ground. Indefinite life is desired with a factor of safety of 1·6 based on endurance strength. What should be the minimum diameter of the shaft if there are no surface discontinuities? Endurance limit is 6,300 kg/sq cm. Size factor is 0·85 and machine surface factor 0·87.*

Ans. 32 *mm.*

4. *A round shaft made of cold finished AISI 1020 steel is subjected to a variable torque whose maximum value is 7,000 kg cm. For a factor*

For ductile materials, we use $A = 0.6$ and $f_{ys} = 0.6\ f_y$.

The equivalent maximum shear stress with ductile materials

$$f_{s.max} = \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2}.$$

The equivalent maximum shear stress is equated to permissible shear stress which is equal to $\frac{f_y \times 0.5}{N}$ where N is the desired factor of safety.

When we are designing with brittle materials, we use the theory of maximum principal stress as the design criterion. Therefore equivalent maximum normal stress will be

$$\tfrac{1}{2}f + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2}$$

and this value of maximum normal stress must be equated to $\frac{f_y}{N}$.

5. In some instances, it may be more economical to design on the basis of an endurance strength for a finite life than for infinite life. A fuel pump pusher rod is to be designed for 10^5 cycles while it is being subjected to a released cycle load of 800 kg.

Test data indicate that the material from which the rod is to be made has a yield strength of 3,800 kg/sq cm and an endurance limit of 2,600 kg/sq cm for reversed loading, but has an endurance strength of 3,500 kg/sq cm for reversed loading of 10^5 cycles.

Suggest the suitable diameter of the pusher rod for both a finite life of 10^5 cycles and for an infinite life. Adopt a factor of safety of 2. Neglect the effect of stress concentration. Use $A = 0.7$, $B = 1$ and $C = 0.88$.

Let d cm be the diameter of the pusher rod.

Since we have released loading $f_a = f_m = \frac{800}{\frac{\pi}{4}d^2} = \frac{1020}{d^2}$ kg/sq cm.

The design equation will be Soderberg equation.

$$\frac{1}{N} = \frac{f_a}{f_y} + \frac{f_m}{f_e\ ABC}$$

For a finite life of 10^5 cycles, by employing Soderberg equation, we get

tively. Use a design factor of 1·8, size factor 0·85 and surface correction factor 0·88.

Use for torsion $A = 0{\cdot}6$ and $f_{s} = 0{\cdot}6\, f_y$.

Ans. 4 cm.

9. A section of a shaft of diameter d is joined to a section of shaft of diameter 1·5d with a fillet which produces an actual stress concentration factor of 1·2 for the shaft in torsion. The material has a yield point in tension of 5,600 kg/sq cm and endurance limit of 2,880 kg/sq cm in reversed torsion. Using a size factor of 0·85, surface finish factor of 0·85 and $A = 0{\cdot}6$, determine the size of the shaft required for a torque which varies from zero to 24,000 kg cm in the shaft at the smaller diameter. Use design factor $N = 2$. *Ans. 50 mm.*

10. If certain steel component has a fatigue limit in reversed bending of 3,600 kg/sq cm and in torsion of 1,800 kg/sq cm and it is computed that when in operation the bending stress will be 3,000 kg/sq cm. Determine by using ellipse quadrant relationship, the permissible value of the torsional shear stress.

Ans. 1,000 kg/sq cm.

2-28. Light weight and minimum dimensions:

The machine element should be sufficiently strong, stiff and wear resistant, while having the minimum possible dimensions and weight. This requirement can be satisfied by employment of light weight rolled sections, and hollow sections, by using high-strength grades of cast iron and light alloys, by introduction of non-metallic materials to replace ferrous and non-ferrous metals and by improving the design of the machine elements.

Let us consider some examples to illustrate the above mentioned points.

1. In the modernisation of an existing piece of machinery, a cast iron beam forming part of the machine frame and having the form of cross section shown in fig. 2-36(a) is to be replaced by a fabricated mild steel beam of the same overall depth as shown in fig. 2-36(b). On the assumption that the safe working tensile stresses for cast iron and mild steel are 2·3 and 9 kg/sq mm respectively.

(i) Design the cross section of the mild steel beam to give a bending strength equal to that of the cast iron beam. The second moment of area

of safety of 1·5 on the Soderberg criterion, determine the diameter of the shaft if (i) the torque is reversed, (ii) the torque varies from zero to maximum and (iii) the torque varies from 3,000 kg cm to a maximum.

Assume $A = 0{\cdot}6$, $B = 0{\cdot}85$ *and* $C = 0{\cdot}87$.

Ultimate strength $= 5{,}500$ *kg/sq cm*

Yield strength $= 4{,}600$ *kg/sq cm*

Ans. (i) 35 mm (ii) 32 mm (iii) 30 mm.

5. *A shaft is made of steel (ultimate tensile strength 7,000 kg/sq cm and yield point 4,200 kg/sq cm) is subjected to a torque varying from 20,000 kg cm anti-clockwise to — 6,000 kg cm clockwise. Calculate the diameter of the shaft if the factor of safety is 2 and it is based on the yield point and the endurance strength in shear.*

Ans. 48 mm.

6. *A cold drawn C — 1025 steel rod of circular section is subjected to a variable bending moment of 6,000 kg cm to 12,000 kg cm as the axial load varies from 2,000 to 4,000 kg. The maximum bending moment occurs at the same instant the axial load is maximum. Determine the required diameter of the rod for a factor of safety 2·25. Neglect any stress concentration and column effect but consider the effect of size and surface.*

Ultimate strength of C — 1025 cold drawn steel is 5,600 kg/sq cm.

Yield strength of C — 1025 cold drawn steel is 2,800 kg/sq cm

Ans 55 mm.

7. *A hot rolled shaft is subjected to a torsional load that varies from 3,200 kg cm clockwise to 1,200 kg cm anti-clockwise as an applied bending moment at a critical section varies from + 4,000 kg cm to — 2,000 kg cm. The shaft is of uniform cross section. Determine the required shaft diameter. The material has an ultimate strength of 5,600 kg/sq cm and yield strength of 4,200 kg/sq cm. Assume the factor of safety to be 2.*

Ans. 4 cm.

8. *A pulley is keyed to a shaft mid way between two anti-friction bearings. The bending moment at the pulley varies from 1,500 kg cm to 4,500 kg cm as the torsional moment in the shaft varies from 500 to 1,500 kg cm. The frequency of the variation of the loads is the same as the shaft speed. The shaft is made of cold drawn steel having an ultimate strength of 5,500 kg/sq cm and a yield strength of 3,100 kg/sq cm. Determine the required diameter for an indefinite life. The stress concentration factor for the key way in bending and torsion may be taken as 1·6 and 1·3 respec-*

$$25410 = I + 200 \times 8{\cdot}15^2$$

or $$I = 25410 - 200 \times 8{\cdot}15^2 = 12{,}130 \text{ cm}^4.$$

Tensile modulus of section $= \dfrac{12130}{8{\cdot}15} = 1{,}500 \text{ cm}^3.$

Bending moment that can be resisted by the section

$$= f \times z = 230 \times 1500$$

$$= 345{,}000 \text{ kg cm.}$$

Approximate second moment of area of mild steel section

$$= \frac{d^2bt}{2} = 25^2 \times 10 \times \frac{t}{2} = 3{,}125\, t \text{ cm}^4.$$

Modulus of section $= b\,dt = 25 \times 10 \times t = 250\,t \text{ cm}^3.$

$\therefore \quad 345000 = 250\,t \times 900$

or $$t = \frac{345000}{250 \times 900} = 1{\cdot}53 \text{ cm};$$ we adopt 1·6 cm as the thickness of the web and flange.

Actual second moment of area of the section

$$= \frac{1}{12}\,[10 \times 25^3 - 8{\cdot}4 \times 21{\cdot}8^3] = 5{,}780 \text{ cm}^4.$$

Rigidity considerations:

Since the deflection, for a given loading and the length, is inversely proportional to EI, flexural rigidity,

$$\frac{\delta_{C.I.}}{\delta_{M.S.}} = \frac{2{\cdot}1 \times 10^6 \times 5780}{0{\cdot}84 \times 10^6 \times 12130} = 1{\cdot}2.$$

Thus the deflection of mild steel beam is less than that of original cast iron beam.

2. *In order to reduce the weight of the control and power plant facilities in a certain aircraft it is planned to use all hollow shafting for power transmission. The hollow shaft is to be 10% larger in diameter than the solid shaft. Determine the percentage weight savings that may be effected through the use of hollow shafting in place of equal strength solid shafting for such an application.*

Let M = bending moment at the critical section

T = twisting moment at the critical section

d = diameter of the solid shaft

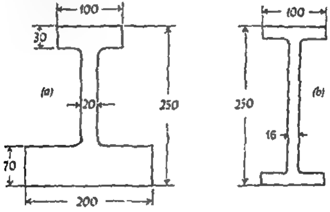
d_o = outer diameter of the hollow shaft

d_i = inside diameter of the hollow shaft.

of the cross section of the steel beam is to be approximated by using the expression $\frac{d^2bt}{2}$ *where*

d = *depth of the beam and*
b = *width of the flange.*

(ii) Compare the deflections of two beams using the true values of their second moments of area. The moduli of elasticity of steel and cast iron are respectively $2{\cdot}1 \times 10^6$ *and* $0{\cdot}84 \times 10^6$ *kg/sq cm.*



(a) Cast iron (b) Mild steel

FIG. 2-36

Area of the cast iron section $= 20 \times 7 + 2 \times 15 + 3 \times 10$
$= 200$ sq cm.

If $\bar{x}$ be the distance of the centre of gravity of the section from the lower base, then

$$200 \times \bar{x} = 140 \times 3{\cdot}5 + 30 \times 14{\cdot}5 + 30 \times 23{\cdot}5$$

or

$$\bar{x} = \frac{140 \times 3{\cdot}5 + 30 \times 14{\cdot}5 + 30 \times 23{\cdot}5}{200}$$

$$= 8{\cdot}15 \text{ cm.}$$

Second moment of area of the cast iron section about the lower base

$$= \int_0^7 20 \times x^2\,dx + \int_7^{22} 2x^2\,dx + \int_{22}^{25} 10x^2\,dx = 25{,}410 \text{ cm}^4.$$

If I be the second moment of area about a parallel axis through the centre of gravity, then

tubing. Space considerations allow the new component to have an outside diameter of 8 cm maximum, but require the length and end fixings of the old member to be retained. The magnesium alloy beam must be as strong as its predecessor and at least as rigid.

Design the cross section of the tube to meet these requirements and determine the percentage reduction in weight that can be expected.

The ultimate tensile strengths of the aluminium and the magnesium alloys are 5,000 and 2,500 kg/sq cm respectively, the moduli of elasticity $0{\cdot}7 \times 10^6$ and $0{\cdot}45 \times 10^6$ kg/sq cm respectively and the densities 2,640 and 1,728 kg/cubic metre respectively.

Let suffixes a and m refer to the aluminium and magnesium alloys respectively.

Then ratio of stresses $\dfrac{f_a}{f_m} = \dfrac{5000}{2500} = 2.$

$\therefore \qquad f_a = 2\, f_m$

and since $\qquad f = \dfrac{My}{I}$

$$\therefore \qquad \frac{M\, y_a}{I_a} = \frac{2M\, y_m}{I_m}$$

$$\therefore \qquad \frac{y_a}{y_m} \times \frac{I_m}{I_a} = 2.$$

But $y_a = 3$ cm and $y_m = 4$ cm.

$$\therefore \qquad \frac{3}{4} \times \frac{I_m}{I_a} = 2$$

or $\qquad I_m = \dfrac{8}{3}\, Ia.$

$$I_a = \frac{\pi}{64} \times 6^4 \text{ and } I_m = \frac{\pi}{64}\,(8^4 - d^4) \text{ where}$$

d is the inner diameter of the magnesium alloy tubing.

$$\therefore \quad \frac{\pi}{64} \times 6^4 \times \frac{8}{3} = \frac{\pi}{64}\,(8^4 - d^4) \text{ because } I_m = \frac{8}{3}\, I_a.$$

$$\therefore \qquad d = 5 \text{ cm.}$$

Rigidity of the tubular component:

Since the deflection of each beam may be taken as equal to a constant multiplied by $\dfrac{W\, l^3}{EI}$, then

$$\frac{\delta_m}{\delta_a} = \frac{Wl^3}{E_m I_m} \times \frac{E_a I_a}{Wl^3} = \frac{E_a I_a}{E_m I_m}$$

We assume that the shafts are made of ductile material for which maximum shear stress is design criterion. For a solid shaft subjected to torsional and bending loads, the shear stress is given by $\frac{16}{\pi d^3}\sqrt{M^2+T^2}$, and for a hollow shaft subjected to the same torsional and bending loads as above, the shear stress will be given by $\frac{16 d_0}{\pi(d_0^4-d_i^4)}\sqrt{M^2+T^2}$

As both the shafts are to be of the same strength, by equating the values of the induced shear stresses, we get

$$\frac{16}{\pi d^3}=\frac{16\, d_0}{\pi(d_0^4-d_i^4)}$$

$$\text{or}\quad 1-\left(\frac{d}{d_0}\right)^3=\left(\frac{d_i}{d_0}\right)^4 \qquad \text{(i)}$$

Let x be the % weight savings to be effected through the use of hollow shafting. For the same length of both shafts and having the same materials, the weights of the shafts are proportional to the areas of cross sections of individual shafts

Hence

$$\frac{\pi}{4}(d_0^2-d_i^2)=\left(1-\frac{x}{100}\right)\frac{\pi}{4}d^2$$

$$\text{or}\ (d_0^2-d_i^2)=\left(1-\frac{x}{100}\right)d^2 \qquad \text{(ii)}$$

Substituting the value of d_i from equation (i) into equation (ii) and solving for x, we get

$$x=\left[1-\left(\frac{d_0}{d}\right)+\sqrt{\left(\frac{d_0}{d_i}\right)^4-\frac{d_0}{d_i}}\right]\times 100 \qquad \text{(iii)}$$

For this example the hollow shaft is to be 10% larger in diameter than the solid shaft so $\frac{d_0}{d}=1.1$

$$\therefore\quad x=[1-1.1+\sqrt{1.1^4-1.1}]\times 100$$
$$=40$$

Thus the hollow shaft will weigh 0.6 of the solid shaft

3. *An aircraft component of 6 cm diameter solid circular cross section subjected to lateral loading has previously been manufactured in an aluminium alloy, but because of the imposition of more stringent weight limitations it is now intended to re-design the beam in magnesium alloy*

low, the main forces being inertia forces. The engineers decided that if a lighter metal could be used, the mass would be decreased and in turn the inertia forces would be decreased and consequently the deflection of the levers would be decreased. Hence they decided to replace the steel lever by aluminium. However the technical director of the company informs that the problem will not be solved by changing the material to aluminium, but the solution lies in redesigning the steel lever. Explain the stand of the technical director whose judgement is correct.

4. A small machine is supported at the end of a cantilever beam 180 cm long. During its operation it exerts a force in this support and must be held within an allowable deflection and the cross section of the cantilever is shown in fig. 2-37. A new model of this machine must extend out to 550 cm and must operate under the same conditions and allowables. It is decided that the new beam will have a width equal to half its depth and a wall thickness equal to 5% of its depth. Suggest the suitable cross section.

Ans. Depth 25 cm.

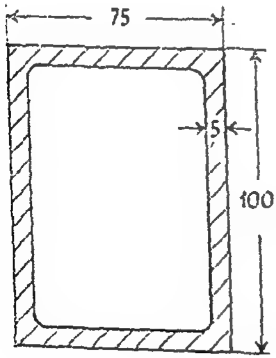


FIG. 2-37

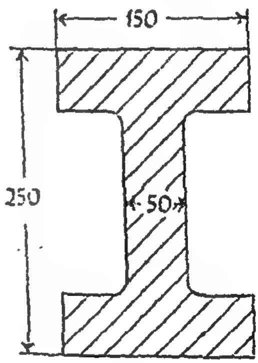


FIG. 2-38

5. A standard 150 mm pipe has been used for main front support of an earth moving scraper. Its main requirement is to resist the torsional load within an allowable angular test. It is desired to replace this pipe section, with a fabricated square box section having the same over all dimensions. Determine the required thickness of plate for this box section to hold it within the same angular twist. Assume bending resistance is sufficient. Outside diameter of the tube 159 mm and inside diameter 144 mm.

Ans. 6 mm.

$$\frac{\delta m}{\delta a}=\frac{7}{4\cdot5}\times\frac{3}{8}=0\cdot384.$$

Thus the deflection of the magnesium alloy beam is only slightly above half that of the aluminium alloy component. In other words it is approximately twice as rigid.

Reduction in weight:

$$\frac{\text{Weight of magnesium alloy}}{\text{Weight of aluminium alloy}}=\frac{\text{Volume}\times\text{density of magnesium}}{\text{Volume}\times\text{density of aluminium}}$$

$$=\frac{1728\ (8^2-5^2)}{2640\ (6^2)}=0\cdot528$$

since lengths of both the beams are the same.

Thus the expected reduction in weight is 0·472 i.e. 47·2%.

Note: This illustrative example shows the advantage of using magnesium alloys. The high $\frac{\text{strength}}{\text{weight}}$ ratio of the magnesium alloy does much to offset the low modulus of elasticity value of the material. The main disadvantages of many of the alloys of magnesium are as under

(i) Low resistance to wear
(ii) Moderate fatigue strength
(iii) Low creep strength

Exercises:

1. The spindle of a drilling machine is subjected to a maximum axial load of 800 kg during operation. Determine the diameter of the solid cast iron post if the tensile stress is limited to 400 kg/sq cm. The distance between the axis of the spindle and the axis of the post is 40 cm.

If the section of the post were to be changed to hollow section with a ratio of outside diameter to inside diameter 2 : 1, what will be the saving in material of the post? The permissible stress in both the cases is the same.

Ans. 9·5 cm, 16·6 l.

2. A machine tool company experimented with bases of different steels to learn if higher strength steel might result in a more rigid base. What will be the conclusion of their experiments?

3. "Associate Engineers" were experiencing some difficulty with a lever which operated at very high speeds. The engineers reasoned the problem in a very logical manner. The actual load on this lever was very

support with slight eccentricity. Consequently there results a bending moment, which causes the bearing to tilt slightly. It is then possible *to calculate the proper second moment of area of the cross section of the bearing support so that the bearing will tilt at the same angle as the ends of the roller under any loading as shown* in fig. 2-40. Both will always be in perfect alignment.

2-30. Temperature stress:

When two materials having different coefficients of expansion are connected together, stresses will be developed in the materials when the temperature is changed. This is due to the restraining action of low coefficient material tending to withhold expansion of the other. For a material firmly held to an original length (corresponding to t_1) the stress induced by a temperature change will be,

$$f = E\alpha\,(t_2 - t_1)$$

where E = modulus of elasticity

α = coefficient of thermal expansion

$t_2 - t_1$ = temperature difference.

The stresses induced will be compressive in nature. If the body is free to expand, no stresses will be set up.

Thermal stresses are very important in the design of pipe lines, large internal combustion engines, steam machinery, etc. Thermal stresses in pipe lines are relieved by incorporating a flexible member such as expansion joint, bend, sleeve, etc.

Various parts of machines and engines are fixed by shrinking on. Suppose a crank is to be shrunk on to a shaft, the crank shaft is made slightly larger than the bore on the crank. When the crank web is heated, it can be put in position on the shaft. On cooling the crank, the contraction binds it firmly on the shaft. In a similar manner tyres are fixed on the wheels of a carriage.

The stresses induced in the material due to shrinkage can be calculated on the assumption that the shaft or boss does not change in diameter. This is not the case as the shaft or connected parts are made of elastic material. However, the assumption made is on safer side.

$$f = \text{stress} = \text{strain} \times \text{modulus of elasticity} \quad \ldots\ldots\ldots\text{(i)}$$

$$\text{Strain} = \frac{\text{difference in two diameters}}{\text{original diameter}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

6. Fig. 2-38 shows a cast iron beam section which is required to resist a torsional loading. In order to reduce the weight of the part it is proposed to replace by a steel beam of section having the same depth and same overall width of the original section. Both the sections have the same torsional resistance. Determine the saving in weight that can be effected by using steel. Explain briefly how you can further reduce the weight.

Ans. Approximately 25%; 80% weight reduction is possible by adopting box section of half its overall dimensions.

7. In as much as the design criteria for most machinery structures is rigidity, strength does not particularly enter as the resistance to deflection is paramount. In old design the cross section of the member is an I section of 500 mm depth and 175 mm wide flange having second moment of area 70,000 cm⁴. Design a steel beam that will be as rigid as cast iron beam and still retain the external dimensions of height and flange width. Also determine the percentage saving in weight by employing steel.

Ans 1 cm thick, 72%.

2-29. Elastic matching:

This term is used when two connecting members are designed so that their angular deflections are equal. This means that they will remain aligned regardless of the value of the load applied.

Consider a roller which is supported by two fixed bearings (not self-aligning bearings). The roller will always deflect somewhat, regardless of how large and rigid it is. The bearing support will not tilt, because the uniform bearing pressure is centred about the centre of gravity of the support. Thus the bearing support will remain horizontal and the end of the roller will tilt with any loading. *This will result in wear and shorter bearing life.*

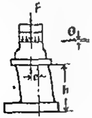


FIG. 2-39

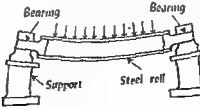


FIG. 2-40

By shifting the bearing support slightly off the centre line of the bearing as shown in fig. 2-39, the bearing force is applied to the

at this position is 50 mm; the compressive stress in the bracket must not exceed 350 kg/sq cm (b) the permissible drilling error 'x' cm if the allowable tensile stress in the bar is 500 kg/sq cm. Ans. (a) 4 cm; (b) 0·0132 cm.

12. Fig. 2-45 shows a double sheave hook block as used on a 3 tonne crane. The pulleys are of 20 cm diameter, the hub width of each pulley being 7·5 cm.

A 5 mm thick gun metal washer is inserted between the two pulleys and also between the pulleys and the side plates. Design the following parts of the block:

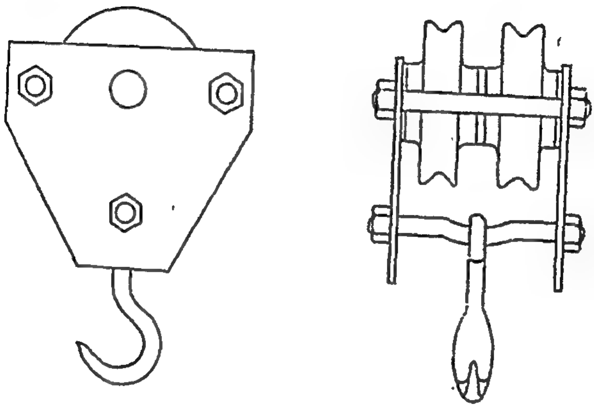


FIG. 2-45

(i) Pin on which the pulleys are mounted
(ii) Pin on which the block is mounted
(iii) Side plates.

$f_t = 700$ kg/sq cm; $f_s = 500$ kg/sq cm; $f_c = 1{,}050$ kg/sq cm and bearing pressure intensity 150 kg/sq cm.

Ans. (a) 4·5 cm (b) 6 cm (c) 8 mm thick × 45 mm wide.

13. A crane hook is circular in cross section with a diameter of 6·25 cm at the dangerous section. The eccentricity (distance from the centre of gravity of the dangerous section to the line of action of the force) is 8·75 cm. Calculate the maximum load that the hook can support safely. Use a design stress of 560 kg/sq cm.

(Sardar Patel University, 1967)

14. (a) Explain the following terms:

(i) Elastic range (ii) Plastic range (iii) Permanent set (iv) Factor of safety.

(b) The punch press similar to one shown in fig. 2-25 has a T section 6 cm long over all along each axis and walls 2 cm thick. The line of action of the tensile load is 10 cm away from the c.g. of the section as shown in figure. If this punch is used to punch holes on a mild steel plate 1·5 mm thick, suggest the largest diameter of the hole that can be punched on this press.

The above stress is induced in the hub if it is assumed that the shaft diameter does not decrease, which condition is approached with large diameter shafts and thin hubs.

Example:

1. A steel collar is to be forced around a 75 mm diameter shaft. Determine the difference between shaft and collar diameter if stress in the collar must not exceed 1,680 kg/sq cm. The modulus of elasticity is $2 \cdot 1 \times 10^6$ kg/sq cm.

The stress in the collar $= \dfrac{E \times \text{difference in two diameters}}{D}$

$\therefore$ Difference in two diameters $= \dfrac{1680 \times 7 \cdot 5}{2 \cdot 1 \times 10^6} = 0 \cdot 006$ cm.

Exercise:

1. A locomotive wheel is 180 cm in diameter. A steel tyre is shrunk on the wheel. Determine the internal diameter of the tyre if after shrinkage the hoop stress in the tyre is 1,050 kg/sq cm. Assume that the wheel is not altered in diameter due to pressure of tyre and that the modulus of elasticity is $2 \cdot 1 \times 10^6$ kg/sq cm. *Ans.* 179·91 *cm.*

EXAMPLES II

1. (a) With suitable examples of machine elements explain how the factor of safety adopted in designing machine elements varies with the nature and type of load imposed on them

(b) Indicate the factor of safety to be used in assuming permissible tensile stress in the following:

(i) Bolts for securing cylinder cover
(ii) I C. engine connecting rod
(iii) Arms of a cast iron flywheel
(iv) Spring of a safety valve
(v) Springs in I C. engine valves
(vi) Rivets in boiler joints
(vii) Flange coupling bolts
(viii) Crankshaft.

2. Show how the permissible stress allowed in a given material varies with the type of machine element for which it is used Illustrate your answer with suitable examples of machine elements manufactured from the same material.

3. Explain why lower safety margins are commonly used in aircraft design where failure may be disastrous, than in many industrial machines where failures endanger no one.

at this position is 50 mm; the *compressive* stress in the bracket must not exceed 350 kg/sq cm (b) the permissible drilling error 'x' cm if the allowable tensile stress in the bar is 500 kg/sq cm. Ans. (a) 4 cm; (b) 0·0132 cm.

12. Fig. 2-45 shows a double sheave hook block as used on a 3 tonne crane. The pulleys are of 20 cm diameter, the hub width of each pulley being 7·5 cm. A 5 mm thick gun metal washer is inserted between the two pulleys and also between the pulleys and the side plates. Design the following parts of the block:

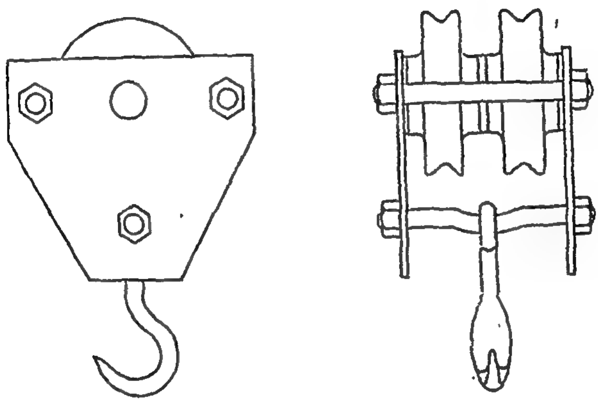


FIG. 2-45

(i) Pin on which the pulleys are mounted
(ii) Pin on which the block is mounted
(iii) Side plates.

$f_t = 700$ kg/sq cm; $f_s = 500$ kg/sq cm; $f_c = 1{,}050$ kg/sq cm and bearing pressure intensity 150 kg/sq cm.

Ans. (a) 4·5 cm (b) 6 cm (c) 8 mm thick $\times$ 45 mm wide.

13. A crane hook is circular in cross section with a diameter of 6·25 cm at the dangerous section. The eccentricity (distance from the centre of gravity of the dangerous section to the line of action of the force) is 8·75 cm. Calculate the maximum load that the hook can support safely. Use a design stress of 560 kg/sq cm. (Sardar Patel University, 1967)

14. (a) Explain the following terms:

(i) Elastic range (ii) Plastic range (iii) Permanent set (iv) Factor of safety.

(b) The punch press similar to one shown in fig. 2-25 has a T section 6 cm long over all along each axis and walls 2 cm thick. The line of action of the tensile load is 10 cm away from the c.g. of the section as shown in figure. If this punch is used to punch holes on a mild steel plate 1·5 mm thick, suggest the largest diameter of the hole that can be punched on this press.

The ultimate strength of the mild steel plate in shear is 3,360 kg/sq cm. The maximum stress in the punch frame should not exceed 600 kg/sq cm.

(Sardar Patel University, 1968)

15. Two shafts A and B are made of the same material and are of the same length. They transmit the same torque under identical conditions. Shaft A is solid while shaft B is hollow with its outer diameter twice the inner. What is the ratio of the weight of shaft A to that of shaft B?

(University of Bombay, 1969)

16. (a) Define: Factor of safety, Stress concentration; Modulus of Elasticity

(b) State what materials are commonly used for the following parts of a steam engine: Cylinder, piston, piston rings, crankshaft, piston rod, crosshead, valve.

(Sardar Patel University, 1969)

17. A machine member is subjected to the following allowable stresses

$f_t = 15$ kg/sq mm $f_c = 10$ kg/sq mm

Take $f_y = 42$ kg/sq mm for the material used

Find the factor of safety by various theories of failure

(Gujrat University, 1969)

18. A cast iron pipe has 250 mm inside diameter and has a wall thickness of 10 mm. The pipe contains water under a head of 100 metre. Calculate

(i) stress due to bending, if the pipe is horizontal, 10 metre long, simply supported at both ends and running full of water

(ii) maximum resultant stress

Take densities of water and cast iron as 1 00 and 7 25 kg/dm³

(Rajsthan University, 1969)

19. (a) State basic considerations in design of machine members Explain the nature and properties of important engineering materials.

(b) Discuss values of factors of safety assumed in designing various machine members for different duties

(Rajsthan University, 1969)

CHAPTER III

CYLINDERS, TANKS AND PIPES

3-1. Introduction:

Cylindrical and spherical tanks are used to store liquids, vapours and gases under pressure. Pipes are used to transmit fluid under pressure. The material of the tank or pipes is subjected to tensile stresses which are at right angles to each other. The thickness of the pressure vessel (i.e. tank or cylinder) must be such that the stresses induced are within limits. As the failure of such a vessel in service involves heavy loss of life and property, the basic specifications of the design, the design formulas and the allowable stresses as well as the rules of governing the erection and operation of these vessels are specified in special regulations.

3-2. Types of vessels:

The vessels are classified as under:

(i) According to geometrical shapes e.g. cylindrical, conical, spherical, etc.

(ii) According to the direction of the forces acting on the walls of the vessels e.g. vessels subjected to internal pressure and external pressure

(iii) According to end head shape e.g. flat head, dished end, convex head

(iv) According to types of service.

The vessels may be made of sheet steel, cast iron and non-ferrous alloys. The heads may be fastened to the shell either permanently (by riveting or welding) or by bolted joints.

Chemical vessels in which the pressure and temperature of the medium may rise owing to chemical reaction may be made of special materials.

3-3. Design of thin cylinders:

Cylindrical pressure vessels fall into either one of the two categories: *thin or thick cylinders*, depending upon whether the

The ultimate strength of the mild steel plate in shear is 3,360 kg/sq cm. *The maximum stress in the punch frame should not exceed 600 kg/sq cm.*

(Sardar Patel University, 1968)

15. Two shafts *A* and *B* are made of the same material and are of the same length. They transmit the same torque under identical conditions. Shaft *A* is solid while shaft *B* is hollow with its outer diameter twice the inner. What is the ratio of the weight of shaft *A* to that of shaft *B*?

(University of Bombay, 1969)

16. (a) Define: Factor of safety, Stress concentration; Modulus of Elasticity

(b) State what materials are commonly used for the following parts of a steam engine: Cylinder, piston, piston rings, crankshaft, piston rod, crosshead, valve

(Sardar Patel University, 1969)

17. A machine member is subjected to the following allowable stresses:

f_s = 15 kg/sq mm f_c = 10 kg/sq mm

Take f_y = 42 kg/sq mm for the material used

Find the factor of safety by various theories of failure

(Gujarat University, 1969)

18. A cast iron pipe has 250 mm inside diameter and has a wall thickness of 10 mm. The pipe contains water under a head of 100 metre Calculate

(i) stress due to bending, if the pipe is horizontal, 10 metre long, simply supported at both ends and running full of water

(ii) *maximum resultant stress*

Take densities of water and cast iron as 1 00 and 7 25 kg/dm³

(Rajsthan University, 1969)

19. (a) State basic considerations in design of machine members Explain the nature and properties of important engineering materials.

(b) Discuss values of factors of safety assumed in designing various machine members for different duties

(Rajsthan University, 1969)

are (i) hoop or circumferential stresses acting across longitudinal sections as shown in fig. 3-1(a), (ii) longitudinal or axial stresses acting across sections at right angles to the longitudinal axis of the cylinder as shown in fig. 3-1(b) and, (iii) radial stresses which are small compared with the previous two and can be neglected. These three stresses are mutually perpendicular and are principal stresses.

Let D = the internal diameter of the cylinder
t = thickness of the cylindrical shell
p = internal (gauge) pressure in the cylinder
f_t = tangential or hoop or circumferencial stress
f_l = axial or longitudinal stress.

Then $$f_t = \frac{pD}{2t} \quad \text{...(i)}$$

and $$f_l = \frac{pD}{4t} \quad \text{...(ii)}$$

Both the above stresses are tensile and are independent of the length of the cylinder. The hoop stress is twice the longitudinal stress. Thus if water in closed pipe freezes, the pipe will rupture along a line running longitudinally along the pipe.

In design of thin cylinders, in order to determine the thickness of the cylinder, we use the formula

$$t = \frac{pD}{2f_t} \quad \text{...(iii)}$$

In constructing large pressure vessels or storage tanks such as boilers, coal bunkers, air receivers, etc. several plates may be used which necessitates the use of welded joints or riveted joints in joining together the ends of the plate. So while designing the thickness of the pressure vessels, we must consider the efficiency of the joints. If η be the efficiency of the longitudinal joint, then

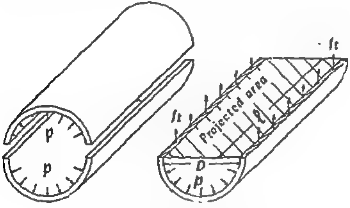
$$t = \frac{pD}{2f_t\,\eta} \quad \text{...(iv)}$$

In designing steam boilers, the thickness calculated from equation (iv) should be compared with the plate thickness set forth by Indian Boiler Regulations. Should the calculated thickness be less than that required by I.B.R., the thickness determined from I. B. R. should be adopted. It should be remembered that the design of boilers should confirm with I. B. R.

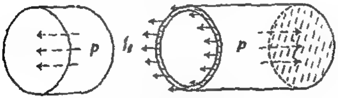
For cylindrical shells, barrels, steam and water drums and domes of boilers, the maximum working pressure per sq in. to be allowed shall be calculated from the following formula:

shell thickness is appreciable or not in relation to the internal diameter of the cylinder. The decision rests with the designer as to the category in which a cylinder shall be placed, and the determining factors are referred to on page 132. The ratio of $\frac{d}{t} = 20$ can be considered a suitable line of demarcation between thin and thick cylinders.

In thin cylinders the stresses may be assumed *uniformly distributed over the wall thickness.* Boilers, pressure vessels, steam pipes, water pipes, etc. are usually treated as thin cylinders.



Failure along longitudinal section
Fig. 3-1(*a*)



Failure across transverse section
Fig. 3-1(*b*)

There are two ways in which the failure of a thin cylinder may occur as shown in fig. 3-1.

Thin cylinder may burst along a longitudinal seam as shown in fig. 3-1(*a*) or it may fail across a transverse section as shown in fig. 3-1(*b*). Due to internal fluid pressure the stresses set up

Exercises:

1. *Find the thickness of a cast iron pipe 45 cm diameter if it is required to sustain 6 kg/sq cm inside pressure. Maximum stress in the pipe material is limited to 140 kg/sq cm.* *Ans.* 10 mm.

2. *A compressed air cylinder for a laboratory use ordinarily carries approximately 70 kg/sq cm pressure at the time of delivery. The inside diameter of such a cylinder is 25 cm. If the steel has a yield point of 2,300 kg/sq cm and a safety factor of 2·5 is adequate, calculate the required wall thickness.* *Ans.* 1 cm.

3. *For use in rural districts fuel gas for home use is frequently stored in cylinders closed by either hemispherical or dished ends. Considering such a tank of 100 cm in diameter to be a thin cylinder made of steel having a permissible tensile stress intensity to be 700 kg/sq cm, determine the thickness if the working pressure is not to exceed 18 kg/sq cm.* *Ans.* 14 mm.

4. *A thin walled cylinder is closed at both ends and contains oil under a pressure of 10 kg/sq cm. The inside diameter of the shell is 60 cm. If the yield point of the material is 2,500 kg/sq cm and a safety factor of 3 is selected, determine the required wall thickness.* *Ans.* 5 mm.

5. *Find the thickness of the metal necessary to make a copper steam pipe 30 cm diameter with a longitudinal brazed seam. The inside pressure is 11 kg/sq cm. The permissible tensile stress intensity in copper is not to exceed 350 kg/sq cm. The efficiency of brazed joint is 78%.* *Ans.* 8 mm.

6. *Find the required shell thickness for a boiler 240 cm in diameter working on 15 kg/sq cm gauge pressure and joint efficiency of 84%. The boiler shell plate is made of mild steel having an ultimate strength in tension of 43 kg/sq mm. A factor of safety 4·5 is adequate.* *Ans.* 23 mm.

7. *A vertical cylindrical gasoline storage tank is 25 metre in diameter. The maximum level of gasoline is not to exceed 18 metre. The specific gravity of gasoline is 0·72. If the yield point of the shell plating is 2,400 kg/sq cm and a factor of safety of 2·5 is adequate, calculate the thickness of the wall of the storage tank. Neglect any localised bending effects.* *Ans.* 18 mm.

8. *A boiler shell, 7 feet mean diameter, is constructed of steel plate having an ultimate tensile strength of 28 tons/sq in. It is subjected to an internal pressure of 250 psig. Calculate the thickness of the shell plates,*

$$\text{W.P} = \frac{(t-2) \times S \times J}{C \times D} \quad \ldots\ldots\ldots(v)$$

where W.P is the working pressure in lb/sq in.

t is the thickness of shell plate in 32nd of an inch.

S is the minimum tensile breaking strength of the shell plate in tons/sq in

J is the percentage strength of the longitudinal seams.

C is 2·75 for double butt strap longitudinal joints.

D is the inner diameter of boiler shell in inch.

Examples:

1. A cylindrical shell of 2·2 metre diameter is constructed of mild steel plate. The shell is subjected to an internal pressure of 8 kg/sq cm gauge. Determine the thickness of the shell plate by adopting a factor of safety 6. The ultimate tensile strength of the steel is 4,700 kg/sq cm. The efficiency of the longitudinal joint may be taken as 78%.

The working stress of the material

$$f_t = \frac{\text{ultimate tensile strength}}{\text{factor of safety}} = \frac{4700}{6} = 783 \text{ kg/sq cm.}$$

The thickness of the plate is given by the formula

$$t = \frac{pD}{2 f_t \eta} = \frac{8 \times 2{\cdot}2 \times 100}{2 \times 783 \times 0{\cdot}78} = 1{\cdot}44 \text{ cm; we adopt } t = 1{\cdot}5 \text{ cm.}$$

2. Determine the thickness for the cylindrical portion of a water-tube vertical boiler of Spencer Hopwood type of 60" diameter and a working pressure of 150 psig. The longitudinal joint is double-riveted butt joint with two butt straps, the efficiency of which may be assumed to lie between 70 to 83%. Ultimate tensile strength of the steel plate is 28 tons/sq in. Assume a factor of safety to be 4.

The thickness of the plate is given by the formula

$$t = \frac{pD}{2 f_t \eta}.$$

We take the lowest value of the efficiency

$$\therefore \quad t = \frac{150 \times 60}{2 \times 7 \times 2240 \times 0{\cdot}7} = 0{\cdot}41''; \text{ we adopt } \frac{7''}{16}.$$

According to Indian Boiler Regulations, the working pressure for a given thickness is given by formula (v) of this page.

$$\therefore \quad \text{W.P.} = \frac{(14-2) \times 28 \times 70}{2{\cdot}75 \times 60} = 143 \text{ psig.}$$

As the working pressure of steam is less than the required one, we increase the thickness of the boiler shell to $\frac{1}{2}''$ which will give working pressure as 167 psig.

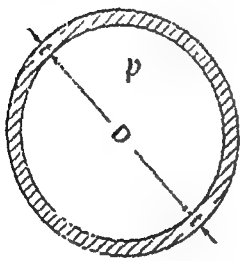
By equating the bursting force to resistance to rupture, we get

$$\frac{\pi}{4}D^2p = \pi D t f_t.$$

or $$t = \frac{Dp}{4f_t} \quad \text{......(ii)}$$

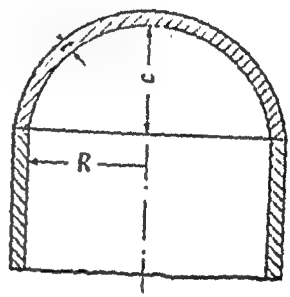
If the shell contains a circumferential joint of efficiency η, the permissible pressure will be reduced and the necessary shell thickness is given by the formula

$$t = \frac{Dp}{4f_t\eta} \quad \text{......(iii)}$$



Spherical shell

Fig. 3-2



Dished end

Fig. 3-3

In boiler construction, we come across many dished ends. The thickness of the dished end may be calculated as follows. In fig. 3-3, R denotes the radius of the circular base, c the camber, t the thickness of the plate and p the pressure of the steam.

If f_t be the permissible tensile stress intensity in the material of the dished end, then

$$f_t = p.\frac{(R^2+c^2)}{4\,t\,c}$$

or $$t = p\,\frac{(R^2+c^2)}{4f_t\,c} \quad \text{......(iv)}$$

When c becomes equal to R, the dished end becomes a hemispherical end and the formula (iv) becomes

$$t = \frac{pR}{2f_t} = \frac{pD}{4f_t}.$$

The above formula agrees with the formula (ii) of this article.

The walls of the vessel have holes for filling it or emptying it and for connection of the pipe fittings. To prevent weakening of

assuming a factor of safety 5. The efficiency of the longitudinal joint is 80%. Check the thickness calculated with the formula given by Indian Boiler Regulations. *Ans.* $1\frac{1}{8}$ *in.*

9. *A steel pipe, 50 cm diameter and 15 mm thick, is closed at the ends by bolted flanges and used as a storage vessel for a fluid at a pressure of 14 kg/sq cm by gauge. Connections to the pipe necessitate, in places, the use of longitudinal joints, the efficiency of which may be taken as 80%. Each end flange is secured with 14 bolts which have core area of 5·6 sq cm. To ensure joint tightness the bolts are screwed up to give a stress 50% greater than that due to pressure. If the pipe and bolt material has an ultimate strength of 43 kg/sq mm, compare the factors of safety for the wall and the bolts.* *Ans.* 2·245:1.

10. *A boiler shell of 150 cm inner diameter is made of mild steel plate of 1 cm thickness. Find the maximum internal pressure that the boiler can be certified for. Assume permissible tensile stress in mild steel to be 700 kg/sq cm.* *Ans.* 7·8 *kg/sq cm by gauge.*

3-4. Design of a thin spherical shell:

As the spherical shells are self supporting, they are found in many engineering applications. When the material of construction is decided upon, the diameter and thickness of the spherical shell are the items to be considered in the design. The diameter of the spherical shell is obtained from the storage capacity of the shell.

If D be the internal diameter of the spherical shell, and V the storage capacity of the shell, then

$$V = \frac{\pi}{6} D^3 \qquad (1)$$

Let p be the internal gauge pressure to which the shell is subjected and t the thickness of the spherical shell. The thickness of the shell can be obtained by equating the force tending to tear the shell apart to the resistance the shell offers to rupture (fig. 3-2).

The force tending to rupture the shell along the diametral plane is $\frac{\pi}{4} D^2 p$.

The area of the shell that resists the rupture of the shell is πDt. If f_t be permissible tensile stress intensity in the material of the shell, resistance of the shell to rupture is $\pi Dt\, f_t$.

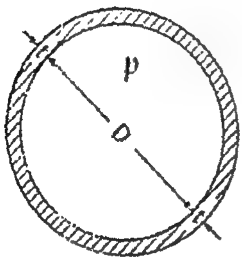
By equating the bursting force to resistance to rupture, we get

$$\frac{\pi}{4}D^2p = \pi D t f_t.$$

or $$t = \frac{Dp}{4f_t} \qquad \text{.......... (ii)}$$

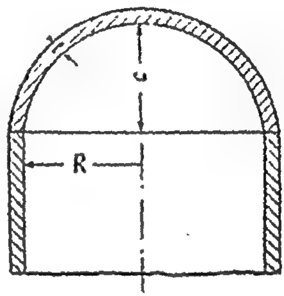
If the shell contains a circumferential joint of efficiency η, the permissible pressure will be reduced and the necessary shell thickness is given by the formula

$$t = \frac{Dp}{4f_t\eta} \qquad \text{.......... (iii)}$$



Spherical shell

Fig. 3-2



Dished end

Fig. 3-3

In boiler construction, we come across many dished ends. The thickness of the dished end may be calculated as follows. In fig. 3-3, R denotes the radius of the circular base, c the camber, t the thickness of the plate and p the pressure of the steam.

If f_t be the permissible tensile stress intensity in the material of the dished end, then

$$f_t = p.\frac{(R^2 + c^2)}{4tc}$$

or $$t = p\frac{(R^2 + c^2)}{4f_t c} \qquad \text{.......... (iv)}$$

When c becomes equal to R, the dished end becomes a hemispherical end and the formula (iv) becomes

$$t = \frac{pR}{2f_t} = \frac{pD}{4f_t}.$$

The above formula agrees with the formula (ii) of this article.

The walls of the vessel have holes for filling it or emptying it and for connection of the pipe fittings. To prevent weakening of

the wall these holes must be reinforced, so as to make the strength of the plate with a hole equal to that of a plate without a hole. The holes are either round or elliptical.

Example:

1. The air receiver consists of a cylinder of 1 metre inside diameter, which is closed by hemispherical ends. The pressure of compressed air inside the cylinder is not to exceed 20 kg/sq cm gauge. If the material is steel whose yield point is 2,500 kg/sq cm and a safety factor of 3·5 is used, calculate the required wall thickness of the cylinder and thickness of the hemispherical ends. Neglect localised effects at the junction of cylinder and hemisphere.

The permissible stress in the material $= \dfrac{2500}{3 \cdot 5}$

$= 715$ kg/sq cm.

The thickness of the cylindrical wall is given by

$t = \dfrac{pD}{2f_t}$ where p is the internal fluid pressure intensity, D the internal diameter of the cylinder and f_t the permissible tensile stress intensity in the material of the shell

On substitution of values, we get

$$t = \frac{20 \times 100}{2 \times 715} = 1 \cdot 4 \text{ cm.}$$

The thickness of the hemispherical ends is given by the formula

$$t_1 = \frac{pD}{4f_t}.$$

On substitution of the values, we get

$$t_1 = \frac{20 \times 100}{4 \times 715} = 0 \cdot 7 \text{ cm; we adopt } t_1 = 8 \text{ mm}$$

Exercises:

1. A bronze spherical pressure vessel of 2·5 metre diameter is installed in a chemical plant. Such a pressure vessel is subjected to internal pressure of 11 kg/sq cm. Calculate the required thickness of the shell if the permissible stress in the bronze material is not to exceed 500 kg/sq cm.

Ans. 1·4 cm.

2. To assist motorists, who have tyre trouble, many service stations bring a small cylindrical tank, closed by hemispherical ends, filled with

the wall these holes must be reinforced, so as to make the strength of the plate with a hole equal to that of a plate without a hole. The holes are either round or elliptical.

Example:

1. The air receiver consists of a cylinder of 1 metre inside diameter, which is closed by hemispherical ends. The pressure of compressed air inside the cylinder is not to exceed 20 kg/sq cm gauge. If the material is a steel whose yield point is 2,500 kg/sq cm and a safety factor of 3·5 is used, calculate the required wall thickness of the cylinder and thickness of the hemispherical ends. Neglect localised effects at the junction of cylinder and hemisphere.

The permissible stress in the material $= \dfrac{2500}{3.5}$

$= 715$ kg/sq cm.

The thickness of the cylindrical wall is given by

$t = \dfrac{pD}{2f_t}$ where p is the internal fluid pressure intensity, D the internal diameter of the cylinder and f_t the permissible tensile stress intensity in the material of the shell.

On substitution of values, we get

$$t = \frac{20 \times 100}{2 \times 715} = 1{\cdot}4 \text{ cm.}$$

The thickness of the hemispherical ends is given by the formula

$$t_1 = \frac{pD}{4f_t}.$$

On substitution of the values, we get

$$t_1 = \frac{20 \times 100}{4 \times 715} = 0{\cdot}7 \text{ cm; we adopt } t_1 = \text{[illegible] mm.}$$

Exercises:

1. A bronze spherical pressure vessel of 2·5 metre diameter is installed in a chemical plant. Such a pressure vessel is subjected to internal pressure of 11 kg/sq cm. Calculate the required thickness of the shell if the permissible stress in the bronze material is not to exceed 500 kg/sq cm.

Ans. 1·4 cm

2. To assist motorists, who have tyre trouble, many service stations bring a small cylindrical tank, closed by hemispherical ends, filled with

In fact the constant c has to also account for rigidity consideration as well as the method of pipe manufacture. For cast pipes the thickness of the pipe determined by calculations based on strength alone is often too small to produce good casting. In such cases the thickness of the castings should not be less than a certain practicable minimum, which depends upon the size of the pipe. The minimum thickness will differ for each material.

The table on page 6 gives the average values of minimum thickness for various castings.

According to Weisbach, the following values of constant c are suggested:

Cast iron	0·9 cm	Zinc	0·4 cm
Copper	0·4 cm	Mild steel	0·3 cm
Lead	0·5 cm		

For determining the thickness of cylindrical part of boilers, the thin cylinder formula is modified as under:

$$t = \frac{pD}{2f_t} + c \text{ cm} \quad \ldots\ldots (v)$$

where $c = 0{\cdot}1$ for t less than or equal to 3 cm, 0·05 for t greater than 3 cm and 0·0 for t greater than 4 cm.

The pipes and their fittings are tested by hydraulic pressure upto the test pressure, which is the maximum pressure to be applied for checking the strength of the pipe. Three types of pressures are distinguished, viz., nominal pressure, working pressure and test pressure. These pressure are covered by Indian Standards.

When the pipes are subjected to a high temperature fluid, while selecting the value of the permissible stress, the *effect of creep* is to be taken into account. *Creep is the slow and continuous deformation of metals under steady stress, which is considerably lower than yield point.* This phenomenon is of vital importance at elevated temperatures. The creep limit is the stress at which the creep rate is equal to a certain value determined by the specifications.

Examples:

1. Determine the thickness of a cast iron pipe to carry 30 cu metre of compressed air per minute at a pressure of 7 kg/sq cm. The velocity of air in the pipe is limited to 8 metre/second.

which does not always coincide with its inside diameter; *as a rule they differ somewhat.*

The main purpose of the pipe installation is to carry a certain quantity of fluid from one point to another point. The velocity of the fluid in the pipe must be specified. When the velocity of the fluid and the quantity to be handled are known, we can determine the nominal bore of the pipe

Let D be the internal diameter of the pipe, Q the quantity of fluid to be carried by volume per unit time and V the velocity of fluid in the pipe per unit time, then,

$$Q=\frac{\pi}{4}D^2V$$

or $$D=\sqrt{\frac{4}{\pi}\times\frac{Q}{V}}=1{\cdot}13\sqrt{\frac{Q}{V}} \quad \text{(i)}$$

If G be the discharge or flow rate measured in kg/hour for gases and tonne/hour for liquids, V be the velocity of flow of the medium in metre/sec and γ the specific weight in kg/cu metre for gases and tonne/cu metre for liquids, then it can be shown that

$$D=18{\cdot}8\sqrt{\frac{G}{\gamma V}}\text{ mm} \quad \text{(ii)}$$

After finding out the minimum value of the inner diameter of the pipe, we select the next larger nominal bore. Then we calculate the thickness of the wall by the equation

$$t=\frac{pD}{2f_t} \quad \text{(iii)}$$

After the thickness has been determined, we select the outside diameter of the pipe from the standards.

The thickness of the pipe wall is selected to suit the pressure of the liquid or gas conveyed by the pipe line, with an allowance for rigidity consideration, and method of pipe manufacture and for wear and corrosion which depend on the material of the pipe and the type of working medium. Hence the design formula will be modified to

$$t=\frac{pD}{2f_t}+c \quad \text{mm} \quad \text{(iv)}$$

where c is the allowance for corrosion added to the design thickness of the wall. If t is less than 6 mm, then c is taken as 1 mm. For values of t greater than 6 mm, c is equal to *0·18 t*.

turbines placed at a vertical depth of 100 metre. Calculate the necessary thickness of the pipe at the turbine if the design stress for the pipe material is not to exceed 700 kg/sq cm. *Ans.* 22 *mm.*

2. *Determine the thickness of a cast iron pipe 25 cm internal diameter to withstand the fluid pressure of 12 kg/sq cm. The stress intensity in the material of the pipe is limited to 250 kg/sq cm.*

Ans. 1·2 *cm.*

3. *A seamless steel pipe is to carry 2,000 cu metre of superheated steam per hour at a pressure of 10 kg/sq cm gauge. The velocity of steam in the pipe is limited to 30 metre/second. Determine the minimum diameter of the steam pipe and suggest the suitable thickness for the pipe, assuming the permissible tensile stress intensity to be 400 kg/sq cm.*

Ans. 16 *cm;* 2 *mm.*

3-6. Design of thick cylinders:

In engineering we come across many cylinders or pressure vessels, which are frequently required to operate under pressures upto 300 kg/sq cm or more. A cannon, while the projectile is travelling the length of its barrel, may be considered a pressure vessel subjected to an internal pressure which may exceed 2,500 kg/sq cm. Under such heavy pressures, the thickness of the wall of the pressure vessel will be relatively large and so the usual assumption regarding the uniform stress distribution in the wall of the pressure vessel is not valid and the theory of thin cylinder cannot be applied.

Several theories have been suggested for the stress distribution in the wall of thick cylinders. Here, we briefly describe the theory which is known as Lamé's theory, and which is based on maximum normal stress[1]. The following assumptions are made:

(i) The material of the cylinder is homogeneous, isotropic and obey's Hooke's law.

(ii) The cylinder is open at ends.

(iii) The pressure intensity, is axi-symmetrical as shown in fig. 3-4(*a*).

1. For further information, students are advised to refer to any standard work on Strength of Materials such as Mechanics of Structures Vol. II by S. B. Junnarkar; Advanced Strength of Materials Vol. II by S. P. Timoshenko, etc.

The amount of air flowing per second $=\frac{30}{60}=0{\cdot}5$ cu metre/sec.

Cross sectional area of the pipe $=\frac{0{\cdot}5}{8}=0{\cdot}0625$ sq metre.

If D be the internal diameter of the pipe, then

$$\frac{\pi}{4}D^2=0{\cdot}0625$$

or $$D=\sqrt{\frac{0{\cdot}0625\times4}{\pi}}=0{\cdot}282 \text{ metre, we adopt 30 cm.}$$

Let us assume the permissible tensile stress intensity in the pipe material to be 150 kg/sq cm. If t be the minimum thickness of the pipe, then

$$t=\frac{pD}{2f_t}=\frac{7\times30}{2\times150}=0{\cdot}7 \text{ cm; we adopt 1 cm.}$$

2. *A steam boiler has 75 sq metre of heating surface and the rate of evaporation is 20 kg/sq metre/hour of heating surface. The pressure of steam generation is 8 kg/sq cm gauge. The specific volume of steam is 0·24 cu metre/kg. Determine the diameter and thickness of the steel steam pipe to carry the steam from this boiler with a velocity of steam in the pipe at 25 metre/sec. The permissible tensile stress intensity in the pipe material is 400 kg/sq cm.*

Amount of steam generated $=75\times20=1{,}500$ kg/hour.
Volume of steam flowing in the pipe

$$=\frac{1500\times0{\cdot}24}{60\times60}=0{\cdot}1 \text{ cu metre/sec.}$$

If D be the diameter of the steam pipe, then

$$\frac{\pi}{4}D^2\times25=0{\cdot}1$$

or $$D=\sqrt{\frac{0{\cdot}1\times4}{25\times\pi}}=0{\cdot}072 \text{ metre; we adopt 75 mm.}$$

If t be the minimum thickness of the pipe, then

$$t=\frac{pD}{2f_t}=\frac{8\times7{\cdot}5}{2\times400}=0{\cdot}075 \text{ cm}$$

This thickness is too small. We adopt 3 mm thick solid drawn steel tube.

Exercises:

1. *Steel pipe 300 cm diameter is used for penstocks in a hydro-electric plant to guide the water from intake at the top of the dam to the*

$b - a$ is the thickness of the cylinder wall.

We see that f_r, the radial stress, is always a compressive stress and f_t, the tangential stress, a tensile one. The maximum value of the tangential stress is at the inner radius and is equal to

$$f_{tmax} = p\left[\frac{b^2 + a^2}{b^2 - a^2}\right] \quad \ldots\ldots\ldots (iii)$$

The maximum value of the tangential stress is always greater than the internal pressure.

The maximum value of the radial stress is p and it occurs at the inner radius. The radial stress vanishes at the outer radius which is not the case with the tangential stress. The tangential stress at the outer radius is $\frac{2pa^2}{b^2 - a^2}$. Fig. 3-4(b) shows the nature of stress distribution in the walls of a thick cylinder when subjected to internal pressure.

The usual design problem is to determine the wall thickness when the internal pressure p and allowable working stress f are known.

From equation (iii), we get

$$t = a\left(\sqrt{\frac{f+p}{f-p}} - 1\right) \quad \ldots\ldots\ldots (iv)$$

where t is the thickness of the cylinder wall and a the inner radius of the cylinder.

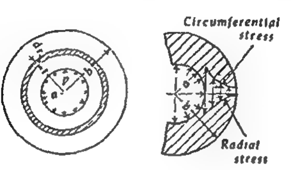
As this equation is based on principal stress in the wall, equation (iv) is applicable to brittle material such as cast iron, cast steel and cast aluminium.

From equation (iv), we see that if the internal pressure p is equal to or greater than f, no thickness of the cylinder wall will prevent failure. Hence it seems that it is impossible to design a cylinder to withstand fluid pressure greater than the allowable working stress for a given material. This difficulty is overcome by using compound cylinders.

Compound cylinders:

The stress distribution of fig. 3-4 shows that the maximum stress occurs at the inside surface when a thick walled cylinder is subjected to internal pressure If the cylinder were to be designed for the maximum stress at the inside surface the material is not used effectively. The direct method of achieving more nearly uniform stress distribution is to subject the material near the inside wall to the initial compressive stresses when the cylinder is not subjected to internal fluid pressure. When the cylinder is loaded, the compressive stresses must be relieved before any tensile stress is developed.

In order to derive an expression for the stresses induced in the material, we cut an element from the wall of the cylinder and by considering statics, geometry and mechanical properties of the material and combining all the three we can derive a differential equation for the radial displacement of the element considered. The solution of the differential equation gives the following



(a)   (b)

Stresses in thick cylinders

Fig. 3-4

general expressions for the tangential and radial stresses at any radius r within the cylinder wall.

$$\left.\begin{aligned} \text{Tangential stress} &= f_t = A + \frac{B}{r^2} \\ \text{Radial stress} &= f_r = A - \frac{B}{r^2} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where A and B are constants which are to be determined from the boundary conditions. These equations are known as Lamé's general equations for the tangential and the radial stresses at any radius r in the wall of a thick cylinder. These stresses are the maximum normal stresses in the wall at the radius r.

In machine design we have to deal with cylinders subjected to internal pressure only. In this case Lamé's general equations for the tangential and radial stresses at any radius r will be

$$\left.\begin{aligned} f_t &= \frac{pa^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \\ f_r &= \frac{pa^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

where a and b are the internal and external radii of the cylinder respectively and p the internal pressure acting in the cylinder.

where a = inner radius of the cylinder
f = permissible tensile stress intensity
p = internal fluid pressure
ν = Poisson's ratio.

This equation is applicable to cylinders having the ends closed or fitted with heads so that axial stresses are induced in the wall material.

Birnie's equation:

This equation is used for open cylinders. The thickness of the cylinder wall is given by the equation, with usual notations,

$$t = a\left[\sqrt{\frac{f+(1-\nu)p}{f-(1+\nu)p}}-1\right] \quad \ldots\ldots\ldots\ldots \text{(x)}$$

This equation is applicable to certain type of pump cylinders, rams, cannons, etc.

Clavarino's equation and Birnie's equation are applicable to ductile materials such as low carbon steel, brass, bronze and aluminium alloys.

As thickness given by Birnie's equation is greater than that given by Clavarino's equation, in case of doubt, Birnie's equation should be used.

The designation of cylinders as *thin* or *thick* depends upon the degree of accuracy the designer requires. The actual percentage error in hoop stress for various ratios of $\frac{d}{t}$ by using thin cylinder equation is given below:

Ratio $\frac{d}{t}$	100	50	20	10	5	2
Error %	1	2	4·8	9·9	18·9	40

Since for many practical purposes an error of more than 5% would be undesirable, the ratio of $\frac{d}{t} = 20$ can be considered a suitable line of demarcation between thin and thick cylinders.

Examples:

1. *The ram of a hydraulic press 20 cm internal diameter is subjected to an internal pressure of 100 kg/sq cm. If the maximum stress in the material of the wall is not to exceed 280 kg/sq cm, find the external diameter.*

The thickness is given by the equation, with usual notations,

$$t = a\left[\sqrt{\frac{f+p}{f-p}}-1\right] = 10\left[\sqrt{\frac{280+100}{280-100}}-1\right] = 4{\cdot}5 \text{ cm.}$$

External diameter $= 20 + 2 \times 4{\cdot}5 = 29$ cm.

There are two techniques commercially adopted for the purpose One method consists of shrinking one cylinder over another cylinder. The inner diameter of the outer cylinder is smaller than the outer diameter of the inner cylinder. The outer cylinder is heated and slipped over the inner cylinder. On cooling, at junctions of two cylinders, contact pressure will be developed which will induce compressive tangential stresses in the material of the inner cylinder and tensile tangential stresses in the material of the outer cylinder. The entire assembly is effective in resisting internal pressure and will withstand higher internal pressure than a single cylinder having the same over all dimensions.

The second technique makes use of the theory of plasticity. Temporary high internal pressure is applied till the plastic state is reached near the inside of the cylinder wall, which results in a residual compressive stress upon the removal of the initial pressure.

Barlow's equation:

From Lamé's equation, we can derive an expression that can be used for thin cylinders.

$$f_{t\max} = p\left(\frac{b^2 + a^2}{b^2 - a^2}\right) \qquad \ldots (v)$$

If t be the thickness of the cylinder wall, then

$$b = a + t \qquad \ldots (vi)$$

On substitution in equation (v), we get

$$f_{t\max} = p\,\frac{(2a^2 + 2at + t^2)}{2at + t^2} \qquad \ldots (vii)$$

As t is small compared to a for thin cylinders, we can neglect t^2 in comparison with other terms; so we get

$$f_{t\max} = p\,\frac{(2a^2 + 2at)}{2at} = \frac{pb}{t}$$

or

$$t = \frac{pb}{f_{t\max}} \qquad \ldots (viii)$$ ✓

Equation (viii) is known as Barlow's equation. This equation is very similar to thin cylinder formula except outer radius b which replaces the inner radius a. As this formula is derived from Lamé's equation, it is slightly more accurate than the thin cylinder formula. This formula is used for high pressure oil and gas pipes.

Clavarino's equation:

This equation is used for closed thick cylinders. The thickness of the cylinder is given by

$$t = a\left[\sqrt{\frac{f + (1 - 2\nu)\,p}{f - (1 + \nu)\,p}} - 1\right] \qquad \ldots (ix)$$

cylinder at a pressure of 150 kg/sq cm gauge. Determine the main overall proportions of the cylinder and indicate the points at which effective sealing will be necessary.

$$\text{Minimum piston area necessary} = \frac{6000}{150} = 40 \text{ sq cm.}$$

If d be the diameter of the piston, then $\frac{\pi}{4} d^2 = 40$

or $d = \sqrt{\frac{40 \times 4}{\pi}} = 7{\cdot}17$ cm; we adopt 7·5 cm.

A compact, strong and rigid form of cylinder is possible by casting in steel. The cylinder will be subjected to repetitions of stress when in operation so there is the possibility of fatigue. By avoiding rapid changes of section through out the working length, the probability of high stress concentrations will be reduced considerably. Assuming a fatigue limit of 30 kg/sq mm and a factor of safety of 4 based on the fatigue limit, the working stress becomes $\frac{30}{4} = 7{\cdot}5$ kg/sq mm i.e., 750 kg/sq cm.

As the cylinder is cast, we employ Lamé's formula.

$$t = \frac{d}{2}\left[\sqrt{\frac{f+p}{f-p}} - 1\right] = \frac{7{\cdot}5}{2}\left[\sqrt{\frac{750+150}{750-150}} - 1\right] = 0{\cdot}83 \text{ cm.}$$

The cylinder bore will be machined to size and the outside surface left as cast. To allow for surface roughness on the outside, for possible surges in pressure and for the fact that easier casting is possible with thicker sections, the cylinder will be made 12 mm thick over the greater part of the cylinder length and enlarged at the ends to accommodate the screwed portions of the socket head screws used for holding the cylinder covers in place. This design form is useful when compact cylinder proportions are desirable. Fig. 3-5 shows the suitable hydraulic cylinder for the purpose.

Other design considerations:

Proper seals are important to prevent leakage and loss of hydraulic power. The sealing material must be compatible with the hydraulic fluid and the operating temperatures.

The seals must be applied to prevent seal "break-off" which contaminates the system. They should also be easy to replace. Sealing devices are used in cylinders to seal piston and piston rod

2. An accumulator is required to store 150 litres of water at a pressure of 200 kg/sq cm. Assuming the length of the stroke to be 3 metre, determine (a) the diameter of the ram, (b) the internal diameter of the cylinder and (c) the thickness of the cylinder wall.

Note: The hydraulic accumulator consists of a loaded plunger working in a vertical cylinder. The load may consist of number of cast iron discs or a tank filled with heavy scrap metal

1 litre = 0·001 cu metre.

If D be the diameter of the ram, then

$$\frac{\pi}{4} D^2 \times 3 = 0{\cdot}001 \times 150$$

or
$$D = \sqrt{\frac{0{\cdot}150 \times 4}{\pi \times 3}} = 0{\cdot}250 \text{ metre; we adopt 26 cm.}$$

Allowing a clearance of 4 cm, the internal diameter of the cylinder will be $4 + 26 + 4 = 34$ cm.

We assume that the cylinder is made of closed grained grey cast iron, for which the stress should not exceed 600 kg/sq cm.

The thickness of the cylinder is given by the formula,

$$t = a\left[\sqrt{\frac{f+p}{f-p}} - 1\right] = \frac{34}{2}\left[\sqrt{\frac{600+200}{600-200}} - 1\right] = 7 \text{ cm.}$$

3. A steel tank for shipping gas is to have an inside diameter of 30 cm and a length 120 cm. The gas pressure is 150 kg/sq cm. The permissible stress is to be 575 kg/sq cm Determine the thickness of the tank.

The tank is made of ductile material and is closed at the ends; therefore, we apply Clavarino's equation for determining thickness of the tank.

With usual notations, the thickness is given by

$$t = a\left[\sqrt{\frac{f + (1-2\nu)p}{f - (1+\nu)p}} - 1\right]$$

$$= 15\left[\sqrt{\frac{575 + (1-0{\cdot}6)\,150}{575 - (1+0{\cdot}3)\,150}} - 1\right]$$

$t = 4{\cdot}35$ cm; we adopt 5 cm.

4. Design of a hydraulic cylinder:

A single acting hydraulic cylinder is to be developed to enable a thrust of 6,000 kg to be exerted through a stroke of 15 cm. The cylinder will be bolted to a steel table. The working fluid is oil, available at the

Most of the cylinder tubes manufactured are low carbon mild steel having an approximate ultimate tensile strength of 50 kg/sq mm minimum and yield stress of 45 kg/sq mm minimum. These values fall to 42 kg/sq mm and 38 kg/sq mm upon stress relieving. For design purposes we make use of the proof stress rather than yield stress, since the latter is not clearly defined for the material. For low-carbon mild steel the normal proof stress of 0·1% gives 45 kg/sq mm and 38 kg/sq mm for the cold drawn and cold drawn stress relieved tubes. Tubings may be threaded internally or externally to take end covers or connections.

For the various sizes of the tubes, manufacturers' catalogues should be referred to.

The factor of safety of 2 on proof stress is employed for rams under internal pressure without shock loads or offset end loads; and a factor of safety between 3 and 5 may be adopted if the tubes have to withstand sudden increases in internal pressure and or offset end loads.

Exercises:

1. A cast steel cylinder for a hydraulic press has an inside diameter of 20 cm. Water pressure is 315 kg/sq cm gauge. Taking the maximum hoop stress as 700 kg/sq cm, determine the outside diameter of the cylinder required by Lamè's formula. *Ans. 32·5 cm.*

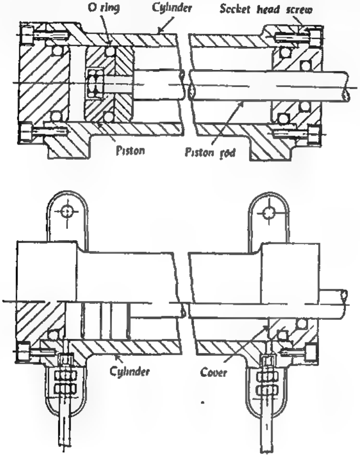
2. A single acting triplex pump has a bore of 8 cm and a stroke of 18 cm. It is required to supply 80 litres of oil per minute at a pressure of 140 kg/sq cm.

Calculate the minimum number of strokes of the pump per minute. Determine the thickness of the cylinder wall using a cast bronze with an apparent factor of safety 6. Ultimate strength for cast bronze 2,500 kg/sq cm. *Ans. 30; 18 mm.*

3. The cylinder of a hydraulic accumulator has an internal diameter of 40 cm. The internal pressure at its maximum is to be 70 kg/sq cm. Determine the thickness of the cylinder if the maximum tensile stress is limited to 280 kg/sq cm. Also, calculate the minimum value of the tensile stress induced. *Ans. 6 cm; 213 kg/sq cm.*

4. A gun metal hydraulic fitting is connected by a 20 cm diameter branch to a container carrying a pressure of 84 kg/sq cm. Connection will be made by a flanged joint. Design the thickness of the gun metal branch allowing a safe stress of 210 kg/sq cm for the gun metal. *Ans. 55 mm.*

assembly, end covers to cylinder and for the piston rod sealing gland. Toroidal seals should be used in *this* design. They form static seals between cylinder tube and its covers and dynamic seals between piston head and cylinder and piston rod and cover. Seals can be seen in fig. 3-5.



Hydraulic Cylinder

Fig. 3-5

The increasing use of fluid power has resulted over the last few years in a corresponding demand for cold drawn tubing from which to manufacture hydraulic cylinders. This kind of tubing is made to higher standards of dimensional accuracy and surface finish than is hot rolled tube, and has higher yield and ultimate tensile strengths as a result of the cold drawing process.

Most of the cylinder tubes manufactured are low carbon mild steel having an approximate ultimate tensile strength of 50 kg/sq mm minimum and yield stress of 45 kg/sq mm minimum. These values fall to 42 kg/sq mm and 38 kg/sq mm upon stress relieving. For design purposes we make use of the proof stress rather than yield stress, since the latter is not clearly defined for the material. For low-carbon mild steel the normal proof stress of 0·1% gives 45 kg/sq mm and 38 kg/sq mm for the cold drawn and cold drawn stress relieved tubes. Tubings may be threaded internally or externally to take end covers or connections.

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3. The cylinder of a hydraulic accumulator has an internal diameter of 40 cm. The internal pressure at its maximum is to be 70 kg/sq cm. Determine the thickness of the cylinder if the maximum tensile stress is limited to 280 kg/sq cm. Also, calculate the minimum value of the tensile stress induced.

Ans. 6 cm; 213 kg/sq cm.

4. A gun metal hydraulic fitting is connected by a 20 cm diameter branch to a container carrying a pressure of 84 kg/sq cm. Connection will be made by a flanged joint. Design the thickness of the gun metal branch allowing a safe stress of 210 kg/sq cm for the gun metal. *Ans.* 55 mm.

5. *A hydraulic testing machine has a maximum capacity of 100 tonnes. The piston diameter is 25 cm. Calculate the wall thickness of the cylinder. Also design the necessary gland, gland bolts and hydraulic seal for the cylinder. Give a neat sketch of the cylinder gland in position.*
(Rajasthan University, 1969)

3·7. Design equation for thick cylinders:

It is possible to derive the simple design equation for thick cylinders *either open or closed.* It is assumed that the cylinder is made of *ductile material for which theory of maximum shear stress is the design criterion.* Let f be the permissible stress intensity for the material of the cylinder. We assume that a suitable factor of safety has been adopted while deciding the value of f. Let k be the thickness ratio i.e. the ratio of the outer diameter of the cylinder to the inner diameter of the cylinder The design equation for a single thick cylinder will be

$$\frac{2p}{f_y}+\frac{1}{k^2}=1 \qquad \ldots \text{(i)}$$

where p is the pressure of the fluid in the cylinder. The above equation is applicable whether the pressure acts at the *inner surface* or at the *outer surface.*

In case of compound cylinders, the design equation becomes

$$\frac{p}{f_y}+\frac{1}{k}=1 \qquad \ldots \text{(ii)}$$

where k is the product of the thickness ratios of the outer cylinder and inner cylinder. For optimum conditions the thickness ratio for the outer and inner cylinders should be equal.

In case of compound cylinder, the building interference will be

$$\frac{\delta R}{R}=\frac{p}{E} \qquad \ldots \text{(iii)}$$

where R = common radius of the two cylinders

E = modulus of elasticity of both the cylinders' material.

The above design equations clearly shows the advantages of compound cylinders over single cylinders.

Let us consider a simple thick cylinder having a thickness ratio 2. Such a cylinder will withstand an internal pressure of $\frac{3}{8}f$. If the internal pressure to be resisted has a value much greater than this, the value of the thickness ratio has to be increased considerably. No single cylinder can be made to avoid yielding at the

bore if the internal pressure approaches half the value of the permissible stress. The higher pressures can be catered for by designing a compound cylinder. In a compound cylinder having the overall thickness ratio of 2, the cylinder will withstand a pressure equal to $\frac{f}{2}$. This pressure will require an infinite value of the thickness ratio for a simple single cylinder. If the compound cylinder be made of two tubes each of which has a thickness ratio of 2, the overall thickness ratio will be 4 and this compound cylinder will withstand an internal pressure of $\frac{3}{4}f$, which pressure is well outside the range of the simple single cylinder.

EXAMPLES III

1. A total load of 40 tonnes has to be exerted by the hydraulic ram (fig. 3-6). The fluid pressure is 40 kg/sq cm. A cast material is used and the stress in tension or compression should not exceed 630 kg/sq cm. Determine the external diameter of the ram if the internal diameter is 62 mm, the diameter of the main cylinder and the thickness of the metal in the cylinder walls. The allowable stress in the bolts holding the cover in position is 350 kg/sq cm. Calculate the diameter of the bolts. Select a suitable pitch circle diameter and obtain the flange thickness t.

Ans. 110 mm; 330 mm diameter cylinder [illegible] thickness 12 mm; 12, M40 bolts on [illegible] mm pitch circle diameter; $t = 42$ mm.

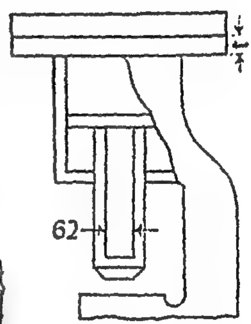

FIG. 3-6

2. In an air operated press the piston rod for operating the cylinder must exert a maximum force of 250 kg. The air pressure in the cylinder is 9 kg/sq cm. Calculate the diameter of the cylinder bore required, assuming that the over-all friction due to stuffing box and piston packing is equivalent to 10% of the maximum force exerted by the piston rod. The cylinder bore should be selected on the basis of 3 mm increment. Also, determine the thickness of the cylinder, assuming that it is a seamless steel tubing. The allowable tensile stress is 200 kg/sq cm.

Ans. 63 mm diameter tube of 2 mm thickness.

3. A hydraulic control for a straight line motion, shown in fig. 3-7, utilises a cylindrical pressure tank A connected to a work cylinder B. The pump maintains a pressure of 35 kg/sq cm in the tank.

(a) Assuming that the tank A is 80 cm diameter, that its joints have strength equal to that of the plate and that the tank is made of steel plates having an allowable tensile strength of 500 kg/sq cm, determine the thickness of the plate required for the tank.

5. *A hydraulic testing machine has a maximum capacity of 100 tonnes. The piston diameter is 25 cm. Calculate the wall thickness of the cylinder. Also design the necessary gland, gland bolts and hydraulic seal for the cylinder. Give a neat sketch of the cylinder gland in position.*
(*Rajasthan University, 1969*)

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In case of compound cylinders, the design equation becomes

$$\frac{p}{f_y}+\frac{1}{k}=1 \qquad \ldots (ii)$$

where k is the product of the thickness ratios of the outer cylinder and inner cylinder. For optimum conditions the thickness ratio for the outer and inner cylinders should be equal

In case of compound cylinder, the building interference will be

$$\frac{\delta R}{R}=\frac{p}{E} \qquad \ldots (iii)$$

where R = common radius of the two cylinders

E = modulus of elasticity of both the cylinders' material.

The above design equations clearly shows the advantages of compound cylinders over single cylinders

Let us consider a simple thick cylinder having a thickness ratio 2. Such a cylinder will withstand an internal pressure of $\frac{3}{8}f$. If the internal pressure to be resisted has a value much greater than this, the value of the thickness ratio has to be increased considerably. No single cylinder can be made to avoid yielding at the

6. In an air operated press the piston rod for the operating cylinder must exert a force of 400 kg. The air pressure in the cylinder is 7 kg/sq cm. Calculate the bore of the cylinder, assuming that overall friction due to stuffing box and piston packing is equivalent to 8% of the maximum force exerted by the piston rod. The cylinder bore should be selected on the basis of 3 mm increment. Also, *determine* the thickness of the cylinder assuming that it is a seamless steel tubing. The allowable stress is 210 kg/sq cm.

(M. S. University of Baroda, 1965)

7. A cast iron pipe is to deliver water at the rate of 2,500 litres/second and at the flow rate of 60 cm/second. The maximum pressure in the pipe is not to exceed 10 kg/sq cm. Determine the diameter of the pipe and its wall thickness, considering the pipe as a thin cylinder. Permissible stress in the cast iron is 175 kg/sq cm.

(University of Bombay, 1967)

8. A cylindrical pressure vessel of 50 cm internal diameter is subjected to an internal fluid pressure of 100 kg/sq cm. Taking $f_t = 400$ kg/sq cm, determine the thickness of the shell of the vessel and also of a hemispherical cover for the same.

(University of Bombay, 1969)

9. A steam engine of 100 H.P. uses 11 kg of steam per horse power hour. The pressure of steam is 7 kg/sq cm and the volume of steam is 0·3 cu metre/kg. Determine the diameter and the thickness of cast iron pipe. The permissible velocity of steam is not to exceed 1,500 metre/minute.

(Sardar Patel University, 1969)

10. Enumerate the various considerations to be taken in the design of castings. State the difference between rational and empirical design giving a suitable example in each case.

(Rajsthan University, 1969)

(b) Assuming a pressure drop of 2 kg/sq cm between the tank and the cylinder, determine the diameter of the piston required to produce an operating force F of 2,500 kg as shown in the figure. Make an allowance for friction in the cylinder and packing equal to 10% of F.

(c) Determine the thickness of the cylinder wall assuming that it is made of C. I. having an allowable tensile stress of 300 kg/sq cm.

(d) Determine the H.P. output of the cylinder during a working stroke assuming the stroke of the piston to be 45 cm and that the time required for working stroke is 5 seconds.

(e) Determine the H.P. of the motor required for continuously operating the pump if the working cycle repeats itself after every 30 seconds and the efficiency of the hydraulic control is 80% and that of the pump is 60%

Ans. (a) 5·6 cm; (b) 12 cm; (c) 7 mm; (d) 3 H P.; (e) 0 21 H.P.

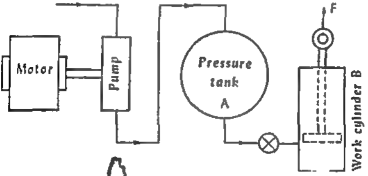


FIG. 3-7

4. The maximum force exerted by a small hydraulic press is 50,000 kg, the working pressure of the fluid being 170 kg/sq cm Determine the minimum diameter of the plunger operating the table Also, suggest the suitable thickness for the cast steel cylinder in which the plunger operates The permissible stress for cast steel is 1,000 kg/sq cm

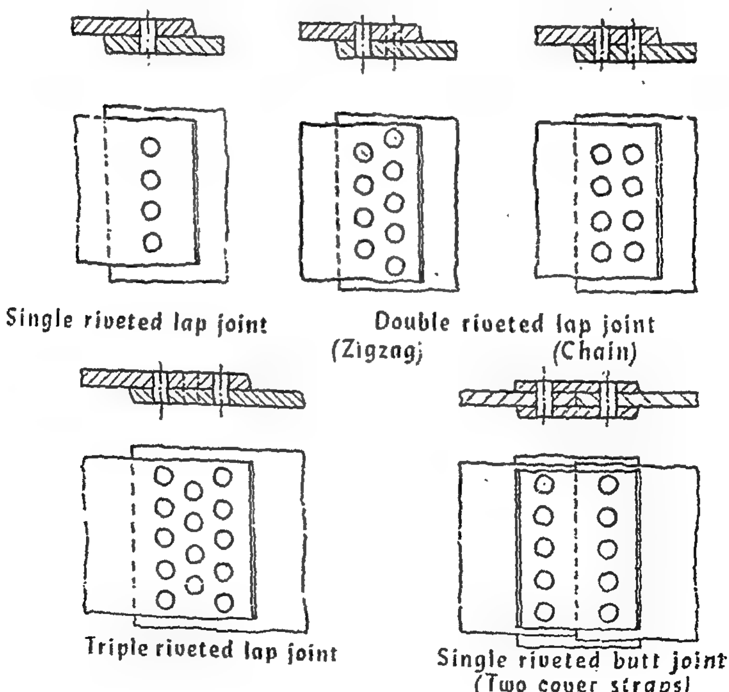
Ans 20 cm; 2 5 cm.

5. In a portable hydraulic riveter, the water under a pressure of 150 kg/sq cm is admitted in the main cylinder through the cover, by way of suitable pipe joint and drives forward the hollow ram, which carries the tool which forms the rivet heads The tool is detachable since the riveter will be required to deal with different sizes and shapes of head The maximum rivet load will be 50,000 kg After forming the rivet, the ram is withdrawn by admitting water under pressure to the inner side of hollow ram through fixed piston rod of 3 cm outer diameter. Determine the diameter of the ram head and thickness of the main cylinder. The permissible tensile stress in the main cylinder wall is limited to 850 kg/sq cm. Efficiency of the ram is 97%

Ans 22 cm; 2 1 cm

In single riveted lap joint or butt joint only one row of rivets is put in each plate to be joined. Two, three or more rows of rivets might be in each plate to be joined and the joints so formed are designated as double, triple or quadrauple riveted joints as the case may be. If the rivets are spaced opposite to each other in adjacent rows, the jont is said to be chain riveted and if the rivets are staggered, a zigzag riveted joint results. The distance, between the centres of adjacent rivets in the same row, is termed the pitch. The perpendicular distance between the centre lines of successive rows is known as a back pitch. The distance between the centres of the rivets in adjacent rows of zigzag riveted joints is termed the diagonal pitch.

Fig. 4-2 shows various types of riveted joints.



Riveted joints
Fig. 4-2(a)

According to purpose, riveted joints are classified as follows:

(i) Strong joints of which strength alone is required, i.e. beams, trusses and other engineering structures

CHAPTER **IV**

RIVETED JOINTS

4·1. *Introduction:*

The parts of machines are connected together by various forms of fastenings. They are classified as follows:

(i) Rivets

(ii) Screws, bolts and nuts

(iii) Pins, keys and cotter.

Rivets: They are used as a permanent fastenings that cannot be dismantled without some part or part of the joint being destroyed.

Examples: Various boiler joints, steel structural joints, ship building work, air-craft manufacture, etc.

Screws, bolts and nuts: They are used as a permanent or removable fastenings to join parts which may be easily separated by unscrewing the fastenings.

Examples: Cylinder cover joint, pipe connections, flanged joints, flange couplings, etc.

Pins, keys and cotter: They are used as permanent or removable fastening to join parts which may be easily separated out by driving or forcing out the fastenings.

Examples: Cottered connections, components of radial valve gear connected by pin joints, connectins of pulley to the shaft, etc.

Now-a-days, in order to join machine parts permanently, welding is used. It has replaced rivets from structural and pressure vessel work. It is used for fabrication of frames, bases and other machine elements made from rolled sections and plates.

Riveted joints are used for connecting two parts in which strength is necessary. In structural connections, strength and rigidity are required. In pressure vessel work, strength, rigidity and prevention of leakage are the essentialities of the joint.

The edges of the plates for boilers and tanks, etc. are usually bevelled to an angle of 80° to facilitate fullering and caulking operation in which the edges are driven by a blunt tool to close the joints. The heads of the rivets are also turned down with a caulking tool to make the joint steam tight. Unless the caulking is done with care, the joint may be seriously injured and also open the joint instead of closing it. Fullering is a better method.

4-6. Design of a riveted joint for boiler work:

In the discussion of the joint dimensions, which follows, the theoretical basis of the design of the riveted joint is given. The rules adopted in practice for the design of boiler joints are specified by Indian Boiler Regulations (I.B.R.). These rules do not agree exactly with those produced by the basic theory; the differences are mainly due to produce leakproof joints.

The resistance to breaking of the joint may be investigated by considering a pitch length of each joint.

Let

f_t = tensile stress in steel plates
f_s = shear stress in steel rivets
d = diameter of rivet hole
t = thickness of the plate
p = pitch of rivets
m = marginal pitch.

The joint may fail in the following ways:

(*a*) *The plate may fracture at the edges* [fig. 4-4(*a*)].

The distance from the edge of the plate to the side of the nearest row of rivet holes must be sufficient to prevent the plate from splitting at the edge during punching or riveting. In practice the distance from the hole centre to the plate edge is made at least equal to $1{\cdot}5d$.

(*b*) *The plate may tear along the weakest section* [fig. 4-4(*b*)].

Tearing resistance of plate between rivet holes is equal to $(p-d)\ t\ f_t$.

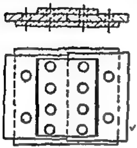
(*c*) *The rivets may shear* [fig. 4-4(*c*)].

The shearing resistance of the joints will depend upon whether the rivet is in single shear or double shear, and number of rivets to be sheared in a joint. *In case of lap joints and butt joints with single cover strap, the rivets will be in single shear, while in case of butt joints with two butt straps the rivets will be in double shear.* Theoretically

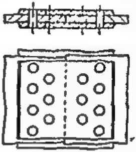
(ii) Tight joints which must guarantee pressure tightness i.e. reserviors, containers, tanks with a low pressure on the walls

(iii) Strong tight joints which must provide both strength and tightness, i.e., steam boilers and gas tanks.

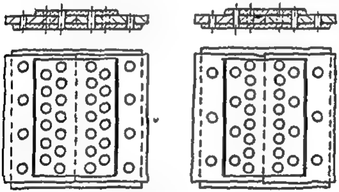
To achieve complete tightness, the joints are caulked.



Double riveted butt joint (Two unequal cover straps)



Double riveted butt joint (Two cover straps)



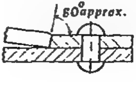
Triple riveted butt joint (Two unequal cover straps)

Riveted joints

Fig. 4-2(*b*)

4-5. Caulking and fullering (Fig. 4-3):

Caulking



Fullering

Fig. 4-3

The edges of the plates for boilers and tanks, etc. are usually bevelled to an angle of 80° to facilitate fullering and caulking operation in which the edges are driven by a blunt tool to close the joints. The heads of the rivets are also turned down with a caulking tool to make the joint steam tight. Unless the caulking is done with care, the joint may be seriously injured and also open the joint instead of closing it. Fullering is a better method.

4-6. Design of a riveted joint for boiler work:

In the discussion of the joint dimensions, which follows, the theoretical basis of the design of the riveted joint is given. The rules adopted in practice for the design of boiler joints are specified by Indian Boiler Regulations (I.B.R.). These rules do not agree exactly with those produced by the basic theory; the differences are mainly due to produce leakproof joints.

The resistance to breaking of the joint may be investigated by considering a pitch length of each joint.

Let

f_t = tensile stress in steel plates
f_s = shear stress in steel rivets
d = diameter of rivet hole
t = thickness of the plate
p = pitch of rivets
m = marginal pitch.

The joint may fail in the following ways:

(a) *The plate may fracture at the edges* [fig. 4-4(a)].

The distance from the edge of the plate to the side of the nearest row of rivet holes must be sufficient to prevent the plate from splitting at the edge during punching or riveting. In practice the distance from the hole centre to the plate edge is made at least equal to $1{\cdot}5d$.

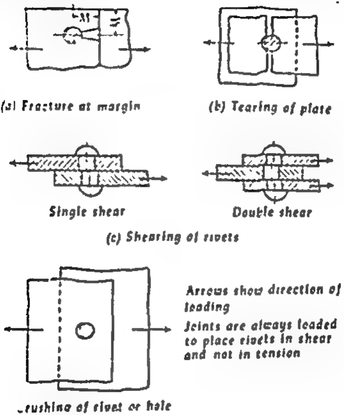
(b) *The plate may tear along the weakest section* [fig. 4-4(b)].

Tearing resistance of plate between rivet holes is equal to $(p - d)\ t\ f_t$.

(c) *The rivets may shear* [fig. 4-4(c)].

The shearing resistance of the joints will depend upon whether the rivet is in single shear or double shear, and number of rivets to be sheared in a joint. *In case of lap joints and butt joints with single cover strap, the rivets will be in single shear, while in case of butt joints with two butt straps the rivets will be in double shear.* Theoretically

the strength of the rivet in double shear will be double than that of a rivet in single shear. But in actual practice it is usual to use a factor of 1·75 to 1·875 instead of two. The Indian Boiler Regulations specify the factor 1·875.



Various possible failures of a riveted joint

Fig. 4-4

The resistance to shearing of one rivet in single shear in the strip equals $\frac{\pi}{4} d^2 f_s$. The resistance of one rivet in double shear in the strip equals $1{\cdot}875 \frac{\pi}{4} d^2 f_s$. From the number of rivets to be sheared in a pitch width, the total resistance of the joint to shearing is known. The table on page 148 gives the number of rivets to be sheared for various kinds of joints for failure.

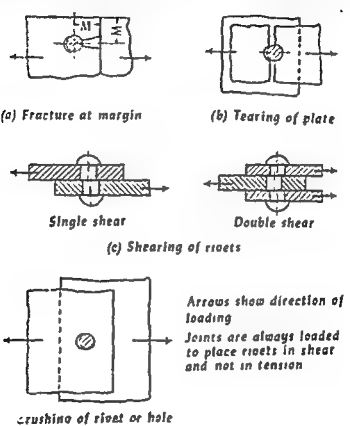
EFFICIENCIES OF RIVETED JOINTS

l of nt	Description of joint	Maximum pitch inches	Thickness of butt strap inches	No. of rivets in shear: Single shear	No. of rivets in shear: Double Shear	Percentage shearing efficiency of joint
igle eted int	Lap joint	1·32t + 1·625	—	1	—	$\frac{A f_s}{pt f_t} \times 100$
	Single butt strap	1·53t + 1·625	1·125t	1	—	$\frac{A f_s}{pt f_t} \times 100$
	Double butt strap	1·75t + 1·625	0·625t	—	1	$\frac{1{\cdot}875\, A f_s}{pt f_t} \times 100$
ouble iveted joint	Lap joint, chain or zigzag arrangement	2·62t + 1·625	—	2	—	$\frac{2A f_s}{pt f_t} \times 100$
	Single butt strap, chain or zigzag arrangement	3·06t + 1·625	1·125t	2	—	$\frac{2A f_s}{pt f_t} \times 100$
	Single butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	4·05t + 1·625	$1{\cdot}125t\,\frac{(p-d)}{(p-2d)}$	3	—	$\frac{3\, A f_s}{pt f_t} \times 100$
	Double butt strap, chain or zigzag arrangement	3·5t + 1·625	0·625t	—	2	$\frac{3{\cdot}75 A f_s}{pt f_t} \times 10$
	Double butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	4·63t + 1·625	$0{\cdot}625t\,\frac{(p-d)}{(p-2d)}$	—	3	$\frac{5{\cdot}625 A f_s}{pt f_t} \times 10$
	Double butt strap, of unequal width chain or zigzag arrangement	5·5t + 1·625	0·75t wide 0·625t narrow	1	1	$\frac{2{\cdot}875 A f_s}{pt f_t} \times 1$
	Double butt strap of unequal width alternate rivet in outer row omitted, chain or zigzag arrangement	4·53t + 1·625	" "	1	2	$\frac{4{\cdot}75\, A f_s}{pt f_t} \times$
Treble riveted joint	Lap joint, chain or zigzag arrangement	3·47t + 1·625	—	3	—	$\frac{3A f_s}{pt f_t} \times 1$
	Lap joint, alternate rivet in outer row omitted, chain or zigzag arrangement	4·14t + 1·625	—	4	—	$\frac{4\, A f_s}{pt f_t} \times 1$
	Double butt strap, chain or zigzag arrangement	4·63t + 1·625	0·625t	—	3	$\frac{5{\cdot}625\, A f_s}{pt f_t}$
	Double butt strap, rivet pitch in the middle row half that in the other rows, chain or zigzag arrangement	5·52t + 1·625	0·625t	—	4	$\frac{7{\cdot}5\, A f_s}{pt f_t}$
	Double butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	6t + 1·625	$0{\cdot}625t\,\frac{(p-d)}{(p-2d)}$	—	5	$\frac{9{\cdot}375\, A f}{pt f_t}$
	Double butt strap, unequal width, chain or zigzag arrangement	4·63t + 1·625	0·75t wide 0·625 narrow	1	2	$\frac{4{\cdot}75\, A}{pt f_t}$
	Double butt straps of unequal width, alternate rivet in outer row omitted, chain or zigzag arrangement	6t + 1·625	0·75t wide 0·625t narrow	1	4	$\frac{8{\cdot}5\, A f}{pt f_t}$

(*d*) *The plates or rivets may crush* [fig. 4-4(*d*)].

If f_c be safe crushing stress for rivets or plates, the crushir resistance of rivets will be dtf_c. If there be n rivets to be crush for the failure of the joint, total resistance of joint to crushing w be $ndtf_c$. The number of rivets, that resist crushing, depe upon the kind of the joint employed.

the strength of the rivet in double shear will be double than that of a rivet in single shear. But in actual practice it is usual to use a factor of 1·75 to 1·875 instead of two. The Indian Boiler Regulations specify the factor 1·875.



Various possible failures of a riveted joint

FIG. 4-4

The resistance to shearing of one rivet in single shear in the strip equals $\frac{\pi}{4}d^2f_s$. The resistance of one rivet in double shear in the strip equals $1{\cdot}875\frac{\pi}{4}d^2f_s$. From the number of rivets to be sheared in a pitch width, the total resistance of the joint to shearing is known. The table on page 148 gives the number of rivets to be sheared for various kinds of joints for failure.

EFFICIENCIES OF RIVETED JOINTS

Kind of Joint	Description of joint	Maximum pitch inches	Thickness of butt strap inches	No. of rivets in shear		Percentage shearing efficiency of joint
				Single shear	Double Shear	
Single riveted joint	Lap joint	$1.52t + 1.625$	—	1	—	$\frac{A f_s}{pt f_t} \times 100$
	Single butt strap	$1.53t + 1.625$	$1.125t$	1	—	$\frac{A f_s}{pt f_t} \times 100$
	Double butt strap	$1.75t + 1.625$	$0.625t$	—	1	$\frac{1.875 A f_s}{pt f_t} \times 100$
Double riveted joint	Lap joint, chain or zigzag arrangement	$2.62t + 1.625$	—	2	—	$\frac{2A f_s}{pt f_t} \times 100$
	Single butt strap, chain or zigzag arrangement	$3.06t + 1.625$	$1.125t$	2	—	$\frac{2A f_s}{pt f_t} \times 100$
	Single butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	$4.05t + 1.625$	$1.125t \frac{(p-d)}{(p-2d)}$	3	—	$\frac{3 A f_s}{pt f_t} \times 100$
	Double butt strap, chain or zigzag arrangement	$3.5t + 1.625$	$0.625t$	—	2	$\frac{3.75 A f_s}{pt f_t} \times 100$
	Double butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	$4.63t + 1.625$	$0.625t \frac{(p-d)}{(p-2d)}$	—	3	$\frac{5.625 A f_s}{pt f_t} \times 100$
	Double butt strap, of unequal width chain or zigzag arrangement	$3.5t + 1.625$	$0.75t$ wide $0.625t$ narrow	1	1	$\frac{2.875 A f_s}{pt f_t} \times 100$
	Double butt strap of unequal width alternate rivet in outer row omitted, chain or zigzag arrangement	$4.63t + 1.625$	" "	1	2	$\frac{4.75 A f_s}{pt f_t} \times 100$
Treble riveted joint	Lap joint, chain or zigzag arrangement	$3.47t + 1.625$	—	3	—	$\frac{3A f_s}{pt f_t} \times 100$
	Lap joint, alternate rivet in outer row omitted, chain or zigzag arrangement	$4.14t + 1.625$	—	4	—	$\frac{4 A f_s}{pt f_t} \times 100$
	Double butt strap, chain or zigzag arrangement	$4.63t + 1.625$	$0.625t$	—	3	$\frac{5.625 A f_s}{pt f_t} \times 100$
	Double butt strap, rivet pitch in the middle row half that in the other rows, chain or zigzag arrangement	$5.52t + 1.625$	$0.625t$	—	4	$\frac{7.5 A f_s}{pt f_t} \times 100$
	Double butt strap, alternate rivet in outer row omitted, chain or zigzag arrangement	$6t + 1.625$	$0.625t \frac{(p-d)}{(p-2d)}$	—	5	$\frac{9.375 A f_s}{pt f_t} \times 100$
	Double butt strap, unequal width, chain or zigzag arrangement	$4.63t + 1.625$	$0.75t$ wide 0.625 narrow	1	2	$\frac{4.75 A f_s}{pt f_t} \times 100$
	Double butt straps of unequal width, alternate rivet in outer row omitted, chain or zigzag arrangement	$6t + 1.625$	$0.75t$ wide $0.625t$ narrow	1	4	$\frac{8.5 A f_s}{pt f_t} \times 100$

(*d*) *The plates or rivets may crush* [fig. 4-4(*d*)].

If f_c be safe crushing stress for rivets or plates, the crushing resistance of rivets will be dtf_c. If there be n rivets to be crushed for the failure of the joint, total resistance of joint to crushing will be $ndtf_c$. The number of rivets, that resist crushing, depend upon the kind of the joint employed.

If the load acting on a riveted joint is of the alternating type and varies between P_{max} and P_{min}, the allowable stress should be multiplied by $\dfrac{1}{1 - \frac{1}{2}\dfrac{P_{min}}{P_{max}}} \leq 1$

where P_{min} and P_{max} should bear their signs.

The following procedure should be adopted for the design of the joint according to I.B.R.:

From the diameter of the boiler shell and the working pressure, we select the kind of the joints for longitudinal and circumferential joints. For longitudinal joint, butt joint is adopted while for the circumferential joint, lap joint is preferred. After selecting the joint, suitable value of the efficiency is assumed.

By using the thin cylinder formula, the thickness of the boiler shell is determined. The calculated thickness of the shell is verified by I.B.R. After thickness has been obtained, the diameter of the rivet is fixed upon.

According to I.B.R., the factor of safety in no case should be less than 4.

If thickness of the plate be greater than *8 mm*, the diameter of a rivet hole is generally taken as $d = 6\sqrt{t}$ *mm*, where t is the thickness of the plate in millimetre. *If plate thickness be less than 8 mm, by equating the shearing resistance of rivets to crushing resistance, the diameter of the rivet hole is determined.*

The pitch of the rivets is calculated by equating the shearing resistance of the joint to the tearing resistance of the plate. We adopt the nearest standard size. The pitch of the rivets should not be less than $2d$ to enable the rivet heads to be formed. The maximum pitch of rivets in the longitudinal joint of the boiler shell has been specified by I.B.R. as follows

$C \times t + 1{\cdot}625''$ = maximum pitch in inches where t is the thickness of the shell plate in inches and C a coefficient as given in the table on page 150.

Should the pitch of the rivets exceed the maximum pitch allowed, the permissible pitch shall be used in place of the actual pitch in determining the percentage of plate section. The percentage greater than 85% is not allowed for any type of riveted joint.

Number of rivets per pitch	Coefficient for lap joint	Coefficient for single butt strapped joint	Coefficient for double butt strapped joint
1	1·31	1·53	1·75
2	2·62	3·06	3·5
3	3·47	4·05	4·63
4	4·17	—	5·52
5	—	—	6·00

The following is the spacing of rows of rivets as specified by I.B.R.:

In lap joints as well as butt joints, in which there are more than one row of rivets and in which there is an equal number of rivets in each row, the distance between the rows of rivets shall be not less than —

$0{\cdot}33p + 0{\cdot}67d$ for zig-zag riveting and $2d$ for chain riveting.

In joints in which the number of rivets in the outer rows is one half of the number in each of the inner rows, and in which the inner rows are chain riveted, the distance between the outer rows and the next rows shall be not less than $0{\cdot}33p + 0{\cdot}67d$ or $2d$ whichever is greater. The distance between the rows in which there are full number of rivets shall be not less than $2d$.

In joints in which the number of rivets in the outer row is one half of the number in each of the inner rows and in which the inner rows are zig-zag, the distance between the outer rows and the next rows shall be not less than $0{\cdot}2p + 1{\cdot}15d$. The distance between the rows in which there are full number of rivets shall be not less than $0{\cdot}165p + 0{\cdot}67d$. In all the above distances p is the pitch of the rivets in outer rows.

From purely theoretical stand point, the combined strength of the cover plates need not be more than that of the drilled plate, but in practice they are made 12·5% stronger for single butt strap and 25% stronger for double butt staps.

The following thicknesses t_1 for the butt straps have been specified by I.B.R.:

The thickness of butt strap in no case shall be less than 1 cm.

$t_1 = 1{\cdot}125t$ for ordinary single butt strap

$t_1 = 1{\cdot}125t\,\dfrac{(p-d)}{(p-2d)}$ for single butt straps every alternate rivet in outer rows being omitted

If the load acting on a riveted joint is of the alternating type and varies between P_{max} and P_{min}, the allowable stress should be multiplied by $\dfrac{1}{1-\frac{1}{3}\dfrac{P_{min}}{P_{max}}} \leq 1$

where P_{min} and P_{max} should bear their signs.

The following procedure should be adopted for the design of the joint according to I.B.R.:

From the diameter of the boiler shell and the working pressure, we select the kind of the joints for longitudinal and circumferential joints. For longitudinal joint, butt joint is adopted while for the circumferential joint, lap joint is preferred. After selecting the joint, suitable value of the efficiency is assumed.

By using the thin cylinder formula, the thickness of the boiler shell is determined. The calculated thickness of the shell is verified by I.B.R. After thickness has been obtained, the diameter of the rivet is fixed upon.

According to I.B.R., the factor of safety in no case should be less than 4.

If thickness of the plate be greater than *8 mm*, the diameter of a rivet hole is generally taken as $d = 6\sqrt{t}$ *mm*, where t is the thickness of the plate in millimetre. *If plate thickness be less than 8 mm, by equaling the shearing resistance of rivets to crushing resistance, the diameter of the rivet hole is determined.*

The pitch of the rivets is calculated by equating the shearing resistance of the joint to the tearing resistance of the plate We adopt the nearest standard size. The pitch of the rivets should not be less than $2d$ to enable the rivet heads to be formed. The maximum pitch of rivets in the longitudinal joint of the boiler shell has been specified by I.B.R. as follows.

$C \times t + 1{\cdot}625'' =$ maximum pitch in inches where $t =$ the thickness of the shell plate in inches and C a coefficient as given in the table on page 150.

Should the pitch of the rivets exceed the maximum pitch allowed, the permissible pitch shall be used in place of the actual pitch in determining the percentage of plate section. The percentage greater than 85% is not allowed for any type of riveted joint.

Number of rivets per pitch	Coefficient for lap joint	Coefficient for single butt strapped joint	Coefficient for double butt strapped joint
1	1·31	1·53	1·75
2	2·62	3·06	3·5
3	3·47	4·05	4·63
4	4·17	—	5·52
5	—	—	6·00

The following is the spacing of rows of rivets as specified by I.B.R.:

In lap joints as well as butt joints, in which there are more than one row of rivets and in which there is an equal number of rivets in each row, the distance between the rows of rivets shall be not less than —

$0{\cdot}33p + 0{\cdot}67d$ for zig-zag riveting and $2d$ for chain riveting.

In joints in which the number of rivets in the outer rows is one half of the number in each of the inner rows, and in which the inner rows are chain riveted, the distance between the outer rows and the next rows shall be not less than $0{\cdot}33p + 0{\cdot}67d$ or $2d$ whichever is greater. The distance between the rows in which there are full number of rivets shall be not less than $2d$.

In joints in which the number of rivets in the outer row is one half of the number in each of the inner rows and in which the inner rows are zig-zag, the distance between the outer rows and the next rows shall be not less than $0{\cdot}2p + 1{\cdot}15d$. The distance between the rows in which there are full number of rivets shall be not less than $0{\cdot}165p + 0{\cdot}67d$. In all the above distances p is the pitch of the rivets in outer rows.

From purely theoretical stand point, the combined strength of the cover plates need not be more than that of the drilled plate, but in practice they are made 12·5% stronger for single butt strap and 25% stronger for double butt staps.

The following thicknesses t_1 for the butt straps have been specified by I.B.R.:

The thickness of butt strap in no case shall be less than 1 cm.

$t_1 = 1{\cdot}125t$ for ordinary single butt strap

$t_1 = 1{\cdot}125t \dfrac{(p-d)}{(p-2d)}$ for single butt straps every alternate rivet in outer rows being omitted

$t_1 = 0{\cdot}625t$ for double butt straps of equal width having ordinary riveting

$t_1 = 0{\cdot}625t \dfrac{(p-d)}{(p-2d)}$ for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

When two unequal width of butt straps are employed, then the thicknesses of butt straps are given as

$t_1 = 0{\cdot}75t$ (wide strap)
$t_1 = 0{\cdot}625t$ (narrow strap).

Single and wide butt straps shall, wherever practicable, be on the inside of the shell. The inner strap plate is thicker than the outer to allow for the considerable wastage which often occurs on inner projecting plates.

The marginal distance is kept $1{\cdot}5d$.

The following procedure is suggested for the design of a circumferential lap joint for a boiler·

(1) The diameter of the rivet will be the same as adopted for a longitudinal joint.

(2) The shear value S, of the rivet is calculated. As the circumferential joint is a lap riveted joint, the rivets are in single shear. Therefore, the shear value S of the rivet will be given by

$$S = \frac{\pi}{4} d^2 f_s.$$

(3) Total shearing load F on the joint is calculated when the inner diameter D of the boiler shell and the pressure p of the steam are known.

$$F = \frac{\pi}{4} D^2 p.$$

(4) By knowing the shear value of a rivet and the total shearing load on the circumferential lap joint, the minimum number of rivets N for the joint can be determined.

$$N = \frac{F}{S}$$

(5) When the efficiency of the longitudinal joint is known, the efficiency of the circumferential lap joint can be

calculated. The strength of the end circumferential joint shall be 50% of that of the longitudinal joint but in no case less than 42% of the strength of the calculated thickness of the plate. The strength of the intermediate circumferential seams must not be less than 62% of the strength of the undrilled plate. This allows a reasonable margin for accidental stresses arising from shell distortion which tend to be more serious towards the middle portion of a longer boiler such as double ended 'Scotch' type of Marine boiler.

From the efficiency of the circumferential lap joint, the pitch of rivets for the lap joint can be obtained.

$$\eta = \frac{p-d}{p}$$

(6) From the pitch of the rivets in the circumferential lap joint, the number of rivets, n, that can be arranged in one row can be obtained.

$$n = \frac{\pi (D+t)}{p}$$

(7) When number of rivets n in a row is known, the kind of joint (single riveted, double riveted, etc.) can be decided upon. If Z be the row of rivets in a circumferential joint, then

$$Z = \frac{N}{n}.$$

(8) After deciding upon the number of rows, the number of rivets and the pitch of the rivets are adjusted. The pitch of the rivets should be such that the leak-proof joint is obtained.

(9) The pitch p' between the rows of rivets is calculated.

(10) Overlap O of the plate can finally be fixed when the pitch between the rows of rivets is known.

$$P = (Z-1)p' + 2 \text{ marginal pitch.}$$

Fig. 4-5 shows the portion of the boiler where circumferential and longitudinal joints meet.

The calculations of the strong tight joints differ from those of the strong joints in that, apart from calculating the rivets for shear, the joint must be checked for slipping of the plates. Slipping causes poor tightness of the joint. Hence this check replaces

$t_1 = 0{\cdot}625t$ for double butt straps of equal width having ordinary riveting

$t_1 = 0{\cdot}625t\,\dfrac{(p-d)}{(p-2d)}$ for double butt straps of equal width having every alternate rivet in the outer rows being omitted.

When two unequal width of butt straps are employed, then the thicknesses of butt straps are given as

$t_1 = 0{\cdot}75t$ (wide strap)
$t_1 = 0{\cdot}625t$ (narrow strap).

Single and wide butt straps shall, wherever practicable, be on the inside of the shell. The inner strap plate is thicker than the outer to allow for the considerable wastage which often occurs on inner projecting plates.

The marginal distance is kept $1{\cdot}5d$.

The following procedure is suggested for the design of a circumferential lap joint for a boiler:

(1) The diameter of the rivet will be the same as adopted for a longitudinal joint.

(2) The shear value S, of the rivet is calculated. As the circumferential joint is a lap riveted joint, the rivets are in single shear. Therefore, the shear value S of the rivet will be given by

$$S = \frac{\pi}{4}\,d^2 f_s.$$

(3) Total shearing load F on the joint is calculated when the inner diameter D of the boiler shell and the pressure p of the steam are known.

$$F = \frac{\pi}{4}\,D^2 p.$$

(4) By knowing the shear value of a rivet and the total shearing load on the circumferential lap joint, the minimum number of rivets N for the joint can be determined.

$$N = \frac{F}{S}$$

(5) When the efficiency of the longitudinal joint is known, the efficiency of the circumferential lap joint can be

With usual notations, the thickness of the boiler shell is given by the formulae,

$$t = \frac{pD}{2f\eta_t} = \frac{14 \times 180}{2 \times 770 \times 0{\cdot}84} = 1{\cdot}95 \text{ cm; we adopt 2 cm.}$$

According to I.B.R., the working pressure allowed for the *minimum tensile strength* of 28 tons/sq in. is 308 psig. Hence we can adopt the thickness $\frac{7}{8}$ in. (Refer page 119.)

The diameter of the rivet is obtained by formula $d = 6\sqrt{t}$ mm.

$d = 6\sqrt{20} = 26{\cdot}8$ mm; from IS 1928-1961 we adopt $28{\cdot}5$ mm.

The joint will be arranged in such a manner that the alternate rivets in outer row will be omitted. The rivets in the outer row will be in single shear, while those in inner rows will be in double shear.

Let p be the pitch between the rivets in outer row. In one pitch length there are five rivets of which four are in double shear while one in single shear. (Refer page 148.)

The tearing resistance of the plate $= (p - d)\ t\ f_t$.

The total shearing resistance of all the rivets will be

$$4 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s + \frac{\pi}{4} d^2 f_s = 8{\cdot}5 \frac{\pi}{4} d^2 f_s.$$

Equating the tearing and shearing resistances of the joint, we get

$$(p - d)\ t f_t = 8{\cdot}5 \frac{\pi}{4} d^2 f_s$$

or

$$p = \frac{8{\cdot}5 \times \frac{\pi}{4} d^2 f_s}{t f_t} + d$$

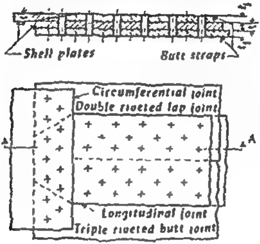
$$= \frac{8{\cdot}5 \times \frac{\pi}{4} \times 2{\cdot}85^2 \times 600}{2 \times 770} + 2{\cdot}85 = 24 \text{ cm.}$$

The maximum permissible pitch in inches, for such a joint according to Indian Boiler Regulations is given by the formula $p = 6t + 1{\cdot}625$, ... $+ 1{\cdot}625 = 6{\cdot}875''$

calculations of the joint for tightness: if there is no slipping the joint is pressure tight.

The requisite tightness of the joint is guaranteed if the following requirement is satisfied:

$$f = \frac{P}{\frac{\pi}{4} d^2 \times N}$$



Boiler shell plate joint

FIG. 4-5

where f is the allowable value of the specific resistance to slipping which depends on the design of the joint, coefficient of friction between the plates and the terminal riveting temperature. N is the number of rivets in a joint.

After making a strong tight joint the boiler vessel is subjected to a hydraulic test for tightness under pressure. Faulty parts of the joints must be caulked.

Design of a boiler joint:

The material of construction for boiler shell and rivets should be the same to avoid thermal stresses and galvanic action. St 34·13 is recommended by Bach for boiler construction. The steel St 34·13 has minimum tensile strength of 34 kg/sq mm and average carbon percentage 0·13%.

$d = 6\sqrt{t}$ mm.

$\therefore \quad d = 6\sqrt{15} = 23{\cdot}2$ mm. From IS: 1928-1961, we adopt 23 mm.

We assume that the rivets in double shear are 1·875 times as strong as those in single shear.

Tearing resistance $= (p - d)\ t\,f_t$.

Shearing resistance $= 3 \times 1{\cdot}875 \times \frac{\pi}{4}\, d^2\, f_s$. (As the joint is triple riveted butt joint, there are three rivets in double shear.)

By equating tearing and shearing resistances, we get the equation for determination of the pitch.

$$(p - d)\ t f_t = 3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s$$

or
$$p = \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s}{t\ f_t} + d$$

$$= \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} \times 2{\cdot}3^2 \times 630}{1{\cdot}5 \times 840} + 2{\cdot}3$$

$$= 14 \text{ cm.}$$

Strap thickness $= 0{\cdot}625 \times$ thickness of the plate
$= 0{\cdot}625 \times 1{\cdot}5 = 0{\cdot}94$ cm; we adopt 1 cm.

Distance between rows of rivet $= 0{\cdot}8p = 0{\cdot}8 \times 14 = 11{\cdot}2$ cm;
we adopt 12 cm.

The lowest of the two efficiencies will be the joint efficiency.

Tearing efficiency $= \dfrac{p - d}{p} = \dfrac{14 - 2{\cdot}3}{14} = 0{\cdot}835$ i.e. 83·5%.

$$\text{Shearing efficiency} = \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s}{p\, t\, f_t}$$

$$= \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} \times 2{\cdot}3^2 \times 630}{14 \times 1{\cdot}5 \times 840}$$

$$= 0{\cdot}835 \text{ i.e. } 83{\cdot}5\%.$$

The efficiency of the joint is 83·5%.

3. *A double riveted, double strap butt joint is to join 2 cm thick plates. The pitch of the rivets in the outer row is to be twice that of the inner row. Zig-zag riveting is to be employed with the following working stresses: $f_s = 630$ kg/sq cm and $f_t = 840$ kg/sq cm. Calculate rivet diameter, rivet pitches in inner and outer rows and the thickness of the butt straps.*

Rivet pitch in the inner row = 8 cm.

$$\text{Thickness of the butt strap} = 0.625t\left[\frac{p-d}{p-2d}\right]$$

$$= 0.625 \times 2\left[\frac{16-2.6}{16-5.2}\right]$$

$$= 1.6 \text{ cm.}$$

4. *A locomotive boiler of 180 cm internal diameter is required to generate steam at 14 kg/sq cm gauge. Calculate the thickness of the shell plate and design the triple riveted longitudinal double butt strap joint with unequal straps.*

Use the following data.

Allowable stress in tension for steel plate 770 kg/sq cm
Allowable stress in shear for rivets 600 kg/sq cm
Allowable stress in compression for steel plate 1,350 kg/sq cm
Efficiency of triple-riveted longitudinal butt joint 84%.

With usual notations, the thickness of the boiler shell is given by the formulae,

$$t = \frac{pD}{2\,f_t\eta_t} = \frac{14 \times 180}{2 \times 770 \times 0{\cdot}84} = 1{\cdot}95 \text{ cm; we adopt 2 cm.}$$

According to I.B.R., the working pressure allowed for the minimum tensile strength of 28 tons/sq in. is 308 psig. Hence we can adopt the thickness $\frac{7}{8}$ in. (Refer page 119.)

The diameter of the rivet is obtained by formula $d = 6\sqrt{t}$ mm.

$d = 6\sqrt{20} = 26{\cdot}8$ mm; from IS 1928-1961 we adopt 28·5 mm.

The joint will be arranged in such a manner that the alternate rivets in outer row will be omitted. The rivets in the outer row will be in single shear, while those in inner rows will be in double shear.

Let p be the pitch between the rivets in outer row. In one pitch length there are five rivets of which four are in double shear while one in single shear. (Refer page 148.)

The tearing resistance of the plate $= (p - d)\; t\, f_t$.

The total shearing resistance of all the rivets will be

$$4 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s + \frac{\pi}{4} d^2 f_s = 8{\cdot}5 \frac{\pi}{4} d^2 f_s.$$

Equating the tearing and shearing resistances of the joint, we get

$$(p - d)\; t f_t = 8{\cdot}5 \frac{\pi}{4}\; d^2 f_s$$

or

$$p = \frac{8{\cdot}5 \times \frac{\pi}{4} d^2 f_s}{t f_t} + d$$

$$= \frac{8{\cdot}5 \times \frac{\pi}{4} \times 2{\cdot}85^2 \times 600}{2 \times 770} + 2{\cdot}85 = 24 \text{ cm.}$$

The maximum permissible pitch in inches, for such a joint according to Indian Boiler Regulations is given by the formula $p = 6t + 1{\cdot}625$, which will be $6 \times \frac{7}{8} + 1{\cdot}625 = 6{\cdot}875''$,

i.e. 17·5 cm.

As the calculated pitch is more than the permissible pitch, we modify the diameter of the rivet and pitch.

We adopt 25 mm diameter rivet at 17 cm pitch.

Assume the rivets to be 1·875 times as strong in double shear as in single shear.

As t is greater than 8 mm, the diameter of the rivet is obtained by the formula $d = 6\sqrt{t}$ mm.

$\therefore \quad d = 6\sqrt{20} = 26{\cdot}8$ mm; from IS 1928 - 1961, we adopt 28·5 mm.

As the joint is double riveted two strap butt joint, there will be three rivets in double shear as the pitch of the rivets in the outer row is twice that of the inner row.

Tearing resistance $= (p - d)\, t f_t$.

Shearing resistance $= 3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s$

By equating the tearing and shearing resistances, we get the rivet pitch in the outer row.

$$(p - d)\, t f_t = 3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s$$

or
$$p = \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} d^2 f_s}{t f_t} + d$$

$$= \frac{3 \times 1{\cdot}875 \times \frac{\pi}{4} (2{\cdot}85)^2 \times 630}{2 \times 840} + 2{\cdot}85 = 16{\cdot}35 \text{ cm}$$

We adopt 16 cm.

Rivet pitch in the inner row $= 8$ cm.

$$\text{Thickness of the butt strap} = 0{\cdot}625\, t \left[\frac{p - d}{p - 2d}\right]$$
$$= 0{\cdot}625 \times 2 \left[\frac{16 - 2{\cdot}85}{16 - 5{\cdot}7}\right]$$
$$= 1{\cdot}6 \text{ cm}$$

4. A locomotive boiler of 180 cm internal diameter is required to generate steam at 14 kg/sq cm gauge. Calculate the thickness of the shell plate and design the triple riveted longitudinal double butt strap joint with unequal straps.

Use the following data.

Allowable stress in tension for steel plate 770 kg/sq cm
Allowable stress in shear for rivets 630 kg/sq cm
Allowable stress in compression for steel plate 1,350 kg/sq cm
Efficiency of triple-riveted longitudinal butt joint 84%

The following design stresses may be assumed:

Tensile 770 kg/sq cm
Shear 630 kg/sq cm
Crushing 1,260 kg/sq cm

The strength of a rivet in double shear may be taken as 1·875 times that of a rivet in single shear.

If t be the thickness of the boiler shell, then

$$t = \frac{pD}{2f_t \times \eta} = \frac{15 \times 180}{2 \times 770 \times 0{\cdot}84}$$
$$= 2{\cdot}09 \text{ cm; we adopt } 2{\cdot}2 \text{ cm.}$$

The diameter of a rivet hole d is given by

$$d = \sqrt{5t} - 0{\cdot}6$$
$= \sqrt{5 \times 2{\cdot}2} - 0{\cdot}6 = 2{\cdot}72$ cm; we adopt diameter of rivet hole as 2·85 cm.

If p be the pitch of the riveted joint, then by equating the shearing resistance of the joint to tearing resistance of the drilled plate, we get

$$(p - 2{\cdot}85) \times 2{\cdot}2 \times 770 = (8 \times 1{\cdot}875 + 3)\frac{\pi}{4} \times 2{\cdot}85^2 \times 630.$$

From the above equation we get $p = 45{\cdot}55$ cm; we adopt 45 cm as the pitch of the longitudinal joint.

Thickness of the wider strap $= 0{\cdot}75 \times 22$
$= 16{\cdot}5$ mm; say 17 mm.

Thickness of the narrower strap $= 0{\cdot}625 \times 22$
$= 13{\cdot}75$ mm; say 14 mm.

The joint is quadruple riveted butt joint with unequal straps.

Transverse pitch $= 2{\cdot}5 \times 2{\cdot}85$
$= 7$ cm.

Marginal pitch $= 1{\cdot}5 \times 2{\cdot}85$
$= 3{\cdot}8$ cm.

Width of the wider strap $= 2\,[3{\cdot}8 + 3 \times 7 + 3{\cdot}8]$
$= 57{\cdot}2$ cm; we adopt 58 cm.

Width of the narrower strap $= 2\,[3{\cdot}8 + 7 + 3{\cdot}8]$
$= 28{\cdot}2$ cm; we adopt 30 cm.

Design of a circumferential lap joint:

Diameter of a rivet hole $= 2{\cdot}85$ cm.

Tearing efficiency of the joint $= \frac{17 - 2.5}{17} = 0.855$

i.e. 85.5%.

$$\text{Shearing efficiency of the joint} = \frac{8.5 \times \frac{\pi}{4} \times 2.5^2 \times 600}{17 \times 2 \times 770}$$

0.955 i.e. 95.5%.

$$\text{Crushing efficiency} = \frac{5 \times 2.5 \times 2 \times 1350}{17 \times 2 \times 770} = 1.29 \text{ i.e. } 129\%.$$

The efficiency of the joint will be 85.5%.

Thickness of wide butt strap $= \frac{3}{4}t = \frac{3}{4} \times 2 = 1.5$ cm.

Thickness of narrow butt strap $= \frac{5}{8}t = \frac{5}{8} \times 2 = 1.25$ cm; we adopt 1.3 cm.

The edge distance = 38 mm.

The distance between the outer rows and next rows shall be not less than $0.2p + 1.15d = 0.2 \times 17 + 1.15 \times 2.5 = 6.28$ cm; we adopt 6.5 cm.

The distance between the rows in which there are full number of rivets shall be not less than $0.165p + 0.67d = 0.165 \times 17 + 0.67 \times 2.5 = 4.5$ cm.

According to I.B.R. single and wide butt straps where practicable will be on the inside of the shell

5. It is desired to specify the main dimensions for a longitudinal and circumferential joints for a boiler of 180 cm diameter generating steam at a pressure of 15 kg/sq cm gauge. The average efficiency of the longitudinal joint may be taken as 84%

Determine the thickness of the boiler shell and the size of the rivet.

It may be assumed that in a longitudinal joint, within one pitch length of the outer rivets, there are 8 rivets in double shear and three in single shear. Thus in a pitch length, between the centres of two rivets in the outermost row has one rivet at pitch, say p, the next inner row, the two rivets at pitch of $\frac{p}{2}$ and the third and fourth rows each four rivets at pitch $\frac{p}{4}$, the rivets on the other side of the joint being a mirror image of the rivets described above. Determine the pitch of the rivets for the longitudinal joint. Determine also the width and thickness for each butt strap for the longitudinal joint. Design also the circumferential joint completely. The transverse pitch may be taken as 2.5 times the diameter of the rivet for all the rows.

The following design stresses may be assumed:

Tensile *770 kg/sq cm*
Shear *630 kg/sq cm*
Crushing *1,260 kg/sq cm*

The strength of a rivet in double shear may be taken as 1·875 times that of a rivet in single shear.

If t be the thickness of the boiler shell, then

$$t = \frac{pD}{2f_t \times \eta} = \frac{15 \times 180}{2 \times 770 \times 0{\cdot}84}$$
$$= 2{\cdot}09 \text{ cm; we adopt } 2{\cdot}2 \text{ cm.}$$

The diameter of a rivet hole d is given by

$$d = \sqrt{5t} - 0{\cdot}6$$
$= \sqrt{5 \times 2{\cdot}2} - 0{\cdot}6 = 2{\cdot}72$ cm; we adopt diameter of rivet hole as 2·85 cm.

If p be the pitch of the riveted joint, then by equating the shearing resistance of the joint to tearing resistance of the drilled plate, we get

$$(p - 2{\cdot}85) \times 2{\cdot}2 \times 770 = (8 \times 1{\cdot}875 + 3)\,\frac{\pi}{4} \times 2{\cdot}85^2 \times 630.$$

From the above equation we get $p = 45{\cdot}55$ cm; we adopt 45 cm as the pitch of the longitudinal joint.

Thickness of the wider strap $= 0{\cdot}75 \times 22$
$= 16{\cdot}5$ mm; say 17 mm.

Thickness of the narrower strap $= 0{\cdot}625 \times 22$
$= 13{\cdot}75$ mm; say 14 mm.

The joint is quadruple riveted butt joint with unequal straps.

Transverse pitch $= 2{\cdot}5 \times 2{\cdot}85$
$= 7$ cm.

Marginal pitch $= 1{\cdot}5 \times 2{\cdot}85$
$= 3{\cdot}8$ cm.

Width of the wider strap $= 2\,[3{\cdot}8 + 3 \times 7 + 3{\cdot}8]$
$= 57{\cdot}2$ cm; we adopt 58 cm.

Width of the narrower strap $= 2\,[3{\cdot}8 + 7 + 3{\cdot}8]$
$= 28{\cdot}2$ cm; we adopt 30 cm.

Design of a circumferential lap joint:

Diameter of a rivet hole $= 2{\cdot}85$ cm.

In a lap joint rivets are in single shear. Hence shear strength of the rivet will be $\frac{\pi}{4} \times 2{\cdot}85^2 \times 630 = 5{,}100$ kg. Shearing load on the joint will be $\frac{\pi}{4} \times 180^2 \times 15 = 48{\cdot}5 \times 10^4$ kg. Hence the minimum number of rivets in the lap joint will be $\frac{48{\cdot}5 \times 10^4}{5100} = 95$.

The efficiency of the lap joint can not be more than 42%. If p' be the pitch of the circumferential joint, then $\frac{p' - 2{\cdot}85}{p'} = 0{\cdot}42$ or $p' = 4{\cdot}86$ cm; we adopt 6·5 cm

The efficiency of the lap joint will be $\frac{6{\cdot}5 - 2{\cdot}85}{6{\cdot}5} = 0{\cdot}562$ i.e. 56·2% which is more than 42%.

Number of rivets that can be arranged in one row in circumferential lap joint $= \frac{\pi\,(180 + 2{\cdot}2)}{6{\cdot}5} = 88$ rivets.

We have to arrange at least 95 rivets in a lap joint, hence we adopt two rows of rivets in each row there being 88 rivets

Pitch between the rows of rivets may be taken as 7 cm.

Over lap of the plate will be $3{\cdot}8 + 7 + 3{\cdot}8 = 14{\cdot}6$ cm

Exercises:

1. How are boiler plates joined?
2. What forms of rivet heads are allowed in boiler construction?
3. What forms of riveted joints are used in boiler construction?
4. Describe and sketch:
(a) single riveted lap joint
(b) double riveted lap joint
(c) double riveted butt joint with double straps
(d) triple riveted butt joint with double straps of unequal width
5. Why is a butt joint preferable to a lap joint?
6. What preparation of plates and butt straps is necessary to ensure good riveted joints?
7. Should rivet holes be punched or drilled?
8. What is caulking and why is it necessary?
9. What are various ways in which a riveted joint may fail?
10. What is meant by single shear and double shear?
11. What is meant by the efficiency of a riveted joint?

Explain how you would find the efficiency of a single riveted lap joint, double riveted lap joint, double riveted butt joint and triple riveted butt joint with two unequal straps and alternate rivets in outer rows being omitted.

12. *What formula is used for calculating the diameter of a rivet?*

13. *Determine the efficiency of a single riveted lap joint having a rivet pitch 8 cm. a rivet diameter of 28·5 mm and a plate thickness of 13 mm?* *Ans.* 50%.

14. *What is the efficiency of a double riveted lap joint having a rivet pitch of 10 cm, a rivet diameter of 25 mm and a plate thickness of 13 mm?* *Ans.* 62%.

15. *Two boiler plates 13 mm thick are connected by a double riveted lap joint having a pitch of 6·5 cm. Determine the least tensile stress in 19 mm rivets of the joint which will enable it to remain tight under a tension of 40 tonne/metre, along the joint, if the coefficient of friction is 0·2.*

16. *Design the longitudinal joint and the girth seam of a marine boiler 3 metre diameter and 2·7 metre length for a working steam pressure of 10·5 atg giving a neat sketch of portions of the joints.*

Calculate also the tearing efficiency, shearing efficiency and crushing efficiency of the joints designed.

17. *Design a double riveted, double cover butt joint for plates 18 mm thick and find the efficiency of the joint.* $f_s = 0{\cdot}8\ f_t$ *and* $f_c = 1{\cdot}3\ f_t$. *Take the strength of a rivet in double shear as 1·875 times its strength in single shear.*

18. *Design a triple riveted double butt strap joint for the longitudinal seam of a boiler 180 cm in diameter when working pressure is 10 kg/sq cm. You may assume ultimate tensile strength of the plate at 4,200 kg/sq cm, crushing strength 6,500 kg/sq cm and shearing strength 3,080 kg/sq cm. Joint efficiency may be assumed as 85%.*

19. *A steel tank 150 cm in diameter is used for storing air at 18 kg/sq cm. Design a double riveted butt joint with unequal cover plates for longitudinal seam and a lap joint for the girth seam. Sketch the intersection of these two joints assuming all details and dimensions.*

20. *A boiler shell 200 cm diameter has to withstand an internal steam pressure of 12 atg, the material is mild steel having a safe tensile strength of 11 kg/sq mm. Assuming an efficiency of 70% of the longitudinal joint, calculate the thickness of the shell plate. Design and draw a sketch of a double riveted butt joint with double cover strap for the longi-*

In a lap joint rivets are in single shear. Hence shear strength of the rivet will be $\frac{\pi}{4} \times 2{\cdot}85^2 \times 630 = 5{,}100$ kg. Shearing load on the joint will be $\frac{\pi}{4} \times 180^2 \times 15 = 48\ 5 \times 10^1$ kg. Hence the minimum number of rivets in the lap joint will be $\frac{48{\cdot}5 \times 10^1}{5100} = 95$.

The efficiency of the lap joint can not be more than 42%. If p' be the pitch of the circumferential joint, then $\frac{p' - 2\ 85}{p'} = 0{\cdot}42$ or $p' = 4{\cdot}86$ cm; we adopt 6·5 cm

The efficiency of the lap joint will be $\frac{6\ 5 - 2{\cdot}85}{6{\cdot}5} = 0\ 562$ i.e. 56·2% which is more than 42%.

Number of rivets that can be arranged in one row in circumferential lap joint $= \frac{\pi\ (180 + 2{\cdot}2)}{6{\cdot}5} = 88$ rivets.

We have to arrange at least 95 rivets in a lap joint; hence we adopt two rows of rivets in each row there being 88 rivets.

Pitch between the rows of rivets may be taken as 7 cm.

Over lap of the plate will be $3\ 8 + 7 + 3{\cdot}8 = 14\ 6$ cm

Exercises:

1. *How are boiler plates joined?*
2. *What forms of rivet heads are allowed in boiler construction?*
3. *What forms of riveted joints are used in boiler construction?*
4. *Describe and sketch:*
(a) *single riveted lap joint*
(b) *double riveted lap joint*
(c) *double riveted butt joint with double straps*
(d) *triple riveted butt joint with double straps of unequal width.*
5. *Why is a butt joint preferable to a lap joint?*
6. *What preparation of plates and butt straps is necessary to ensure good riveted joints?*
7. *Should rivet holes be punched or drilled?*
8. *What is caulking and why is it necessary?*
9. *What are various ways in which a riveted joint may fail?*
10. *What is meant by single shear and double shear?*
11. *What is meant by the efficiency of a riveted joint?*

For plates in tension ....1,400 kg/sq cm
For rivets in shear ....1,050 kg/sq cm
For crushing of rivets and plates
Single shear ....2,240 kg/sq cm
Double shear ....2,800 kg/sq cm

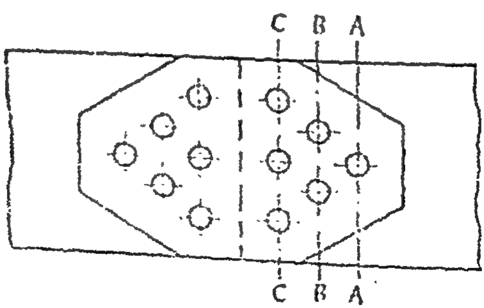
The following table, due to Rotscher, will be of much use in design of cylindrical storage tanks:

Thickness of the plate t mm	2	3	4	5—6	6—8	8—12	11—15
Diameter of the shank of the rivet d' mm	8	9	10	12	14	16	20
Diameter of the hole of the rivet d mm	8·4	9·5	11	13	15	17	21
Pitch of rivets p 3d+5 mm	29	32	35	38	47	56	65
Marginal pitch m mm	16	17	17	18	21	25	30
Stiffner angle iron	40×45×5			45×45 ×7	50×50 ×9	75×75 ×12	80×80 ×12

For exhaust pipes, chimneys, etc. the pitch of the rivets p may be $5d$.

4·9. Lozenge Joint:

Fig. 4-6 shows a riveted jont for two plates of a girder for roof or bridge work and such a joint is known as lozenge joint.



Lozenge joint
Fig. 4-6

This is the joint of greatest economy in which the section of the bar is not reduced by more than one rivet hole. The plates to

tudinal seam of the above boiler. The safe shear stress of the rivets is 8 kg/sq mm and the safe crushing strength of the rivet is 16 kg/sq mm. Rivet in double shear is 1·75 times as strong as a rivet in single shear. Calculate the diameter of the rivets, the pitch of the rivets and the efficiency of the joint.

21. A locomotive boiler shell 200 cm in diameter and under an internal pressure of 12 kg/sq cm, is to be made of steel plate, of ultimate tensile strength of 3,850 kg/sq cm, and a factor of safety of 5. The efficiency of the treble riveted butt joint is 85% and of double riveted circumferential lap joint is 70%. Determine the thickness of the shell and design the suitable longitudinal and circumferential joints.

22. Design and prepare working drawings of a treble riveted butt joint suitable for the longitudinal seam and a double-riveted lap joint for the circumferential seam of a Lancashire boiler 2·4 metre diameter and working pressure of 12 kg/sq cm by gauge. Plates and rivets are to be made of mild steel having ultimate strength in tension and shear of 43 and 33 kg/sq mm respectively. Factor of safety 5. The longitudinal joint efficiency of 85%. Resistance of rivets in double shear equals to 1·875 times that of rivets in single shear. Indicate how you will make the joints steam and water tight after riveting.

4·8. Joints for Storage Tanks:

The chief requirement for the riveted joints in case of ordinary tanks, coal bunkers, bins for bulk material are the strength and rigidity rather than leakage. The usual proportions for the riveted joints approach to the proportions from the theoretical consideration of equal strength.

The thickness of the plate is calculated from the thin cylinder formula. The diameter of the rivet is calculated by formula $d = 6\sqrt{t}$ mm where t is the thickness of the plate in mm. The pitch of the joint is calculated by equating the strength of the joint in tension to the strength of the rivets in shear. As there is no consideration of caulking, the pitches permitted in such joints are larger than that employed in design of pressure vessels. Joints for storage tanks are single or double riveted lap joints. The edges of the storage tanks are stiffened by angle irons.

The permissible stress values are greater than that employed in the design of pressure vessels. The values commonly adopted in such joints are given below:

The permissible values of the stresses for the design of riveted connections in structural engineering are higher than those used in pressure vessel design. The centres of the rivets are not less than $1\frac{1}{2}d$, from the edge of the plate, where d is the diameter of the rivets.

The following proportions will be of much use in design of riveted connections in structural engineering:

Thickness of plate t mm
Diameter of rivet hole $d = t + 10$ mm
Diameter of a shank of a rivet $d' = d - 1$ mm
Pitch of rivets $p \geq 2{\cdot}5d < 6d$
Marginal pitch in a direction of load $m_1 \geq 2d < 6d$
Marginal pitch sideways $m_2 \geq 1{\cdot}5d < 4d$.
IS: 1929-1961 recommends the following sizes for rivets for general purpose:

Diameter of the hole d mm	13·5	15·5	17·5	19·5	21·5	23·5	25·5	29	32	35	38	41	44	50
Diameter of the shank of a rivet d' mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48

In design for lighter construction such as in aeroplane industry, duralumin is used. The permissible stress for duralumin is 0·4 or 0·5 of the yield point. The following table gives the yield point stresses for duralumin:

Tensile (plate):	2,700 kg/sq cm
Shear (rivet):	1,800 kg/sq cm
Crushing:	4,100 kg/sq cm.

The usual proportions for the riveted joints for lighter construction are given below:

The rivet heads are formed cold so as to strain the rivets in shear.
Diameter of the rivet hole $d = 1{\cdot}5t + 2$ mm
Pitch of rivets $p = 2{\cdot}5d$ to $6d$
Marginal pitch $d_m = 2d$
Distance between the rows of rivets $= 2{\cdot}5d$ to $3d$
Diameter of a shank of a rivet $d' = d - 0{\cdot}1$ mm for $d \leq 10$ mm
$= d - 0{\cdot}2$ mm for $d \geq 10$ mm.

Example:

1. Mild steel tie bars, for a bridge structure, 35 cm wide and 2 cm thick are to be connected by a double cover butt joint. Design this joint allowing safe working stresses as follows:

$f_t = 9$ *kg/sq mm*, $f_s = 7{\cdot}5$ *kg/sq mm*; $f_c = 15$ *kg/sq mm.*

The diameter of the rivets is obtained by the formula $d = 6\sqrt{t}$ mm.

$\therefore\ d = 6\sqrt{20} = 26{\cdot}8$ mm; we adopt 25·5 mm as the rivet diameter.

be connected are either butted together and riveted as shown in fig. 4-6 to a single or double butt strap or the plates are simply lapped one over the other. By arranging the rivet as shown in fig. 4-6 the joint is made of uniform strength.

Let us analyse such a joint.

Let b and t be respectively the width and thickness of the plate to be connected and d be the diameter of the rivet. We assume that the plates are connected by single butt strap.

Before the joint can fail across BB, one rivet must be sheared and the plate must tear through a section of area $t\,(b-2d)$.

Resistance along $BB = \frac{\pi}{4} d^2 f_s + t\,(b-2d) f_t$.

Similarly before the joint can fail across CC, three rivets must be sheared and the plate must tear through a section of area $t\,(b-3d)$. Resistance along $CC = 3 \times \frac{\pi}{4} d^2 f_s + t\ (b-3d) f_t$.

In such a joint, as the section of the plate decreases, the number of rivets to be sheared increases.

The following procedure is followed.

From the thickness of the plate, the diameter of the rivet is determined and the kind of joint (lap, single butt strap, double butt strap) is decided upon. Shearing resistance as well as the bearing resistance of the rivet is calculated. The lower of the two values is taken in order to fix the number of rivets. It is assumed that the resistance of a rivet in double shear is 1.75 times that in single shear in order to allow for possible eccentricity of load or defective workmanship.

The strap thickness t_1 equals $0{\cdot}75t$ approximately for double cover plate joints. For single cover plate joints, $t_1 = 1{\cdot}25t$ approximately.

For this joint, as we proceed from outer row of rivets to inner row of rivets, the resistance of the joint to failure increases. The weakest section will be the outermost row, which has been weakened by one rivet diameter. In general, the efficiency of lozenge joint can be written as $\frac{(b-d)}{b}$ where b and d are the width of the plate and the diameter of the rivet respectively.

The permissible values of the stresses for the design of riveted connections in structural engineering are higher than those used in pressure vessel design. The centres of the rivets are not less than $1\frac{1}{2}d$, from the edge of the plate, where d is the diameter of the rivets.

The following proportions will be of much use in design of riveted connections in structural engineering:

Thickness of plate t mm

Diameter of rivet hole $d = t + 10$ mm

Diameter of a shank of a rivet $d' = d - 1$ mm

Pitch of rivets $p \geq 2{\cdot}5d < 6d$

Marginal pitch in a direction of load $m_1 \geq 2d < 6d$

Marginal pitch sideways $m_2 \geq 1{\cdot}5d < 4d$.

IS: 1929-1961 recommends the following sizes for rivets for general purpose:

Diameter of the hole d mm	13·5	15·5	17·5	19·5	21·5	23·5	25·5	29	32	35	38	41	44	50
Diameter of the shank of a rivet d' mm	12	14	16	18	20	22	24	27	30	33	36	39	42	48

In design for lighter construction such as in aeroplane industry, duralumin is used. The permissible stress for duralumin is 0·4 or 0·5 of the yield point. The following table gives the yield point stresses for duralumin:

Tensile (plate):	2,700 kg/sq cm
Shear (rivet):	1,800 kg/sq cm
Crushing:	4,100 kg/sq cm.

The usual proportions for the riveted joints for lighter construction are given below:

The rivet heads are formed cold so as to strain the rivets in shear.

Diameter of the rivet hole $d = 1{\cdot}5t + 2$ mm

Pitch of rivets $p = 2{\cdot}5d$ to $6d$

Marginal pitch $d_m = 2d$

Distance between the rows of rivets $= 2{\cdot}5d$ to $3d$

Diameter of a shank of a rivet $d' = d - 0{\cdot}1$ mm for $d \leq 10$ mm

$= d - 0{\cdot}2$ mm for $d \geq 10$ mm.

Example:

1. *Mild steel tie bars, for a bridge structure, 35 cm wide and 2 cm thick are to be connected by a double cover butt joint. Design this joint allowing safe working stresses as follows:*

$f_t = 9$ *kg/sq mm*, $f_s = 7{\cdot}5$ *kg/sq mm*, $f_c = 15$ *kg/sq mm.*

The diameter of the rivets is obtained by the formula

$d = 6\sqrt{t}$ mm.

$\therefore$ $d = 6\sqrt{20} = 26{\cdot}8$ mm; we adopt 25·5 mm as the rivet diameter.

We assume that the resistance of a rivet in double shear is 1·75 times that in single shear. Resistance of plate to tearing at outer row $= (35 - 2{\cdot}55)\ 2 \times 900$
$= 58{,}500$ kg.

Shearing resistance of one rivet $= 1{\cdot}75 \times \frac{\pi}{4} \times 2{\cdot}55^2 \times 750$
$= 6{,}700$ kg.

Bearing resistance of one rivet $= 2 \times 2{\cdot}55 \times 1500$
$= 7{,}650$ kg.

As the shearing resistance of the rivet is less than the bearing resistance, we use the former in deciding upon the number of rivets. Equating the tensile strength of the plate to the shearing resistance of n rivets, we get

$$58500 = n \times 6700$$

or $$n = \frac{58500}{6700} = 8{\cdot}2.$$

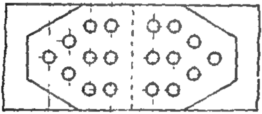


Fig. 4-7

Hence 9 rivets may be used and they may be arranged as shown in fig. 4-7.

The thickness of the butt straps will be $0{\cdot}75t = \frac{3}{4} \times 2 = 1{\cdot}5$ cm.

We investigate the strength of the joint at four critical sections AA, BB, CC and DD. Although the joint might fracture at any one of the sections AA, BB, CC or DD, it cannot fracture along BB without shearing one rivet in double shear, along CC, without shearing three rivets in double shear and along DD without shearing six rivets in double shear.

Along AA, joint has a strength of $(35 - 2{\cdot}55) \times 2 \times 900$
$= 58{,}500$ kg.

Along BB joint has a strength of $(35 - 2 \times 2{\cdot}55) \times 2 \times 900 + 1 \times 6700 = 60{,}500$ kg as the fracture along BB cannot take place without shearing one rivet in double shear.

Similarly we can determine the strength along CC and DD.

Strength along $CC = (35 - 3 \times 2{\cdot}55) \times 2 \times 900 + 3 \times 6700$
$= 69{,}300$ kg.

Strength along $DD = (35 - 3 \times 2{\cdot}55) \times 2 \times 900 + 6 \times 6700$
$= 89{,}400$ kg.

Shearing resistance of all rivets $= 9 \times 6700 = 60{,}300$ kg.

The lowest strength of the joint is along AA.

$$\text{Efficiency of the joint} = \frac{(b-d)}{b} = \frac{(35 - 2{\cdot}55)}{35}$$
$$= 0{\cdot}928 \text{ i.e. } 92{\cdot}8\%.$$

Note: It should be noted that if instead of diamond form of joint, had we adopted chain riveting with three rows of three rivets in each, the least strength of the joint would be $(35 - 3 \times 2{\cdot}55) \times 2 \times 900 = 49{,}200$ kg, which gives an efficiency of $\frac{35 - 3 \times 2{\cdot}55}{35} = 0{\cdot}782$ i.e., $78{\cdot}2\%$.

Exercises:

1. Design a diamond, double cover butt joint for a tie bar of 25 mm thickness subjected to an axial load of 35 tonnes. Maximum tensile and shear stresses are limited to 11 kg/sq mm and 8·5 kg/sq mm respectively.

Ans. Use 30 mm diameter 3 rivets on each side, assuming 1 and 2 rivets in rows; 15 cm wide.

2. Two lengths of flat steel bar 2 cm thick, are to be connected by a double cover butt joint to carry a load of 40 tonnes. Determine the diameter and number of rivets required for the joint. Also, determine the width of the bar.

Assume a rivet in double shear to be 1·75 times stronger than that in single shear.

Maximum tensile stress = 9 kg/sq mm
,, shear stress = 7·5 kg/sq mm
,, crushing stress = 15 kg/sq mm.

Ans. Use 27 mm diameter 6 rivets on each side, assuming 1, 2, 3 rivets in rows; 15 cm wide.

3. A tie member in a roof truss has to carry an axial load of 46 tonnes. The member is a flat bar 15 mm thick and of constant width. Design and sketch a double cover butt joint.

We assume that the resistance of a rivet in double shear is 1·75 times that in single shear. Resistance of plate to tearing at outer row $= (35 - 2\cdot55)\ 2 \times 900$

$= 58{,}500$ kg.

Shearing resistance of one rivet $= 1\cdot75 \times \frac{\pi}{4} \times 2\cdot55^2 \times 750$

$= 6{,}700$ kg.

Bearing resistance of one rivet $= 2 \times 2\cdot55 \times 1500$

$= 7{,}650$ kg.

As the shearing resistance of the rivet is less than the bearing resistance, we use the former in deciding upon the number of rivets. Equating the tensile strength of the plate to the shearing resistance of n rivets, we get

$$58500 = n \times 6700$$

or $$n = \frac{58500}{6700} = 8\cdot2.$$

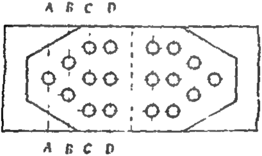


Fig. 4-7

Hence 9 rivets may be used and they may be arranged as shown in fig. 4-7.

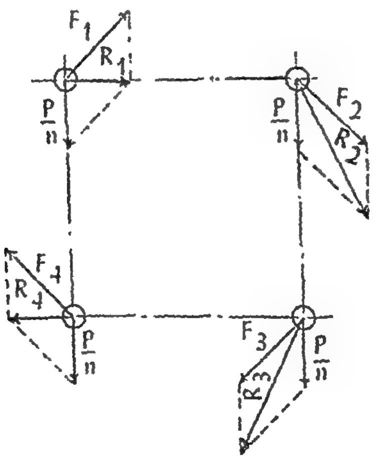
The thickness of the butt straps will be $0\cdot75t = \frac{3}{4} \times 2 = 1\cdot5$ cm

We investigate the strength of the joint at four critical sections AA, BB, CC and DD. Although the joint might fracture at any one of the sections AA, BB, CC or DD, it cannot fracture along BB without shearing one rivet in double shear, along CC, without shearing three rivets in double shear and along DD without shearing six rivets in double shear.

Along AA, joint has a strength of $(35 - 2\cdot55) \times 2 \times 900$ $= 58{,}500$ kg.

Let n be the number of rivets in the group. Due to $P_1 = P$, the direct shear load coming upon each rivet is the same and is equal to $F = \frac{P}{n}$, the direction of direct shear load being parallel to the applied load. Due to the turning moment Pe, the secondary load produced in any rivet in the group is proportional to the distance that the rivet is from the centre of gravity of the group and is at right angles to the line joining the centre of gravity of the rivet group to the centre of the rivet. Therefore, the resisting moment of the rivet about the centre of rotation varies as the square of the moment arm. Let F_1 represent the secondary shear load induced in the rivet situated at a distance l_1 from the centroid of the rivet group. If l_2, l_3, etc. represent the distances from the centre of gravity G to the rivets 2, 3 ,etc., then the external moment Pe being equal to the summation of the resisting moments due to n rivets, we get

$$Pe = \frac{F_1}{l_1}\left[l_1^2 + l_2^2 + l_3^2 + \ldots\ldots + l_n^2 \right].$$



Resultant shear loads on eccentrically loaded rivets.

FIG. 4-10

From the position of the rivets, the distances l_1, l_2.......are known and hence, secondary shear loads on all rivets are calculated. The primary and secondary shear loads are added vectorially to determine the resultant load R on the rivet as shown in fig. 4-10. The heavily loaded rivet will be one in which the included angle

Maximum tensile stress 15 kg/sq mm
,, shear stress 9 kg/sq mm
,, crushing stress 18 kg/sq mm.

Determine the efficiency of the joint.

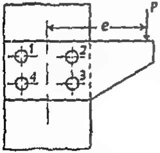
Ans. Use 23 mm diameter, 9 rivets on each side, assuming 1, 2, 3 and 3 rivets in rows; 15 cm wide.

4. *Two 15 mm thick tie plates are connected by a lap riveted joint. The tie carries a load of 15,000 kg. Calculate (a) width of the plate, (b) diameter and number of rivets, (c) rows of rivets and (d) efficiency of the joint. Suggest also the arrangement of rivets in rows. Take* $f_t = 770$ *kg/sq cm,* $f_s = 610$ *kg/sq cm and* $f_c = 1,360$ *kg/sq cm.*

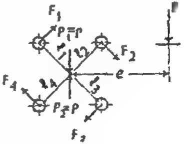
Ans. (a) 18 cm wide (b) Use 23 mm diameter 7 rivets (c) Three rows 2, 3, 2 rivets (d) 74·5%.

4·10. Eccentric loads on riveted connection:

In general the line of action of the load must pass through the centre of gravity of the rivet areas Manytimes we come across connections in which the load is not applied through the centroid of the rivet formation as shown in fig. 4-8. In such connections all rivets are not equally loaded The direct shear on each rivet will be accompanied by a secondary shear caused by the tendency of the force to twist the joint about the centre of gravity.



Eccentrically loaded rivets
Fig. 4-8



Secondary shear loads
Fig. 4-9

The following procedure for the design of a rivet is adopted: Generally all the rivets are of the same size. The centre of gravity of the rivet group is determined. Two equal and opposite forces to P are introduced at the centre of gravity of group of rivets as shown in fig. 4-9. We denote these forces by P_1 and P_2.

If d cm be the diameter of each rivet, then

$$\frac{\pi}{4} d^2 \times 630 = 7{\cdot}45 \times 1000$$

or $$d = \sqrt{\frac{7{\cdot}45 \times 1000 \times 4}{\pi \times 630}} = 3{\cdot}84 \text{ cm};$$

we adopt 41 mm diameter rivet.

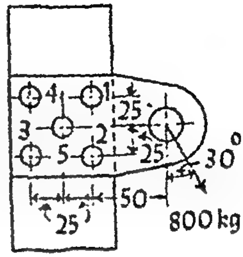


FIG. 4-12

2. *Find the size of rivets required for the bracket shown in fig. 4-12. The plate is 3 mm thick and the rivets which are to be all of the same size, are in single shear. Take 18 kg/sq mm as the permissible shear stress.*

Direct load on each rivet $= \frac{800}{5} = 160$ kg.

Distance of line of action of 800 kg load from centroid will be $7{\cdot}5 \cos 30° = 6{\cdot}5$ cm.

Distance of rivets 1, 2, 3 and 4 from centroid $= 2{\cdot}5\sqrt{2}$
$= 3{\cdot}54$ cm.

Distance of rivet 5 from centroid = 0.

Load in rivets 1, 2, 3 and 4 due to torque will be equal to

$$\frac{800 \times 6{\cdot}5}{4 \times 3{\cdot}54} = 340 \text{ kg.}$$

Load in rivet 5 due to torque = 0.

The resultant force on each rivet can be found out graphically or mathematically as the angle can be calculated. The maximum force on any rivet is to be on rivet 1 and is equal to 497 kg, from which the diameter of the rivet may be obtained.

$$\therefore \quad \frac{\pi}{4} d^2 \times 1800 = 497$$

or $$d = \sqrt{\frac{497 \times 4}{1800 \times \pi}} = 0{\cdot}525 \text{ cm; we adopt 6 mm.}$$

Check for crushing:

$$\text{Crushing stress} = \frac{\text{load}}{\text{area}} = \frac{497}{0{\cdot}6 \times 0{\cdot}3} = 2{,}760 \text{ kg/sq cm.}$$

This stress is well below $2f_s = 2 \times 1800 = 3{,}600$ kg/sq cm; therefore the rivets will withstand crushing.

between the primary and secondary shear is the least. The heavily loaded rivet becomes the critical one for determining the strength of the connections. When the permissible shear stress is known, the diameter of the rivet can be specified.

Examples:

1. Fig. 4-11 shows a column to which a bracket is riveted, carrying a load of 10 tonnes at a distance of 25 cm from the centre line of the column. Examine the distribution of load among the rivets. If the maximum shear stress in the rivet is limited to 630 kg/sq cm determine the diameter of the rivet.

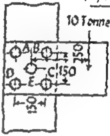


Fig. 4-11

The centroid of the rivet formation is at the centre rivet E, and the centres of all the rivets are equidistant from the centroid of group of rivets and is equal to $7{\cdot}5\sqrt{2} = 10{\cdot}6$ cm. There are 5 rivets.

Direct shear load per rivet $= \frac{10}{5} = 2$ tonnes.

Due to eccentricity of 25 cm, the load has a tendency to turn the plate in clockwise direction about the central rivet E. The turning moment $= 10 \times 25 = 250$ tonne cm.

The turning tendency of the plate is resisted by four rivets A, B, C and D. As all these rivets are situated at equal distances from E, the secondary shear load induced in each of these rivets will be of the same magnitude and the direction of each of these forces on each rivet will be at right angles to the moment arm of that rivet. If F be the magnitude of secondary shear load on each rivet, then

$$4F \times 10{\cdot}6 = 250$$

or $$F = \frac{250}{4 \times 10{\cdot}6} = 5{\cdot}9 \text{ tonnes.}$$

The magnitude of resultant load on each rivet will not necessarily be same as it depends not only on the magnitudes of primary and secondary shear forces but also on the included angle between the two. The maximum resultant load will be on rivets B and C. The included angle between two shear forces 2 tonnes and 5·9 tonnes is 45°. Therefore, the maximum resultant load R on the rivet is given by

$$R = \sqrt{2^2 + 5{\cdot}9^2 + 2 \times 2 \times 5{\cdot}9 \cos 45^\circ} = 7{\cdot}45 \text{ tonnes.}$$

Rectangular arrangement:

In this arrangement, the distance of the centre of gravity of each corner rivet from the centre of gravity of group of rivets will be $\frac{12 \cdot 5}{2}\sqrt{1+4} = 6 \cdot 25\sqrt{5} = 14$ cm.

If q be the magnitude of secondary shear load in a rivet situated at a unit distance from the centre of gravity of group of rivets, then

$$4q \times 14^2 + 2q \times 6 \cdot 25^2 = 6000 \times 25$$

or $$q = \frac{6000 \times 25}{4 \times 14^2 + 2 \times 6 \cdot 25^2} = 174 \text{ kg.}$$

Maximum secondary shear load $= 174 \times 14$
$= 2{,}436$ kg.

Angle of inclination between the primary and secondary shear load on a rivet which is nearest to the line of action of load

$$= \tan^{-1} \frac{6 \cdot 25}{12 \cdot 5} = 26°\text{-}34'.$$

Hence the maximum resultant shear load in the rectangular pattern as shown in fig. 4-13 will be

$$R = \sqrt{1000^2 + 2436^2 + 2 \times 1000 \times 2436 \cos 26°\text{-}34'} = 3{,}420 \text{ kg.}$$

Triangular arrangement:

In this arrangement the centre of gravity of group of rivets will be at the centroid of the triangle. The length of the median of the triangle will be $\sqrt{25^2 - 12 \cdot 5^2} = 21 \cdot 6$ cm. As the centroid trisects the medians, the three rivets are situated at a distance of 7·2 cm from the group of rivets and the remaining three at a distance of 14·4 cm.

If q be the magnitude of the secondary shear load in a rivet situated at a unit distance from the centre of gravity of group of rivets, then

$$3q\,[14 \cdot 4^2 + 7 \cdot 2^2] = 6000 \times 25$$

or $$q = \frac{6000 \times 25}{51 \cdot 8 \times 3 \times 5} = 193 \text{ kg.}$$

Maximum shear load on the rivet $= 193 \times 14 \cdot 4$
$= 2{,}780$ kg.

Angle of inclination between the primary and secondary shear load on a rivet which is nearest to the line of action of load $= 30°$. Hence the maximum resultant shear load in the triangular arrangement as shown in fig. 4-13 will be

3. A bracket supporting a vertical load of 6,000 kg is to be riveted to an adjacent member. The number of rivets thought to be necessary is provisionally fixed at six, but the rivet arrangement pattern has yet to be decided. Fig. 4-13 shows three possible patterns, the line of action of the load being in each case 25 cm from the vertical centre line of the rivets. Determine which of these patterns will enable the smallest diameter of the rivet to be used and what that diameter will be if the rivets are in single shear and the permissible shear stress intensity is limited to 400 kg/sq cm.

We assume that the rivets are fitted in reamed holes. The rivets in each arrangement are subjected to primary shear load and secondary shear load. The primary shear load on each rivet in each pattern is the same and its magnitude will be $\frac{6000}{6} = 1{,}000$ kg. The direction of this load is vertically downwards. Let us consider the secondary shear load coming in each arrangement.

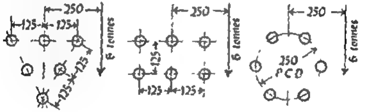


Fig. 4-13

Circular arrangement:

As all the rivets in this arrangement are situated at equal distances from the centre of gravity of group of rivets, the magnitude of the secondary shear load on each rivet will be the same; however the line of action of the secondary shear load will be at right angle to the line joining the centre of gravity of the group of rivets and the centre of gravity of the rivet under consideration. If Q be the secondary shear load on each rivet, then

$$Q \times 6 \times 12{\cdot}5 = 6000 \times 25 \text{ or } Q = \frac{6000 \times 25}{6 \times 12{\cdot}5} = 2{,}000 \text{ kg.}$$

The resultant load will be maximum on a rivet which is nearest to the line of action of the load. As the line joining the centre of gravity of this rivet and the centre of gravity of group of rivets is horizontal, the line of action of the secondary shear load will be vertical. Hence the maximum resultant shear load in circular pattern as shown in fig. 4-13 will be $1000 + 2000 = 3{,}000$ kg.

Adopt reasonable working stresses. The diagonal pitch should be estimated from the expression $0{\cdot}65p + 0{\cdot}35d$.

Sketch a portion of each joint showing the main dimensions.

2. A cylindrical pressure vessel is to be of diameter 1·5 metre and length 2·5 metre and is to be of riveted construction using high tensile alloy steel plates. The greatest working internal pressure is to be 42 kg/sq cm.

Determine a suitable thickness for the plates and design a treble riveted butt joint for the longitudinal seams. Make the spacing of the rivets in the outer rows double than that in the inner rows.

Describe all the ways in which the joint might fail and calculate its efficiency in each case.

Give a fully dimensioned sketch of a small portion of the joint showing several pitch lengths.

The working stresses are:

Tension in plates = 24 kg/sq mm

Shear in rivets = 11 kg/sq mm

Bearing on rivets = 22 kg/sq mm.

3. A double cover butt joint is made up of two 15 mm thick mild steel plates riveted together. The joint is subjected to a tensile load of 14 tonnes. Assuming that the plates are not weakened by more than one rivet hole in the outermost rows, design the joint and draw completely dimensioned sketch of the joint. Calculate also the efficiency of the joint designed. Assume the following safe stresses for the plates and rivets.

$f_t = 7{\cdot}5$ kg/sq mm, $f_s = 6$ kg/sq mm and $f_c = 13$ kg/sq mm.

4. Fig. 4-16 shows a bracket fixed on a steel column by means of 3 rivets. Derive an expression for the maximum load on a rivet in terms of P, e, l_1 and l_2 due to the overturning effect of the force P.

Calculate the size of the rivets used if $P = 1{,}500$ kg, $e = 50$ cm, $l_1 = 5$ cm and $l_2 = 40$ cm.

Take permissible tensile and shear stresses as 700 and 560 kg/sq cm respectively.

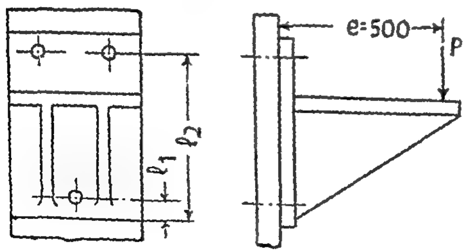


Fig. 4-16

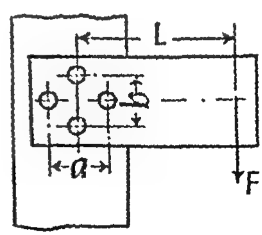


Fig. 4-17

5. Fig. 4-17 shows a 15 mm plate riveted to a vertical member by rivets 19 mm diameter. $a = 10$ cm, $b = 15$ cm and $L = 45$ cm. What load F this connection will carry if the design stress in shearing for the rivets is 600 kg/sq cm and the design stress in crushing for the plate and the rivets is 1,100 kg/sq cm?

$$R = \sqrt{1000^2 + 2780^2 + 2 \times 2780 \times 1000 \times \cos 30^\circ}$$
$$= 3{,}680 \text{ kg.}$$

Thus the circular arrangement will be better. Hence this arrangement should be adopted.

If d be the diameter of the rivet, then $\frac{\pi}{4} d^2 \times 400 = 3000$

or $d = \sqrt{\frac{3000 \times 4}{\pi \times 400}} = 3{\cdot}09$ cm, we adopt 33 mm diameter rivets.

Exercises:

1. *Fig. 4-14 shows a riveted joint with an eccentric load. The rivets are 18 mm in diameter. Find the shear stress in most heavily loaded rivet. If the plate is 1 cm thick, find the crushing stress on the rivet.*

Ans *1,110 kg/sq cm; 1,570 kg/sq cm.*

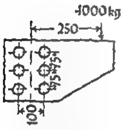


Fig. 4-14

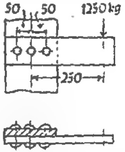


Fig. 4-15

2. *Find the diameter of the rivet for the riveted joint shown in fig. 4-15 the maximum shearing stress on the most heavily loaded rivet is to be 560 kg/sq cm.* *Ans* *3,750 kg; 3 cm.*

EXAMPLES IV

1. A cylindrical boiler of 250 cm diameter is to be of riveted steel plate construction and is to operate at a maximum steam pressure 10 kg/sq cm gauge.

Determine a suitable thickness for the plates forming the shell. The steel has an ultimate tensile strength of 45 kg/sq mm.

Design the main longitudinal and circumferential riveted joints, using treble-riveted butt joints with two cover straps for longitudinal joints and double riveted lap joints for circumferential joints.

a suitable diameter for the rivets, explaining the method used and the assumptions involved. Also, determine a suitable diameter for the pulley spindle and a suitable thickness for the plates.

Assume working stresses of 6 kg/sq mm for shear and 15 kg/sq mm for bearing. Neglect bending effects. Graphical methods may be used if desired.

9. Two lengths of flat steel bar 2 cm thick are to be connected by a double cover butt joint to carry a load of 40 tonnes. Determine the diameter and number of rivets required for the joint. Also, determine the width of the bar.

Assume a rivet in double shear to be 1·75 times stronger than that in single shear.

Assume the following values of the permissible stresses:

$f_t = 945$ kg/sq cm

$f_s = 790$ kg/sq cm

$f_c = 1{,}580$ kg/sq cm.

Draw a dimensioned sketch of the joint designed by you.

(Sardar Vallabhbhai Vidyapeeth, 1965)

10. A gusset plate is attached to a vertical member by means of 5 rivets in a vertical line. The load line is inclined at 30° to the vertical line of the rivets and passes through the second rivet from the bottom. Find the size of the rivet if $f_s = 400$ kg/sq cm and the pitch of the rivets is 100 mm.

(M S. University of Baroda, 1965)

11. Design and draw a double riveted butt joint for the longitudinal seam of a vertical water tube boiler of a Spencer Hopwood type. The internal diameter of the boiler shell is 120 cm. The working pressure of a boiler is 10·5 kg/sq cm gauge. Also, calculate the efficiency of the boiler joint designed by you.

The following values of the permissible stresses may be adopted:

$f_t = 840$ kg/sq cm

$f_c = 1{,}400$ kg/sq cm

$f_s = 560$ kg/sq cm.

(M. S. University of Baroda, 1965)

12. Design and prepare working drawings of a treble riveted double-strap butt joint suitable for the longitudinal seam of a Lancashire boiler 2 m diameter and working pressure of 10·5 kg per sq cm gauge. Plates and rivets are to be made of mild steel having ultimate strength in tension, bearing and shear, 4,200 kg per sq cm, 6,300 kg per sq cm, and 2,800 kg per sq cm respectively. Factor of safety 5. The efficiency of the joint may be assumed as 85%. The pitch in the outer row of rivets is to be double than that of the inner rows. Resistance of rivets in double shear is equal to 1·75 times that of rivets in single shear. Check the efficiency of the joint.

(Sardar Vallabhbhai Vidyapeeth, 1966)

13. Design the longitudinal and circumferential joints of a cylindrical boiler shell 2·5 metre diameter and made of 26 mm steel plate. The longitudinal joint is to be a butt joint with two cover plates and the circumferential joint a lap joint. The working pressure in the boiler is to be 16 kg/sq cm gauge. For plates and rivets take

6. Find the load P, fig. 4-18, that the riveted connection shown can take if the stress in the rivets is not to exceed 1,100 kg/sq cm. The size of the rivets used is 25 mm diameter.

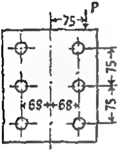


Fig. 4-18

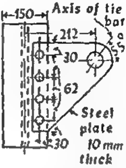


Fig. 4-19

7. Fig. 4-19 shows a cleat 1 cm thick to connect a tie-bar, inclined at 55° to the horizontal to the flange of an 20 × 15 × 16 kg section used as a vertical column. The cleat is to be riveted to the section with four rivets of equal diameter in single shear as shown. The mean flange thickness of the section is 16 5 mm.

The maximum pull in the tie bar is 2 5 tonnes and this may be assumed to act in the plane of shear of rivets

Determine the force on each rivet under this maximum load and hence decide the diameter required, using the following working stresses:

Shear: 6 kg/sq mm

Bearing: 15 kg/sq mm.

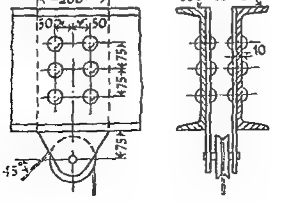


Fig. 4-20

8. Fig 4-20 shows an arrangement for supporting a pulley carrying a wire rope hoisting cable. The tension in the cable is to be 4,500 kg. Determine

CHAPTER V

BOLTS, NUTS AND SCREWS

5-1. Introduction:

The screw thread is a form obtained by cutting a continuous helical groove on the cylindrical surface. The threaded portion engages with a corresponding threaded hole in the nut or machine part, the two elements form what is called a screw pair.

Screws have two general purposes in engineering:

(a) to act as fastening to secure one member to other member,

(b) to transmit power.

The first of these purposes requires strong threads of a low efficiency and they should not be loosened during the service. Screws used for transmission of power must have a high efficiency so as to reduce power loss during power transmission to a minimum.

5-2. Definitions:

The major diameter is the largest diameter of the screw thread. It is often referred to as the outside diameter, crest diameter or full-diameter on external threads. The size of the screw is specified by its nominal major diameter.

The minor diameter is the smallest diameter of the thread. It is often referred to as the root diameter or core diameter on external threads.

The pitch is the axial distance from a point on a screw thread to a corresponding point on the adjacent thread.

$$\text{Pitch} = \frac{1}{\text{number of threads per unit length of screw}}.$$

The lead is a distance which a screw thread advances axially in one turn. On a single thread screw, the lead and pitch are identical; on a double thread screw, the lead is twice the pitch; on a triple thread screw the lead is three times the pitch and so on.

5-3. Forms of screw threads:

Screw threads commonly adopted by engineers are British Standard Whitworth (B.S.W.), British Standard Fine (B.S.F.), British Standard Pipe (B.S.P.), British Association (B.A.), American National Standard, Lowenherz, Metric Screw threads — Systéme International (S.I.), Acme, Square and Buttress. Fig. 5-1 shows the thread proportions for various kinds of threads. Whitworth thread is the standard British thread. The proportions for B.S.W., B.S.F. and

f_t = 1,200 kg/sq cm
f_s = 800 kg/sq cm
f_c = 1,600 kg/sq cm.

(Bombay University, 1967)

14. Design and prepare the drawing of a triple riveted, double straps (unequal) butt joint suitable for the longitudinal seam of a Lancashire boiler, 2·4 metre in diameter and working at 11 kg/sq cm gauge pressure. The efficiency of the triple riveted longitudinal butt joint should not be less than 85%.

The safe tensile stress of the steel plate is 800 kg/sq cm.

The safe bearing stress of the steel plate and the rivets is 1,600 kg/sq cm and the safe shear stress for the rivets is 600 kg/sq cm.

(Sardar Patel University, 1968)

15. In a structure, three flats carry tensile loads of 12 tonnes, 16 tonnes and 20 tonnes respectively. Their lines of action meet at a point. A gusset plate joins the flats through 15 mm rivets. Design the joint and draw two fully dimensioned views of the joint

f_s = 0·8 tonne/sq cm, f_t = 1 tonne/sq cm and f_b = 2 tonne/sq cm.

(Bombay University, 1969)

pipes, the threads are specified by inner diameter of the pipe. B.A. threads are used in instrument work. Lowenherz threads are used in Germany on measuring, precision and physical apparatus. Acme, square, trapezoidal and buttress threads are used for power transmmission such as screw jacks, lead screw of a lathe,

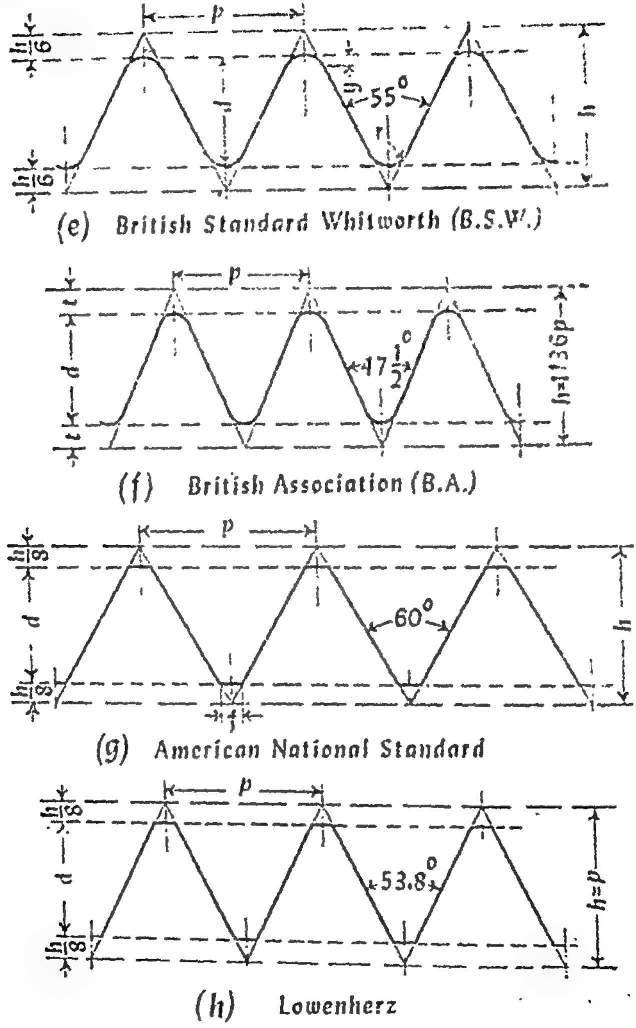
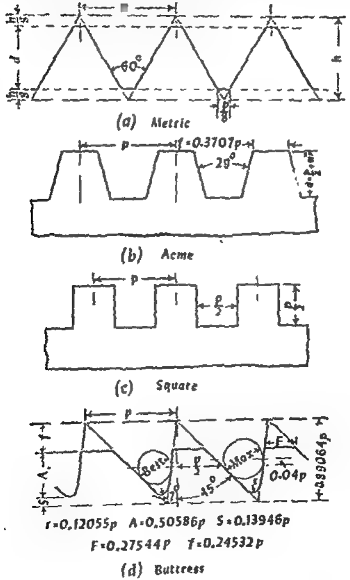


Fig. 5-1(b)

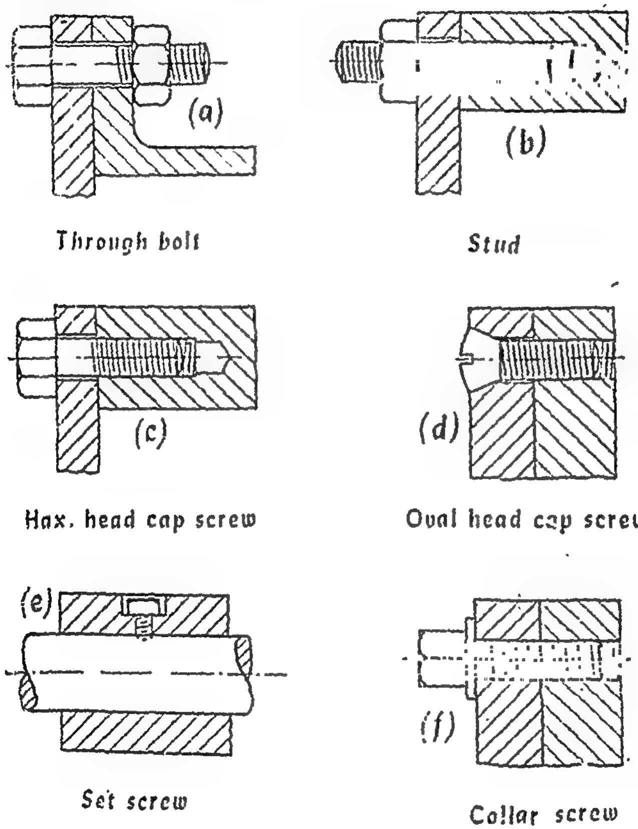
vices, presses, etc. Acme and trapezoidal threads are cheaper to manufactu than square threads, but less efficient than square threads. Acme threads a sometimes used with a split nut in order to facilitate the engagement and disc gagement of the nut. Both these threads can transmit power in any directic

B.S.P. (both parallel and tapered) are the same. In B.S.F. threads finer pitches are used. These threads are used where small axial adjustments are necessary such as on bolts for piston rod — and connecting rod ends, in automobile work, etc. In B.S.P. threads finer pitches are provided. These threads are used for steel and iron piping and for tubes carrying fluids. For external threaded



Various kinds of screw threads
Fig. 5-1(*a*)

bolts, etc. The head and nut may be rough or finished. The head and nut are square or hexagonal. When a through bolt is put under tension by axial loading, it should be easy fit in the holes. If the load acts perpendicular to the axis of the bolt tending to slide one of the connected parts along the other, thus causing a shear load on the bolt, the holes should be reamed and the shank should be finished all over its length. Such bolts are known as coupling bolts. They are costlier to manufacture. Another form of a bolt known as carriage bolt is chiefly used for wood construction.



Various kinds of screwed fastenings

Fig. 5-3

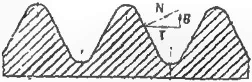
Special kinds of bolts are used in automobile work. The heads and nuts of these bolts are hexagonal in shape and are smaller in size. Finer pitches are adopted. The head of automobile bolts are slotted in order to work with a screw driver. The nuts are recessed or castellated in order to facilitate the locking by means of cotter pins. When through bolts are used, no threading is required in parts to be connected.

Buttress threads are used to transmit power in one direction. It has got advantages of both trapezoidal and square threads. Acme or trapezoidal threads are used in machine tools where disengaging nut is required.

The dimensions for screw threads in metric system for general purposes, are specified in IS: 1362-1962.

5-4. Advantages of square threads over V threads:

The force acting on a screwed rod is axial which is supported by the reaction, which is normal to the surface of the threads, between surfaces of the threads on the screw and nut.



Forces acting on a triangular thread

FIG. 5-2

Let T be the axial tensile load acting on the screw, N the normal pressure on the threads, which is also the measure of friction and B the bursting force on the nut, as shown in fig 5-2 From the force triangle, we see that bursting *force on the nut increases with the angle of thread and the friction between the* nut also increases with the angle of threads In square threads the sides of the threads are parallel and the bursting force on the nut vanishes and the axial tension in the screw is practically equal to the normal pressure on the threads For these reasons, square threads are adopted for transmission of power.

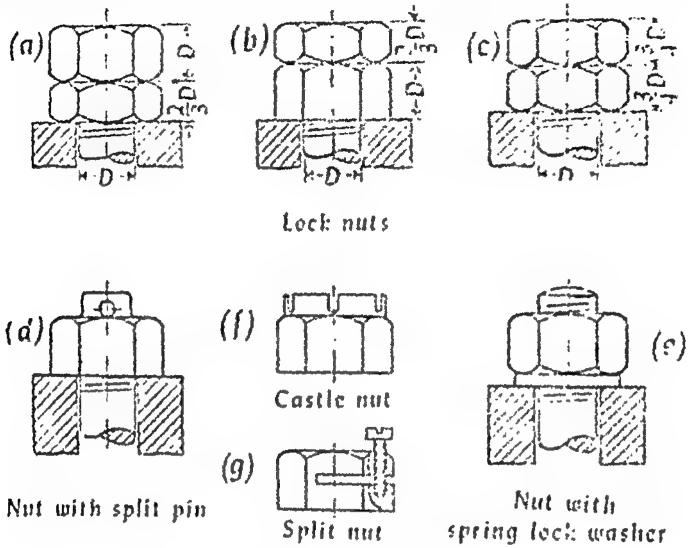
5-5. Screw fastenings:

The following important screw fastenings are met with in construction of machines (See fig. 5-3):

(a) Through bolts
(b) Tap bolts and cap screws
(c) Machine screws
(d) Set screws
(e) Studs.

Through bolts: A through bolt is shown in fig. 5-3(*a*). It is a cylindrical bar with an integral head at one end with threads at the other end. The nut is provided at the threaded end The cylindrical part of the bolt is known as shank. The shank of the bolt may be rough or finished. The bolt is known as machine bolt when the shank is rough. Different minor characteristics which through bolts may possess to fit them to a specific usage create different names by which they become known, such as machine bolts, eye bolts, carriage bolts, automobile

put all the load on the thin nut which is not desirable. In actual practice the compromise is obtained by keeping the total thickness to be the same and making both the nuts of the same thickness as shown in fig. 5-4(c).



Locking devices
FIG. 5-4

There are many other ways of locking the nut of which the most common method is to employ the taper pin or split pin [fig. 5-4(*d*)]. The pin passes through the bolt just above the nut. Strictly speaking this arrangement does not lock the nut, it simply prevents the nut from screwing off. Some of the common arrangements are: use of spring lock washer [fig. 5-4(*e*)], castellated nut [fig. 5-4(*f*)] and split nut [fig. 5-4(*g*)]. Use of castellated nut is much favoured in automobile industry. More information on various locking devices can be obtained from a standard work on Machine Drawing.

5-7. Washers:

The function of a washer is to provide sufficient and suitable bearing area for a nut or a bolt head. They are also used when the hole through which bolt passes is much larger than the bolt, so as to provide sufficient bearing area. They are used when screwing down nuts against wood or stone, to distribute the load over larger bearing area.

Washers are thin circular plates made with a central hole slightly larger than the bolt diameter. They are specified by a nominal diameter d, which is meant to be the diameter of the bolt with which the washer is to be used. The thickness of the washer is usually taken to be equal to $0{\cdot}15d$. The outside diameter of the washer is twice the nominal diameter but the relationship between these two dimensions is not constant for all nominal sizes.

Tap bolts and cap screws

They do not require nut but screw directly into one of the pieces to be connected. The holding power is due to the pressure exerted by the head of the screw. As the head of the tap bolt is larger than that of the cap screw, the holding power of the latter is less than that of the former. The head of the cap screw is rounded on the top. Cap screws make an excellent fastening for machine parts that do not require frequent removal.

Machine screws:

Very small cap screws are known as machine screws. The heads of these screws are slotted so that they can be tightened by means of a screw driver. They are used in assembly of small machines such as type-writers, jigs, carburettors, etc.

Set screws.

The function of set screws is to prevent the relative motion of machine parts such as collars on the shaft, hub of the pulley on the shaft, etc. A set screw is screwed through a tapped hole in one part until the end or the point of the screw is pressed against the other part. The holding power of a set screw is the frictional resistance set up at the point. They are used for transmission of light loads. They are made with square heads or without heads. The points of set screws are hardened. Set screws are commonly used for connecting pulleys or gears transmitting light load. In order to get the appropriate size of the set screw for a given diameter D of the shaft, the following empirical formula is used:

Diameter of a set screw $= 0.125D + 8$ mm

Stud:

A stud is a bolt in which the head is replaced by a threaded end. It passes through one of the parts to be connected and is screwed into the other part. Thus the stud always remains in position when two parts are disconnected. Clearance between the threads and hole facilitates the removal of the part without injury to the free end of the stud. With this construction, the wear and crumbling of the threads in a weak material are avoided.

Studs are employed for connecting heads of cylinders in engines and pumps.

The dimensions for various kinds of studs are recommended by IS: 1862-1961.

5-6. Locking devices for nuts:

In machine parts subjected to rapid movement there is a considerable tendency for a nut to work loose owing to constant vibration. This tendency is prevented by using the lock nut which is an extra nut, which is screwed down tightly on the ordinary nut. As the duty of the lock nut is only to jamb the first nut and not to take much or any of the stresses, it may be much thinner than the usual standard. As a rule the thin nut is generally made equal to half the bolt diameter. As the top nut practically takes the whole load, thick nut should be there at the top as shown in fig. 5-4(*a*) and the thin nut should be inside. This arrangement is not practically convenient as ordinary spanners are frequently too thick to admit of fitting on the thin nut the thick nut. The obvious way of overcoming this difficulty is to arrange the thin nut on the top as shown in fig. 5-4(*b*) which will

where f_t is the permissible tensile stress intensity in the material of the bolt. The diameter at the root of the thread will be obtained from the above equation. The next step will be to refer to the table of metric screw threads and the most suitable standard diameter will be obtained.

If the external load acting on the machine part is to be resisted jointly by n bolts, we have

$$\text{external load} = n \times \frac{\pi}{4} d_c^2 \times f_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Since the table of threads gives the area at the root of the threads, the calculations for the size of the bolt will be simplified if we solve the equation for the minimum area of the bolt. If tables for the metric screw threads are not available, the following relation is employed to calculate nominal diameter d from core diameter d_c in case of coarse threads.

$$d_c = 0{\cdot}84d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Examples:

1. Find the safe load in kg which the following bolts will bear, assuming a safe tensile stress of 400 kg/sq cm.

M5, M10, M16, M20, M27, M30 and M39.

If a bolt is not initially stressed, the maximum safe axial load which may be applied to it is given by:

cross sectional area at bottom of thread × permissible stress.

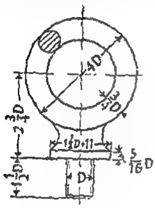
$\therefore$ P = safe load in kg = $400 \times a_c$.

a_c can be seen from tables for Metric threads which are given in Appendix III. The safe load for each bolt is given in table below:

Diameter d mm	Cross sectional area at the bottom of the thread = a_c sq cm	$P = 400 \times a_c$ kg
5	0·142	56·8
10	0·58	232
16	1·57	628
20	2·45	980
27	4·59	1,836
30	5·61	2,242
39	9·76	3,904

5-8. Eye bolt:

This bolt consists of a ring of circular section material having screwed shank forged integral with it. Eye bolts are used for lifting and transporting heavy pieces. A tapped hole is generally provided above the centre of gravity of heavy part such as cylinder frame, turbine casing, frames of heavy electric motors or generators, etc. so that an eye bolt can be inserted for lifting purposes. They are also used for the suspension of lifting tackles The average proportions for an eye bolt are shown in fig 5-5



Eye bolt
FIG. 5-5

Example.

1 A 2,000 kg gear box is provided with a steel (as rolled B1113) eye bolt for use in moving it. What size bolt should be used if (a) coarse threads are used? (b) fine threads are used?

The ultimate strength of the steel used will be 4,900 kg/sq cm. We adopt a factor of safety of 6. Therefore allowable stress will be $\frac{4900}{6} = 815$ kg/sq cm Minimum core area at the bottom of the thread required will be $\frac{2000}{816} = 2{\cdot}45$ sq cm. For coarse threads we adopt M22 which has a stress area of 3·03 sq cm while for fine threads we adopt M20 × 1·5 having a stress area of 2·72 sq cm.

Exercise:

1. A motor weighing 2,000 kg is lifted by a wrought iron eye bolt which is screwed into the frame. Decide upon a design factor and determine the size of the eye bolt if (a) coarse threads are used and (b) fine threads are used? Ultimate strength 3.400 kg/sq cm

Ans. Design factor 6, M24; M22 × 1·5

Design note: · Fine threads are not recommended for brittle materials.

5-9. Efficiency of threads:

When we consider thread friction only, the efficiency of V-threaded screw is given by the expression,

$$\eta = \frac{\cos\theta - \mu\tan\alpha}{\cos\theta + \mu\cot\alpha} \qquad \text{(i)}$$

As the leverage is 10, the weight at the end of the lever will be $\frac{576}{10} = 57{\cdot}6$ kg.

Load on the fulcrum will be $576 - 57{\cdot}6 = 518{\cdot}4$ kg.

Fulcrum of the lever loaded safety valve is subjected to axial tensile load (fig. 12-23).

Minimum cross sectional area required at the bottom of the threads $= \frac{518{\cdot}4}{400} = 1{\cdot}3$ sq cm.

We assume fine threads. We adopt $M16 \times 1{\cdot}5$ which has 1·67 sq cm area at the bottom of the threads. The pitch of the threads is 1·5 mm.

5. Suggest the suitable size for the pillar of a boiler stop valve, if the maximum compressive load on the spindle of the stop valve is 2,000 kg.

The pillars are made of mild steel. The compressive load in the spindle is transmitted by the bridge to the two pillars which are subjected to tensile load. The pillars are screwed into the body cover at lower end and the top screwed portion receives the nut. We assume permissible tensile stress intensity as 500 kg/sq cm.

Load on each pillar $= \frac{2000}{2} = 1{,}000$ kg.

Minimum cross sectional area at the bottom of the thread $= \frac{1000}{500} = 2$ sq cm.

We adopt coarse threads on 20 mm diameter, which will provide 2·45 sq cm area at the bottom of the thread.

Note: For rigidity, the diameter on the plain portion of the pillar may be increased to 22 mm. The pillars are provided with collars at top and bottom as shown in figure of boiler stop valve in chapter XIX. The lower collars have two flats machined on them to take a standard spanner.

Exercises:

1. Calculate the axial tensile load that can be safely applied to a M36 bolt, allowing a safe tensile stress of 550 kg/sq cm.

Ans. 4,500 kg.

2. A ball bearing company give in their tables the following approximate safe load on coarse threaded bolts:

2. *The cap of a connecting rod end is secured by two bolts. If the maximum pull in the connecting rod is 6,000 kg, find the diameter of the bolts if the tensile stress is not to exceed 350 kg/sq cm. Assume fine threads.*

The load shared by each bolt $= \frac{6000}{2} = 3{,}000$ kg.

Minimum area at the bottom of the thread required

$$= \frac{3000}{350} = 8{\cdot}57 \text{ sq cm.}$$

From IS: 1362-1962 we adopt $M36 \times 3$ which has 8·65 sq cm area at the bottom of the thread and 3 mm pitch.

3. *The area of the valve face of a D-slide valve may be taken as a rectangle 25 cm by 20 cm, the pressure of steam to be 9 kg/sq cm by gauge, coefficient of friction between valve and seat to be 0·2. The stress at the bottom of the screw thread is not to exceed 200 kg/sq cm. Determine the diameter of a steel valve spindle, assuming the engine to be non-condensing.*

Area of valve face $= 25 \times 20 = 500$ sq cm

Normal force acting on the valve $= 9 \times 500 = 4{,}500$ kg.

Frictional resistance to the motion of valve $= 4500 \times 0{\cdot}2$
$= 900$ kg.

The force in the valve rod will be due to frictional resistance to the motion of the valve.

$\therefore$ Maximum force in the valve rod $= 900$ kg.

Minimum cross sectional area of valve spindle $= \frac{900}{200}$
$= 4{\cdot}5$ sq cm.

Assuming the diameter of the valve spindle to be 3 cm, the cross sectional area at the bottom of the threads will be 5·61 sq cm if coarse threads are provided.

4. *A lever loaded safety valve has a diameter of 7 cm and the blow off pressure is 15 kg/sq cm. The fulcrum of the lever is screwed into the cast iron body of the cover. Suggest the suitable size of the threaded part of the fulcrum if the permissible tensile stress intensity is limited to 400 kg/sq cm. The leverage ratio is 10.*

The valve is required to blow at a pressure of 15 kg/sq cm.

Load on the valve $= \frac{\pi}{4} \times 7^2 \times 15 = 576$ kg.

As the leverage is 10, the weight at the end of the lever will be $\frac{576}{10} = 57{\cdot}6$ kg.

Load on the fulcrum will be $576 - 57{\cdot}6 = 518{\cdot}4$ kg.

Fulcrum of the lever loaded safety valve is subjected to axial tensile load (fig. 12-23).

Minimum cross sectional area required at the bottom of the threads $= \frac{518{\cdot}4}{400} = 1{\cdot}3$ sq cm.

We assume fine threads. We adopt $M16 \times 1{\cdot}5$ which has 1·67 sq cm area at the bottom of the threads. The pitch of the threads is 1·5 mm.

5. *Suggest the suitable size for the pillar of a boiler stop valve, if the maximum compressive load on the spindle of the stop valve is 2,000 kg.*

The pillars are made of mild steel. The compressive load in the spindle is transmitted by the bridge to the two pillars which are subjected to tensile load. The pillars are screwed into the body cover at lower end and the top screwed portion receives the nut. We assume permissible tensile stress intensity as 500 kg/sq cm.

Load on each pillar $= \frac{2000}{2} = 1{,}000$ kg.

Minimum cross sectional area at the bottom of the thread $= \frac{1000}{500} = 2$ sq cm.

We adopt coarse threads on 20 mm diameter, which will provide 2·45 sq cm area at the bottom of the thread.

Note: For rigidity, the diameter on the plain portion of the pillar may be increased to 22 mm. The pillars are provided with collars at top and bottom as shown in figure of boiler stop valve in chapter XIX. The lower collars have two flats machined on them to take a standard spanner.

Exercises:

1. *Calculate the axial tensile load that can be safely applied to a M36 bolt, allowing a safe tensile stress of 550 kg/sq cm.*

Ans. 4,500 *kg.*

2. *A ball bearing company give in their tables the following approximate safe load on coarse threaded bolts:*

Diameter in mm	*Safe load in kg*
6	70
8	125
12	300
14	400

Upon what safe stress are these loads calculated?

Ans. *350 kg/sq cm.*

3. If the eye-bolt not initially stressed is subjected to a direct load of 3,500 kg, determine the nominal diameter of the eye bolt. Assume that the permissible tensile stress intensity in the bolt material is not to exceed 550 kg/sq cm. *Ans.* *M33.*

4. The cover of a hydraulic cylinder is secured by two studs. The internal diameter of the cylinder is 5 cm and internal water pressure 60 kg/sq cm. Taking safe stress of studs as 180 kg/sq cm, give the outside diameter of the nearest practical stud. *Ans.* *M24.*

5. The cover of a main cylinder of a hydraulic machine is subjected to a total load of 62,000 kg. It is closed by means of studs. The number of studs used should be a multiple of four and it is better to use a large number of small studs than a fewer of large size in order to give a more uniform clamping pressure around the circumference. It was decided to use 16 studs. Suggest the suitable size of the studs if the permissible stress is not to exceed 600 kg/sq cm. *Ans.* *M33.*

6. An electric motor weighing 900 kg is to be provided with an eye bolt which is screwed into cast iron frame for lifting purposes.

- (a) *What size of bolt should be used, assuming the permissible tensile stress intensity in the bolt material to be 400 kg/sq cm? Use coarse threads.*
- (b) *How far should the bolt extend into casting?*

Ans. (a) *M20;* (b) *3 cm.*

7. The tension member of a jib crane is subjected to a tensile force of 4,500 kg. This member is round with coarse threads on each end. Determine the diameter of the rod if the permissible stress intensity be 850 kg/sq cm. *Ans.* *M30.*

8. A connecting rod cap is fixed by two studs. The maximum tensile load on the connecting rod is 700 kg. The studs are made of nickel steel having an endurance limit of 3,150 kg/sq cm. Assume the factor of safety to be 4.

As the leverage is 10, the weight at the end of the lever will be $\frac{576}{10} = 57{\cdot}6$ kg.

Load on the fulcrum will be $576 - 57{\cdot}6 = 518{\cdot}4$ kg.

Fulcrum of the lever loaded safety valve is subjected to axial tensile load (fig. 12-23).

Minimum cross sectional area required at the bottom of the threads $= \frac{518{\cdot}4}{400} = 1{\cdot}3$ sq cm.

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The pillars are made of mild steel. The compressive load in the spindle is transmitted by the bridge to the two pillars which are subjected to tensile load. The pillars are screwed into the body cover at lower end and the top screwed portion receives the nut. We assume permissible tensile stress intensity as 500 kg/sq cm.

Load on each pillar $= \frac{2000}{2} = 1{,}000$ kg.

Minimum cross sectional area at the bottom of the thread $= \frac{1000}{500} = 2$ sq cm.

We adopt coarse threads on 20 mm diameter, which will provide 2·45 sq cm area at the bottom of the thread.

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Ans. 4,500 *kg.*

2. *A ball bearing company give in their tables the following approximate safe load on coarse threaded bolts:*

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Upon what safe stress are these loads calculated?

Ans. 350 kg/sq cm.

3. *If the eye-bolt not initially stressed is subjected to a direct load of 3,500 kg, determine the nominal diameter of the eye bolt. Assume that the permissible tensile stress intensity in the bolt material is not to exceed 550 kg/sq cm.* *Ans.* M33.

4. *The cover of a hydraulic cylinder is secured by two studs. The internal diameter of the cylinder is 5 cm and internal water pressure 60 kg/sq cm. Taking safe stress of studs as 180 kg/sq cm, give the outside diameter of the nearest practical stud.* *Ans* M24.

5. *The cover of a main cylinder of a hydraulic machine is subjected to a total load of 62,000 kg. It is closed by means of studs. The number of studs used should be a multiple of four and it is better to use a large number of small studs than a fewer of large size in order to give a more uniform clamping pressure around the circumference. It was decided to use 16 studs. Suggest the suitable size of the studs if the permissible stress is not to exceed 600 kg/sq cm.* *Ans.* M33.

6. *An electric motor weighing 900 kg is to be provided with an eye bolt which is screwed into cast iron frame for lifting purposes.*

(a) *What size of bolt should be used, assuming the permissible tensile stress intensity in the bolt material to be 400 kg/sq cm? Use coarse threads*

(b) *How far should the bolt extend into casting?*

Ans. (a) M20; (b) 3 cm.

7. *The tension member of a jib crane is subjected to a tensile force of 4,500 kg. This member is round with coarse threads on each end. Determine the diameter of the rod if the permissible stress intensity be 850 kg/sq cm.* *Ans.* M30.

8. *A connecting rod cap is fixed by two studs. The maximum tensile load on the connecting rod is 700 kg. The studs are made of nickel steel having an endurance limit of 3,150 kg/sq cm. Assume the factor of safety to be 4.* [illegible]

Design and prepare a dimensioned free hand sketch of the stud with a suitable nut. *Ans.* *M16.*

9. *Find out the diameter of the screwed end of a piston rod for a double acting steam engine having a cylinder of 30 cm diameter and the maximum pressure of the entering steam not exceeding 8·4 kg/sq cm gauge. Permissible stress intensity is not to exceed 420 kg/sq cm.*

Ans. *4·5 cm.*

10. *The face of a D slide valve is 40 cm × 25 cm. The effective steam pressure on the valve is 8 kg/sq cm. If the coefficient of friction between the valve and the cylinder face is 0·2, determine the pull or thrust in the valve rod and design and sketch the valve rod end fitted to the valve.*

Ans. *1,600 kg; 30 mm diameter based on 300 kg/sq cm permissible stress.*

5-13. Stresses due to combined load:

In case of steam engine cylinder cover joints, the bolts are subjected to initial tightening load as well as to steam load. *The resultant load on each bolt will depend upon relative elastic yielding of the bolt and the connected members.* If the connected members are very yielding as compared with the bolt, the resultant load on bolt will be approximately the sum of the external load and initial tension. This will be the case when there will be a soft gasket between the cylinder cover and the flange. *If the bolt is very yielding as compared with the connected members, the resultant load will be either the external load or the initial tension whichever is greater.* This will be the case when there is metal to metal contact between the cylinder flange and cover, or when there is a hard gasket between the connected parts. The metal to metal joint is used for very high pressures. The metal surfaces must be very precisely ground to prevent leakage. This joint is very costly. So when the surfaces are machine finished but not ground, a thin inelastic gasket made of copper or lead sheet or a thin asbestos sheet is introduced which does not change the conditions materially.

The resultant load on a bolt in general is given by the equation

$$P = P_1 + KP_2 \qquad \text{(i)}$$

where P = resultant load on the bolt

P_1 = initial tightening load on the bolt

P_2 = external load on the bolt

Diameter in mm	*Safe load in kg*
6	70
8	125
12	300
14	400

Upon what safe stress are these loads calculated?

Ans. 350 kg/sq cm.

3. *If the eye-bolt not initially stressed is subjected to a direct load of 3,500 kg, determine the nominal diameter of the eye bolt. Assume that the permissible tensile stress intensity in the bolt material is not to exceed 550 kg/sq cm.* *Ans.* M33.

4. *The cover of a hydraulic cylinder is secured by two studs. The internal diameter of the cylinder is 5 cm and internal water pressure 60 kg/sq cm. Taking safe stress of studs as 180 kg/sq cm, give the outside diameter of the nearest practical stud.* *Ans.* M24.

5. *The cover of a main cylinder of a hydraulic machine is subjected to a total load of 62,000 kg. It is closed by means of studs. The number of studs used should be a multiple of four and it is better to use a large number of small studs than a fewer of large size in order to give a more uniform clamping pressure around the circumference. It was decided to use 16 studs. Suggest the suitable size of the studs if the permissible stress is not to exceed 600 kg/sq cm.* *Ans* M33

6. *An electric motor weighing 900 kg is to be provided with an eye bolt which is screwed into cast iron frame for lifting purposes.*

(a) *What size of bolt should be used, assuming the permissible tensile stress intensity in the bolt material to be 400 kg/sq cm? Use coarse threads*

(b) *How far should the bolt extend into casting?*

Ans (a) M20; (b) 3 cm.

7. *The tension member of a jib crane is subjected to a tensile force of 4,500 kg. This member is round with coarse threads on each end. Determine the diameter of the rod if the permissible stress intensity be 850 kg/sq cm.* *Ans.* M30.

8. *A connecting rod cap is fixed by two studs. The maximum tensile load on the connecting rod is 700 kg. The studs are made of nickel steel having an endurance limit of 3,150 kg/sq cm. Assume the factor of safety to be 4.*

Examples:

1. *A cylinder cover of a steam engine is secured by 12 studs. The cylinder is 30 cm diameter and has a steam pressure of 11 kg/sq cm by gauge. Calculate the diameter of studs assuming the permissible stress intensity to be 280 kg/sq cm.*

In connecting cover to the engine cylinder the studs are preferred to bolts because the absence of bolt heads allows the flanges to be made narrower as a result the bending stresses at the root of the flange will be decreased. There will be no interference with the lagging round the cylinder. The depth of the tapped hole should be 1·25*d* for steel casting, 1·5*d* to 1·75*d* for cast iron and 1·75*d* to 2*d* for aluminium.

Let p = pressure of steam in the cylinder
n = number of studs for the joint
f_t = permissible tensile stress intensity in bolt material
D = diameter of the cylinder
d_c = diameter of the bolt at the bottom of the thread.

We have

$$\frac{\pi}{4} d_c^2 \times f_t \times n = \frac{\pi}{4} \times D^2 p$$

or

$$d_c = D \sqrt{\frac{p}{n \times f_t}}.$$

On substitution of values, we get

$$d_c = 30 \sqrt{\frac{11}{12 \times 280}} = 1{\cdot}72 \text{ cm.}$$

From IS: 1862 – 1961; we adopt M18 studs.

2. *The head of a steam engine cylinder 60 cm diameter is subjected to a steam pressure of 13 kg/sq cm. The head is held in place by 16 M39 bolts. A copper gasket is used to make the joint steam tight. Determine the probable stress in the bolt. Take $K = 0{\cdot}25$.*

The resultant load on the bolt $= P_1 + KP_2$
where P_1 = initial tightening load and
P_2 = applied load on the bolt.

Initial tightening load $= 284d = 284 \times 39 = 11{,}076$ kg.

$$\text{Applied load on each bolt} = \frac{\frac{\pi}{4} \times 60^2 \times 13}{16} = 2{,}300 \text{ kg.}$$

$$\text{Resultant load on the bolt} = 11076 + 0{\cdot}25 \times 2300$$
$$= 11{,}652 \text{ kg.}$$

$K = \frac{k_b}{k_b + k_c}$ where k_b is the stiffness constant for the bolt and k_c is the stiffness constant for compressed member.

The following table gives the values of

$K = \frac{k_b}{k_b + k_c}$ used by many designers while making preliminary calculations for bolt design.

Type of joint	K
Metal to metal joint	0·00 to 0·10
Hard copper gasket	0·25 to 0·50
Soft copper gasket	0·50 to 0·75
Soft thick gasket	0·75 to 1·00

From the above discussion it is clear that it is not very easy to calculate the stresses in bolts in a flanged joint. The difficulty will be further increased as the initial tightening load will not be known. In such circumstances the following design procedure can be suggested:

We allow the nominal stress for the bolt material. In order to get the resisting force for all the bolts, we assume that bolts are called upon to resist a force equal to the full internal working pressure acting over the area of the circle just touching the inner side of the bolt holes. *The pitch of the bolt is generally not more than 5 times the diameter of the bolt in order to make the joint steam tight. It is not less than 3 times the diameter of the bolt in order to have sufficient room for tightening the nut by a wrench.* Generally the number of bolts are multiples of 2 or 3.

Sometimes the cylinder cover is spigoted into the cylinder and if the fit is a good one, we may assume that bolts are called upon to resist a force due to steam or gas pressure acting only on the cross sectional area of the cylinder.

In important joints, we should design the bolt as described in the earlier part of the article. In such design (accurate design) the higher permissible stress may be allowed depending on the quality of the steel.

Fig. 5-6 shows the Soderberg line. The sum of the average and variable stress is shown at point *A* of the fig. 5-6. Since the point *A* falls below the working stress line, the bolt is safely loaded.

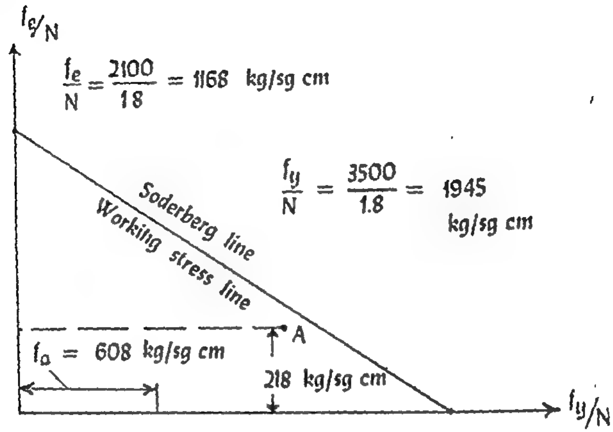


FIG. 5-6

It should be noted that the stress concentration factor is applied only to a variable stress components and not to the average stress which may be considered as static.

Design note:

Stress concentration at the root of a standard coarse thread is very high. Photoelastic tests indicate stress concentration factors as high as 5·6. This may not be too serious for bolts made of ductile material when subjected to static loads. However the stress concentration factor has been found to reduce the endurance limit of standard coarse threads by factors ranging from 2 to 4. Therefore the fluctuating stress in a threaded bolt must be multiplied by a suitable stress concentration factor.

4. *The cylinder head of a 25 cm × 45 cm Freon compressor is attached by 10 studs made of C 1118 as rolled, the gas pressure being 14 kg/sq cm by gauge. The initial tension in the bolts, assumed to be equally loaded such that a cylinder pressure of 28 kg/sq cm by gauge is required for the joint to be on the point of opening. The bolted parts are cast steel and for the first calculations, it will be satisfactory to assume the equivalent diameter of the compressed parts to be twice the bolt size. For a design factor of 2, suggest the suitable size of the stud in accordance with Soderberg criterion.*

From table, the cross sectional area at the bottom of threads for $M39$ bolt $= 9{\cdot}76$ sq cm.

$\therefore$ Probable stress in the bolt $= \dfrac{11652}{9{\cdot}76} = 1{,}193$ kg/sq cm.

3. The external load applied to a bolt fluctuates between zero and 800 kg. The ratio of the cm deflection per kg of load for the bolt to that for the members is 3. The endurance limit of the bolt material in reversed axial loading is 2,100 kg/sq cm and the yield point is 3,500 kg/sq cm. The root area of the thread is 1·15 sq cm. Assume a stress concentration factor of 2·5 and a factor of safety 1·8 based on the yield strength of the material. The stress concentration factor takes into account the effect of surface and size.

Determine the minimum initial tightening load that must be applied to prevent separation. Plot the Soderberg working stress diagram for the material and determine if the bolt is safely loaded based on an initial load as determined earlier.

The fatigue strength of a bolt depends upon the maximum and minimum loads to which it is subjected. When the external load P_2 is fluctuating, the initial tightening load P_1 should be sufficient to prevent separation with a reasonable factor of safety. Separation will be pending when the resultant load P is equal to external bolt load P_2. Then

$$P = P_1 + K P_2$$

or $P_1 \geq P_2\ (1 - K)$ to prevent separation. When no separation occurs, the load may vary between P_1 and $P_1 + K P_2$. The initial tightening load must be greater than or equal to

$800\ (1 - \frac{1}{4}) = 600$ kg.

Maximum resultant load $= 600 + \dfrac{800}{4} = 800$ kg.

Minimum resultant load $= 600 + 0 = 600$ kg.

Mean load $= \dfrac{800 + 600}{2} = 700$ kg.

Variable load $= 800 - 600 = 200$ kg.

Average stress $= \dfrac{700}{1{\cdot}15} = 608$ kg/sq cm.

Variable stress component $= \dfrac{100}{1{\cdot}15} = 87$ kg/sq cm.

$K \times f_r = 2{\cdot}5 \times 87 = 218$ kg/sq cm.

75 cm pitch circle diameter. Determine the diameter of the bolt allowing 350 kg/sq cm as the permissible tensile stress intensity in the bolt material.

Ans. M39 bolts.

3. *The cylinder head of an air compressor is held in place by steel studs. The cylinder bore is 40 cm and the maximum air pressure is 9 kg/sq cm by gauge. The head to cylinder contact surfaces are ground together, no packing being necessary. Determine the number and sizes of studs to be used if 1,000 kg/sq cm is the permissible tensile stress intensity in bolt material.* *Ans. M16; 12 studs.*

4. *The cylinder of a portable hydraulic riveter is 22 cm in diameter. The pressure of the fluid is 140 kg/sq cm by gauge. Determine the suitable thickness of the cylinder wall assuming that the maximum permissible stress is not to exceed 1,050 kg/sq cm. Assuming 16 studs have been used to connect the cover to the cylinder, determine the size of studs. Allowable stress intensity in the bolt material is limited to 630 kg/sq cm.*

Ans. 20 mm; M39 studs.

5. *The diameter of the air cylinder of an air operated arbor press is 20 cm in diameter and the cylinder assembly is held together by six tension bolts which are of the length of the cylinder. The maximum operating air pressure in the cylinder is 9 kg/sq cm and because of a back load on the piston rod 4 kg/sq cm pressure must be maintained when the press is off. Experience with previous designs indicates that the gasket must be pre-loaded with 2,000 kg force to prevent air leakage. Suggest the suitable diameter for the tension bolts which have rolled threads and are made of steel having 5,400 kg/sq cm as the yield strength. Assume the factor of operated safety to be 2·8 and* $k_c = 3k_b$. *Ans. M8.*

6. *A cast iron cylinder head is fastened to a 500 mm inside diameter cylinder by means of 8 bolts. Consider the bolt to be extremely flexible as compared to the bolted parts. For an internal pressure of 14 kg/sq cm, what is the axial force on each bolt if the bolts were tightened just enough to prevent the joint opening under a pressure of 20 kg/sq cm?*

Ans. 4,900 kg.

7. *A through bolt is used to fasten two plates with a gasket between the two plates. It is known that the ratio of deflection of the bolt per unit load to the deflection of the bolted part per unit load is 1. What percentage of the applied load to the plates will be added to the initial tightening load of the bolt? Assume that the plates will not separate under load.*

Ans. 75% of the applied load goes into bolt.

The diameter of the compressor cylinder is 25 cm and the stroke 45 cm. For AISI C 1118, ultimate strength is 5,200 kg/sq cm and yield strength 3,200 kg/sq cm.

$$\text{Force on head} = \frac{\pi \times 25^2}{4} \times 14 = 6{,}900 \text{ kg.}$$

$$\text{Force per stud} = \frac{6900}{10} = 690 \text{ kg.}$$

$k_e = 3\ k_b$

Initial load in stud $= Q \times P_2 \times \dfrac{k_e}{k_b + k_e}$ where Q is the factor *that depends upon the pressure required for the joint to open out.*

$$P_1 = \frac{28}{14} \times 690 \times \frac{3\,k_b}{3k_b + k_b} = 690 \times 2 \times \tfrac{3}{4} = 1{,}035 \text{ kg.}$$

Minimum load $= 1{,}035$ kg.

Maximum load $= 1035 + \dfrac{690}{4} = 1{,}515$ kg

Average load $= \dfrac{1035 + 1515}{2} = 1{,}275$ kg.

Variable component of load $= 240$ kg.

We assume a stress concentration factor of 2·8 and the value of endurance strength for reversed axial loading as half the value of ultimate strength. If A sq cm be the area at the bottom of the thread, according to Soderberg criterion, we get,

$$\frac{1}{2} = \left[\frac{1275}{A \times 3200} + \frac{2{\cdot}8 \times 240}{0{\cdot}5 \times 5200 \times A}\right]$$

or $\quad A = 2\left[\dfrac{1275}{3200} + \dfrac{2{\cdot}8 \times 240}{2600}\right] = 1{\cdot}32$ sq cm.

We adopt coarse threads. From tables we adopt M 16 studs

Exercises:

1. *Steam engine cylinder has 30 cm bore. The maximum steam pressure is 9 kg/sq cm by gauge. Assume safe stress in studs to be 220 kg/sq cm, determine the diameter of studs and also their number.*

Ans. M20; 12 studs.

2. *A 60 cm diameter pressure vessel is to be used for the preservative treatment of lumber. The treating cycle employs pressure upto 12 kg/sq cm by gauge. The vessel is to use a head fastened with 12 bolts on*

meter which is less than the root of the threads. Using a factor of safety equal to 2, what mechanical property of the material and what value of that property would you specify?

5-15. Screwed boiler stays:

Most boilers have one or more large flat plates. When flat plates are subjected to pressure, they will try to bulge out. The bulging tendency of flat plates in boilers is prevented by members known as stays.

The kinds of stays are: (a) gusset stays, (b) diagonal stays, (c) longitudinal stays, (d) girder stays and (e) stay bolts.

Here, we are concerned with screwed boiler stays of the types longitudinal through stays and stay bolts. Through stays run right through the boiler and are liable to sag in the middle. As a result their use in a modern Lancashire boiler fades away and it is confined to more compact cylindrical marine and loco-type boilers. The main difference between through stays and stay bolts is in length only. When through stays are short, they are called stay bolts. They are commonly found in water legs of locomotive boilers.

Let A = area of the plate supported by the stay

p = pressure of steam acting on the plate

f_t = safe tensile stress allowed in stay.

Minimum cross sectional area at the bottom of the thread of the screwed stay $= \dfrac{A \times p}{f_t}$ ....(i)

When core area of the screwed stay is known, the diameter for the stay rod can be specified.

The support afforded by screwed through stay is spread over an increased area by using a thick washer under the outer nut. This thick washer is riveted to the boiler end plate.

According to I.B.R. the working pressure for the screwed stays is to be calculated from the appropriate one of the following two formulas

$$\text{W.P.} = \frac{C}{A}\left(D - \frac{1 \cdot 28}{N}\right)^2 \quad \text{..............................(ii)}$$

where

W.P. = the working pressure in lb per square inch

D = diameter of stays over threads in inches

D_1 = diameter of body of stay at its smallest part in inches

N = number of threads of stay per inch

A = area in sq in. supported by one stay

C = 7,100 f[illegible] or special wrought iron screw stays to combustic[illegible] fire boxes

8. The external load applied to a bolted joint fluctuates between zero and 800 kg. The bolt is tightened with an initial load of 650 kg. The root area of the bolt is 1·15 sq cm. The ratio of deflection per unit load for the bolt to that for members is 3.

Determine the maximum and minimum bolt loads. Determine the average stress and the variable stress, assuming a stress concentration factor of 3 which includes surface and size effects. Plot the Soderberg working stress diagram and determine if the bolt is safely loaded for a factor of safety of 1·8. The material has a yield point of 2,800 kg/sq cm, and an endurance limit in reversed axial loading of 1,400 kg/sq cm

Ans. 850 kg, 650 kg; 655 kg/sq cm; 87 kg/sq cm. The bolt is safely loaded.

9. A cast iron Diesel Engine cylinder head is held on by 8 stud bolts with coarse threads. These bolts are made of AISI 3140 steel. Assume that the compressed material has an equivalent diameter twice the bolt size. The maximum cylinder pressure is 50 kg/sq cm and the bore of the engine is 20 cm. The initial load in the bolt is such that a cylinder pressure of 100 kg/sq cm brings the joint to opening For a design factor of 2, suggest the suitable size of the bolt according to Soderberg criterion. Take the stress concentration factor as 3 8. Endurance strength in reversed axial loading 5,200 kg/sq cm and yield strength as 9,400 kg/sq cm.

$k_c = 1{\cdot}15\ k_b$.　　*Ans. M16*

5-14. Bolts of uniform strength:

Sometimes the bolts are subjected to shock or impact loads as in connecting rod bolts, fastening of power hammers and presses In such cases the bolts should be designed to absorb impact energy as well as to resist torque.

If a bolt of the useful form having a full sized shank and threaded end is used to support an impact load, the larger part of the energy will be absorbed in the threaded portion as the stresses are higher in the threaded portion and the energy absorbed per cubic centimetre is proportional to the square of the stress. If the shank of the bolt is reduced in diameter as shown in fig. 5-7(*a*) shank of the bolt will undergo a higher stress and hence will absorb a greater proportion of the energy thus relieving the material at the section near the thread. A reduction in the shank area corresponding to the thread root area will result in design of bolt of uniform strength. Instead of turning down the body, the area of

stay if the permissible tensile stress intensity in the stay due to steam pressure alone is limited to 500 kg/sq cm. Show by sketches how the ends of the stays are connected to the end plates of the boiler. Ans. [illegible]

5-16. Bolts subject to shear:

Screws subjected to shearing stress should be avoided as far as possible by the use of dowel pins fitted accurately into place after the screws have been fitted. Where it is not possible to use dowel pins, bolts should be fitted tightly into reamed holes.

A bolted joint to withstand shear is designed by the formula

$$A = \frac{P}{r \cdot f_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (?)$$

where P = total load on bolts

r = number of bolts

f_s = working shear stress in bolts

A = area of the cross section of the body of the bolt.

Having determined the area, the diameter of the bolt can be obtained. *The plane of shear should never be across threaded portion of the shank.* In the best design, the diameter of the shank is made slightly larger than the threaded part of the bolt to avoid injury to the thread. When bolt is subjected to both tension and shear as in coupling bearing, the diameter of the shank is approximated from shear load and threaded part from tension load. A diameter slightly larger than that required for either shear or tension can be assumed and stress due to combined shear and tension should be checked by the formula

$$f_{s\,max} = \frac{1}{2}\sqrt{f_t^2 + 4f_s^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (?)$$

$$f_{t\,max} = \frac{1}{2}f_t + \frac{1}{2}\sqrt{f_t^2 + 4f_s^2} \quad \ldots\ldots (?)$$

Maximum values of shear stress [illegible] not exceed the permissible limits.

Example

[illegible] *An overhung gear mounted* [illegible] *wheel is connected to the flange by* [illegible] *diameter of 20 cm. If the* [illegible] *500 kg/sq cm, suggest* [illegible]

Torque to be transmitted [illegible]

$C = 8{,}640$ for steel longitudinal or cross stays fitted with nuts
$C = 4{,}700$ for copper screw stays to fire boxes

Where stays are made with enlarged ends and the body of the stay is smaller in diameter than at the bottom of the thread, the working pressure shall be calculated from the formula

$$\text{W.P.} = \frac{CD_1^2}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Examples:

1. Find the diameter of copper screwed stays in a boiler. Each stay supports an area equal to a rectangle of 15 × 10 cm. The pressure of steam is 11 kg/sq cm by gauge. The permissible tensile stress intensity in the stay material is limited to 280 kg/sq cm.

The area supported by each stay = 15 × 10 = 150 sq cm.

The load on each stay = 150 × 11 = 1,650 kg.

$$\text{Minimum area at the bottom of the thread} = \frac{1650}{280}$$
$$= 5.9 \text{ sq cm}$$

From IS: 1362-1962, for coarse series, we adopt M33 diameter stay.

2. The longitudinal bar stays of a short boiler are pitched at 35 cm horizontally and vertically. The steam pressure is 7 kg/sq cm by gauge. Find the diameter of the stay, the material being mild steel having the safe tensile stress as 500 kg/sq cm.

Area of plate supported per stay = 35 × 35 = 1,225 sq cm.

Load on longitudinal bar stay = 1225 × 7 = 8,757 kg

$$\text{Minimum area at the bottom of the thread} = \frac{8757}{500}$$
$$= 17.5 \text{ sq cm.}$$

From table, we adopt 6 cm diameter stays.

Exercises:

1. A flat surface in a marine boiler is constructed of 2 cm plates with wrought iron screwed stays 15 cm apart horizontally and vertically. The pressure of steam is 5 kg/sq cm. If the safe stress for the screw is 280 kg/sq cm, suggest the suitable diameter for the stay. Ans. M27.

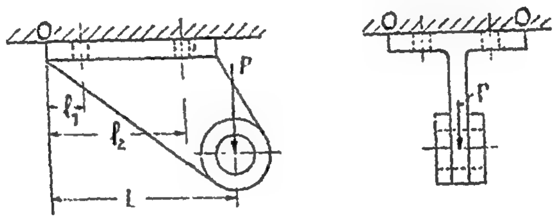
2. The longitudinal stay of a boiler 5 metre long supports an area 40 cm × 40 cm on each end plate. The working pressure of the boiler is 10 kg/sq cm by gauge. Determine the diameter of the longitudinal

to the axis of the bolt, shear as well as tensile stresses are induced. Shear stresses can be avoided by fitting dowel pins correctly.

(a) ***When load is parallel to the bolt axis:—***

(i) **Rectangular base:**

Bolts holding a bracket as shown in fig. 5-8 are not efficient. The load tends to rotate the bracket about the edge OO, thus stretching each bolt by the amount depending upon its distance from the tilting edge. All the bolts are not equally stressed. However, for convenience and economy, all the bolts are made of the same size.



Fastening of a bracket

FIG. 5-8

As the flange is heavy, it may be considered to be a rigid body. In order to determine the force in any bolt, the following procedure is followed:

Let W be the load in a bolt, situated at a unit distance from the tilting edge due to the turning tendency of the bracket.

Load in a bolt at a distance l_1 from the tilting edge will be Wl_1. The moment of this load about the tilting edge will be Wl_1^2. There are two bolts situated at a distance l_1 from the tilting edge. Thus the resisting moment due to these two bolts will be $2Wl_1^2$. Similarly the resisting moment due to two bolts situated at a distance l_2 from the tilting edge will be $2Wl_2^2$. By equating the moment of the screw loads about the tilting axis to the moment of load P about the same axis, we have

$$PL = 2W\ (l_1^2 + l_2^2) \qquad \text{(i)}$$

From equation (i), W, the load in a bolt situated at a unit distance can be calculated.

$$\therefore\quad W = \frac{PL}{2(l_1^2 + l_1^2)} \qquad \text{(ii)}$$

Pitch circle radius $= \frac{20}{2} = 10$ cm.

Shearing load on all the bolts $= \frac{17900}{10} = 1{,}790$ kg.

Shear load on each bolt $= \frac{1790}{4} = 447$ kg.

Area of cross section of the body of the bolt $= \frac{447}{300}$

$= 1{\cdot}49$ sq cm.

The suitable diameter of the bolt will be 1·4 cm.

Exercises:

1. A single plate clutch transmits 20 metric h.p. at 1,200 r.p.m. The driving unit is connected to the driving shaft by means of four medium carbon steel bolts placed on a 12 cm pitch circle diameter. Determine the suitable diameter of the bolt if the permissible shear stress is limited to 140 kg/sq cm.

Ans M18.

2. What is the maximum horse power that can be transmitted safely by a flange coupling with 6-1 cm diameter bolts on a 8 cm diameter bolt circle when revolving at 1,000 r p.m ? Use a design stress of 210 kg/sq cm.

Ans. 55 5 h p

3. A mild steel bolt M24 is subjected to a direct pull of 1,200 kg and to a shearing force of 1,000 kg Find the maximum tensile and shearing stresses for the bolt.

Ans. 500 kg/sq cm; 331 kg/sq cm.

4. A shaft transmits 125 h.p. at a speed of 270 r p m. The ends are connected by a flange coupling with a flange thickness of 2 cm and bolts on 20 cm bolt circle. Make calculations to determine the number of bolts needed and to check the flange thickness. Use a design stress of 240 kg/sq cm for bolts in shear and 1,000 kg/sq cm for the flange in bearing

Ans 6 bolts.

5-17. Bolts under eccentric loading:

In many engineering applications the bolts are subjected to an eccentric loading. There are two possible cases: (i) the direction of the load is parallel to the axis of the bolt (ii) the direction of the load is perpendicular to the axis of the bolt. Bolts are subjected to tensile loading only when the direction of the load is parallel to the axis of the bolt. When the load is perpendicular

$l_2 = a - b\cos(\theta + 60)$ $\qquad$ $l_5 = a + b\cos(\theta + 60)$
$l_3 = a + b\cos(60 - \theta)$ $\qquad$ $l_6 = a - b\cos(60 - \theta)$

Substituting these values in (iii), we get

$$W = \frac{PL}{(6a^2 + 3b^2)} \quad \text{.......(iv)}$$

$$\text{Load in a bolt situated at } 1 = \frac{PL(a - b\cos\theta)}{(6a^2 + 3b^2)} \quad \text{.......(v)}$$

Now to determine the maximum load for a given moment PL and dimensions a and b, it is evident that this occurs when $\cos\theta$ is minimum i.e., $\cos\theta = -1$, which is the case when $\theta = 180°$.

$$\text{Maximum load in a bolt will be } \frac{PL\ (a + b)}{3(2a^2 + b^2)} \quad \text{.......(vi)}$$

In general if we have n bolts equally spaced along a pitch circle radius b, maximum load in a bolt will be

$$\frac{2PL\ (a + b)}{n\ (2a^2 + b^2)} \quad \text{.......(vii)}$$

The maximum value should be used if the direction of load can change with relation to the bolts as in the case of a pillar crane. If the direction of load is fixed, the maximum load on the bolt can be reduced by positioning the bolts so that two of them will be equally stressed as shown in fig. 5-10.

The maximum load for the bolt in such a case will be

$$\frac{2PL}{n} \times \frac{(a + b\cos\frac{180}{n})}{(2a^2 + b^2)} \quad \text{.......(viii)}$$

When the maximum load for a bolt is known, the size of the bolt can be obtained.

(b) When load is perpendicular to the axis of the bolt:

Fig. 5-11 shows the connections by means of bolts in which the direction of the load is perpendicular to the bolt axis. Fig. 5-11(a) represents a solid cast iron flanged bearing frequently found on heavy machine tools. Due to power transmitted the bearing is subjected to a load P which tends to produce shearing stress in each of the bolts. If the bolts are fitted in reamed holes, they are assumed to be stressed equally. If rough bolts are used in clearance holes, the shear load will be distributed between two bolts. Generally dowel pins are used to take shear load. In fig. 5-11(a), two dowel pins are shown. Hence bolts should be designed to carry tensile load only.

When load in a bolt situated at a unit distance is known, the loads in individual bolts can be obtained.

The load in bolts situated at a distance l_1 from the tilting edge will be Wl_1 and in bolts situated at a distance l_2 from the tilting edge will be Wl_2.

In general, the most heavily loaded bolts are those at the greatest distance from the edge of turning. When the maximum load in a bolt is known, the size of a bolt for any allowable stress can be obtained. In our case the maximum load for a bolt will be

$$\frac{PL\ l_2}{2(l_1^2 + l_2^2)}.$$

(ii) Circular base:

Columns or machine members are frequently made with a circular base similar to that shown in fig. 5-9 in which a represents

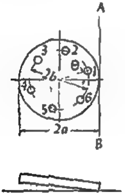


Fig. 5-9

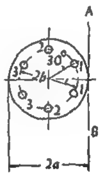


Fig. 5-10

the radius of the column flange and b the radius of the bolt pitch circle. To simplify the discussion, a round flange bearing with six bolts will be analysed. We number the bolts as shown. Adopting a notation similar to that used with rectangular base, and by equating the external moment PL by the sum of the moments of the bolt loads about the axis AB, we get

$$PL = W(l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_5^2 + l_6^2)$$

or $$W = \frac{PL}{(l_1^2 + l_2^2 + l_3^2 + l_4^2 + l_5^2 + l_6^2)} \quad \text{...............(iii)}$$

From the geometry of the figure

$l_1 = a - b\cos\theta$ $\qquad\qquad$ $l_4 = a + b\cos\theta$

When equivalent loads are known, the size of the bolt can be specified when allowable stresses are known.

The projecting lip is provided for rectangular bases to take the direct shear as shown in fig. 5-11(*b*).

The methods discused in article 4-10 in connection with rivets may be applied to eccentrically loaded bolted assemblies also.

Examples:

1. The pillar crane shown in fig. 5-12 is fastened to the foundation by 12 bolts spaced equally on a bolt circle of 180 cm diameter. Determine the size of the bolt when a load of 4,000 kg acts at a radius of 800 cm. The diameter of the pillar flange is 210 cm. The allowable stress intensity for the bolt material is 900 kg/sq cm.

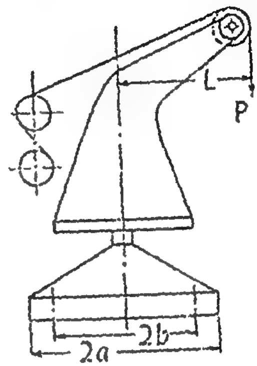


FIG. 5-12

The maximum load on a bolt of a pillar crane foundation will be

$$T = \frac{2P(L-a)}{n} \times \frac{(a+b)}{(2a^2+b^2)}$$

where

T = maximum load on a bolt
P = maximum value of eccentric load
L = the eccentricity of the load
a = radius of the pillar flange
n = number of foundation bolts
b = radius of bolt circle.

We have $(L-a) = 800 - 105 = 695$ cm.

On substitution of values, we get

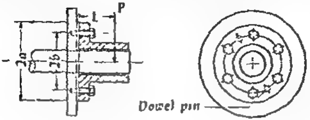
$$T = \frac{2 \times 4000 \times 695}{12} \times \frac{(105+90)}{(2 \times 105^2 + 90^2)} = 6{,}500 \text{ kg.}$$

Minimum cross sectional area required at the bottom of the thread $= \dfrac{6500}{900} = 7{\cdot}22$ sq cm.

From IS: 1362-1962, we adopt *M* 36.

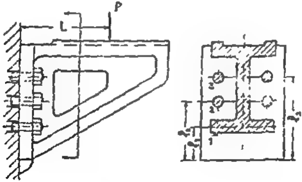
2. What load is the crane runway bracket shown in fig. 5-13 capable of supporting, if the stresses produced in the two 25 mm bolts used in fastening the bracket to the roof truss is limited to 700 kg/sq cm?

Due to eccentric load P the bearing is subjected to an external moment PL, which must be balanced by an equal moment due to the tension set up in bolts The maximum load on each bolt can be obtained by the procedure explained earlier.



Fastening of a circular base bracket and a flanged bearing
FIG 5-11(a)

The diameter of the bolt is assumed and direct shear stress is calculated if no dowel pins are ued. Similarly, we calculate the maximum tensile load coming upon the bolt which is farthest from the tilting edge. Two stresses are combined and the resultant stress should not exceed the permissible limit.



Fastening of a rectangular base bracket and a flanged bearing
FIG. 5-11(b)

The other procedure will be to determine the equivalent tensile load $T_E = \frac{1}{2} \{T + \sqrt{T^2 + 4Q^2}\}$ ....(ix)
or equivalent shear load $Q_E = \frac{1}{2} \{\sqrt{T^2 + 4Q^2}\}$ ....(x)

where T = maximum tensile load coming on any bolt
Q = average shear load coming on a bolt.

There are two bolts; one at 3·7 cm from the tilting edge and the other at 33·7 cm from the tilting edge. The distance between the tilting edge and the line of action of the load is 45 cm. Let P be the maximum load that a bracket can support. *We assume that the bolt nearer to the tilting edge carries no load, the assumption being on the safer side.*

The probable cross sectional area at the bottom of 25 mm bolt having coarse threads will be 3·54 sq cm. As the permissible stress in the bolt material is limited to 700 kg/sq cm, the maximum load in the bolt should not exceed, $3{\cdot}54 \times 700 = 2{,}478$ kg. By equating the external tilting moment to the moment due to bolt load, we get

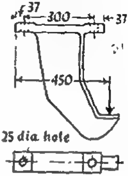


FIG. 5-13

$$2478 \times 33{\cdot}7 = P \times 45$$

or $$P = \frac{2478 \times 33{\cdot}7}{45} = 1{,}860 \text{ kg}$$

3. *A bearing similar to one shown in fig. 5-11(a) is fastened to the frame of a machine by means of four cap screws spaced equally on 25 cm pitch circle. The bearing flange is 35 cm in diameter. The load of 3,500 kg is located at 12·5 cm from the frame. Assuming that the cap screws are relieved from all shearing action by the use of two 20 mm dowel pins, suggest the suitable diameter for the cap screw, assuming 550 kg/sq cm as the allowable stress intensity in the material of cap screw.*

We assume that the direction of load is fixed and the bolts are so located that two of them will be equally stressed

With usual notation the maximum load on a cap screw is given by

$$T = \frac{2PL}{n}\left\{\frac{a + b\cos 180/n}{2a^2 + b^2}\right\}.$$

On substitution of values, we get

$$T = \frac{2 \times 3500 \times 12{\cdot}5}{4}\left\{\frac{17{\cdot}5 + 12{\cdot}5\cos 45^\circ}{2 \times 17{\cdot}5^2 + 12{\cdot}5^2}\right\} = 1{,}250 \text{ kg}.$$

As the permissible stress intensity is not to exceed 550 kg/sq cm, the minimum cross sectional area at the bottom of the thread

each bolt and the proper diameter for these bolts. Permissible tensile stress intensity in the bolts is 850 kg/sq cm. *Ans.* M12.

5. *The screw of a shaft straightener as shown in fig. 5-15 is subjected to a maximum load of 2,000 kg. Determine the size of the two bolts required to fasten the straightener to the base, assuming the allowable stress to be 550 kg/sq cm. Neglect tightening up stresses.* *Ans.* M 30.

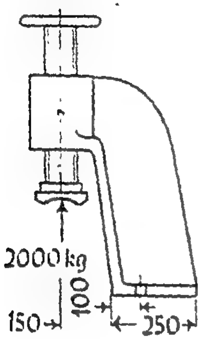


Fig. 5-15

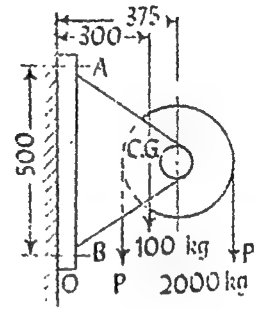


Fig. 5-16

6. *A pulley bracket is supported by four bolts two at A and two at B as shown in fig. 5-16. The weight of the pulley and the bracket equals 100 kg and the load on the rope, P, is 2,000 kg. Assuming that the bracket is held against the wall and prevented from tipping about O by two bolts at A (i.e. neglecting the bolts at B) and using an allowable tensile stress in the bolts as 400 kg/sq cm, determine the size of bolts required.* *Ans.* M27.

7. *In the machine frame shown in fig. 5-17(a), the guide post is bolted to the base by means of eight bolts, four on each side. Determine the proper size of bolts assuming 700 kg/sq cm as the permissible tensile stress intensity in the bolt material.*

Ans. 30 *mm.*

8. *Fig. 5-17(b) shows the column of a press secured to the bottom of the block by 8 bolts. The load P = 5,000 kg lies in the plane of the radial arm but is inclined at an angle of 30° to the vertical. Suggest the suitable diameter of the bolt, if the maximum permissible tensile and shear stresses are limited to 850 and 650 kg/sq cm respectively.*

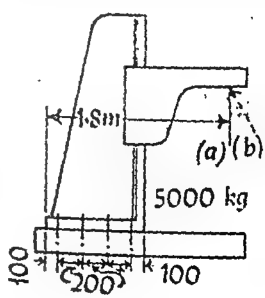


Fig. 5-17(a) and (b)

The permissible tensile stress intensity is 850 kg/sq cm.

$\therefore$ Minimum cross sectional area at the bottom of the thread

$$= \frac{2686}{850} = 3{\cdot}16 \text{ sq cm.}$$

From metric table, we adopt M 24 bolt having 3·53 sq cm area at the bottom of the thread

In fact area of 3·16 sq cm is required for the shank of the bolt and not at the bottom of the threads as the core section is not subjected to shear load.

Exercises:

1. *The pillar crane shown in fig. 5-12 is fastened to the foundation by 8 bolts spaced equally on a bolt circle, the diameter of which is 2b. Derive an expression for the maximum load coming upon any bolt, assuming the diameter of the pillar flange to be 2a, in terms of load P and eccentricity L. Determine the size of the bolts having given the following data:*

$P = 5{,}000$ kg; $L = 500$ cm, $2a = 180$ cm, $2b = 160$ cm. Allowable tensile stress intensity is 700 kg/sq cm

Ans $\dfrac{P(L-a)}{4}\left(\dfrac{a+b}{2a^2+b^2}\right)$, *M30.*

2. *The crane runway bracket shown in fig 5-13 carries a maximum load of 1,500 kg. If the bracket is connected to the roof truss by two coarse threaded bolts and if the permissible tensile stress intensity in the bolt material is limited to 630 kg sq cm, determine the nominal diameter of the bolt.* *Ans M27.*

3. *A bearing similar to one shown in fig 5-11(a) is fastened to the frame by 6 bolts spaced equally on a 25 cm bolt circle. The bearing flange diameter is 30 cm and the load of 4,200 kg is applied at 27 5 cm from the frame. Determine the size of the bolts and tensile, shear and resultant stresses produced on each of the bolts (a) with two bolts located in the vertical plane of symmetry of the bearing and (b) with two bolts located in the horizontal plane of symmetry of the bearing. Assume the permissible tensile stress intensity in the bolt material to be 850 kg/sq cm.* *Ans. M20.*

4. *The inside diameter of the stator of a 100 KW, 960 r.p.m. motor is 100 cm. Starting torque may be assumed to be 200 per cent of the running torque. The maximum belt pull on the motor shaft is 900 kg. The shaft centre is 62·5 cm above the base line. There are four foundation bolts spaced 65 cm on centres axially and 110 cm on centres normal to the axis. The width of the base is 125 cm. Determine the load on*

If D cm be the diameter of the shaft on which the set screw of diameter d is pressed, then

$d = 0{\cdot}125D + 0{\cdot}8$ cm ....(i)

The tangential force at the surface of a shaft in kg is given by

$F = 132d^{2\cdot3}$ kg ....(ii)

If T be the torque transmitted by a set screw in kg cm, then

$T = \dfrac{FD}{2}$ kg cm ....(iii)

Metric h.p. transmitted $= \dfrac{TN}{71620}$ ....(iv)

where N is the speed of the shaft in r.p.m.

Example:

1. *A 15 cm pulley is fastened to a 3 cm shaft by a set screw. If a net tangential force of 30 kg is applied to the surface of the pulley, what size screw should be used when the load is steady?*

Force on set screw $= 30 \times \dfrac{15}{3} = 150$ kg. Let us adopt a design factor of 3. Holding force required $= 150 \times 3 = 450$ kg.

If d cm be the diameter of the set screw, then $450 = 132d^{2\cdot3}$

or $d = \sqrt[2.3]{\dfrac{450}{132}} = 1{\cdot}157$ cm; we adopt 12 mm.

Exercises:

1. *A lever 40 cm long is fixed to the spindle 50 mm diameter by means of a set screw. Suggest the suitable size of the set screw for a design factor of 3.20 kg will be the maximum force that will be applied at the end of a lever.* *Ans.* 2·5 *cm.*

2. *A 30 cm gear is mounted on a 5 cm shaft and is held in place by a 12 mm set screw. For a design factor of 4, what would be the tangential load that could be applied to the teeth?* *Ans.* 8·3 *kg.*

EXAMPLES V

1. For ease of transportation, heavy flywheels are cast in pieces and assembled at the site. In such a type of flywheel the cross sectional area of the rim is 70 sq cm and the maximum hoop stress in the rim is limited to 110 kg/sq cm.

9. Calculate the size of the bolts for the three hole bracket as shown in fig. 5-18. Use coarse threads and a design stress of 270 kg/sq cm. *Ans. M20.*

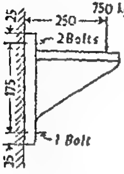


Fig. 5-18

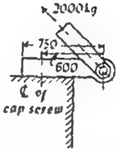


Fig. 5-19

10. Fig. 5-19 shows a cap screw subjected to a load of 2,000 kg inclined at an angle 45°. Determine the size of the screw if the tensile stress intensity in the material is limited to 560 kg/sq cm and shear stress intensity in the material to 450 kg/sq cm *Ans. 50 mm.*

5-18. Design of a nut:

The calculation of the stresses in bending, shear and compression that are set up in the threads of a bolt and nut of mild steel are not required if the effective height of a nut is made equal to the nominal diameter of the bolt When a nut is made of weaker material than the bolt, the nut should have larger thicknesses such as $1\frac{1}{2}d$ for gun metal, $2d$ for cast iron and $2\frac{1}{2}d$ for aluminium alloys

Where cast iron or aluminium is used, V threads are permissible for permanent fastenings because frequent screwing and unscrewing may damage the threads. When bolts are to be screwed and unscrewed frequently, screwed in steel bushing for cast iron and cast in bronze or monel metal insert should be used for aluminium and it should be drilled and tapped in place.

5-19. Power transmitting capacity of set screws:

They are used only where the force transmitted is low. Mr. B.H.D. Pinckney gives the following expression for the safe holding power of a cup or flat pointed set screws in terms of tangential force at the surface of the shaft.

(a) Find the distance of the load from the centre of the pillar in the position 'XX' so that a load of 6 tonnes may be lifted without stressing the bolt in excess of 600 kg/sq cm. (b) Determine, also, the stress if the load is applied on a line YY of the boom, at the same distance as in (a). See fig. 5-22.

7. The pillar crane shown in fig. 5-23 is fastened to the foundation by 12 bolts spaced equally on a bolt circle of diameter $2b$. Derive an expression for the maximum load coming upon any one bolt, assuming the diameter of the pillar flange as $2a$.

Determine the size of the foundation bolts using the following data:

Load $P = 8$ tonnes; radius $L = 5$ metre; diameter of pillar flange circle = 210 cm; bolt circle diameter = 180 cm and permissible tensile stress on bolts = 4·5 kg/sq mm.

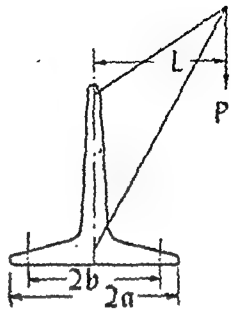


FIG. 5-23

8. Fig. 5-24 gives incomplete details of a cast iron hanger bracket. The journal diameter is 55 mm and it is 125 mm long. If the weight of the bracket may be neglected, find (a) the maximum value of the tensile stress in section AA. (b) the size of the four supporting bolts if the safe stress is 630 kg/sq cm.

The loading may be taken as vertical and equal to 20 kg/sq cm of bearing area.

Complete the design, including the bearing and give sketches showing the main dimensions. Ans. 115 kg/sq cm; M18

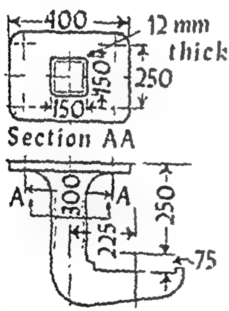


FIG. 5-24

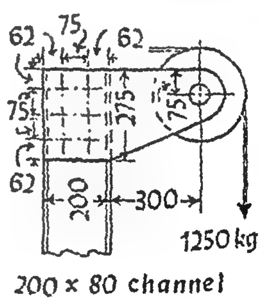


FIG. 5-25

9. Discuss the uses and relative merits of bolting, riveting and welding as means of fastening. Explain why in some bolts part of the shank is turned down to the core diameter of the threads.

The pulley bracket shown in fig. 5-25 is to be fastened to the back of the vertical channel stanchion by six bolts in single shear, spaced as shown. A horizontal rope passes over the pulley and supports a counter balance weight weighing 1,250 kg. The rope force may be assumed to act in the plane of the bolted joint

Suggest the suitable size of the bolts, two in number, required to join two sections at the rim if the tensile stress in the rim is limited to 500 kg/sq cm.

Ans. M36.

2. A symmetrical cast iron balance weight is attached to the crank web by two long bolts, which pass through clear holes and therefore incapable of carrying any shear force. Suggest the suitable size of the bolt if the permissible tensile stress intensity in the bolt material is limited to 350 kg/sq cm. The distance between the axis of the crankshaft and the centre of gravity of the balance weight is 30 cm. The crank shaft rotates at 360 r p m. Ans. M20.

3. Design giving a neat sketch an eye bolt to lift the upper casing, of an experimental Parson's reaction turbine, weighing 1,500 kg. Find how far the screwed end of the eye bolt should be inserted in the cast steel body for which permissible shear stress may be taken as 300 kg/sq cm. The permissible tensile stress intensity may be taken as 600 kg/sq cm.

Ans. 22 mm diameter with 25 mm threaded length.

4. A hanger bracket for a line shaft is fixed by eight bolts as shown in fig 5-20. The axis of the shaft is 10" (25 cm) below the ceiling and bolt load is equivalent to a horizontal pull of 2,400 lb (1,200 kg). The thickness of hanger flange is 5/8" (1·5 cm) and that of the ceiling beam is 3/4" (18 mm).

Design and draw a working (free hand) sketch of one of the bolts. Permissible stresses in bolts are 5,000 psi (350 kg/sq cm) and 4,000 psi (280 kg/sq cm) in tension and shear respectively. (Gujarat University, 1953)

FIG. 5-20

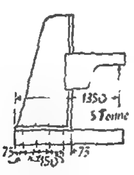


FIG. 5-21

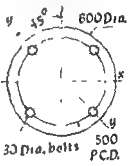


FIG. 5-22

5. In fig. 5-21 the column of a drill press bolted to a cast iron base by cap screws is shown. Under the action of the load 5 tons (5 tonnes) the screws are elongated in proportion to the distance of their centres from the tilting edge. Assuming that the base and column are infinitely rigid, determine a suitable size for the screw from B.S.W. table (Metric table) using an allowable stress value in tension for the material of the bolt of 12,500 psi (875 kg/sq cm). If you use a formula, prove it. (Roorkee University, 1953)

6. A pillar crane has a circular base 60 cm diameter and is fixed to the concrete base by 4 M30 bolts equally spaced on a bolt circle 500 mm diameter.

If the bolts of size M 7 for the cover are used, find the number of bolts required. The allowable stress for the bolts is 350 kg/sq cm.

(Sardar Patel University, 1968)

15. A standard hexagonal headed M48 bolt and nut has 5 mm pitch V threads. Assuming the threads to be sharp at the root and crest, calculate the strength of the bolt and the nut for every manner of failure and state the maximum load it can take.

Permissible stresses may be taken as 400 kg/sq cm in tension and 250 kg/sq cm in shear.

Determine the greatest shearing force set up on a bolt and hence select a suitable bolt diameter, using a working shear stress of 6·5 kg/sq mm.

Ans 1,423 kg; M20 bolt.

10. In fig 5-26 is shown a round rod *R* forming part of the swivelling crane hook which has to carry a maximum load of 20 tonnes. Calculate the dimensions of the rod *R*, the supporting block *S* with two end bearing *B-B* and the collar *C*.

Assume a tensile stress of 11 kg/sq mm for the screwed portion of the rod, bearing pressure of 140 kg/sq cm for the collar and for the end bearings and tensile stresses of 7.5 kg/sq mm for the supporting block. Make a neat dimensioned sketch of the end view of the assembly without bearings.

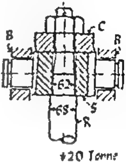


FIG. 5-26

11. A shaft is supported on a wall bracket by 3 bolts. Find the size of the bolts required for a permissible tensile stress of 400 kg/sq cm. The maximum force exerted by the shaft on the bracket is 2,000 kg and it acts vertically downwards at a distance of 300 mm from the wall. Assume bolts to be arranged in a triangular fashion with two bolts 100 mm below shaft centre and third bolt 400 mm below shaft centre. The lower edge of the bracket is 50 mm below the lower bolt. Sketch the arrangement

(M. S University of Baroda, 1965)

12. A pedestal bearing is supported by a J hanger. A vertical load of 1,500 kg is acting down on the bearing. The square base of the hanger is fixed with ceiling by 4 bolts. The distance between the centre lines of the bolts is 20 cm. The load line is 4 cm away from the vertical line through the c g of the base of the hanger. Find a suitable diameter for the bolts. The tensile stress for the bolt material is limited to 350 kg/sq cm.

(Sardar Vallabhbhai Vidyapeeth, 1966)

13. Calculate the diameter of the screwed end of a piston rod for a double acting steam engine:

Cylinder diameter 30 cm

Maximum steam pressure at entrance 8 kg sq cm

Condenser pressure 0·136 kg/sq cm

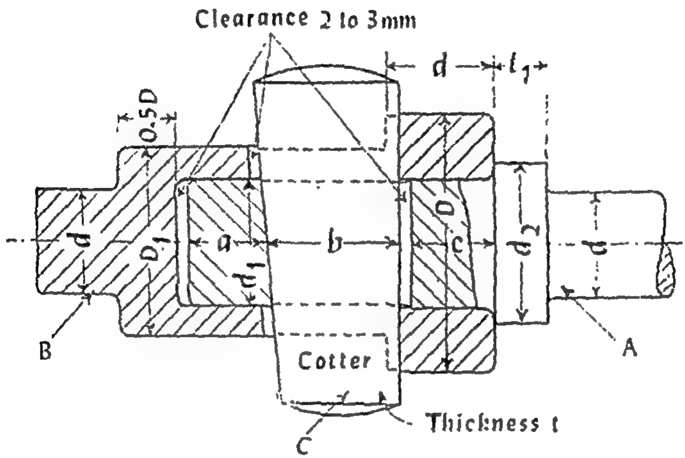
Permissible stress in the rod 250 kg sq cm.

(Bombay University, 1967)

14. In an air operated press the piston rod for operating the cylinder must exert a maximum force of 400 kg. The air pressure in the cylinder is 7 kg/sq cm gauge. Calculate the diameter of the cylinder bore required, assuming that the overall friction due to stuffing box and piston packing is equivalent to 10% of the maximum force exerted by the piston rod. The cylinder bore should be selected on the basis of 5 mm increment. Also determine the thickness of the cylinder, assuming that it is a seamless steel tubing. The allowable tensile stress for the steel tubing is 200 kg/sq cm.

6-2. Design of cottered Joints (Fig. 6-1):

We assume that the rod end, socket end and cotter are made of the same material. Let f_t, f_s and f_c be permissible tensile, shearing and crushing stress intensities respectively. Let P be the tensile force to which the joint is subjected. The failure of such a joint may take place in any one of the following ways discussed below:



Cotter joint
FIG. 6-1

(a) **Tension failure of rods at diameter** d:

The area that resists tearing of the rods is $\frac{\pi}{4}d^2$ and the internal resistance of the rod will be $\frac{\pi}{4}d^2 f_t$. By equating external force P to internal resistance of the rod, we get

CHAPTER VI

COTTER AND KNUCKLE JOINTS

(A) DESIGN OF COTTERED JOINTS

6-1. Introduction:

A cotter is a flat wedge-shaped piece of steel which is used to connect rigidly rods which transmit motion and to work in the direction of their length, without rotation. Such a joint may be subjected to tensile and/or compressive forces along the axes of the rods. Examples of cottered connections are piston rod and crosshead of an engine, pump or compressor, valve rod and its stem, a strap end and its connecting rod, a rod connecting a steam cylinder to a base plate in a marine engine, a piston rod and its extension as a tail or pump rod, etc.

When cotter joint is used to connect two rods, one of the rods is made with a socket end, *B* as shown in fig. 6-1, into which the other end *A* fits or a separate sleeve fits over the end of each rod as shown in fig. 6-2. The cotter *C* is driven tightly through the sleeve and rod ends or through socket and rod ends. The slots are made little longer than the cotter. The cotter tapers in width and not in thickness and the usual taper is 1 in 24. The relative positions of the slots are such that the driving in of the cotter tends to force the rod into the socket. The taper of the slot as well as on the cotter is usually on one side. Clearances between the cotter and slots in the rod end and socket allow the driven cotter to draw together the two parts of the joint until the socket end comes in contact with the collar on the rod end as shown in fig. 6-1. The bearing edges of the cotter and rods are generally made semi-circular as this gives a better surface than square edges, and allows the cotter holes to be drilled. The draw of the cotter need not exceed 3 mm.

The advantage of this joint is that it can be quickly and easily made or taken apart and the parts always occupy exactly the same relative positions after re-assembly.

The crushing resistance is $d_1 t f_c$. By equating the crushing resistance to external load, we get

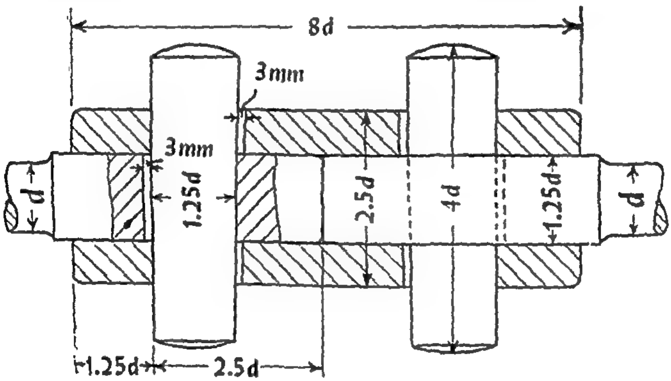
$$P = d_1 t f_c \quad \text{......(viii)}$$

(h) Crushing failure of socket or cotter:

The area that resists crushing is $(D - d_1)\ t$. The crushing resistance is $(D - d_1)\ t f_c$. By equating the crushing resistance to external load, we get

$$P = (D - d_1)\ t f_c \quad \text{......(ix)}$$

The cotter joint as illustrated in fig. 6-1 may also be used for a compressive load, thus producing compression in the rod in place of tension. When a cottered joint is used for compressive load it is necessary that collar should be investigated.



Cotter joint with a sleeve

FIG. 6-2

(i) Failure of the collar by crushing:

The area that resists the crushing of the collar is $\frac{\pi}{4}\ (d_2^2 - d_1^2)$.

The crushing resistance of the collar is $\frac{\pi}{4}\ (d_2^2 - d_1^2)\ f_c$. By equating the crushing resistance of the collar to external load, we get

$$P = \frac{\pi}{4}\ (d_2^2 - d_1^2)\ f_c \quad \text{......(x)}$$

From this equation, the diameter d_2 can be obtained.

(j) Failure of the collar by shearing:

The area that resists shearing of the collar is $\pi d_1 t_1$. The

$$P = (\frac{\pi}{4} d_1^2 - d_1 t) f_t \quad \text{(ii)}$$

From this equation, d_1, the inner diameter of the socket can be obtained when the thickness of the cotter is known. The thickness of the cotter is given by the equation,

$$t = 0{\cdot}31 \ d \quad \text{(iii)}$$

(c) Tension failure of socket across the slot:

The area that resists the tearing of the socket across the slot is $\frac{\pi}{4} (D_1^2 - d_1^2) - (D_1 - d_1)t$. The internal resistance of the socket across the slot will be

$$\left\{\frac{\pi}{4} (D_1^2 - d_1^2) - (D_1 - d_1) \ t\right\} f_t.$$

$$\therefore \quad P = \left\{\frac{\pi}{4} (D_1^2 - d_1^2) - (D_1 - d_1) \ t\right\} f_t \quad \text{(iv)}$$

From equation (iv), D_1, the outer diameter of the socket can be obtained.

(d) Shear failure of the cotter:

The cotter is in double shear. The area that resists the shear of the cotter is $2bt$. The internal resistance of cotter to double shear is $2bt f_s$. Therefore, the strength equation will be

$$P = 2bt \ f_s. \quad \text{(v)}$$

From this equation, we determine the width of the cotter.

(e) Shear failure of the rod end:

The rod end is in double shear. The area that resists the shearing of the rod end is $2ad_1$. The internal resistance of the rod end to shear is $2ad_1 f_s$.

$$\therefore \quad P = 2ad_1 f_s \quad \text{(vi)}$$

From this equation, dimension a can be calculated

(f) Shear failure of the socket end:

The socket end is in double shear. The area that resists the shearing of the socket is $2c \ (D - d_1)$. The internal resistance of the socket to shearing is $2c \ (D - d_1) f_s$.

$$\therefore \quad P = 2c \ (D - d_1) f_s \quad \text{(vii)}$$

(g) Crushing failure of rod or cotter:

The area that resists the crushing of a rod or a cotter is $d_1 t$.

In practice the following proportions in terms of d are generally adopted when all components of the cottered joint are of steel, where d is the diameter of the rod, the sizes being dimensioned to nearest commonly adopted millimetre sizes.

$d_1 = 1{\cdot}21d$	$a = c = 0{\cdot}75d$
$d_2 = 1{\cdot}5d$	$h = 1{\cdot}3d$
$D = 2{\cdot}4d$	$t = 0{\cdot}31d$
$D_1 = 1{\cdot}75d$	$t_1 = 0{\cdot}45d$

Draw of the cotter 2 to 3 mm.

Taper of the cotter 1 in 24.

If the rod and cotter are both made of steel or both of wrought iron, the tensile stress may be taken as one and quarter times the shearing stress and the compressive stress may be taken as equal to twice the tensile stress.

$$f_s = 0{\cdot}8\, f_t$$
$$f_c = 2\, f_t.$$

Example:

1. *Design a cottered joint like that of fig. 6-1 to resist safely a load of 4,000 kg that acts along the coincident axes of the rods connected by the cotter. The material of the cotter and rods will permit the following safe stresses:*

$$f_t = 500 \text{ kg/sq cm};\ f_c = 1{,}050 \text{ kg/sq cm};\ f_s = 400 \text{ kg/sq cm}.$$

If d be the diameter of the rod, and as the rod proper is in direct tension, we have

$$4000 = \frac{\pi}{4} d^2 \times 500$$

or $$d = \sqrt{\frac{4 \times 4000}{\pi \times 500}} = 3{\cdot}2 \text{ cm; we adopt } 3{\cdot}5 \text{ cm.}$$

The maximum tensional stress in the rod end is at the section through the cotter hole, and we have

$$P = \left(\frac{\pi}{4} d_1^2 - t\, d_1\right) f_t.$$

In practice t is made equal to $\dfrac{d_1}{4}$.

If this value is substituted in the above equation, we get

$$d_1 = \sqrt{\frac{4P}{2{\cdot}14 f_t}} = \sqrt{\frac{4 \times 4000}{2{\cdot}14 \times 500}} = 3{\cdot}87 \text{ cm; we adopt } 4 \text{ cm}$$

and $t = \dfrac{d_1}{4} = \dfrac{4}{4} = 1$ cm.

The greatest tensile stress in the socket is at the section through the cotter hole, and

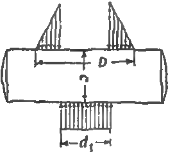
shearing resistance of collar is $\pi d_1 t_1 f_s$. By equating the shearing resistance of collar to external load, we get

$$P = \pi d_1 t_1 f_s. \qquad \text{(xi)}$$

From this equation, the thickness of the collar t_1 can be determined.

To ensure that the cotter does not work back, the taper should not exceed 1 in 24. If a greater taper is required a locking device must be provided.

Cotters are frequently bent when being driven into position. When bending occurs, the bending moment cannot be correctly estimated, as the distribution of pressure on the bearing surface is unknown. However, by making simplifying assumptions, the bending stresses in the cotter can be estimated. The bearing load on the cotter in the rod end is assumed uniformly distributed while in the socket end it is uniformly varying over the length as shown in fig. 6-3.



Loading of a cotter in a cotter joint
FIG. 6-3

The bending moment will be maximum at the centre of the *cotter and its magnitude is* $\frac{P}{2}\left[\frac{d_1}{4} + \frac{(D-d_1)}{6}\right]$

and the maximum bending stress is (fig. 6-3)

$$f_t = \frac{\text{bending moment}}{\text{modulus of section}} = \frac{\frac{P}{2}\left[\frac{d_1}{4} + \frac{(D-d_1)}{6}\right]}{\frac{tb^2}{6}}.$$

Tightening of cotter introduces initial stresses that are difficult to estimate. In many cases, it is necessary to alter the proportions obtained from theoretical analysis in order to meet the conditions of service.

thickness of the collar are taken as 6 cm and 1·5 cm respectively. The induced crushing stress intensity and shear stress intensity shall not exceed 255 and 212 kg/sq cm respectively. These values are within permissible limits.

Exercises:

1. *Define a cotter.*

2. *Give several examples of the use of cotters in machine construction. Of what material are they usually made?*

3. *Give sketches showing the usual proportions for a cottered joint suitable for securing tie rods.*

4. *What is the taper usually given to cotters?*

5. *What are the advantages of rounding off the edges of the cotter?*

6. *What is meant by the draw of cotter?*

7. *Make a neat sketch showing the views of a cotter joint and write equations showing the strength of a joint for the most probable methods of failure.*

8. *Design a cottered joint like that of fig. 6-1 to safely resist a load of 3,500 kg which acts along the axes of the rods connected by the cotters. The material of the cotter and rods will permit the following safe stresses:*

$f_t = 560$ *kg/sq cm;* $f_s = 450$ *kg/sq cm;* $f_c = 980$ *kg/sq cm.*

Ans. $d = 3$ *cm;* $d_1 = 3{\cdot}8$ *cm;* $d_2 = 4{\cdot}5$ *cm;* $D = 7{\cdot}2$ *cm;* $b = 4$ *cm;* $D_1 = 5{\cdot}3$ *cm;* $a = c = 2{\cdot}4$ *cm;* $t = 1$ *cm;* $t_1 = 1{\cdot}4$ *cm.*

9. *A rod 4 cm diameter is to be pierced with a slot 5·5 cm long and 1 cm wide. Find the diameter to which the rod must be increased at the slot in order that there shall be no loss of strength.*

Ans. 4·8 *cm.*

10. *Design a cottered joint to connect two mild steel rods of equal diameter transmitting an axial force of 2,500 kg which is subject to slow reversals of direction. Tensile stress in the material is limited to 500 kg/sq cm. The shear stress has the value* $\frac{4}{5}$ *of the permissible tensile stress. The bearing pressure between the cotter and the rods is limited to 600 kg/sq cm.*

Ans. $d = 3$ *cm;* $d_1 = 3{\cdot}5$ *cm;* $D = 7$ *cm;* $a = c = 2{\cdot}5$ *cm;* $t = 1{\cdot}5$ *cm;* $b = 3$ *cm.*

11. *A cottered joint is required to carry a load of 7,000 kg. Design and set out to scale full size, the transverse section through which the cotter*

$$P = \left\{\frac{\pi}{4}(D_1^2 - d_1^2) - (D_1 - d_1)\, t\right\} f_t.$$

From which

$$4000 = \left\{\frac{\pi}{4}(D_1^2 - 4^2) - 1\,(D_1 - 4)\right\} 500.$$

The equation for determination of D_1 will be

$D_1^2 - 1{\cdot}27 D_1 - 21{\cdot}1 = 0.$

Solving this affected quadratic equation by the algebraic formula, we get

$D_1 = 5{\cdot}3$ cm; we adopt 6 cm.

The cotter is in double shear; therefore

$P = 2bt\, f_s$

or $b = \dfrac{P}{2t f_s} = \dfrac{4000}{2 \times 1 \times 400} = 5$ cm.

Checking for crushing between the rod and cotter, we have

$$f_c = \frac{P}{d_1 t} = \frac{4000}{4 \times 1} = 1{,}000 \text{ kg/sq cm.}$$

The above value of f_c is less than the assumed safe crushing stress of 1,050 kg/sq cm. Therefore, the design is safely dimensioned for this area in crushing.

The diameter D is obtained from the crushing of cotter and socket. The force between the cotter and socket is

$P = (D - d_1)\, t\, f_c$

or $D = \dfrac{P}{t f_c} + d_1 = \dfrac{4000}{1 \times 1050} + 4$

$= 3{\cdot}8 + 4 = 7{\cdot}8$ cm; we adopt 8 cm.

To obtain a, by using the formula $P = 2ad_1 f_s$

we get $a = \dfrac{P}{2d_1 f_s} = \dfrac{4000}{2 \times 4 \times 400} = 1{\cdot}25$ cm

and $c = \dfrac{P}{2\,(D - d_1)\, f_s} = \dfrac{4000}{2\,(8 - 4)\,400} = 1{\cdot}25$ cm.

This completes the design of the joint except for the taper of the cotter which is usually 1 in 24.

The diameter and thickness of the collar on the rod end are determined by the compressive load, since the only load on the collar when the joint is in tension is that due to driving in the cotter. The magnitude of this load is unknown, but it is at least as large as the compressive load on the rod. The diameter and

thickness of the collar are taken as 6 cm and 1·5 cm respectively. The induced crushing stress intensity and shear stress intensity shall not exceed 255 and 212 kg/sq cm respectively. These values are within permissible limits.

Exercises:

1. *Define a cotter.*

2. *Give several examples of the use of cotters in machine construction. Of what material are they usually made?*

3. *Give sketches showing the usual proportions for a cottered joint suitable for securing tie rods.*

4. *What is the taper usually given to cotters?*

5. *What are the advantages of rounding off the edges of the cotter?*

6. *What is meant by the draw of cotter?*

7. *Make a neat sketch showing the views of a cotter joint and write equations showing the strength of a joint for the most probable methods of failure.*

8. *Design a cottered joint like that of fig. 6-1 to safely resist a load of 3,500 kg which acts along the axes of the rods connected by the cotter. The material of the cotter and rods will permit the following safe stresses:*
$f_t = 560$ *kg/sq cm;* $f_s = 450$ *kg/sq cm;* $f_c = 980$ *kg/sq cm.*
Ans. $d = 3$ *cm;* $d_1 = 3{\cdot}8$ *cm;* $d_2 = 4{\cdot}5$ *cm;* $D = 7{\cdot}2$ *cm;* $b = 4$ *cm;* $D_1 = 5{\cdot}3$ *cm;* $a = c = 2{\cdot}4$ *cm;* $t = 1$ *cm;* $t_1 = 1{\cdot}4$ *cm.*

9. *A rod 4 cm diameter is to be pierced with a slot 5·5 cm long and 1 cm wide. Find the diameter to which the rod must be increased at the slot in order that there shall be no loss of strength.*
Ans. 4·8 *cm.*

10. *Design a cottered joint to connect two mild steel rods of equal diameter transmitting an axial force of 2,500 kg which is subject to slow reversals of direction. Tensile stress in the material is limited to 500 kg/sq cm. The shear stress has the value* $\frac{4}{5}$ *of the permissible tensile stress. The bearing pressure between the cotter and the rods is limited to 600 kg/sq cm.*
Ans. $d = 3$ *cm;* $d_1 = 3{\cdot}5$ *cm;* $D = 7$ *cm;* $a = c = 2{\cdot}5$ *cm;* $t = 1{\cdot}5$ *cm;* $b = 3$ *cm.*

11. *A cottered joint is required to carry a load of 7,000 kg. Design and set out to scale full size, the transverse section through which the cotter*

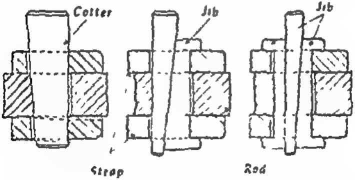
passes, and also determine a suitable width for the cotter. The cotter thickness is $\frac{1}{4}$ × diameter of the rod at the cotter hole. The tensile stress in the rod and socket is 400 kg/sq cm and the shear stress in the cotter is 350 kg/sq cm. State the crushing stress on the rod and cotter, and also give the diameter of the socket collar if the crushing stress is to have the same value on both bearing faces of the cotter.

12. *If the diameter of the rod in a cottered joint is 4 cm, find the dimensions of the cotter. Take $f_s = 0.8 f_t$ and $f_b = 2f_t$.*

Ans. 1·6 cm; 6 cm.

6-3. Gib and Cotter:

When a cotter is used for the purpose of connecting a thin strap and a thicker rod as in the case of steam engine connecting rods as shown in fig. 6-4, the friction between the cotter and the straps causes the strap to open out as shown by the dotted lines in fig. 6-4 when the cotter is driven in. This may be prevented by the use of a gib as shown in fig. 6-5. The gib and cotter are parallel along their outside edges and taper on their inside edges.



Cotter without gib Use of gib Cotter with double gibs
FIG. 6-4 FIG. 6-5 FIG. 6-6

Sometimes a small set screw is fitted, a screwing through the rod jamming against the cotter and prevents it loosening or if this is not possible the gib is extended by a screwed rod fastening the cotter with the top and bottom nuts. The gib also gives larger surface for the cotter to slide on. To make the sliding surface same on each side of the cotter, two gibs are used as shown in fig. 6-6. Thus, the fitting of a gib serves the following purposes:

As the width of the strap is 6 cm, the thickness of the strap at the thinnest part will be taken as 1·3 cm. The area provided at the thinnest part of the strap is $6 \times 2 \times 1{\cdot}3 = 15{\cdot}6$ cm, thus inducing 225 kg/sq cm as the tensile stress intensity.

The thickness of the gib and cotter is taken as one-fourth the width of the strap which gives us 1·5 cm as the thickness of gib and cotter.

If t_2 be the thickness of the strap across the cotter holes, by equating the area of the thinnest part of the strap to area of the cross section of the strap at the cotter hole, we get

$$2t_2\ (6 - 1{\cdot}5) = 2 \times 1{\cdot}3 \times 6$$

or $$t_2 = \frac{2 \times 1{\cdot}3 \times 6}{2 \times 4{\cdot}5} = 1{\cdot}73 \text{ cm; we adopt } 1{\cdot}8 \text{ cm.}$$

The thickness of the strap at the crown is taken as 1·5 times the thickness of the strap at the thinnest part.

$$\text{The thickness of the strap at the crown} = 1{\cdot}5 \times 1{\cdot}3$$
$$= 1{\cdot}95 \text{ cm; we adopt 2 cm.}$$

If B be the total width of cotter and gib combined, its value can be obtained by considering the shear failure of the cotter and gib. As the cotter and gib are in double shear, we get

$$2Btf_s = P$$

or $$B = \frac{P}{2t\,f_s} = \frac{3500}{2 \times 1{\cdot}5 \times 175} = 6{\cdot}67 \text{ cm; we adopt 7 cm.}$$

The width of the gib is taken as 4 cm, while that of the cotter is taken as 3 cm.

Note: If an oil hole is provided in the strap, weakening effect of the hole should be considered while determining the thickness of the strap at the thinnest part.

Exercises:

1. *Explain the use of a gib.*

2. *Show two methods by which a cotter may be prevented from slackening back.*

3. *Sketch a connecting rod end with strap, gib and cotter.*

4. *The big end of a connecting rod as shown in fig. 6-7 is subjected to a maximum load of 7,000 kg. Calculate the diameter of the circular part of the rod adjacent to the strap end if the permissible tensile stress is limited*

$$P = 2bt_1f_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

where f_t is the permissible tensile stress intensity in the material of the strap. From equation (i) the thickness of the strap at the thinnest part can be calculated. The thickness of the strap at the cotter is increased such that the area of the cross section of the strap at the cotter hole is not less than the area of the strap at the thinnest part.

The thickness t of the cotter is assumed to be $\frac{1}{4}$ × width of the strap. The combined width B of the cotter and gib is obtained by considering the shearing of the gib and cotter. *As the cotter is in double shear the equation for determination of* B will be

$$P = 2Btf_s \ldots\ldots\ldots\ldots \text{(ii)}$$

where f_s is the permissible shear stress intensity for the material of the cotter and gib.

The following are the usual proportions for the common strap end

t_1 = thickness of the strap at the thinnest part

$t_2 = 1{\cdot}15t_1$ to $1{\cdot}5t_1$ $\qquad l_1 = 2t_1$

$t_3 = 1{\cdot}2t_1$ to $1{\cdot}3t_1$ $\qquad l_2 = 2{\cdot}5t_1$

t = thickness of the cotter $= \dfrac{b}{4}$

Width of cotter $= 0{\cdot}45B$, width of gib $= 0{\cdot}55B$

Example:

1. The big end (common strap end type) of a connecting rod as shown in fig. 6-7 is subjected to a maximum load of 3 500 kg. The diameter of the circular part of the rod adjacent to the strap end is 6 cm. Determine (a) the width of the strap end, (b) the thickness of the strap at the thinnest part, at the cotter hole and at the crown, and (c) the width and thickness of gib and cotter. Safe tensile stress in the material of the strap is limited to 225 kg/sq cm. Safe shear stress value in cotter and gib is not to exceed 175 kg/sq cm.

The width of the strap is generally equal to or slightly greater than the diameter of the adjacent end of the round part of the rod. The width of the strap is taken to be 6 cm. The connecting rod is subjected to a tensile load of 3,500 kg. As the permissible tensile stress intensity is limited to 225 kg/sq cm, the minimum cross sectional area to be provided at the thinnest part of the strap will be $\dfrac{3500}{225} = 15{\cdot}5$ sq cm.

is obtained by considering the shearing of the cotter. The taper of the cotter may be 1 in 30.

Example:

1. A small engine has a crosshead attached to the piston rod by a cotter, the end of the rod fitting in a tapered socket formed in the crosshead. All the parts are of the same material, suitable working stresses being 225 kg/sq cm in tension and in shear and 800 kg/sq cm in crushing. The piston is 25 cm in diameter and the effective steam pressure on piston 7 kg/sq cm. Determine (a) diameter of the rod through the cotter hole, (b) width and thickness of the cotter, (c) diameter of the socket through the cotter hole and (d) the diameter of the enlarged end of the socket.

The area of the piston of a steam engine $= \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 25^2$

$= 491$ sq cm.

$\therefore$ Maximum load on piston $= 491 \times 7 = 3{,}437$ kg.

We assume the thickness of the cotter to be $\frac{1}{4}$ the diameter of the piston rod at the cotter.

If d be the diameter of the piston rod at the cotter, then

$$P = \left(\frac{\pi}{4} d^2 - dt\right) f_t = \left(\frac{\pi}{4} d^2 - \tfrac{1}{4} d^2\right) f_t = 0{\cdot}535\ d^2 f_t.$$

$$\therefore \quad d = \sqrt{\frac{P}{0{\cdot}535 \times f_t}} = \sqrt{\frac{3437}{0{\cdot}535 \times 225}} = 5{\cdot}35 \text{ cm; we adopt } 55 \text{ mm.}$$

The thickness of the cotter will be equal to $\frac{55}{4} \approx 15$ mm.

The cotter is in double shear. If b be the width of the cotter, then

$$P = 2btf_s.$$

$$\therefore \quad b = \frac{P}{2tf_s} = \frac{3437}{2 \times 1{\cdot}5 \times 225} = 5{\cdot}1 \text{ cm; we adopt } 55 \text{ mm.}$$

If D_1 be the diameter of the socket through the cotter hole, then

$$\left\{\frac{\pi}{4}(D_1^2 - 5{\cdot}5^2) - 1{\cdot}5\ (D_1 - 5{\cdot}5)\right\} 225 = 3437.$$

After simplification, we get

$D_1^2 - 1{\cdot}91 D_1 - 39{\cdot}2 = 0$

Solving we get $D_1 = 7{\cdot}28$ cm; we adopt 75 mm.

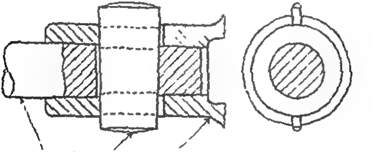
As the area provided in crushing by the socket is too small, we have the end of the socket enlarged to diameter D_2 which is

to 250 kg/sq cm. Also, determine the width of the strap end and the thickness of the strap at the thinnest part, at the cotter hole and at the crown. If the shear stress value in the cotter and the gib is not to exceed 150 kg/sq cm, suggest the suitable cross sectional dimensions for the gib and cotter.

Ans. Diameter of circular part 6 cm; other dimensions can be fixed from proportions given on page 235.

6·4. Connection of piston rod to crosshead:

In small engines the piston rod is often screwed into the boss of the crosshead and secured by a check nut. This arrangement is convenient for the lengthwise adjustment. The most general method is to use a cotter. This method is very expensive and inconvenient from the manufacturing point of view; and furthermore it does not admit of the same convenient lengthwise adjustment. Fig. 6-8 shows a common construction employed in fastening piston rod to crosshead in which the piston rod is tapered to resist the thrust instead of being provided with a collar for the purpose. The taper on the piston rod may be from 1 in 24 to 1 in 12.



Connection of piston rod and crosshead by means of a cotter
FIG. 6-8

The diameter d of the parallel part of the piston rod is calculated from the column formula. The tapered length of the piston rod is taken to be $2{\cdot}2d$. When tapered length of the piston rod is known, diameter, d_1, at the tapered end of the piston rod is known. The bearing area provided will be $\frac{\pi}{4}(d^2 - d_1^2)$

The thickness of the cotter is usually taken to be one-fourth the diameter of the piston rod at the cotter. The width of the cotter

is obtained by considering the shearing of the cotter. The taper of the cotter may be 1 in 30.

Example:

1. A small engine has a crosshead attached to the piston rod by a cotter, the end of the rod fitting in a tapered socket formed in the crosshead. All the parts are of the same material, suitable working stresses being 225 kg/sq cm in tension and in shear and 800 kg/sq cm in crushing. The piston is 25 cm in diameter and the effective steam pressure on piston 7 kg/sq cm. Determine (a) diameter of the rod through the cotter hole, (b) width and thickness of the cotter, (c) diameter of the socket through the cotter hole and (d) the diameter of the enlarged end of the socket.

The area of the piston of a steam engine $= \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 25^2$

$= 491$ sq cm.

$\therefore$ Maximum load on piston $= 491 \times 7 = 3{,}437$ kg.

We assume the thickness of the cotter to be $\frac{1}{4}$ the diameter of the piston rod at the cotter.

If d be the diameter of the piston rod at the cotter, then

$$P = \left(\frac{\pi}{4} d^2 - dt\right) f_t = \left(\frac{\pi}{4} d^2 - \frac{1}{4} d^2\right) f_t = 0{\cdot}535\ d^2 f_t.$$

$$\therefore \quad d = \sqrt{\frac{P}{0{\cdot}535 \times f_t}} = \sqrt{\frac{3437}{0{\cdot}535 \times 225}} = 5{\cdot}35 \text{ cm; we adopt } 55 \text{ mm.}$$

The thickness of the cotter will be equal to $\frac{55}{4} \approx 15$ mm.

The cotter is in double shear. If b be the width of the cotter, then

$$P = 2btf_s.$$

$$\therefore \quad b = \frac{P}{2t f_s} = \frac{3437}{2 \times 1{\cdot}5 \times 225} = 5{\cdot}1 \text{ cm; we adopt } 55 \text{ mm.}$$

If D_1 be the diameter of the socket through the cotter hole, then

$$\left\{\frac{\pi}{4}(D_1^2 - 5{\cdot}5^2) - 1{\cdot}5\ (D_1 - 5{\cdot}5)\right\} 225 = 3437.$$

After simplification, we get

$$D_1^2 - 1{\cdot}91 D_1 - 39{\cdot}2 = 0$$

Solving we get $D_1 = 7{\cdot}28$ cm; we adopt 75 mm.

As the area provided in crushing by the socket is too small, we have the end of the socket enlarged to diameter D_2 which is

obtained on the basis of crushing. The equation for determination of D_2 is

$$P = t\,(D_2 - d)\,f_c$$

$$\therefore \quad D_2 = \frac{P}{f_c\,t} + d = \frac{3137}{800 \times 1{\cdot}5} + 5{\cdot}5 = 8{\cdot}36 \text{ cm; we adopt 85 mm}$$

Exercises:

1. Give sketches showing how you would fix a piston rod to a cross-head by means of a cotter.

2. The piston rod of a steam engine is 12 cm in diameter and carries a load of 30,000 kg. It is connected to crosshead by means of a cotter of 3 cm thickness. Calculate

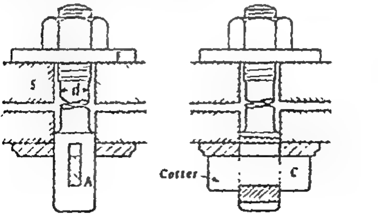
(i) *the width of the cotter if the permissible shear stress in the cotter is limited to 400 kg/sq cm*

(ii) *the length of the rod in the crosshead after the cotter if the permissible stress in shear is limited to 150 kg/sq cm.*

Ans. 13 cm; 8·5 cm.

3. Design and draw a cotter joint for connecting piston rod to the crosshead of a double acting steam engine having cylinder diameter of 30 cm and the effective steam pressure at the beginning of the piston stroke is 7 kg/sq cm. The thickness of the cotter is to be 0·3 times the diameter of the rod at the point where cotter is located. The allowable stresses for the material of the cotter and the rod are:

$f_t = 550$ *kg/sq cm,* $f_s = 420$ *kg/sq cm, and* $f_c = 850$ *kg/sq cm*



Cotter foundation bolt

FIG. 6-9

6-5. Cotter foundation bolts:

Cotters are often used as fastenings in such cases as foundation and holding down bolts for machines and engines, where it is not possible or convenient to use an ordinary bolt or stud. An example of such cotter foundation bolt is shown in fig. 6-9, where the bolt is dropped down from above and the cotter is driven in from the side, and the whole arrangement is tightened by screwing down the nut.

In fig. 6-9, F is the flange to be bolted down, S the stone bed, A the bolt and C the cotter.

The illustrative example explains the principle underlying the design of acotter foundation bolt.

Example:

1. *A foundation bolt with a circular end is secured to the floor by means of a steel cotter. If the pull on the bolt is 14,000 kg, design the bolt and the cotter.*

We assume the material of the bolt and the cotter to be the same. Let us assume the following values of the permissible stresses:

$f_t = 550$ kg/sq cm; $f_s = 420$ kg/sq cm; $f_c = 1{,}100$ kg/sq cm.

Let d be the diameter of the bolt.

$$\therefore \quad \frac{\pi}{4} d^2 \times 550 = 14000$$

or $$d = \sqrt{\frac{14000 \times 4}{550 \times \pi}} = 5{\cdot}7 \text{ cm; we adopt 6 cm.}$$

Let d_1 be the diameter of the enlarged end of the rod and the thickness of the cotter be $\frac{d_1}{4}$.

From the strength equation, we get

$$\left(\frac{\pi}{4} d_1^2 - \frac{d_1^2}{4}\right) 550 = 14000.$$

From the above equation we shall have $d_1 = 6{\cdot}9$ cm; we adopt $d_1 = 7$ cm.

$t = \frac{1}{4} \times 7 = 1{\cdot}75$ cm; we adopt $1{\cdot}8$ cm.

In order to determine the width of the cotter, we consider the shearing of the cotter. If b be the width of the cotter, the equation for determination of b will be

$$2 \times 1{\cdot}8 \times b \times 420 = 14000$$

or $$b = \frac{14000}{2 \times 1{\cdot}8 \times 420} = 9{\cdot}3 \text{ cm; we adopt } b = 9{\cdot}5 \text{ cm.}$$

Let us check the dimensions for the compressive stresses. Compressive stresses induced between the cotter and the rod will be $\frac{14000}{7 \times 1.8} = 1{,}110$ kg/sq cm.

As this value exceeds the permissible limit, we increase the diameter of the enlarged end of the rod to 7·5 cm and the thickness of the cotter to 2 cm. The induced compressive stresses will be reduced to $\frac{14000}{7.5 \times 2} = 935$ kg/sq cm, which is within safe limits.

The length of the cotter must be such that no excessive bearing pressure is induced between cotter and foundation. The length of the enlarged end of the rod beyond socket is taken as 4·5 cm. With this dimension the shear stress shall not exceed 223 kg/sq cm which is within safe limits.

Exercises:

1. Show the use of a cotter in connection with a foundation bolt for fastening down an engine bed.

2. Fig. 6-10 shows two views of a cottered connection. Find the maximum load P that may be applied so that the following stresses are not exceeded: Tensile 350 kg/sq cm, shear 210 kg/sq cm and bearing pressure 800 kg/sq cm. Neglect the effect of stress concentration.

Ans 1 890 kg.

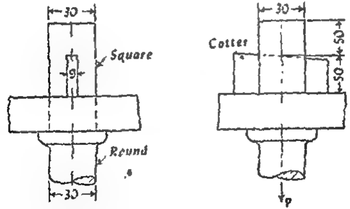


Fig. 6-10

3. A foundation bolt with a circular end is secured by means of a cotter. Determine (a) the diameter of the enlarged end of the rod at the

slot, (b) the thickness of the cotter and (c) the width of the cotter in terms of the diameter of the bolt. Assume shearing stress on the cotter to be ¾th, and bearing pressure twice the tensional stress in the bolt. Find the dimensions if the diameter of the bolt be 5 cm.

Ans. *Diameter of the enlarged end of the rod at the slot = 1·22d*
Thickness of the cotter = 0·26d
Width of the cotter = 1·33d.

4. *In a cottered foundation bolt, the diameter of the bolt through which the cotter passes is 5 cm and the thickness of the cotter 1·4 cm. Determine the width of the cotter. What is the bearing stress between the cotter and the bolt? Take* f_s = *500 kg/sq cm and* f_t = *600 kg/sq cm.*

Ans. *5·5 cm; 1,090 kg/sq cm.*

5. *Determine the weakest part of the cottered bolt shown in fig. 6-11. Diameter of bolt 45 mm; thickness of cotter 14 mm; depth of cotter 50 mm; distance from cotter to end of the bolt 65 mm. Assume the following values of the permissible stresses:* f_t = *630 kg/sq cm;* f_s = *500 kg/sq cm;* f_c = *950 kg/sq cm.*

Ans. *Crushing failure of the cotter.*

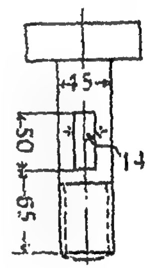


Fig. 6-11

(B) DESIGN OF A KNUCKLE JOINT

6-6. Introduction:

Knuckle joints or forked joints are used to connect two rods or bars, that are under the action of tensile loads, although if the joint is guided the rods may support a compressive load. This joint permits a small amount of flexibility or angular movement if necessary. This joint can be readily disconnected for adjustments or repairs. Such joints are very common in both, machines and structures. Common uses are with valve and eccentric rods, air brake arrangement on locomotive, the reversing gear on any steam engine, remote control of steam valves, lever and rod connections of many kinds and tension links in bridge structures.

Fig. 6-12 shows a knuckle joint. The eye is formed at the end of a rod and fork or double eye is formed on the end of the

Let us check the dimensions for the compressive stresses. Compressive stresses induced between the cotter and the rod will be $\frac{14000}{7 \times 1{\cdot}8} = 1{,}110$ kg/sq cm.

As this value exceeds the permissible limit, we increase the diameter of the enlarged end of the rod to 7·5 cm and the thickness of the cotter to 2 cm. The induced compressive stresses will be reduced to $\frac{14000}{7{\cdot}5 \times 2} = 935$ kg/sq cm, which is within safe limits.

The length of the cotter must be such that no excessive bearing pressure is induced between cotter and foundation. The length of the enlarged end of the rod beyond socket is taken as 4·5 cm. With this dimension the shear stress shall not exceed 223 kg/sq cm which is within safe limits.

Exercises:

1. Show the use of a cotter in connection with a foundation bolt for fastening down an engine bed.

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Ans. 1,890 kg.

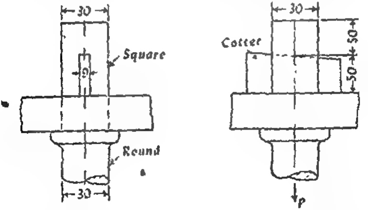


Fig. 6-10

3. A foundation bolt with a circular end is secured by means of a cotter. Determine (a) the diameter of the enlarged end of the rod at the

In the above table, d denotes the diameter of the rod, d_1 the diameter of the knuckle pin, D the outside diameter of the eye, A the thickness of the fork and B the thickness of the eye.

If a knuckle pin is loose in the forks, it is subjected to bending. Considering the load to be uniformly distributed along the eye and uniformly varying over the forks, the bending moment will be maximum at the mid section of the pin, its value being

$$M = \frac{P}{2}\left[\frac{B}{4} + \frac{A}{3}\right],$$ and the maximum bending stress is

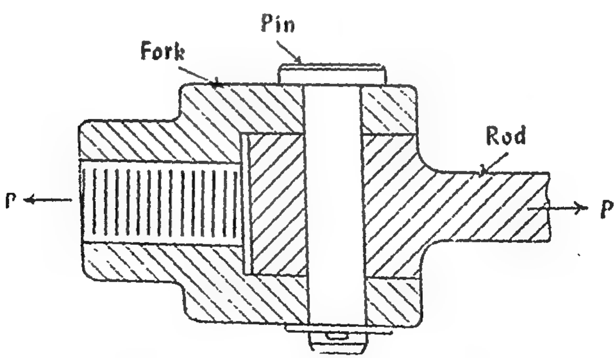
$$f_t = \frac{\frac{P}{2}\left[\frac{B}{4} + \frac{A}{3}\right]}{\frac{\pi}{32} d_1^3}$$ from which

$$d_1 = \sqrt[3]{\frac{16P\left(\frac{B}{4} + \frac{A}{3}\right)}{\pi f_t}}.$$

The proportions of the joint in terms of d, diameter of the rod, are given below:

$$A = 0{\cdot}75d;\ B = 1{\cdot}25d;\ D = 2d;\ d_1 = d.$$

After adopting the dimensions of various parts of the joints in terms of the rod diameter d, the joints will be represented in terms of the rod diameter as shown in fig. 6-12.

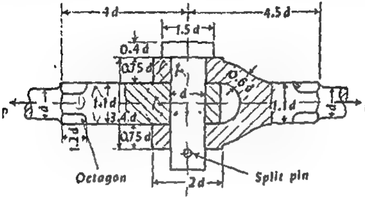


Another form of a knuckle joint
Fig. 6-13(a)

No part of the joint will be weaker than the rod with the above proportion. The pin could be made somewhat smaller as can be seen from the stress analysis, but making the knuckle pin of

other rod. The eye fits between the fork or the double eye. Two parts are connected by a turned pin passing through both and may be secured by a split pin or a tapered pin.

The tapered pin may be secured by a thin nut screwed upto a shoulder on the end of a pin. The knuckle pin may be prevented from rotating in the fork by means of a small stop pin.



Knuckle joint
FIG. 6-12

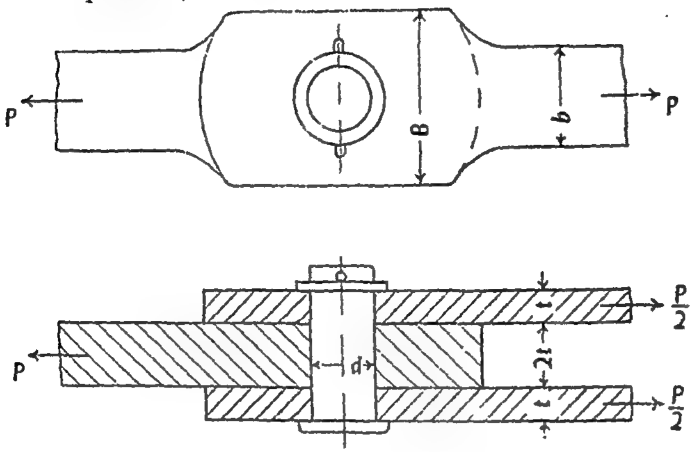
In designing such a joint the proportions of the pin and eyes are made to be of equal strength with the rod. The methods of failure of the joint are indicated in a tabular form in which the strength of the joint for each method of failure may be specified. In the following analysis stress concentration is neglected.

No.	Kind of failure	Area resisting failure	Resistance of the part to failure
1	Tension failure of solid rod	$\frac{\pi}{4}d^2$	$\frac{\pi}{4}d^2 f_t$
2	Tension failure of eye	$(D-d_1)\,B$	$(D-d_1)\,B f_t$
3	Tension failure of fork end	$2\,(D-d_1)\,A$	$2\,(D-d_1)\,A f_t$
4	Shear failure of knuckle pin	$2\frac{\pi}{4}d_1^2$ (Pin is in *double* shear)	$2\frac{\pi}{4}d_1^2 f_s$
5	Shear failure of eye	$(D-d_1)\,B$	$(D-d_1)\,B f_s$
6	Shear failure of fork	$2(D-d_1)\,A$	$2(D-d_1)\,A f_s$
7	Crushing failure of pin in eye	$d_1 B$	$d_1 B f_c$
8	Crushing failure of pin in fork	$2d_1 A$	$2d_1 A f_c$

(b) The thickness, t, of each side link is obtained by considering the crushing of the pin.

(c) The width of each link, b, is calculated by considering the tensional failure of the link.

(d) The width, B, of the enlarged end of the link is obtained by considering the tearing across hole. B will be at least equal to $(b + d)$.



Suspension link
FIG. 6-14

Examples:

1. *A knuckle joint is required for a rod which has to withstand a tensile load of 10,000 kg. Find the diameters of the rod and the pin. Safe working stress both in tension and shear is 800 kg/sq cm. Suggest the suitable dimensions for the entire joint.*

If d be the diameter of the rod, then

$$\frac{\pi}{4} d^2 f_t = P$$

or $$d = \sqrt{\frac{4P}{\pi f_t}} = \sqrt{\frac{4 \times 10000}{\pi \times 800}} = 4 \text{ cm.}$$

We assume the following dimensions for the joint:

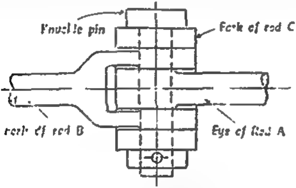
Diameter of the pin 4 cm
Thickness of the eye 5 cm
Outside diameter of the eye 8 cm
Thickness of the fork 3 cm
Diameter of the head of the knuckle pin 6 cm
Thickness of the head of the pin 1·0 cm.

the same size as the rod provides a margin of strength to resist bending action which occurs when the pin is somewhat loose fit or becomes worn.

From strength analysis the dimension A should be equal to one-half B. However the dimension A is greater than one-half B in order to prevent deflection or spreading of the forks which would introduce bending of the pin

Another form of a knuckle joint is shown in fig. 6-13(a).

The knuckle connection is also employed for uniting more than two rods in a common joint. Such an arrangement is met with in toggle press. Two rods are forked and third has got an eye formed at the end of the rod. Fig. 6-13(b) shows such a connection. The



Knuckle joint for connecting three rods
FIG. 6-13(b)

dimensions of the joint can be calculated from first principles. The pin will be longer and so in order to resist the bending stresses it shall be of slightly larger diameter.

6-7. Joint of suspension links:

Suspension links or plate link chains are used in suspension bridges. Fig. 6-14 shows the joint of such a link. In order to allow for weakening effect of the hole in the links, the width of each link is increased at least by the amount equal to the diameter of the pin.

The following procedure should be adopted for the design of such a joint:

(a) The diameter, d, of the pin should be calculated by considering the double shear of the pin.

If b be the width of link, then

$$b \times 2t \times f_t = P$$

or $$b = \frac{P}{2tf_t} = \frac{30 \times 1000}{2 \times 1{\cdot}5 \times 1200} = 8{\cdot}34 \text{ cm; we adopt } 8{\cdot}5 \text{ cm.}$$

The knuckle pin is in double shear. The diameter of the pin is obtained by the equation

$$2 \times \frac{\pi}{4}\, d^2 f_s = P$$

or $$d = \sqrt{\frac{2P}{\pi f_s}} = \sqrt{\frac{2 \times 30000}{\pi \times 800}} = 4{\cdot}86 \text{ cm. We adopt 5 cm.}$$

The width of the link at the centre line of the knuckle pin is $8{\cdot}5 + 5 = 13{\cdot}5$ cm.

In order to know the crushing resistance, we determine the projected area of the pin which is $5 \times 3 = 15$ sq cm and the corresponding crushing stress will be

$$\frac{30000}{15} = 2{,}000 \text{ kg/sq cm.}$$

Note: Generally in suspension links the value of the permissible crushing stress is 50% more than the tensile stress. In this case it is 67% more. The crushing stress can be reduced either by increasing the diameter of the pin, or increasing the thickness of the side link or both.

3. *An eye is forged at one end of one half of a tie rod and a fork at the end of the other half. A pin is passed through the two sides of the fork and through the eye. The pull in the tie rod is 12,000 kg. If the tensile stress is not to exceed 625 kg/sq cm and the shearing stress 280 kg/sq cm, determine the diameters of the tie rod and the pin.*

If d be the diameter of the tie rod, then $\frac{\pi}{4}\, d^2 f_t = P$.

$$\therefore \quad d = \sqrt{\frac{4 \times 12000}{\pi \times 625}} = 4{\cdot}95 \text{ cm, say 5 cm.}$$

If d_p be the diameter of the knuckle pin, then $2 \times \frac{\pi}{4}\, d_p^2 f_s = P$.

$$\therefore \quad d_p = \sqrt{\frac{2 \times 12000}{\pi \times 280}} = 5{\cdot}23 \text{ cm, say } 5{\cdot}5 \text{ cm.}$$

4. *Determine the leading dimensions of the link shown in fig. 6-15 which is to operate under a tensile load P that varies between 3,000 and*

Let us check the stresses in various sections of the joint. The area of the section of the eye of the rod by a plane at right angles to the axis of the rod and containing the axis of the pin is $(8-4)5 = 20$ sq cm. The tensile stress in the eye of the rod is $\frac{10000}{20} = 500$ kg/sq cm.

The area of the fork of the rod by a plane at right angles to the axis of the rod containing the axis of the pin is $2 \times 3\ (8-4) = 24$ sq cm.

The tensile stress at this section is, therefore,

$$\frac{10000}{24} = 417 \text{ kg/sq cm.}$$

The knuckle pin which is 4 cm in diameter, is subjected to shearing stress at the two sections just inside the fork. The shearing stress in the pin is, therefore,

$$\frac{10000}{2 \times \frac{\pi}{4} \times 4^2} = 398 \text{ kg/sq cm.}$$

There are crushing stresses between the connecting pin and the sides of the holes in the eye and the fork. For the eye the bearing area is $4 \times 5 = 20$ sq cm and corresponding crushing stress is

$$\frac{10000}{20} = 500 \text{ kg/sq cm}$$

For the fork the bearing area on the pin is $3 \times 2 \times 4 = 24$ sq cm and the corresponding crushing stress is

$$\frac{10000}{24} = 417 \text{ kg/sq cm}$$

Thus, we see that the stresses are within limits and the joint is safe.

2. *The suspension link in a steel structure is subjected to a maximum load of 30 tonnes. The thickness of each side link is 1·5 cm. Determine (a) the width of the link at the middle, (b) the diameter of the knuckle pin and (c) the width of the link at the centre line of the pin.*

Safe stresses should not be greater than 1,200 kg/sq cm in tension and 800 kg/sq cm in shear.

Also, calculate the crushing stress on the pin.

If we consider the shear failure of eye and fork, the magnitude of the shear stress is 667 kg/sq cm, which is less than the permissible value which is 700 kg/sq cm.

Exercises:

1. *Mention some of the applications of the knuckle joint in engineering.*

2. *Make a neat sketch showing two views of a knuckle joint, and write equations showing the strength of the joint for the most probable methods of failure.*

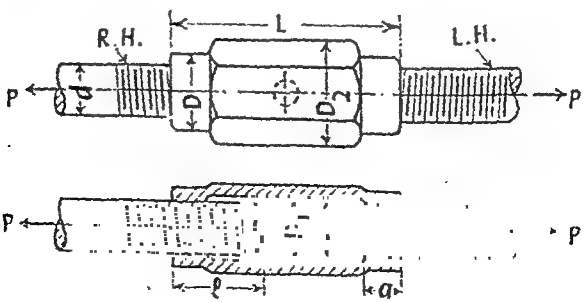
3. *Give two views with figures and dimensions of a knuckle or forked joint, suitable for connecting two rods of 3 cm diameter.*

4. *Make a working drawing of the end of a suspension link to take a load of 15 tonnes; the width of the link is 15 cm. Use a working tensile stress of 8 kg/sq mm. If the diameter of the pin is 10 cm, to which shear stress is the pin subjected? Also, calculate the crushing stress on the pin.*

Ans. $t = 7$ *mm; 190 kg/sq cm; 1,070 kg/sq cm.*

6-8. Design of a coupler or turnbuckle:

This is a machine part which is used for connecting two members which are subjected to tensile loading and which require slight adjustment in length. One end of each bar is threaded, threads on each of them being of opposite nature i.e. one threaded bar has right hand threads cut on it while the other has left hand threads. When the coupler is given one complete rotation either both ends of the bars approach or recede by the amount equal to twice the lead depending upon the direction of rotation.



Turnbuckle

Fig. 6-16(*a*)

Fig. 6-16 shows the types of turnbuckle in general use. There is an obvious advantage in using the form of coupling

1,000 kg several times each hour. Safe tensile stress of 9 kg/sq mm and shear stress of 7 kg/sq mm may be assumed for the material of the link.

Note: The link can be considered to be subjected to a static load. It should be remembered that the static loading does not mean that the load carried by the part never varies in any manner. By static loading we understand that the load variations are relatively few and these variations are so slow that neither fatigue nor impact should be considered.

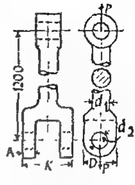


Fig. 6-15

The leading dimensions of the link will be, the rod diameter d_1, the knuckle pin diameter d_2, the eye diameter D and the width of the fork K.

If d_1 be the diameter of the rod, then

$$\frac{\pi}{4}\, d_1^2\, f_t = P.$$

$$\therefore \qquad d_1 = \sqrt{\frac{4P}{\pi f_t}} = \sqrt{\frac{4 \times 4000}{\pi \times 900}} = 2\cdot 38 \text{ cm; we adopt } 2\cdot 5 \text{ cm.}$$

The knuckle pin is in double shear. We have

$$2 \times \frac{\pi}{4}\, d_2^2 \times f_s = P.$$

$$\therefore \qquad d_2 = \sqrt{\frac{2P}{\pi f_s}} = \sqrt{\frac{2 \times 4000}{\pi \times 700}} = 1\cdot 92 \text{ cm, we adopt } 2 \text{ cm.}$$

We assume that there is no bending of the pin; so the diameter of the knuckle pin obtained from the shear strength is adopted. The diameter of the eye D is taken as twice the diameter of knuckle pin, 4 cm. The width of the fork, K, is taken as thrice the diameter of knuckle pin, 6 cm. With these dimensions we check the tensile stresses in the eye and the fork end.

The area of the section of the eye of the link by a plane at right angles to the axis of the link and containing the axis of the pin is $(D - d_2) \times$ thickness of the eye. The tensile stress in the eye of the link will be $\frac{4000}{3 \times 2} = 667$ kg/sq cm which is less than the permissible value.

There are compressive stresses between the knuckle pin and sides of the holes in the eye and the fork. This value is the same for both and its magnitude is 667 kg/sq cm.

Area that resists the tearing of the nut $= \frac{\pi}{4}(D^2 - d^2)$.

The materials of the rod and coupler may be different; so we adopt permissible tensile stresses as f_t and f_t' respectively.

If f_t' be the permissible tensile stress intensity in the material of the nut, then

$$\text{load} = P = \frac{\pi}{4}(D^2 - d^2) f_t' \quad \text{.........(iii)}$$

From the above equation, as d is known, outside diameter of the coupler at nut ends can be calculated.

(d) The outside diameter of the coupler at the middle is calculated from tensile stress considerations.

Let D_1 be the diameter of the hollow portion from inside and D_2 be the diameter of the coupler at the outside.

The area that resists the tearing of the nut in the middle

$$= \frac{\pi}{4}(D_2^2 - D_1^2).$$

$$\text{Load} = \frac{\pi}{4}(D_2^2 - D_1^2) f_t' \quad \text{.........(iv)}$$

In the above equation, we have two unknown D_1 and D_2.

The inside diameter of the coupler is taken as $d + 6$ mm. After D_1 is known, from the above equation D_2 can be calculated.

(e) The total length of the coupler can be calculated when the amount of adjustment required is known. Generally, the total length of the coupler is taken 6 times the diameter of the rod.

The outside portion of the coupler is given the hexagonal shape so that it can be turned by a spanner. Sometimes a tommy bar may be used for the same purpose and this is inserted in a hole in the coupling and is indicated in fig. 6-16(a) by the dotted circle.

Example:

1. *The pull in a tie rod of an iron roof truss is 10 tonnes. Calculate the diameter of the rod if the greatest intensity of stress is not to exceed 750 kg/sq cm. Find the dimensions of coupling joint for the above tie rod assuming the safe shearing stress to be 300 kg/sq cm.*

(a) **Diameter of the rod:**

The load is of 10 tonnes and the permissible tensile stress intensity is not to exceed 750 kg/sq cm.

shown in fig. 6-16(*b*) in which two ends of the rods can be seen.

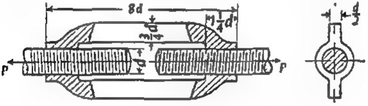
The following is the procedure in the design of a coupler:

(a) The diameter of the rod is calculated from the tensile stress considerations

Let d_c be the core diameter of the rod and f_t be the permissible tensile stress intensity at the core section.

$$\text{Load} = P = \frac{\pi}{4} d^2 f_t \; . \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

From the above equation core diameter is calculated and we can specify the diameter of the rod, when kinds of threads are known. Generally coarse threads are used.



Turnbuckle

FIG. 6-16(*b*)

(b) The length of the screwed portion of the coupler at each end is determined from the considerations of the shearing of the threads in the nut.

Let l_1 be the length of the screwed portion in the coupler, f_s the shearing stress in the nut and d the diameter of the rod.

Area that resists the shearing of the threads in the nut is equal to $\pi l_1 d$.

$$\text{Load} = P = \pi l_1 d f_s \ldots\ldots \qquad \ldots \text{(ii)}$$

From the above equation the length of the screwed portion of the coupler can be calculated.

The calculated value comes out to be very low. In actual practice l_1 varies from d to $1{\cdot}25d$ for steel nuts and $1{\cdot}5d$ to $2d$ for cast iron and softer material nuts, so the calculated value of l_1 should be modified.

(c) The outside diameter of the coupler at the nut portion can be calculated from the tensile stress considerations.

Let D be the outside diameter of the coupler at the ends.

Area that resists the tearing of the nut $= \frac{\pi}{4}(D^2 - d^2)$.

The materials of the rod and coupler may be different; so we adopt permissible tensile stresses as f_t and f_t' respectively.

If f_t' be the permissible tensile stress intensity in the material of the nut, then

$$\text{load} = P = \frac{\pi}{4}(D^2 - d^2)f_t' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

From the above equation, as d is known, outside diameter of the coupler at nut ends can be calculated.

(d) The outside diameter of the coupler at the middle is calculated from tensile stress considerations.

Let D_1 be the diameter of the hollow portion from inside and D_2 be the diameter of the coupler at the outside.

The area that resists the tearing of the nut in the middle

$$= \frac{\pi}{4}(D_2^2 - D^2_1).$$

$$\text{Load} = \frac{\pi}{4}(D_2^2 - D^2_1)f_t' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

In the above equation, we have two unknown D_1 and D_2.

The inside diameter of the coupler is taken as $d + 6$ mm. After D_1 is known, from the above equation D_2 can be calculated.

(e) The total length of the coupler can be calculated when the amount of adjustment required is known. Generally, the total length of the coupler is taken 6 times the diameter of the rod.

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(a) **Diameter of the rod:**

The load is of 10 tonnes and the permissible tensile stress intensity is not to exceed 750 kg/sq cm.

$\therefore$ Minimum cross sectional area necessary at the bottom of the thread $= \frac{10000}{750} = 13{\cdot}35$ sq cm.

This is the area at the core section. We adopt coarse threads. From the table, we adopt M 50.

(b) Length of the screwed portion of the nut at each end:

Let l be the length of the screwed portion at each end.

The shear area provided $= \pi \times 5 \times l$ sq cm.

$\therefore$ $\pi \times 5 \times l \times 300 = 10000.$

From the above equation, we get $l = 2{\cdot}13$ cm.

This length is too small. We adopt the length of the screwed portion at the nut equal to $1{\cdot}20 \times$ diameter of the rod. In our case it will be $1{\cdot}2 \times 5 = 6$ cm. The bearing stresses on the *threads will be much reduced.*

(c) Outside diameter of the coupler at the nut portion:

Let D be the outside diameter of the coupler at the ends. We get

$\frac{\pi}{4}(D^2 - d^2) \times f_t =$ load, where d is the diameter of the rod.

On substitution of the values, we get

$$\frac{\pi}{4}(D^2 - 5^2) \times 750 = 10000$$

or $D = \sqrt{\frac{10000 \times 4}{750 \times \pi} + 5^2} = 6{\cdot}5$ cm; we adopt 7 cm.

(d) Outside diameter of the coupler at the middle:

The diameter of the hollow portion from inside $= 6$ cm

Let D_2 be the outside diameter of the coupler at the middle;

then we get $\frac{\pi}{4}(D_2^2 - 6^2) \times 750 = 10000$

or $D_2 = \sqrt{\frac{10000 \times 4}{750 \times \pi} + 6^2} = 7{\cdot}28$ cm; we adopt 8 cm

Total length of coupler we take as 40 cm.

Exercises:

1. The pull in C. I. turn buckle of the stay rope of an electric post is 1,350 kg. Design the turnbuckle assuming the safe tensile stress f_t

Find, to the nearest size of a mm the sizes of the section of the links and determine the shear and crushing stress on link pins and the maximum tensile stress in the links. The stress in the plain section of the link is 350 kg/sq cm and the slack side tension may be taken as zero.

8. Fig. 6-20 shows a turnbuckle used to connect two round rods in a steel structure. The maximum pull is estimated to be 5 tonnes. Design the turnbuckle assuming that the working stresses for the material are noted below:

Tension=7 kg/sq mm; shear=3 kg/sq mm and compression=9 kg/sq mm.

Make a neat dimensioned sketch of the turnbuckle.

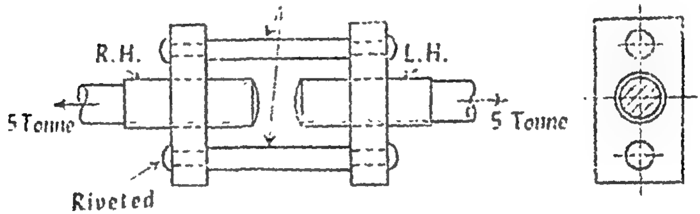


Fig. 6-20

9. The pull in the turnbuckle of a stay rope of an electric distribution post is 1,000 kg (see fig. 6-21). Design and draw a suitable turnbuckle, assuming safe tensile stress for rods of M. S. as 7·5 kg/sq mm and for C. I. 3 kg/sq mm, safe shear stress for threads to be 3 kg/sq mm. Use coarse threads. Modify result to practical values.

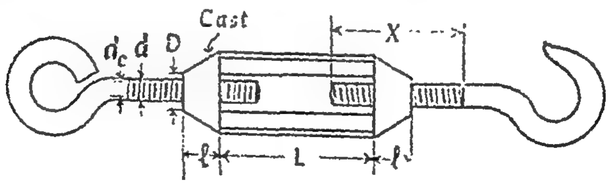


Fig. 6-21

10. Sketch and design a connecting rod with strap, gib and cotter for maximum load of 3,000 kg. Allowable bearing pressure for crank pin 60 kg/s cm.

The strap and cotter are made of mild steel for which the permissible stres are 420 kg/sq cm in tension and 320 kg/sq cm in shear. Give two views (plan a elevation) of the connecting rod end designed.

(Gujarat University, 19

$\therefore$ Minimum cross sectional area necessary at the bottom of the thread $= \frac{10000}{750} = 13{\cdot}33$ sq cm.

This is the area at the core section. We adopt coarse threads. From the table, we adopt M 50.

(b) Length of the screwed portion of the nut at each end:

Let l be the length of the screwed portion at each end.

The shear area provided $= \pi \times 5 \times l$ sq cm.

$$\therefore \quad \pi \times 5 \times l \times 300 = 10000$$

From the above equation, we get $l = 2{\cdot}13$ cm.

This length is too small. We adopt the length of the screwed portion at the nut equal to $1{\cdot}20 \times$ diameter of the rod. In our case it will be $1{\cdot}2 \times 5 = 6$ cm. The bearing stresses on the threads will be much reduced.

(c) Outside diameter of the coupler at the nut portion:

Let D be the outside diameter of the coupler at the ends. We get

$\frac{\pi}{4}(D^2 - d^2) \times f_t =$ load, where d is the diameter of the rod.

On substitution of the values, we get

$$\frac{\pi}{4}(D^2 - 5^2) \times 750 = 10000$$

or $D = \sqrt{\frac{10000 \times 4}{750 \times \pi} + 5^2} = 6{\cdot}5$ cm, we adopt 7 cm.

(d) Outside diameter of the coupler at the middle:

The diameter of the hollow portion from inside = 6 cm.

Let D_2 be the outside diameter of the coupler at the middle; then we get $\frac{\pi}{4}(D_2^2 - 6^2) \times 750 = 10000$

or $D_2 = \sqrt{\frac{10000 \times 4}{750 \times \pi} + 6^2} = 7{\cdot}28$ cm, we adopt 8 cm.

Total length of coupler we take as 40 cm.

Exercises:

1. The pull in C. I. turn buckle of the stay rope of an electric post is 1,350 kg. Design the turnbuckle assuming the safe tensile stress f_t

CHAPTER VII

SHAFTS, KEYS AND COUPLINGS

7-1. Introduction:

A shaft is a rotating member which transmits power from one point to another point. It may be divided in two main groups: *transmission shafts* and *machine shafts.* Shafts which are used to transmit power between the source and the machines absorbing power are called transmission shafts. Such shafts carry machine parts such as gears and pulleys and, therefore, they are subjected to bending in addition to twisting. *Line shafts, countershafts, head shafts and all* factory shafts are included in this group. Machine shafts are those which form an integral part of the machine itself, the common example being the crankshaft.

A spindle is a short revolving shaft that imparts motion either to a cutting tool or to a work piece. Drill press spindles and lathe spindles are examples of each type.

Shafts are usually cylindrical, but occasionally they may be square or cross shaped in section. They are solid in cross section. Sometimes hollow shafts are also used.

Another machine part which is shaft like in appearance is called an *axle*, which does not necessarily rotate. It simply acts as a support for some rotating body such as a hoisting drum, a car wheel or a rope sheave. In general, it is subjected to transverse loads and are stressed principally in bending.

7-2. Materials and design stresses:

The material commonly adopted for shafts is mild steel. When greater strength is required, as in high speed machinery, an alloy steel such as nickel, nickel chromium or chrome vanadium steel is used. When resistance to corrosion is desired, some copper alloys are used. Shafts are made by hot rolling and finished to size by cold drawing or by turning and grinding. Cold drawing produces a shaft of fairly uniform diameter which is stronger than that produced by hot rolling and finished by turning and grinding. The cold rolled shaft has higher residual stresses which cause dis-

11. Design a suitable joint for fastening piston rod to the crosshead of a steam engine having a cylinder diameter of [illegible] cm and having a steam pressure of 12 kg/sq cm by gauge. Assume suitable safe working stresses mentioning the material in view. Give two dimensioned views of the joint.

(Bombay University, 1967)

12. Design and prepare the drawing of a cotter joint for fastening the piston rod to the cross head of a steam engine having a cylinder diameter of 30 cm and a mean pressure of 10 kg/sq cm gauge. The rod end fitted in the cross-head is tapered. The thickness of the cotter is to be 0 3 times the smaller diameter of the rod at cotter hole

The safe stresses are

Tensile	520 kg/sq cm
Shear	420 kg/sq cm
Bearing	840 kg/sq cm.

(Sardar Patel University, 1968)

13. Design and sketch a turnbuckle with right and left hand threaded pull rods to take a pull of [illegible] tonnes

$f_t = 800$ kg/sq cm; $f_s = 400$ kg/sq cm; $f_c = 900$ kg/sq cm.

(Bombay University, 1969)

14. A cottered bolt is subjected to a tensile load of 6,000 kg. The thickness of the cotter is approximately equal to 0 3 × diameter of the bolt. Assume the following values of the permissible stresses

$f_t = 600$/kg sq cm

$f_s = 450$ kg/sq cm

$f_c = 900$ kg/sq cm.

Suggest the diameter of the bolt and section of the cotter

(Sardar Patel University, 1970)

(d) Twisting moment varies in magnitude and direction.

(e) Combined bending and twisting.

Careful study of the straining action will lead to the proper choice of the working stresses when the material of construction is known.

7-3. Design of axles:

The axles are made of St 50·11 (the minimum ultimate strength of the material is 50 kg/sq mm and the average carbon content is 0·11%).

The axle is subjected to transverse loads. We determine the reactions at the supports due to the applied load and calculate the maximum bending moment on the axle either graphically or mathematically.

If M be the maximum bending moment acting on the axle of diameter d, then

$$M = \frac{\pi}{32} d^3 \times f = \frac{d^3 f}{10{\cdot}2}.$$

$$\therefore \quad f = \frac{10{\cdot}2M}{d^3} \qquad \text{(i)}$$

$$\text{Also} \quad d = \sqrt[3]{\frac{10{\cdot}2M}{f}} = 2{\cdot}169 \sqrt[3]{\frac{M}{f}} \qquad \text{(ii)}$$

where f equals the fibre stress in tension and compression. We have determined the diameter of the axle where the maximum bending moment occurs. The diameter at any other point can be obtained from the maximum diameter if it is remembered that the diameters at any two points should be proportional to the cube roots of the bending moments at those points. If the section is not circular, it is still convenient to design a cylindrical axle and then to replace the cylindrical section by equivalent section of any other form. For the rotating axle the cylindrical form is the only form which gives equal strength in all directions.

If the axle were to be cylindrical and hollow, its diameter can be obtained by the equation

$$D = 2{\cdot}169 \sqrt[3]{\frac{M}{f(1-k^4)}} \qquad \text{(iii)}$$

where D = outside diameter of the cylindrical axle and

$$k = \frac{\text{inside diameter}}{\text{outside diameter}}.$$

tortion of shaft when it is subjected to machining operations such as slotting or milling. Machine shafts are generally forged.

The following are the common sizes of transmission shafts (Dimensions are in millimetres.):

25	50	90	160	260	360	460
30	55	100	180	280	380	480
35	60	110	200	300	400	500
40	70	125	220	320	420	
45	80	140	240	340	440	

The lengths of shafting will not exceed 7 metre on account of transport difficulties.

For shafting purchased under definite physical specifications the allowable tensile stress is taken equal to 60 per cent of the elastic limit in tension but not more than 36 per cent of its ultimate strength. The maximum allowable shear stress is taken equal to 50 per cent of the allowable tensile stress which gives a design stress equal to 30 per cent of the elastic limit in tension and not to exceed 18 per cent of the ultimate strength in tension. In shafts with keyways the allowable stresses are 75% of the values just given

The ultimate tensile strength for commercial steel shafting may range from 3,150 kg/sq cm for hot rolled and turned low carbon steel to upwards of 4,900 kg/sq cm for cold finished low carbon steel. Corresponding stresses at the elastic limit would be about 1,575 kg/sq cm and 3,850 kg/sq cm.

The following stresses are usually adopted for the design of a shaft.

f_t = 1,120 kg/sq cm maximum permissible tensile or compressive stress

f_s = 560 kg/sq cm maximum permissible shear stress.

Before selecting the proper value of the working stresses, the careful study of the straining action should be made The more frequently occuring straining actions are as follows:

(a) Straining action chiefly a bending moment constant in direction during the revolution of a shaft. The flexural stress on each fibre ranges from $+f_t$ to $-f_t$ in each revolution of the shaft.

(b) Twisting moment constant in direction, stress varies from f_s to 0 infrequently.

(c) Twisting moment constant in direction but f_s varies in magnitude; stress ranges from f_s to 0 frequently.

on which guide roller is mounted. The permissible stress in the axle material is limited to 1,000 kg/sq cm. *Ans.* 55 *mm.*

Note: To reduce the deflection and hence the flexural stresses, the guide pulley should be mounted near the supports. It is kept in position on the axle by means of positioning collars as shown in figure. The diameter of the positioning collar may be taken as twice the diameter of the axle.

3. *An axle is supported on two end journals and carries a load of 7,000 kg at a point 60 cm from one journal and 120 cm from the other journal. Determine the diameter of the axle at the point of application of load. The stress allowed is 600 kg/sq cm.* *Ans.* 16 *cm.*

4. *A pair of wheels of a railway waggon carries a load of 6 tonnes, 4 on one wheel and 2 on the other, centre of the axle boxes are 190 cm and gauge of rails is 150 cm. Determine the diameter of the axle at the wheel. Safe stress 7·5 kg/sq mm.* *Ans.* 105 *mm.*

7-4. Design of shafts on the basis of strength:

Shafts may be subjected to torsional, bending or axial loads or to a combination of these loads.

If the load is torsional, the principal stress induced is shear. When a pure torque acts on a circular shaft, the relation between applied moment and internal stress is given by the equation

$$\frac{T}{J}=\frac{f_s}{r} \qquad \text{...(i)}$$

where

T = the applied torque

J = polar second moment of cross sectional area about axis of rotation

f_s = torsional shear stress

r = distance from axis to the outermost fibre.

For round solid shafts, equation (i) becomes

$$f_s=\frac{16T}{\pi d^3} \qquad \text{...(ii)}$$

or $$d=1{\cdot}72\sqrt[3]{\frac{T}{f_s}} \qquad \text{...(iii)}$$

For hollow shafts,

$$f_s=\frac{16\,TD}{\pi(D^4-d_i^4)} \qquad \text{...(iv)}$$

The usual value of k is taken as 0·5; then we have

$$D = 2{\cdot}169 \sqrt[3]{1{\cdot}065 \frac{M}{f}} \quad \text{..............................(iv)}$$

The following permissible values are adopted for axle:
Stationary axles: 600 to 1,000 kg/sq cm
Rotating axles: 300 to 600 kg/sq cm

Example:

1. A rope sheave of a balance weight for the cage is supported on an axle 25 cm long. The maximum tension in the rope is 1,500 kg. Determine the diameter of the axle if it is made of St 50·11. Assume the suitable factor of safety.

The ultimate tensile strength of the St 50·11 is 50 kg/sq mm i.e. 5,000 kg/sq cm.

We assume the factor of safety to be 8. The permissible stress will be $= \frac{5000}{8} = 625$ kg/sq cm

Maximum load on the axle $= 2 \times 1500 = 3{,}000$ kg.

Assuming that the load acts at the middle of the span, the maximum bending moment at mid span will be

$$\frac{3000 \times 25}{4} = 18{,}750 \text{ kg cm}$$

If d cm be the diameter of the axle, then

$$18750 = \frac{\pi}{32} d^3 \times 625.$$

$$\therefore \quad d = \sqrt[3]{\frac{18750 \times 32}{625 \times \pi}} = 6{\cdot}73 \text{ cm, we adopt 70 mm.}$$

Exercises:

1. The trunnions of a mixing machine have an effective length of 30 cm and the weight which comes on each trunnion is 1,250 kg. What should be the diameter if the fibre stress is not to exceed 350 kg/sq cm? *Ans.* 8 *cm.*

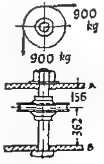


Fig. 7-1

2. Fig. 7-1 shows the arrangement for turning the wire rope of a crane through 90°, by means of a guide roller. The maximum tension in the wire rope is 900 kg. Determine the diameter of the axle

componens, from which the horizontal and vertical components of the reactions may be computed. Horizontal and vertical shear diagrams can be drawn for the load and reaction components and the bending moment diagrams may be drawn for horizontal and vertical loads. The resulting bending moment M in the shaft is found by combining the horizontal and vertical moments M_h and M_v as follows:

$$M = \sqrt{M_h^2 + M_v^2} \quad \ldots\ldots (x)$$

Since rotating shafts are subjected to reversal of stresses, fatigue factor must be used. When bending and torsional loads are subject to variations, the stresses will be greater than those calculated by static conditions. In order to account for increase in stresses, we employ shock factor. Thus, when a shaft is subjected to combined bending and torsion, combined shock factor K_t and fatigue factor K_m are applied respectively to computed twisting moment T and computed bending moment M.

The following table gives the values for K_t and K_m:

	K_t	K_m
Stationary shafts:		
Gradually applied loads	1	1
Suddenly applied loads	1·5 to 2	1·5 to 2
Rotating shafts:		
Gradually applied or steady loads	1	1
Suddenly applied loads, minor shocks only	1·0 to 1·5	1·5 to 2
Suddenly applied loads heavy shocks	1·5 to 3	2·0 to 3

Shafts subjected to compression combined with twisting moment are frequently met with in machinery. Common examples of such shafts are the propeller shafts of ships and shafts for driving worm gears. Occasionally vertical shafts carrying heavy rotating parts are subjected to combined twisting and compression.

Let P be the axial thrust in a shaft of diameter d, which is subjected to a twisting moment T.

Direct compressive stress $= \dfrac{4P}{\pi d^2}$.

Maximum shear stress $= \dfrac{16T}{\pi d^3}$.

Maximum value of the principal stress will be

$$\frac{2}{\pi d^2}\left[P + \sqrt{P^2 + \frac{64T^2}{d^2}}\right] \quad \ldots\ldots (xi)$$

$$f_s = \frac{16T}{\pi D^3 (1 - k^4)}$$

or $D = 1{\cdot}72 \sqrt[3]{\dfrac{T}{f_s (1 - k^4)}}$ ....(v)

where

d = diameter of the solid shaft
D = outside diameter of the hollow shaft
d_1 = inside diameter of the hollow shaft
k = ratio of inside diameter to outside diameter.

A rotating shaft carrying pulleys, sheaves, gears and sprockets is subjected to both bending and twistig when used for the transmission of power. In designing shafts subjected to combined bending and torsion, several formulas based upon different theories are advocated by various investigators. IV

The results of various investigators are given below.

(a) The maximum normal stress theory (Rankine's theory): This theory is not considered for ductile materials. It gives good results for brittle materials.

$M_e = \frac{1}{2}\ (M + \sqrt{M^2 + T^2}) = \frac{\pi}{32} d^3 f_t$ . ... (vi)

(b) The maximum shear stress theory (Guest's theory): This theory is considered for ductile materials.

$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} d^3 f_s$ .. ..(vii)

(c) The maximum strain theory (St. Venant's theory):

$M_e = \frac{1}{2}(1 - \nu)\ M + \frac{1}{2}(1 + \nu)\ \sqrt{M^2 + T^2}$... ....(viii)

where ν is Poisson's ratio

When $\nu = 0{\cdot}3$, we have

$M_e = 0{\cdot}35\ M + 0{\cdot}65 \sqrt{M^2 + T^2} = \frac{\pi}{32} d^3 f_t$ . .....(ix)

This theory is often used by designers in European countries

In many instances the bending actions on a shaft are not all in one plane; in such cases the resulting bending moment at a given point on the shaft is obtained as the vector sum for the two component moments at that point.

In order to determine the bending moments at various points on the shafts, all loads can be resolved into horizontal and vertical

When the angle of twist is measured in degrees, the equation (i) is modified to

$$\theta = \frac{584lT}{Gd^4} \quad \ldots\ldots (ii)$$

where l = length of shaft
T = torque on shaft
G = modulus of rigidity
d = shaft diameter.

The permissible value of the angle of twist depends on the particular application. Widely used specifications for shafts state that the angle of twist must not exceed 1 degree for a length of shaft equal to 20 times the diameter. In line shafts 2·5 to 3·5 degrees per metre may be taken as the limiting value. The torsional rigidity of machine tool shaft should be very high. The twist in drive shafts of machine tools should not exceed 0·26 degree per metre.

Lateral rigidity:

In order to maintain proper bearing clearances or gear teeth alignment diameter of the shaft is decided from lateral deflection. The shaft is generally of variable cross section so the usual formulas of mechanics of materials cannot be used and the graphical method is employed. It is desirable to limit the permissible maximum transverse deflection of the line shaft. The ratio of maximum deflection to length of the shaft between bearing supports should not exceed $\frac{1}{1200}$.

7-6. Design of Hollow and Square Shafts:

The material of the solid shaft is not used effectively as the centre portion of the shaft does not contribute much to the torque transmitting capacity of the shaft; therefore, hollow shafts are used for large transmission of power. A hollow shaft has greater strength and stiffness than solid shaft of equal weight. In addition the removal of the core from the centre of large shafts increases their reliability.

The formulas derived for the solid shafts can be used for the hollow shaft by taking the suitable value of the polar second moment of cross section area about axis of rotation in equations

If the maximum shear stress theory is the design criterion, then the maximum shear stress f_s is given by

$$f_s = \frac{2}{\pi d^3}\sqrt{P^2 + \frac{64T^2}{d^2}} \quad \ldots\ldots \quad \text{(xii)}$$

In order to determine d from equations (xi) or (xii), we assume a trial value for d somewhat larger than that required for the twisting moment alone and stresses are evaluated. If the calculated value of the stress does not come near the allowable maximum, make a second calculation and so on.

The above analysis holds good for the short shafts. For long shafts the buckling effects should be considered.

Empirical design of shafts:

In many cases it is not easy to predict the loads on the shafts particularly if the member has more than two bearings, which involves the theory of continuous beams. For these reasons, empirical formula is adopted for the design of shaft in terms of horse power transmitted and the speed of shaft rotation in revolutions per minute.

$$d = \sqrt[3]{\frac{H.P. \times C}{R.P.M.}}\ \text{cm} \quad \ldots\ldots \text{(xiii)}$$

where C is a constant with a value 810 for transmission shafts subjected to torsion only, 1,290 for line shafting subjected to limited bending loads and 2,180 for main or head shafts subjected to heavy bending loads.

The distance between bearings on line shafts and countershafts is limited by excessive deflection and bearing wear

7·5. Design of Shaft on the Basis of Rigidity:

In many cases the shaft is to be designed from rigidity point of view. We should consider torsional rigidity as well as lateral rigidity.

Torsional rigidity:

The angle of twist, θ, in radians for a circular shaft of uniform cross section is given by

$$\theta = \frac{Tl}{JG} \quad \ldots\ldots \quad \text{(i)}$$

The pump efficiency is 80%; therefore the horse power of the driving motor will be $\frac{66 \cdot 6}{0 \cdot 8} = 83 \cdot 5$ h.p. $hp = \frac{2\pi NT}{75 \times 60}$

$$\text{Torque} = \frac{71620 \times \text{h.p.}}{\text{speed}} = \frac{71620 \times 83 \cdot 5}{900} = 6{,}650 \text{ kg cm.}$$

As the maximum torque is 30% more than the average torque, the shaft should be designed for a torque $1 \cdot 3 \times 6650 = 8{,}650$ kg cm. If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 550 = 8650$$

or
$$d = \sqrt[3]{\frac{8650}{500} \times \frac{16}{\pi}} = 4 \cdot 3 \text{ cm; we adopt } 4 \cdot 5 \text{ cm.}$$

2. *The power developed by an automobile engine running at 2,400 r.p.m. is transmitted to a motor car shaft through a 5:1 gearing. The shaft consists of a steel tube 4 cm outside diameter and 3 mm thick. If the maximum stress induced in the tube be 300 kg/sq cm, determine the power developed in the engine cylinders.*

The inside diameter of the tube $= 4 - 2 \times 0 \cdot 3 = 3 \cdot 4$ cm.

Torque acting on the motor car shaft

$$= \frac{\pi}{16}\left[\frac{4^4 - 3 \cdot 4^4}{4}\right] 300$$
$$= 1{,}795 \text{ kg cm.}$$

As the gear ratio is 5:1, the torque acting on the engine shaft will be equal to $\frac{1795}{5} = 360$ kg cm.

$$\text{H.P.} = \frac{T \times N}{71620} = \frac{360 \times 2400}{71620} = 12 \text{ h.p.}$$

3. *A cast iron pulley of 900 mm diameter overhangs the nearest bearing by 240 mm. It is driven from below by an induction motor. The belt ends are vertical and parallel. The tension on the tight side of the belt is 350 kg and that on slack side of the belt is 150 kg. The pulley weighs 80 kg. Suggest the suitable diameter for the shaft of the pulley if the value of the permissible shear stress is limited to 600 kg/sq cm.*

The shaft is subjected to bending and twisting. Bending load on the shaft is due to its own weight and due to belt pull. As the belt ends are parallel and vertical, the resultant bending load on the shaft will be $350 + 150 + 80 = 580$ kg.

$$\frac{T}{J} = \frac{f_s}{r} \text{ and } \theta = \frac{Tl}{JG}.$$

For a hollow shaft to be equal in torsional strength to a solid shaft their resisting moments must be equal. If both shafts are made of the same material and are stressed to the same intensity, then, if d is the diameter of the solid shaft, we have

$$\frac{\pi}{16}\left(\frac{D^4 - d_i^4}{D}\right) = \frac{\pi}{16} d^3.$$

$\therefore$ $$D^3 = \frac{d^3}{1-k^4} \text{ where } k = \frac{d_i}{D}.$$

If $$k = 0{\cdot}5,$$

$$D = 1{\cdot}022\, d \quad \ldots \text{(i)}$$

From equation (i), we see that a hollow shaft having inside diameter one-half of outside diameter would be only 2·2% larger than a solid shaft of equal strength but there would be a reduction in weight of 22·6%. If both shafts are of the same diameter, the hollow shaft would be 93 7% as strong as the solid shaft, yet the reduction in weight would be 25%.

According to St. Venant the moment of resistance to twisting of square shaft of side a is given by

$$T = \tfrac{2}{9} a^3 f_s \quad \ldots \text{(ii)}$$

Note: For detailed discussion for the torque transmitting capacity of non-circular shafts, standard text books on Advanced Strength of Materials should be referred

Examples:

1. A circulating water pump of a condenser is of the centrifugal type and connected directly to a three phase induction motor running at 900 r.p.m. The pump delivers 30,000 litres per minute against the dynamic head of 10 metre. Determine the horse power of the motor required to drive the pump, if the pump efficiency is 80%. If the maximum torque on the motor shaft is 30% more than the average torque, determine the diameter of the motor shaft if the permissible shear stress in the shaft material is not to exceed 550 kg/sq cm.

As the quantity of water delivered is 30,000 litres/min = 30 cu metre against a dynamic head of 10 metre, the water horse power will be

$$\frac{30 \times 10 \times 1000}{75 \times 60} = 66{\cdot}6 \text{ h.p.}$$

According to maximum shear stress theory, we get

$$500 = \frac{1}{2}\sqrt{\left(\frac{1730}{d^2}\right)^2 + \left[\frac{9750}{d^3}\right]^2 \times 4}$$

Thus by solving the above equation we get $d = 3$ cm.

5. *The uniform shaft shown in fig. 7-2 carries belt pulleys at A and B with vertical belts. It is supported in self-aligning bearings at C and D. The shaft transmits 10 h.p. at 400 r.p.m. The tension on the tight side of belt A is 200 kg and that on the tight side of belt B is 90 kg. Pulley A weighs 20 kg and pulley B 40 kg. Estimate a suitable diameter for the shaft, adopting a working shear stress of 420 kg/sq cm. The equivalent torque formula $T_e = \sqrt{M^2 + T^2}$ may be used. Compute suitable lengths for the bearings at C and D if the bearing pressure must not exceed 10 kg/sq cm.*

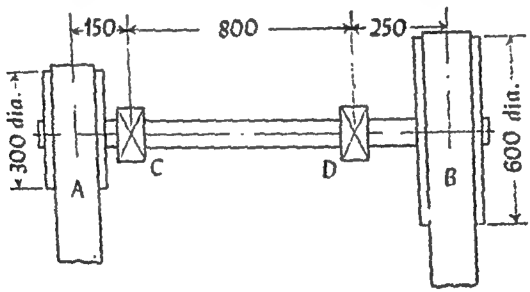


FIG. 7-2

The shaft transmits 10 h.p. at 400 r.p.m.

$\therefore$ Torque acting on the shaft $= \dfrac{71620 \times \text{h.p.}}{\text{speed}}$

$$= \frac{71620 \times 10}{400} = 1{,}792 \text{ kg cm.}$$

Net effective tension at pulley $A = \dfrac{1792}{15} = 119{\cdot}5$ kg.

Tension on tight side of belt A is 200 kg.

$\therefore$ Tension on slack side of belt $A = 200 - 119{\cdot}5 = 80{\cdot}5$ kg.

Total belt pull vertically downwards $= 200 + 80{\cdot}5 = 280{\cdot}5$ kg.

Weight of pulley $A = 20$ kg.

Total bending load on shaft at $A = 280{\cdot}5 + 20 = 300{\cdot}5$ kg.

Net effective tension at pulley $B = \dfrac{1792}{30} = 59{\cdot}7$ kg.

Bending moment on the shaft where it enters the bearing

$$= 580 \times 24$$
$$= 13{,}920 \text{ kg cm.}$$

The shaft is subjected to twisting moment due to power transmitted. The twisting moment acting on the shaft is equal to $(350 - 150)\ 45 = 9{,}000$ kg cm.

From the data given it seems that the maximum shear stress is the design criterion.

Equivalent twisting moment $T_e = \sqrt{13920^2 + 9000^2}$
$= 16{,}600$ kg cm.

If d cm be the diameter of the shaft, then

$$\frac{\pi}{16} d^3\ 600 = 16600$$

or
$$d = \sqrt[3]{\frac{16600}{600} \times \frac{16}{\pi}}$$
$$= 5{\cdot}21 \text{ cm; we adopt 55 mm.}$$

4. *A cold drawn monel propeller shaft for a launch is to transmit 400 h.p. at 1,500 r.p.m. without being subjected to significant bending moment and the slenderness ratio is less than 40. The efficiency of the propeller is 70% at 30 knots. (1 knot = 1·85 km/hour.) Determine the shaft diameter based on maximum shear stress theory. Permissible shear stress may be taken as 500 kg/sq cm.*

$$\text{Torque} = \frac{71620 \times 400}{1500} = 1{,}910 \text{ kg cm.}$$

$$\text{Velocity} = \frac{30 \times 1{\cdot}85 \times 1000}{60 \times 60} = 15{\cdot}4 \text{ metre/sec.}$$

If F be the axial load on the shaft, then

$$\frac{F \times 15{\cdot}4}{75} = 400 \times 0{\cdot}7 \quad \text{or} \quad F = \frac{400 \times 0{\cdot}7 \times 75}{15{\cdot}4} = 1{,}360 \text{ kg.}$$

If d cm be the diameter of the solid shaft, then

$$f_c = \frac{1360}{\frac{\pi}{4} d^2} = \frac{1730}{d^2} \text{ kg/sq cm.}$$

$$f_s = \frac{1910}{\frac{\pi}{16} d^3} = \frac{9750}{d^3} \text{ kg/sq cm.}$$

6. *Three pulleys A, B and C are mounted on a shaft and are at distances of 1,200 mm, 2,100 mm and 2,700 mm respectively from the left hand bearing. The bearings are 3,600 mm apart. Pulley A is 50 cm, B 75 cm and C 37·5 cm in diameter. A power unit supplies 20 h.p. to A and machinery takes 12 h.p. from B and 8 h.p. from C. A horizontal drive is arranged to A, while the drive from B has to be vertically downwards. The drive from C is taken off at 45° to drive A and in a downward direction. The speed of the shaft is 200 r.p.m. and the allowable shear stress in the shaft is 320 kg/sq cm. The angle of lap of belt on pulley is 180° in each case, and the coefficient of friction between belt and pulley is 0·32. Obtain the shaft diameter.*

Fig. 7-3(*a*) shows the arrangement of pulleys on the shaft 360 cm long. *D* and *E* are the bearings.

The shaft rotates at 200 r.p.m.

$$\text{Torque on shaft at } A = \frac{71620 \times 20}{200} = 7{,}162 \text{ kg cm.}$$

$$\text{Torque on pulley } B = \frac{71620 \times 12}{200} = 4{,}297 \text{ kg cm.}$$

$$\text{Torque on pulley } C = \frac{71620 \times 8}{200} = 2{,}865 \text{ kg cm.}$$

The torque diagram for the shaft is shown in fig. 7-3(*b*). The angle of lap of the belt on each pulley is 180° and the coefficient of friction between the belt and the pulley is 0·32.

If T_1 and T_2 be the tensions in the tight and slack sides of the belt respectively, then

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0{\cdot}32\pi} = 2{\cdot}73.$$

Pulley A:

Torque = 7,162 kg cm; radius = 25 cm.

$\therefore \quad T_1 - T_2 = \dfrac{7162}{25} = 286$ kg. But $T_1 = 2{\cdot}73\ T_2$.

$\therefore \quad 2{\cdot}73 T_2 - T_2 = 286$ kg.

$\therefore \quad T_2 = 166$ kg; $T_1 = 166 + 286 = 452$ kg.

$T_1 + T_2 = 452 + 166 = 618$ kg.

Pulley B:

Torque = 4,297 kg cm; radius = 37·5 cm.

$\therefore \quad T_1 - T_2 = \dfrac{4297}{37{\cdot}5} = 114$ kg. But $T_1 = 2{\cdot}73 T_2$.

Tension on tight side of belt B is 90 kg.

$\therefore$ Tension on slack side of belt $B = 90 - 59.7 = 30.3$ kg.

Total belt pull vertically downwards $= 90 + 30.3 = 120.3$ kg.

Weight of pulley $B = 40$ kg.

$\therefore$ Total bending (vertically downwards) load on shaft at B equals $90 + 30.3 + 40 = 160.3$ kg.

The shaft is subjected to twisting moment due to power transmitted and is subjected to bending due to vertical belt pulls and weight of the pulleys. The twisting moment is constant from A to B, while the bending moment varies over the entire length. The bending moment will be maximum either at C or at D. Therefore, the critical section for the design will be either at C or at D.

Bending moment at $C = 300.5 \times 15 = 4{,}500$ kg cm.

Bending moment at $D = 160.3 \times 25 = 4{,}007$ kg cm.

The bending moment at C is maximum, therefore, equivalent twisting moment at C is to be determined in order to determine the shaft diameter.

$T_e = \sqrt{4500^2 + 1792^2} = 4{,}830$ kg cm.

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 420 = 4830$$

or $$d = \sqrt[3]{\frac{4830 \times 16}{420 \times \pi}} = 3.9 \text{ cm, we adopt 4 cm.}$$

In order to determine the suitable length for the bearings at C and D, we calculate the reactions at the bearings by taking moments about C and D. We denote reactions at C and D by R_C and R_D respectively.

$R_C \times 80 = 300 \times 95 - 160.3 \times 25$

$R_D \times 80 = 160.3 \times 105 - 300 \times 15$

$\therefore$ $R_C = 304$ kg; $R_D = 156.3$ kg.

If l_1 be the length of bearing at C, then

$$l_1 = \frac{304}{4 \times 10} = 7.6 \text{ cm, we adopt 8 cm}$$

If l_2 be the length of the bearing at D, then

$$l_2 = \frac{156.3}{4 \times 10} = 3.95 \text{ cm; we adopt 4.5 cm.}$$

$\therefore \quad 2{\cdot}73T_2 - T_2 = 114$ kg.

$\therefore \quad T_2 = 66$ kg; $\quad T_1 = 66 + 114 = 180$ kg.

$\therefore \quad T_1 + T_2 = 180 + 66 = 246$ kg.

Pulley C:

Torque $= 2{,}865$ kg cm; radius $= 18{\cdot}75$ cm.

$\therefore \quad T_1 - T_2 = \dfrac{2865}{18{\cdot}75} = 153$ kg. $\quad$ But $T_1 = 2{\cdot}73T_2$.

$\therefore \quad 2{\cdot}73T_2 - T_2 = 153$ kg.

$\therefore \quad T_2 = 88{\cdot}5$ kg; $\quad T_1 = 88{\cdot}5 + 153 = 241{\cdot}5$ kg.

$\therefore \quad T_1 + T_2 = 88{\cdot}5 + 241{\cdot}5 = 330$ kg.

The shaft is subjected to twisting moment due to power transmitted and to bending due to the pull in the belts. The weight of the pulley is neglected. As all loads are not parallel, we resolve them into two components along two perpendicular directions — horizontal and vertical. Fig 7-3(*c*) shows the bending loads on the shaft along these two directions.

The horizontal loading on the shaft is as under:

At pulley *A*, 618 kg, at pulley *B*, 0 kg and at pulley *C*, 330 cos 45° = 234 kg.

The vertical loading on the shaft will be 0 kg at *A*, 246 kg at *B* and 234 kg at *C*.

Fig. 7-3(*d*) shows bending moment diagrams for the horizontal and vertical loading and resultant loading.

Horizontal loading:

The horizontal component of reaction at *D* is determined by taking moment for all the horizontal forces about *E*.

$$R_{DH} = \frac{618 \times 240 + 234 \times 90}{360} = 470{\cdot}5 \text{ kg.}$$

Similarly,

$$R_{EH} = \frac{618 \times 120 + 234 \times 270}{360} = 381{\cdot}5 \text{ kg.}$$

Bending moment for horizontal loading:

B.M. at $D = 0$

B.M. at $A = 470{\cdot}5 \times 1{\cdot}2 = 564{\cdot}6$ kg metre

B.M. at $B = 470{\cdot}5 \times 2{\cdot}1 - 618 \times 0{\cdot}9 = 431{\cdot}8$ kg metre

B.M. at $C = 381{\cdot}5 \times 0{\cdot}9 = 343{\cdot}4$ kg metre

B.M. at $E = 0$.

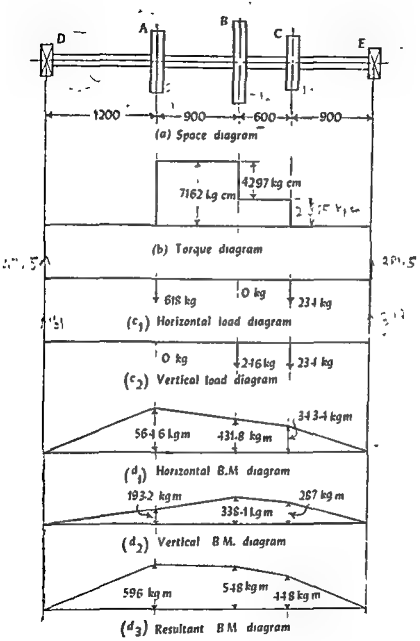

D
A
B
C
E
1200
900
600
900
(a) Space diagram
4297 kg cm
7162 kg cm
(b) Torque diagram
618 kg
0 kg
234 kg
(c_1) Horizontal load diagram
0 kg
246 kg
234 kg
(c_2) Vertical load diagram
343·4 kgm
564·6 kgm
431·8 kgm
(d_1) Horizontal B.M diagram
193·2 kgm
287 kgm
338·1 kgm
(d_2) Vertical B M. diagram
596 kgm
548 kgm
448 kgm
(d_3) Resultant B M diagram

FIG. 7-3

diameter of the gear shaft if the allowable shear stress is 300 kg/sq cm. This value allows for the weakening effect of the key way.

$$\text{Torque on the gear shaft} = \frac{71620 \times 100}{140}$$

$$= 51{,}200 \text{ kg cm.}$$

$$\text{Tangential tooth load } F = \frac{\text{torque}}{\text{radius of pitch circle}} = \frac{51200}{60}$$

$$= 854 \text{ kg.}$$

Pressure angle $= 20°$.

$$\therefore \quad \text{Maximum tooth load } Q = \frac{F}{\cos 20°} = \frac{854}{0{\cdot}9397} = 908 \text{ kg.}$$

The force F is the driving force and puts the torque on the gear shaft while Q produces bending effect. The maximum tooth load Q is inclined at an angle of 20° with the horizontal. The weight of the gear will act downwards at the centre of the gear. For the bending of the shaft the greatest possible resultant of maximum tooth load and weight must be considered. The resultant of these two forces is found out either by a formula or by a parallelogram of forces. The angle between the two forces is 70°. The resultant of these two forces is

$$R = \sqrt{908^2 + 200^2 + 2 \times 908 \times 200 \cos 70°} = 1{,}000 \text{ kg.}$$

Note: The bending moment on the gear shaft $= \dfrac{R \times l}{4}$ where l is the length of the shaft between the bearings. This formula gives a higher bending moment as a result we obtain a stronger shaft. In the above formula the reactions are assumed to act at the centres of the bearings.

$$\text{The maximum bending moment on the shaft} = \frac{1000 \times 40}{4}$$

$$= 10{,}000 \text{ kg cm.}$$

The twisting moment at the middle of the shaft $= 51{,}200$ kg cm. The shaft will be under bending and shear stresses, and the former will be alternating in nature. The equivalent twisting moment by Guest formula (assuming the failure of the shaft due to maximum shear stress) will give us

$$T_e = \sqrt{51200^2 + 10000^2} = 52{,}000 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 300 = 52000$$

or

$$d = \sqrt[3]{\frac{52000 \times 16}{300 \times \pi}} = 9{\cdot}59 \text{ cm; we adopt 10 cm.}$$

Vertical loading:

The vertical component of reaction at D is determined by taking moment for all the vertical forces about E.

$$R_{DV} = \frac{234 \times 90 + 246 \times 150}{360} = 161 \text{ kg.}$$

Similarly,

$$R_{EV} = \frac{234 \times 270 + 246 \times 210}{360} = 319 \text{ kg.}$$

Bending moment for vertical loading:

B.M. at $D = 0$

B.M. at $A = 161 \times 1{\cdot}2 = 193{\cdot}2$ kg metre

B.M. at $B = 161 \times 2{\cdot}1 = 338{\cdot}1$ kg metre

B.M. at $C = 319 \times 0{\cdot}9 = 287$ kg metre

B.M. at $E = 0$.

The resultant bending moments for salient points are calculated as follows:

B.M. at $D = 0$

B.M. at $A = \sqrt{564{\cdot}6^2 + 193{\cdot}2^2} = 596$ kg metre

B.M. at $B = \sqrt{431{\cdot}8^2 + 338{\cdot}1^2} = 548$ kg metre

B.M. at $C = \sqrt{343{\cdot}4^2 + 287^2} = 448$ kg metre.

Thus, from the resultant bending moment values and the twisting moments we see that the critical section is at the pulley A where the maximum values of the bending moment and twisting moment occur.

B.M. $= 596 \times 100 = 59{,}600$ kg cm.

T.M. $= 7{,}162$ kg cm.

$\therefore$ Equivalent twisting moment $= \sqrt{59600^2 + 7162^2}$

$= 60{,}100$ kg cm.

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16}\, d^3 \times 320 = 60100$$

or $$d = \sqrt[3]{\frac{60100 \times 16}{\pi \times 320}} = 9{\cdot}87 \text{ cm; we adopt } 10 \text{ cm.}$$

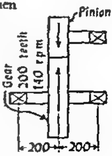


FIG. 7-4

7. *A cast gear wheel (fig. 7-4) is driven, by a pinion and transmits 100 h.p. at 140 r.p.m. The gear has 200 machine cut teeth having pressure angle of 20°. Gear weighs 200 kg. The pitch circle diameter of the gear is 120 cm. Determine the*

diameter of the gear shaft if the allowable shear stress is 300 kg/sq cm. This value allows for the weakening effect of the key way.

$$\text{Torque on the gear shaft} = \frac{71620 \times 100}{140}$$

$$= 51{,}200 \text{ kg cm.}$$

$$\text{Tangential tooth load } F = \frac{\text{torque}}{\text{radius of pitch circle}} = \frac{51200}{60}$$

$$= 854 \text{ kg.}$$

Pressure angle $= 20°$.

$$\therefore \text{ Maximum tooth load } Q = \frac{F}{\cos 20°} = \frac{854}{0{\cdot}9397} = 908 \text{ kg.}$$

The force F is the driving force and puts the torque on the gear shaft while Q produces bending effect. The maximum tooth load Q is inclined at an angle of 20° with the horizontal. The weight of the gear will act downwards at the centre of the gear. For the bending of the shaft the greatest possible resultant of maximum tooth load and weight must be considered. The resultant of these two forces is found out either by a formula or by a parallelogram of forces. The angle between the two forces is 70°. The resultant of these two forces is

$$R = \sqrt{908^2 + 200^2 + 2 \times 908 \times 200 \cos 70°} = 1{,}000 \text{ kg.}$$

Note: The bending moment on the gear shaft $= \frac{R \times l}{4}$ where l is the length of the shaft between the bearings. This formula gives a higher bending moment as a result we obtain a stronger shaft. In the above formula the reactions are assumed to act at the centres of the bearings.

$$\text{The maximum bending moment on the shaft} = \frac{1000 \times 40}{4}$$

$$= 10{,}000 \text{ kg cm.}$$

The twisting moment at the middle of the shaft = 51,200 kg cm. The shaft will be under bending and shear stresses, and the former will be alternating in nature. The equivalent twisting moment by Guest formula (assuming the failure of the shaft due to maximum shear stress) will give us

$$T_e = \sqrt{51200^2 + 10000^2} = 52{,}000 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 300 = 52000$$

or

$$d = \sqrt[3]{\frac{52000 \times 16}{300 \times \pi}} = 9{\cdot}59 \text{ cm; we adopt 10 cm.}$$

8. *A steel spindle 15 mm diameter and 60 cm long transmits 2 h.p. at 1,000 r.p.m. It is found that the angle of twist is excessive. It is desired to reduce the angle of twist to one-quarter of its original value by fitting a sleeve of different material over the spindle and connecting the two rigidly at the ends. Determine the outside diameter of the sleeve.*

For the spindle, $G = 0{\cdot}84 \times 10^6$ *kg/sq cm.*

For the sleeve, $G = 0{\cdot}35 \times 10^6$ *kg/sq cm.*

Original spindle:

$$\text{Torque transmitted} = \frac{71620 \times \text{h.p.}}{\text{speed}} = \frac{71620 \times 2}{1000} = 143{\cdot}2 \text{ kg cm.}$$

$$\text{Angle of twist in radian} = \frac{Tl}{GJ} = \frac{143{\cdot}2 \times 60}{0{\cdot}84 \times 10^6 \times \frac{\pi}{32}(1{\cdot}5)^4}$$

$$= 2{\cdot}06 \times 10^{-2} \text{ radian.}$$

As the angle of twist is to be reduced to one-quarter of its original value, the angle of twist for the sleeved shaft will be $\frac{2{\cdot}06 \times 10^{-2}}{4} = 0{\cdot}0052$ radian. The applied torque will be shared by the original spindle and the sleeve. The angle of twist of both will be the same. As the angle of twist is proportional to the torque transmitted and as the angle of twist is reduced to $\frac{1}{4}$th of its original value, the torque transmitted will be reduced to one-fourth of the original value. The torque transmitted by spindle will be $\frac{143{\cdot}2}{4} = 35{\cdot}8$ kg cm.

The torque transmitted by sleeve $= 143{\cdot}2 - 35{\cdot}8 = 107{\cdot}4$ kg cm.

Angle of twist of sleeve $= 0{\cdot}0052$ radian.

$$\therefore \quad 0{\cdot}0052 = \frac{107{\cdot}4 \times 60}{0{\cdot}35 \times 10^6 \times J}, \text{ or } J = 3{\cdot}55 \text{ cm}^4.$$

If D cm be the outside diameter of the sleeve, then

$$\frac{\pi}{32}(D^4 - 1{\cdot}5^4) = 3{\cdot}55.$$

$\therefore \quad D = 2{\cdot}54$ cm; we adopt $2{\cdot}6$ cm.

The shear stresses induced in the spindle and sleeve will be $54{\cdot}5$ and $35{\cdot}3$ kg/sq cm respectively.

Variable stress component due to bending

$$= \frac{[45900 - (-15300)]}{2d^3}$$

$$= \frac{30600}{d^3} \text{ kg/sq cm.}$$

We assume that the endurance limit is half the ultimate tensile strength i.e. $5600 \times 0{\cdot}5 = 2{,}800$ kg/sq cm. We assume size correction factor as 0·85 and surface correction factor as 0·88.

The equivalent normal stress is $f_a + \frac{f_y}{f_e} \cdot \frac{K_f \times f_m}{BC}$

$$f_e \text{ normal} = \frac{15300}{d^3} + \frac{4000}{2800} \times \frac{1{\cdot}7 \times 30600}{0{\cdot}85 \times 0{\cdot}88} \times \frac{1}{d^3}$$

$$= \frac{115300}{d^3} \text{ kg/sq cm.}$$

After determining the value of the equivalent normal stress, let us determine the value of the equivalent shear stress.

$$\text{Maximum shear stress} = \frac{16 \times 1500}{\pi d^3} = \frac{7635}{d^3} \text{ kg/sq cm.}$$

$$\text{Minimum shear stress} = \frac{16 \times 500}{\pi d^3} = \frac{2545}{d^3} \text{ kg/sq cm.}$$

$$\text{Mean shear stress} = \frac{7635 + 2545}{2d^3} = \frac{5090}{d^3} \text{ kg/sq cm.}$$

$$\text{Variable stress component} = \frac{7635 - 2545}{2d^3} = \frac{2545}{d^3} \text{ kg/sq cm.}$$

We assume that yield strength in shear is equal to 0·6 of the yield strength in tension. We assume $A = 0{\cdot}6$ and the size factor as 0·85 and surface factor as 0·88.

$$f_e \text{ shear} = \frac{5090}{d^3} + \frac{0{\cdot}6 \times 4000}{5600 \times 0{\cdot}6 \times 0{\cdot}5} \times \frac{1{\cdot}4 \times 2545}{0{\cdot}85 \times 0{\cdot}88\, d^3}$$

$$= \frac{12390}{d^3} \text{ kg/sq cm.}$$

$$\text{Allowable shear stress} = \frac{4000 \times 0{\cdot}5}{1{\cdot}8} = 1{,}100 \text{ kg/sq cm.}$$

Design note:

It should be noted that we have used a factor 0·6 for pure torsional shear to determine the yield strength but we have used a factor 0·5 for combined stress shear, according to maximum shear theory of failure.

kg cm. The shock and fatigue factor for twisting is 1·5, the shaft should be designed for a twisting moment of $1\cdot5 \times 20000 = 30{,}000$ kg cm.

Equivalent bending moment, according to principal stress theory (Rankine's theory), is given by

$$M_e = \tfrac{1}{2}\,\{M + \sqrt{M^2 + T^2}\} = \tfrac{1}{2}\,\{28400 + \sqrt{28400^2 + 30000^2}\}$$
$$= 35{,}000 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{32}\,d^3 \times 1150 = 35000$$

$$\therefore \qquad d = \sqrt[3]{\frac{35000}{1150} \times \frac{32}{\pi}} = 6\cdot77 \text{ cm.}$$

Equivalent twisting moment, according to maximum shear stress theory (Guest's theory), is given by

$$T_e = \sqrt{M^2 + T^2} = \sqrt{28400^2 + 30000^2} = 41{,}400 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16}\,d^3 \times 560 = 41400$$

$$\therefore \qquad d = \sqrt[3]{\frac{41400 \times 16}{560 \times \pi}} = 7\cdot22 \text{ cm.}$$

We adopt 7·5 cm diameter shaft.

10. A pulley is keyed to a shaft mid-way between two anti-friction bearings. The bending moment at the pulley varies from 1,500 kg cm clockwise to 4,500 kg cm anticlockwise as the torsional moment in the shaft varies from 500 to 1,500 kg cm. The shaft is made of cold drawn steel having an ultimate strength of 5,600 kg/sq cm and a yield strength of 4,000 kg/sq cm. The frequency of the variation of the loads is the same as the shaft speed. Determine the required diameter of the shaft for an indefinite life. The stress concentration factor for the keyway in bending and torsion may be taken as 1·7 and 1·4 respectively. Use a design factor of 1·8.

Let d cm be the diameter of the solid shaft.

Let us first of all determine the equivalent normal stress.

$$\text{Maximum stress due to bending} = \frac{4500 \times 32}{\pi d^3} = \frac{45900}{d^3} \text{ kg/sq cm.}$$

$$\text{Minimum stress due to bending} = -\frac{1500 \times 32}{\pi d^3} = -\frac{15300}{d^3} \text{ kg/sq cm.}$$

$$\text{Mean stress due to bending} = \frac{(45900 - 15300)}{2d^3} = \frac{15300}{d^3} \text{ kg/sq cm.}$$

20 h.p. is supplied at the first load, a power of 12 h.p. is taken off at the second load and the remainder at the third load. Determine the suitable diameter for the shaft if the permissible shear stress is limited to 500 kg/sq cm. Draw the bending moment and torque diagrams for the shaft.

6. *An automobile transmission shaft is required to transfer 60 h.p. at 500 r.p.m. The outside diameter must not exceed 5 cm and the maximum shear stress is not to exceed 800 kg/sq cm. Compare the weights of hollow and solid shafts which would just meet these requirements.*

Ans. 2·4 : 1.

7. *A 20 h.p., 1,400 r.p.m. motor drives a centrifugal pump through a single set of 3:1 reduction gear. The load may be considered to be suddenly applied with minor shocks for which the combined shock and fatigue factor may be taken as 1·5. Determine the diameters of the pump and motor shafts if the permissible stress intensity is not to exceed 450 kg/sq cm.*

Ans. 4 cm; 3 cm.

8. *The shaft of a 40 h.p., 800 r.p.m. direct current motor is 75 cm from centre to centre of the bearings. The magnetic pull on the armature is equivalent to a uniformly distributed total load of 625 kg distributed over middle one-third of the length of the shaft. Suggest the suitable diameter for the shaft if the permissible value of the shear stress in the shaft material is not to exceed 420 kg/sq cm. Also, calculate the maximum value of the tensile stress in the shaft.*

Ans. 5 cm; 860 kg/sq cm.

9. *A hoist with 120 cm diameter drum lifts a cage by means of a wire rope that winds on the drum. The drum is driven by an electric motor through a double reduction gear. Determine the diameter of shaft if the permissible shear stress is limited to 350 kg/sq cm.*

The maximum acceleration of the cage is limited to 1 metre/sec^2. The weight of the system is given as follows:

	Speed R.P.M.	Weight kg	Radius of gyration, cm
Rotor of motor and pinion	1,440	226	11·5
Intermediate gear	340	363	23
Low speed gear	75	900	69
Drum and shaft	75	1,140	61
Cage	—	1,800	—
Rope rising	—	450	—

Ans. 55 mm.

$$\therefore \quad 1100 = \sqrt{\left(\frac{115300}{2d^3}\right)^2 + \left(\frac{12390}{d^3}\right)^2} = \frac{69000}{d^3}$$

or $$d = \sqrt[3]{\frac{69000}{1100}} = 3{\cdot}98 \text{ cm; we adopt 45 mm diameter.}$$

Exercises:

1. Determine the diameter of a solid steel shaft to transmit 120 h.p. at a speed of 290 r.p.m. if the angle of twist per metre length is not to exceed 0·1°. The modulus of rigidity of the shaft material is 0·84 × 10⁶ kg/sq cm. *Ans.* 12 cm.

2. The shaft running at 120 r.p.m. transmits 625 h.p. The working conditions to be satisfied by the shaft are (a) the shear stress must not exceed 560 kg/sq cm, (b) the angle of twist must not be more than 1° on a length of 16 diameters. The modulus of rigidity is 0·85 × 10⁶ kg/sq cm. *Ans.* 16 cm.

3. A shaft 90 cm between bearings supports a 60 cm pulley 30 cm to the right of the left hand bearing and the belt drives a pulley directly below. Another pulley 45 cm in diameter is located 20 cm to the left of the right hand bearing and the belt is driven from a pulley horizontally to the right. The angle of contact for both the pulleys is 180° and the tension ratio is 2·2. The maximum tension in the belt on a 60 cm diameter pulley is 225 kg. Determine the suitable diameter for a solid shaft allowing $f_t = 630$ kg/sq cm and $f_s = 420$ kg/sq cm. *Ans.* 5 cm

4. A mild steel shaft transmitting 20 h.p. at 280 r.p.m. is supported on two bearings 75 cm apart and has keyed to it a pulley and a gear. The power is supplied to the shaft at a pulley of 45 cm diameter and the belt ends are horizontal and the tension ratio is 2. The pulley is keyed at a distance 20 cm to the right of the left hand bearing. A 15 cm diameter 14½° involute gear, located at 15 cm to the right of the right hand bearing, delivers power to a gear directly below the shaft. Calculate the diameter of the shaft assuming working stresses to be 700 kg/sq cm in tension and 560 kg/sq cm in shear. Draw the bending moment diagrams

5. A shaft 150 cm long is supported at the ends by journal bearings and rotates at 125 r.p.m. A vertical load of 800 kg is applied 30 cm from the left hand bearings, a load of 1,200 kg acting downward and forward at an angle of 45° with the horizontal is applied midway between bearings and a load of 1,000 kg acting downward and forward at an angle of 60° with the horizontal and 25 cm to the left of right hand bearing. Power of

is 10 cm wide and 6 mm thick and its angle of lap is 180°. The pulley may be taken to weigh 50 kg.

Assuming there are no transverse forces to the right of B, determine the necessary size of shaft, using the maximum shear stress criterion. The working shear stress is to be 400 kg/sq cm. Comment on this value of the working stress. Make a neat proportioned sketch of the pulley suitable for C giving leading dimensions. Provision is to be made for shifting the belt on to a loose pulley on the machine.

Design and sketch a key for securing the pulley to the shaft.

Ans. 70 mm diameter shaft.

13. *Fig. 7-6 shows a solid mild steel cross-shaft in a brake mechanism. The shaft is supported at each end in bearings having spherical seating. The two levers CD and EF actuate horizontal links which exert equal pulls whose maximum value is such as to require a total operating force P of value 750 kg on the driving lever AB. Draw the diagram of resultant bending moment for the shaft and determine a suitable diameter for it, using maximum shear stress criterion for the design. Also, decide suitable dimensions for the levers. State the values of the maximum forces on the bearings.*

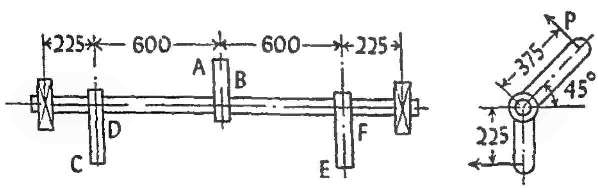


Fig. 7-6

14. *A ship requires 300 h.p. to drive it at a forward speed of 5 metre/sec and at this speed the propeller rotates at 180 r.p.m. The horizontal propeller shaft is not liable to buckle. Find the diameter of the shaft if the maximum allowable shear stress is 450 kg/sq cm and the twist in degree per metre if the modulus of rigidity $C = 0{\cdot}8 \times 10^6$ kg/sq cm.*

Ans. 90 mm.

15. *Determine the diameter for a hollow shaft having inside diameter 0·6 times outside diameter. The maximum allowable shear stress for shaft is 850 kg/sq cm. The shaft is driven by a 90 cm diameter overhung pulley placed vertically below it. The weight of the pulley is 60 kg. The tension on the tight and slack side of the belt are 290 kg and 100 kg*

10. The power output of a gas engine is absorbed by means of a friction brake, the brake drum having an effective diameter (allowing for belt thickness) of 75 cm, the angle of lap being 180° and the coefficient of friction between belt and drum being 0·35. The brake drum is overhung on the crankshaft, its central plane lying 22·5 cm out from the end of the nearest bearing. When the engine is delivering its maximum of 15 h.p. at 200 r.p.m., calculate the tensions in the belt on each side of the drum.

Deduce a satisfactory diameter for the shaft allowing a maximum value of principal stress of 420 kg/sq cm and neglecting the weight of the drum. Design a suitable key for fixing the brake drum to the shaft, assuming a maximum shear stress in the key of 330 kg/sq cm, a bearing pressure of 840 kg/sq cm and width of key = ¼ diameter of the shaft.

Ans. 70 *cm.*

11. A shaft 90 cm long is to be supported at each end in short swivel bearings held in C brackets attached to a wall. The shaft is driven by a belt at a point 25 cm from one end and drives on to a second belt 25 cm from the other end, the power transmitted being 5 h.p. at 500 r.p.m. The both pulleys are 20 cm diameter with a belt lap of 180°.

Determine the diameter of the shaft, which may be treated as simply supported with concentrated loads. Neglect the weights of the pulleys and shaft. Take the coefficient of friction for the belts on the pulleys as 0·4 and the safe shear stress as 310 kg/sq cm. Make fully dimensioned sketches of one bearing with its support bracket.   *Ans.* 40 *mm.*

12. A solid horizontal steel shaft is to run in self aligning bearings A and B as shown diagrammatically in fig. 7-5. It is driven from the right of B, and A is a terminal bearing.

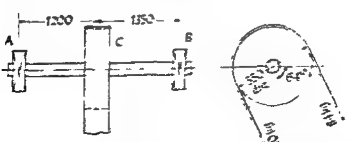
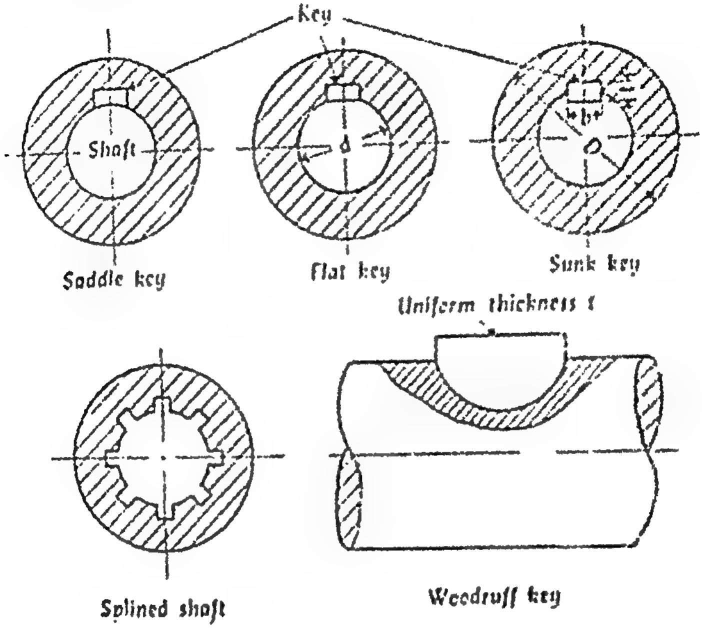


Fig. 7-5

From the pulley C which is 530 mm diameter, a belt drive supplies power to a machine, the belt being inclined at 60° to the horizontal. The maximum belt tensions are 180 kg and 85 kg as shown. The belt

The holding power of the sunk key is due to the resistances which they offer in shear and compression.



Various forms of keys
Fig. 7-7

The square key is square in section while in rectangular key the thickness is smaller than the width. They are designed so that one-half of the key is in the shaft while the other half is in the hub. The fitting of the key should be done very accurately. It should accurately fit at the sides while theoretically it should touch the top and bottom with a light pressure. These keys may be with tapered sides to facilitate the insertion and removal of the key. The taper occurs on the hub side. The insertion of the tapered key sets up a high pressure between the shaft and the hub which produces bursting pressure on the hub at the same time a large rictional force that is helpful in transmission of power. When end of keyseat is inaccessible, then in order to permit easy l of the key the gib head is provided.

l relative motion between a shaft and the hub is keys or splines are used. The working clear-

respectively. The overhung is 25 cm. Assume angle of lap of belt on the pulley to be 180°. *Ans.* 105 mm.

16. *A short stub shaft, made of SAE 1035 as rolled receives 30 H.P. at 300 r.p.m. via a 30 cm spur gear, the power being delivered to another shaft through a flexible coupling. The gear is keyed midway between the bearings. The pressure angle of the gear teeth is 20° and the design factor is 1·5 on maximum shear stress theory with varying stresses. Bearings are 40 cm apart.*

Neglecting the radial component of the tooth load, determine the shaft diameter. Considering both the tangential and radial components, compute the shaft diameter. Is the difference enough to change your choice of shaft size? *Ans.* 40 mm.

7-7. Form of Keys:

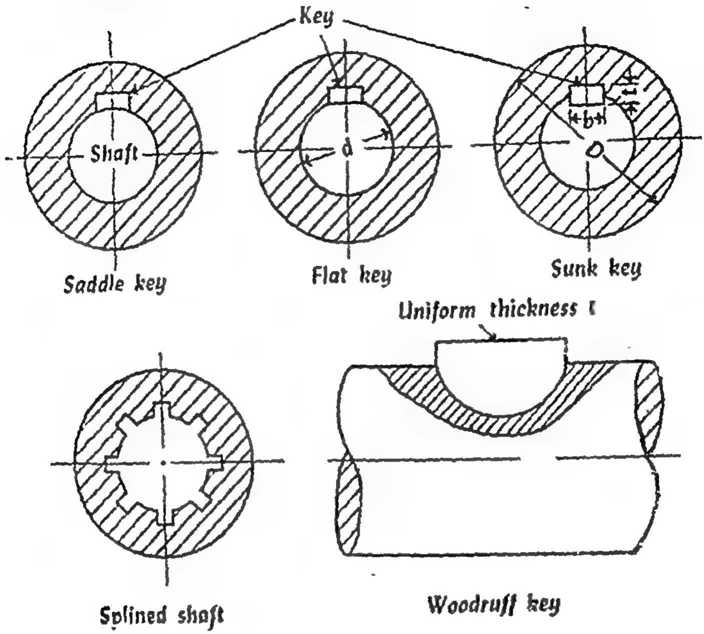
A key is a device which is used for connecting two machine parts for preventing relative motion of rotation with respect to each other. In many applications the key prevents the lengthwise relative motion also. The connected parts act as a single unit. A groove called a keyway or keyseat is usually cut into the shaft and the hub of the part to be connected. As the keyway is cut, the shaft is weakened due to reduction of metal near the circumference and due to stress concentration at the corners. Keys are generally made from cold rolled mild steel bars. The commonly adopted forms of keys may be divided into four classes: (1) saddle key, (2) tangent key, (3) sunk key, (4) round key and taper pins.

7-8. Keys:

Saddle key as shown in fig. 7-7 is used for light services. A keyway is provided only in the hub of the part to be attached and the key is hollowed to fit the shaft. The holding power of the key is due to friction forces set up. Sometimes to increase the holding power, the surface of the key is left flat and the shaft is planed off to accommodate the key. The key is called a flat key (fig. 7-7). Tangent keys are fitted to withstand torsion in one direction only. Sunk keys (fig. 7-7) are designed to fit in a sunk keyway whose bed is parallel to the axis of the shaft. Sunk keys are of the following types:

(i) Rectangular or square key (ii) Gib headed key
(iii) Feather key (iv) Woodruff key.

The holding power of the sunk key is due to the resistances which they offer in shear and compression.



Various forms of keys
FIG. 7-7

The square key is square in section while in rectangular key the thickness is smaller than the width. They are designed so that one-half of the key is in the shaft while the other half is in the hub. The fitting of the key should be done very accurately. It should accurately fit at the sides while theoretically it should touch the top and bottom with a light pressure. These keys may be with tapered sides to facilitate the insertion and removal of the key. The taper occurs on the hub side. The insertion of the tapered key sets up a high pressure between the shaft and the hub which produces bursting pressure on the hub at the same time a large frictional force that is helpful in transmission of power. When one end of keyseat is inaccessible, then in order to permit easy removal of the key the gib head is provided.

When axial relative motion between a shaft and the hub is necessary, feather keys or splines are used. The working clear-

ances are allowed in the sliding keyway at the top and at the sides. In some types the key is so constructed that it can be made fast to the hub while in other types it is fastened by countersunk screws to the shaft.

Splined shafts are extensively used in automobile industry. It has a number of key like projections, as shown in fig. 7-7, integral with it, equally spaced round the circumference. These projections engage with corresponding recesses in the splined hub.

Woodruff keys as shown in fig. 7-7 resemble segments of circles in shape. The shaft is carefully milled to accommodate the circular portion of the key and to allow the key to extend into the hub a distance of one-half the thickness of the key. Among advantages of this type of key is the ease of removal from the shaft. The extra depth in the shaft gives a deep base for the key and prevents any tendency to tip over. It is successfully used on tapering shaft ends. When considerable power is to be transmitted, two or more keys can be placed end to end having a common key-seat in the hub. Woodruff keys find their applications in machine tools, automobiles and aircraft constructions.

Round keys are of circular section and fit in holes drilled partly in the shaft and partly in the hub while the taper pins are inserted in through holes drilled in shafts and hubs.

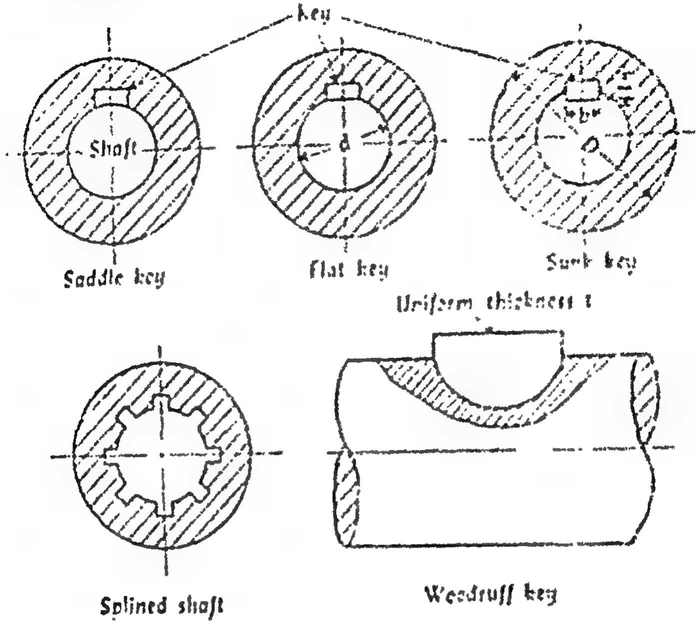
7-9. Design of sunk keys:

If T be the torque transmitted by the shaft, then the tangential force at the shaft radius will be $\frac{T}{d/2} = \frac{2T}{d}$ where d is the diameter of the shaft. Due to power transmitted shearing and crushing failures of the key are likely to take place. We assume that half the thickness of the key is in the shaft and the remaining half in the hub (see fig. 7-7). Let l, b and t respectively be the length, breadth or width and thickness of the key.

The area that resists the shearing of the key is $l\,b$. The tangential shearing force at the shaft radius is $P = \frac{2T}{d}$.

$\therefore$ Shear stress induced in the key $= \frac{P}{l\,b} = \frac{2T}{d\,l\,b}$ and this value should not exceed the permissible value of the stress.

The holding power of the sunk key is due to the resistances which they offer in shear and compression.



Various forms of keys
Fig. 7-7

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When axial relative motion between a shaft and the hub is necessary, feather keys or splines are used. The working clear-

The area that resists the crushing of the key is $\frac{lt}{2}$.

$\therefore$ Crushing stress induced in the key $= \frac{P}{lt/2} = \frac{4T}{t\,d\,l}$ and this value should not exceed the permissible value of the stress.

When the key is fitted on all the four sides, the permissible crushing stress is more than twice the permissible stress in shear. In this case, generally, we check the key for shear strength only. When the key is not fitted on all the four sides, the permissible crushing stress is about 1·7 times the permissible shear stress and the key must be checked for crushing also. When the key is made of the same material as the shaft, the length of the key if the keyed member transmits practically the whole shaft torque, is determined by equating the shear strength of the key to the torsional shear strength of the shaft.

The usual design procedure for the key is to obtain the dimensions of the key and then to check for the stresses.

Let U be the unit of proportion.

$$U = d + 13 \text{ mm}$$

$$\text{Width of the key} = \frac{U}{4}$$

$$\text{Thickness of the key} = \frac{U}{6}$$

$$\text{Length of the key} = 1{\cdot}5U.$$

If the connected element transmits only a portion of the torque of the shaft, the length of the key is reduced.

7-10. Effect of keyways in sunk keys:

The effect of cutting the keyway in the shaft is to reduce the torsional strength of the shaft. This is due to the removal of the material and serious stress concentrations at the re-entrant corners of keyways. According to the results of tests conducted by H.F. Moore the ratio of the presumable torsional strength of a solid circular shaft having an ordinary keyway to the strength of the same sized shaft without keyway may be expressed by the ratio.

$$1 - 0{\cdot}2\,\frac{b}{d} - 1{\cdot}1\,\frac{h}{d},$$

where h is the depth of keyway and b the width, and d the diameter of shaft.

Generally, it is assumed that by cutting the keyway the torsional strength of the shaft is reduced by 25%, which value is higher than that given by the above ratio.

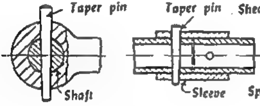
The effect of cutting the keyway is to increase the angle of twist for that portion of the shaft. Since at the portion of the shaft, hub of the connected

element tends to stiffen at that point so the effect of keyway is nullified. If the keyway is long, then we must consider its effect. The angle of twist is increased in the ratio C expressed by the equation

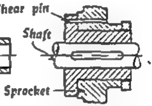
$$C = 1 + 0.4\frac{b}{d} + 0.7\frac{h}{d}.$$

7-11. Taper pin:

When small torque is to be transmitted, the shaft and the connected elements are fastened by taper pins as shown in fig. 7-8. Hollow shafts are connected by means of a sleeve and a taper pin. Standard taper for the taper pins is 1 in 48. The pin holes should be drilled and reamed with a taper reamer after the parts have been assembled. The mean diameter of the taper pin can be obtained from first principles as under.



Taper pins
FIG. 7-8



Use of a shear pin
FIG. 7-9

If T be the torque transmitted by the taper pin of mean diameter d_1 and d be the diameter of the shaft, then the force tending to shear the pin will be $\frac{T}{d/2} = \frac{2T}{d}$. If f_s be the permissible value of the shear stress in the pin, then, as the pin is in double shear, we get

$$2 \times \frac{\pi}{4} d_1^2 \times f_s = \frac{2T}{d}$$

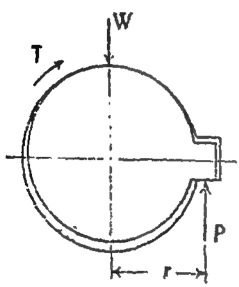
or
$$d_1 = \sqrt{\frac{4T}{\pi d f_s}}.$$

Such pins are used as a safety device against overloading of the connected parts. The pin will shear off before any damage is done to the connected costlier parts. In such connections the pin is known as the shear pin and the *diameter is calculated by considering the ultimate shear strength.* Fig. 7-9 shows such an application.

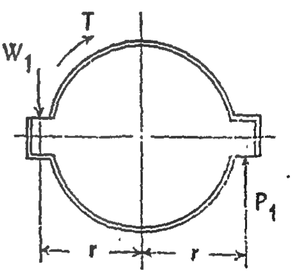
7-12. Feather keys:

They are used when the connected part is required to be moved axially along the shaft when the power is being transmitted. The key is, generally, tight in the shaft and the clearance is being provided between the key and the hub keyway. The force required to slide the connected part along the shaft depends upon the number of keys. The use of two feather keys equally spaced requires half the axial force than the use of one key. The feather key is very tight in the shaft and for stress analysis purpose it may be considered rigidly fixed in the shaft.

In fig. 7-10, the key is fixed in the shaft which rotates in the direction of the arrow and drives the connected element. The torque is transmitted by a force P at the key and a similar force F acting at the circumference of the shaft. If d be the diameter of the shaft, and T is the torque being transmitted, then $T = \frac{Pd}{2}$ or $P = \frac{2T}{d}$.



Shaft with one feather key
FIG. 7-10



Shaft with two feather keys
FIG. 7-11

The axial force required to move the hub axially $= W\mu + P\mu = 2\,P\mu$ where μ is the coefficient of sliding friction and $P = W$.

Where two feather keys are used and if they are well fitted so that each takes an equal share of the load, the shaft will assume the position as shown in fig. 7-11.

Torque $= P_1 \times d$

or $\quad P_1 = \frac{T}{d}$, which is half the pressure for the case of fig. 7-10.

Force to slide outer member along the shaft will be equal to $P_1\mu + W'_1\mu = 2P_1\mu$ which is half of that for the case of fig. 7-10.

From the above analysis the advantage of splined shaft is seen when the connected part is required to be moved along the shaft.

The size of the splines may be calculated as for the key or obtained from the table given below:

D is the diameter of the shaft on which splines are cut.

Number of splines	Width of the spline	Height of the spline	Root diameter of the shaft
4	0·241D	0·075D	0·850D
6	0·250D	0·050D	0·900D
10	0·156D	0·045D	0·910D
16	0·098D	0·045D	0·910D

Examples:

1. A propeller of an outboard motor has 3 cm outside diameter hub which fits on 2 cm diameter shaft. The hub and shaft are fastened by a brass shear pin. If the overload occurs at the propeller, the pin will shear thus avoiding damage to the rest of the mechanism. Calculate the diameter of the shear pin which will fail at a torque of 860 kg cm with an ultimate shear strength of 2,800 kg/sq cm.

The brass shear pin will be a taper pin passing through the hub and the shaft. The pin will be in double shear and the areas that resist the shear of the pin will lie along the surfaces of the shaft.

Let d cm be the diameter of the brass shear pin. The rupturing torque for the shear pin will be $2 \times \frac{\pi}{4} d^2 \times 2800 \times 1$ kg cm.

$$\therefore \quad 860 = 2 \times \frac{\pi}{4} d^2 \times 2800$$

$$\text{or} \qquad d = \sqrt{\frac{860 \times 4}{2\pi \times 2800}} = 0{\cdot}44 \text{ cm.}$$

Note: Here we do not round off the dimension to the next higher value as in that case the pin may not shear at 860 kg cm torque.

2. A rectangular sunk key 14 mm wide × 10 mm thick × 75 mm long is required to transmit 12,000 kg cm torque from a 5 cm diameter solid shaft. Determine whether the length is sufficient or not if the permissible shear stress and crushing stress intensities are limited to 560 and 1,680 kg/sq cm respectively.

$$\text{Tangential load at the shaft radius} = \frac{12000}{2 \cdot 5} = 4{,}800 \text{ kg.}$$

$$\text{Area that resists the shearing of the key} = 7 \cdot 5 \times 1 \cdot 4$$
$$= 10 \cdot 5 \text{ sq cm.}$$

$$\text{Induced shear stress intensity} = \frac{4800}{10 \cdot 5} = 457 \text{ kg/sq cm,}$$

which is less than the permissible value of 560 kg/sq cm.

$$\text{Area that resists the crushing of the key} = 7 \cdot 5 \times 0 \cdot 5$$
$$= 3 \cdot 75 \text{ sq cm.}$$

$$\text{Induced crushing stress intensity} = \frac{4800}{3 \cdot 75} = 1{,}280 \text{ kg/sq cm,}$$

which is less than the permissible value of 1,680 kg/sq cm.

As the induced stress intensities are less than the permissible one, the design is safe.

3. *A flywheel of weight 3,000 kg is keyed to a shaft 125 mm diameter. The shaft drives rollers for rolling plates. When in operation each plate takes $1\frac{1}{2}$ sec to pass through and speed drops from 70 r.p.m. to 50 r.p.m. during that time. The radius of gyration of the flywheel is 90 cm. Determine the torque necessary for operation, the shear force on the key and the shear stress in the shaft. Discuss other aspects of the design that could be considered. Also, suggest the suitable dimensions for the key.*

$$\text{Moment of inertia of flywheel} = \frac{3000}{9 \cdot 81} \times 0 \cdot 9^2$$
$$= 248 \text{ kg metre sec}^2.$$

$$\text{Change in speed of the flywheel} = \frac{(70 - 50)\, 2\pi}{60}$$
$$= 2 \cdot 1 \text{ radian/sec.}$$

This change of speed takes place in 1·5 seconds, therefore, the angular retardation of the flywheel, assuming it to be uniform, will be $\frac{2 \cdot 1}{1 \cdot 5} = 1 \cdot 4$ radian/sec^2.

$$\text{Torque necessary for operation} = 248 \times 1 \cdot 4 = 347 \text{ kg metre}$$
$$= 34{,}700 \text{ kg cm.}$$

$$\text{Shear force on the key} = \frac{34700 \times 2}{12 \cdot 5} = 5{,}620 \text{ kg.}$$

If f_s be the shear stress intensity in the shaft material, then

$$\frac{\pi}{16} \times 12 \cdot 5^3 \times f_s = 34700$$

or

$$f_s = \frac{34700 \times 16}{\pi \times 12 \cdot 5^3} = 90 \text{ kg/sq cm.}$$

Force to slide outer member along the shaft will be equal to $P_1\mu + W_1\mu = 2P_1\mu$ which is half of that for the case of fig. 7-10.

From the above analysis the advantage of splined shaft is seen when the connected part is required to be moved along the shaft.

The size of the splines may be calculated as for the key or obtained from the table given below:

D is the diameter of the shaft on which splines are cut

Number of splines	Width of the spline	Height of the spline	Root diameter of the shaft
4	0 241D	0 075D	0 850D
6	0 250D	0 050D	0 900D
10	0 156D	0 045D	0 910D
16	0 098D	0 045D	0 910D

Examples:

1. A propeller of an outboard motor has 3 cm outside diameter hub which fits on 2 cm diameter shaft. The hub and shaft are fastened by a brass shear pin. If the overload occurs at the propeller, the pin will shear thus avoiding damage to the rest of the mechanism. Calculate the diameter of the shear pin which will fail at a torque of 860 kg cm with an ultimate shear strength of 2,800 kg/sq cm.

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Note: Here we do not round off the dimension to the next higher value as in that case the pin may not shear at 860 kg cm torque

2. A rectangular sunk key 14 mm wide × 10 mm thick × 75 mm long is required to transmit 12,000 kg cm torque from a 5 cm diameter solid shaft. Determine whether the length is sufficient or not if the permissible shear stress and crushing stress intensities are limited to 560 and 1,680 kg/sq cm respectively.

maximum tension in the tight side of the belt will be limited to $20 \times 0.6 = 12$ kg/cm width.

The friction tension ratio $\frac{T_1}{T_2}$ equals $e^{\mu\theta} = e^{0.4\pi} = = 3.5$.

The tension in the slack side $= \frac{12}{3.5} = 3.43$ kg/cm width.

$\therefore$ Net pull $= T_1 - T_2 = 12 - 3.43 = 8.57$ kg/cm width.

H.P. transmitted $= \frac{8.57 \times \pi \times 0.75 \times 300}{4500} = 1.34.$

$\therefore$ Width of belt $= \frac{11.1}{1.34} = 8.3$ cm; we adopt 10 cm.

The maximum torque of the motor $= \frac{71620 \times 11.1}{300}$

$= 2{,}645$ kg cm.

Shear force on the key $= \frac{2645}{6} \times 2 = 882$ kg.

The width of the key is 1·3 cm.

If l be the length of the key, then

$882 = l \times 1.3 \times 420$

or $\qquad l = \frac{882}{1.3 \times 420} = 1.6$ cm.

This length is too small for the pulley and, therefore, we adopt the length of the key equal to the length of the hub which will be near about 8·5 cm, i.e. 85 mm.

Exercises:

1. A belt pulley running at 200 r.p.m. transmits 100 h.p. The pulley is connected to a 80 mm diameter shaft by means of a rectangular sunk key of dimensions 22 × 18 × 100 mm. Determine the value of the stresses induced in the shaft and key.

Ans. 357 kg/sq cm; 407 kg/sq cm; 1,000 kg/sq cm.

2. A shaft and key are made of the same material and the key width is ¼ shaft diameter. Considering shear only, determine the minimum length of key in terms of shaft diameter. The shearing strength of the key material is 60% of its crushing strength, determine the thickness of the key to make the key equally strong in shear and crushing. Ans. 1·57d; 0·3d.

3. A 40 cm long lever is fixed to a 4 cm diameter shaft by means of a taper pin passed through its hub perpendicular to the axis, the mean diameter of the pin is 1 cm. A force of 12 kg is applied at the end of a lever.

We assume the shear stress intensity in the key as 300 kg/sq cm, width of key 3 cm and thickness of key 2 cm. If l be the length of the key, then the maximum load that can be resisted by key will be $3 \times 300 \times l = 900\,l$ kg. Equating this to shear force on the key, we get

$$900\,l = 5620$$

or $$l = \frac{5620}{900} = 6{\cdot}25 \text{ cm.}$$

This length of the key seems to be too small and the length of the key should be at least equal to the length of the hub which we take to be 18 cm. The shear stress intensity in the key will be $\frac{5620}{18 \times 3} = 104$ kg/sq cm. The diameter of the hub of the fly-wheel we take to be 22 cm. The shear stress induced in the hub material during the rolling operation can be obtained by the formula

$$\frac{\pi}{16}\left[\frac{22^4 - 12{\cdot}5^4}{22}\right] f_s = 34700$$

or $$f_s = \frac{34700 \times 22 \times 16}{\pi\,[22^4 - 12{\cdot}5^4]} = 18{\cdot}5 \text{ kg/sq cm.}$$

The material of the hub is cast iron for which the allowable shear stress may be taken as 150 kg/sq cm.

4. A driving pulley 75 cm diameter rotates at 300 r.p.m. and is keyed to a 60 mm shaft. The shaft is attached to a pump which is required to deliver 1,000 litres of water per minute against a head of 30 metre. The coefficient of friction between belt and pulley is 0·4 and the belt has an angle of lap on the pulley of 180°. If the overall efficiency of the pump is 60%, determine the size of a suitable driving motor. A belt 6 mm thick is to be used on the pulley. Obtain the width of belt for a safe stress of 20 kg/sq cm. A key 13 mm × 10 mm is to be used to secure the pulley on the shaft; calculate the length of key for a shear stress of 420 kg/sq cm.

$$\text{Water horse power} = \frac{1000 \times 30}{4500} = 6{\cdot}66.$$

The efficiency of the pump is 60%, therefore horse power of the driving motor will be $\frac{6{\cdot}66}{0{\cdot}6} = 11{\cdot}1$

Let us consider h.p. transmitted by 1 cm width of the belt. The thickness of the belt is 6 mm and a safe stress 20 kg/sq cm; the

9. *Draw the sketches of various types of keys used and indicate where each one is preferred. Give your reasons for the choice. A belt pulley is fastened to a 75 mm diameter shaft running at 200 r.p.m. by means of a key 16 mm wide and 125 mm long. Allowable stress for the shaft and key materials are 400 kg/sq cm in shear and 1,000 kg/sq cm in bearing. Find the horse power transmitted and the depth of the key required.*

Ans. 92·5 *h.p.*; 1 *cm.*

10. *If a key and shaft are made from the same material, determine the necessary length of the key for equal strength of the shaft and key, The key is rectangular with a width $\frac{d}{4}$ and a height of $\frac{3d}{16}$. The shaft is under torsion only.*

Ans. Key length $= 1{\cdot}57d$.

✓ **7-13. Force and Shrink Fits: (Driving Fits on Solid Shafts)**

This is a particular case of the theory of thick cylinders, which has been discussed in chapter 3. This is an elementary design application in which these fits are used for connecting hubs and shafts, sometimes in addition to keys, when an especially rigid connection is desired.

Let us consider a hub of nominal external radius r_2 and internal radius r_1 forced onto a solid shaft of nominal radius r_1.

Let E_1 and μ_1 be young's modulus and Poisson's ratio for the material of the shaft and let E_2 and μ_2 be the corresponding values for the hub. Let p be the shrinkage pressure at the contact surface. This pressure is called the *interface pressure.*

The tangential and radial stresses due to the force fitting of the hub on the shaft are given by the general expressions

$$\text{Tangential stress} = f_t = A + \frac{B}{r^2}$$

$$\text{Radial stress} = f_r = A - \frac{B}{r^2}$$

where A and B are constants which are to be determined from the boundary conditions. Both the above equations apply to the solid shaft as well as to the hub. The boundary conditions for both the elements will be different; hence constants A and B will be different for the shaft and the hub.

For a solid shaft if the constant B were to be other than zero, at the centre the tangential and radial stresses will be infinite. The stresses should remain finite and hence for the solid shaft,

Is the pin safe if the shearing stress on the pin is limited to 600 kg/sq cm? What torsional shear stress is produced in the shaft?

Ans. Safe; 38·3 kg/sq cm.

4. *The transmission gears of an automobile are carried on 50 mm—6 spline shaft and slide when under load. The hub length of each gear is 4 cm. Determine the pressure on the spline if the horse power transmitted at 2,000 r.p.m. is 80. The following proportions may be adopted.*

The width of the spline is $\frac{D}{4}$ *and the thickness of the spline* $\frac{D}{10}$, *where D is the diameter of the shaft.*

Ans. 306 kg/sq cm.

5. *A 40 cm gear transmitting 60 h p at 120 r.p.m is to be fastened to a shaft having splines. The hub is 1·5 times the shaft diameter D. Determine the spline dimensions if the width of the spline is* $\frac{D}{4}$ *and the thickness of the spline* $\frac{D}{10}$. *The permissible shear stress intensity in the shaft material is not to exceed 560 kg/sq cm and the crushing intensity on the spline is limited to 85 kg/sq cm.*

Ans 100 mm diameter; 6 spline shaft.

6. *Calculate horse power transmitted by a shaft of 15 mm diameter running at 300 r.p.m Two feather keys are fitted to the shaft and the axial force required to slide the outer member along the shaft is 15 kg. The coefficient of sliding friction is 0 15.*

Ans 0 314 h p.

7. *In a plate rolling mill the plates take 3 sec to pass through a pair of rollers. To maintain the speed as constant as possible, a flywheel weighing 4 tonnes is keyed to a 15 cm diameter shaft, but the speed drops from 65 r.p.m. to 50 r.p.m. The radius of gyration of the flywheel is 100 cm. Obtain the torque applied to the shaft, the shear stress induced in the shaft and the shear force on the key. Comment on assumptions that you have made in your approach to the design.*

Ans. 216 kg metre, 32·4 kg/sq cm; 2,880 kg.

8. *A propeller is fastened by means of a sunk key to the shaft subjected to torsion only. Find the ratio of sectional areas of shaft and key for equality of strength of key and shaft. Find the length of key 2 cm wide for a shaft 7 cm in diameter. Take allowable shear stresses for key and shaft as 550 and 660 kg/sq cm respectively.* *Ans. 12 cm.*

9. *Draw the sketches of various types of keys used and indicate where each one is preferred. Give your reasons for the choice. A belt pulley is fastened to a 75 mm diameter shaft running at 200 r.p.m. by means of a key 16 mm wide and 125 mm long. Allowable stress for the shaft and key materials are 400 kg/sq cm in shear and 1,000 kg/sq cm in bearing. Find the horse power transmitted and the depth of the key required.*

Ans. 92·5 h.p.; 1 cm.

10. *If a key and shaft are made from the same material, determine the necessary length of the key for equal strength of the shaft and key, The key is rectangular with a width* $\frac{d}{4}$ *and a height of* $\frac{3d}{16}$. *The shaft is under torsion only.*

Ans. Key length = 1·57d.

7-13. Force and Shrink Fits: (Driving Fits on Solid Shafts)

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The tangential and radial stresses due to the force fitting of the hub on the shaft are given by the general expressions

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where A and B are constants which are to be determined from the boundary conditions. Both the above equations apply to the solid shaft as well as to the hub. The boundary conditions for both the elements will be different; hence constants A and B will be different for the shaft and the hub.

For a solid shaft if the constant B were to be other than zero, at the centre the tangential and radial stresses will be infinite. The stresses should remain finite and hence for the solid shaft,

the constant B is zero and hence we deduce that *at all radii for the solid shaft the hoop and radial stresses are equal and of the same type i.e. compressive, and the value of this stress is that of the necessary interface pressure to ensure no slipping at the interface.* If therefore the fit allowance on the nominal interface diameter is δ then this force fit allowance must be given by the decrease in diameter of the shaft added to the increase in internal diameter of the hub.

If p be the corresponding interface pressure which is compresive, then we get at the interface

Tangential stress in shaft $= -p$

Radial stress in shaft $= -p$

Radial stress in hub at interface $= -p$

Tangential stress in the hub $= p\,\dfrac{r_2^2 + r_1^2}{r_2^2 - r_1^2}$

Circumferential strain in the shaft (compressive)

$$= \frac{1}{E_1}(p - \mu_1 p) = \frac{p}{E_1}(1 - \mu_1)$$

Circumferential strain in hub $= \dfrac{p}{E_2}\left[\dfrac{r_2^2 + r_1^2}{r_2^2 - r_1^2} + \mu_2\right]$

$$\therefore \quad \frac{\delta}{2r_1} = p\left\{\frac{1}{E_1} - \frac{\mu_1}{E_1} + \frac{1}{E_2}\left[\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right] + \frac{\mu_2}{E_2}\right\} \qquad (i)$$

Thus when the interface pressure is given, the diametral interference can be obtained or vice versa

In practice $\dfrac{\delta}{2r_1}$ is usually of the order $\dfrac{1}{2000}$.

If the hub is cast iron and the shaft is steel, the modulus of elasticity for steel is twice that for cast iron and Poisson's ratio for steel and cast iron are virtually the same and its value may be taken as 0·27.

If hub and shaft both are made of the same material the

$$\frac{\delta}{2r_1} = \frac{p}{E}\left[1 + \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}\right] \quad \ldots \quad \ldots \quad \ldots \ldots \quad (ii)$$

When the interface pressure intensity is known, the torque that the fit will transmit and the force needed to make the fit may be estimated. If l be the length of the fit and D the diameter at the interface, the contact area is πDl. The normal force is $p\pi Dl$ and the frictional force is $\mu p\pi Dl$ and the moment of the frictional

force about the axis of the shaft is $\mu p \pi D l \times \frac{D}{2}$. Thus the torque that can be transmited by this fit is

$$T = \frac{\mu p \pi D^2 l}{2} \quad \text{.................................(ii)}$$

If N is the design factor then the fit can be used to transmit

$$\frac{T}{N} = \frac{\mu p \pi D^2 l}{2N}$$

Joints of this type may be assembled by force or shrink fits. The mean values of the coefficient of friction for steel and cast iron parts are

$$\mu = 0{\cdot}08 \text{ for a force fit}$$
$$= 0{\cdot}14 \text{ for a shrink fit.}$$

The axial force required to press the hub onto the shaft is the product of the area $\pi D l$, the interface normal pressure p and the coefficient of friction μ. Thus the axial force $F = \mu p \pi D l$....(iii)

The strength of a press fitted joint may be defined as its ability to resist an axial thrust or torque or the joint action of both forces, which tend to displace one part relative to the other.

The load carrying capacity of a press-fitted joint depends primarily on the amount of interference δ i.e. the difference between the actual diameters of the shaft and hole measured before assembly.

When the joint is subjected to an axial thrust P, then

$$P \leq \mu p \pi D l \quad \text{.................................(iv)}$$

If the joint transmits the torque T, then

$$T \leq \frac{\mu p \pi D^2 l}{2} \quad \text{.................................(v)}$$

If the joint transmits the torque T and simultaneously resists the axial thrust P, then

$$\sqrt{\left(\frac{2T}{D}\right)^2 + P^2} \leq \mu p \pi D l \quad \text{.........................(vi)}$$

Example:

1. A 15 cm diameter steel shaft is to have a press fit with 30 cm outside diameter by 25 cm long hub of cast iron. The maximum tangential stress is to be 350 kg/sq cm. The coefficient of friction may be taken as 0·12 and Poisson's ratio 0·3. Determine the safe torque capacity of the

the constant B is zero and hence we deduce that *at all radii for the solid shaft the hoop and radial stresses are equal and of the same type i.e. compressive, and the value of this stress is that of the necessary interface pressure to ensure no slipping at the interface.* If therefore the fit allowance on the nominal interface diameter is δ then this force fit allowance must be given by the decrease in diameter of the shaft added to the increase in internal diameter of the hub.

If p be the corresponding interface pressure which is compresive, then we get at the interface

Tangential stress in shaft $= -p$

Radial stress in shaft $= -p$

Radial stress in hub at interface $= -p$

Tangential stress in the hub $= p\,\dfrac{r_2^2+r_1^2}{r_2^2-r_1^2}$

Circumferential strain in the shaft (compressive)

$$= \frac{1}{E_1}(p-\mu_1 p) = \frac{p}{E_1}(1-\mu_1)$$

Circumferential strain in hub $= \dfrac{p}{E_2}\left[\dfrac{r_2^2+r_1^2}{r_2^2-r_1^2}+\mu_2\right]$

$$\therefore\quad \frac{\delta}{2r_1} = p\left\{\frac{1}{E_1}-\frac{\mu_1}{E_1}+\frac{1}{E_2}\left[\frac{r_2^2+r_1^2}{r_2^2-r_1^2}\right]+\frac{\mu_2}{E_2}\right\} \qquad (i)$$

Thus when the interface pressure is given, the diametral interference can be obtained or vice versa

In practice $\dfrac{\delta}{2r_1}$ is usually of the order $\dfrac{1}{2000}$.

If the hub is cast iron and the shaft is steel, the modulus of elasticity for steel is twice that for cast iron and Poisson's ratio for steel and cast iron are virtually the same and its value may be taken as 0·27.

If hub and shaft both are made of the same material the

$$\frac{\delta}{2r_1} = \frac{p}{E}\left[1+\frac{r_2^2+r_1^2}{r_2^2-r_1^2}\right] \qquad \ldots (ii)$$

When the interface pressure intensity is known, the torque that the fit will transmit and the force needed to make the fit may be estimated. If l be the length of the fit and D the diameter at the interface, the contact area is πDl. The normal force is $p\pi Dl$ and the frictional force is $\mu p\pi Dl$ and the moment of the frictional

2. A gear with 20 teeth of module 12 mm is pressed on a 120 mm diameter solid shaft. The gear is cut from a steel blank having a uniform thickness of 12 cm. In service the gear must transmit a peak torque of 60,000 kg cm but must pass a static torque test at twice this value without movement. Estimate the interference required between bore and shaft and the maximum assembly force. Tooth dedendum may be taken as 1·25 times the module.

$\mu = 0{\cdot}15$ *and* $E = 2{\cdot}1 \times 10^6$ *kg/sq cm.*

Ans. 20,000 *kg;* 0·005 *cm.*

7-14. Couplings—Introduction:

Whenever lengths of shafting exceeds some 7 metre in length, it is made up of two or more lengths. In such conditions it becomes necessary to join the ends of two shafts in such a manner that both the shafts act as the same unit. The elements which join two shafts are known as *couplings or clutches.* Couplings are permanent connections while clutches are such that the operator can connect or disconnect the other shaft at will, as in automobiles, while in motion.

Shafts to be connected may have collinear axes, intersecting axes or parallel axes at a relatively small distance. Oldham coupling is used to connect two parallel shafts when they are at a very small distance apart. Hooke's coupling is used to connect two shafts having intersecting axes. When the axes are in the same straight line, rigid and flexible couplings of various constructions are employed. In this chapter, we shall consider some of the couplings both of rigid and flexible types. Each coupling should as far as possible satisfy the requisite requirements of a good coupling.

The important requisites of good coupling are:

(a) It must transmit the full torque of the shaft.

(b) It must keep the shafts in perfect alignment.

(c) It must be easy to assemble or dissemble.

(d) The bolt heads, key heads, nuts and other projecting parts should be protected by suitable flanges, rims or cover plates.

7-15. Sleeve coupling or muff coupling:

This is a simple coupling which is used to connect two shafts rigidly. Fig. 7-12 shows one form of sleeve coupling. It consists

assembly with a design factor of 10. What will be the axial force required to press the hub on the shaft? Also suggest the suitable value of the diametral interference.

The maximum tangential stress occurs at the bore of the outer member and it is tensile. Let p be the interface pressure.

$$\therefore \quad 350 = p\,\frac{15^2 + 7{\cdot}5^2}{15^2 - 7{\cdot}5^2}$$

$$\text{or} \qquad p = 350\left[\frac{15^2 - 7{\cdot}5^2}{15^2 + 7{\cdot}5^2}\right] = 210 \text{ kg/sq cm.}$$

Axial force required to press the hub on the shaft

$= 210 \times \pi \times 15 \times 25 \times 0{\cdot}12$

$= 30{,}000$ kg.

Torque required to shift the hub relative to shaft $= 30000 \times \dfrac{15}{2}$

$= 225{,}000$ kg cm.

Safe torque that can be transmitted with a design factor of 10 will be $\dfrac{225000}{10} = 22{,}500$ kg cm.

We assume that the modulus of elasticity of cast iron is half that of steel, which is $2{\cdot}1 \times 10^6$ kg/sq cm.

$$\therefore \quad \delta = 210 \times 15\left[\frac{1}{2{\cdot}1 \times 10^6} - \frac{0{\cdot}3}{2{\cdot}1 \times 10^6} + \frac{1}{1{\cdot}05 \times 10^6} \times \left(\frac{15^2 + 7{\cdot}5^2}{15^2 - 7{\cdot}5^2}\right) + \frac{0{\cdot}3}{1{\cdot}05 \times 10^6}\right]$$

$$= \frac{210 \times 15}{10^6}\left[\frac{1}{2{\cdot}1} - \frac{0{\cdot}3}{2{\cdot}1} + \frac{1}{1{\cdot}05} \times \frac{5}{3} + \frac{0{\cdot}3}{1{\cdot}05}\right]$$

$= 0{\cdot}00692$ cm.

Exercises:

1. A bronze disc 5 cm thick is to be pressed onto a steel shaft which has a diameter of 10 cm and the disc has an outside diameter of 30 cm. When assembled the fit must be capable of withstanding a torque of 81,000 kg cm. Determine the allowance which must be made to obtain this force fit. The coefficient of friction for the two surfaces is 0·3.

E for steel $2{\cdot}1 \times 10^6$ *kg/sq cm*

E for bronze $0{\cdot}945 \times 10^6$ *kg/sq cm*

μ may be taken as 0·25 for both metals.

Ans. 0·0064 *cm.*

2. *A gear with 20 teeth of module 12 mm is pressed on a 120 mm diameter solid shaft. The gear is cut from a steel blank having a uniform thickness of 12 cm. In service the gear must transmit a peak torque of 60,000 kg cm but must pass a static torque test at twice this value without movement. Estimate the interference required between bore and shaft and the maximum assembly force. Tooth dedendum may be taken as 1·25 times the module.*

$\mu = 0{\cdot}15$ *and* $E = 2{\cdot}1 \times 10^6$ *kg/sq cm.*

Ans. 20,000 kg; 0·005 cm.

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7-15. Sleeve coupling or muff coupling:

This is a simple coupling which is used to connect two shafts rigidly. Fig. 7-12 shows one form of sleeve coupling. It consists

Following are the usual proportions for cast iron sleeve coupling:

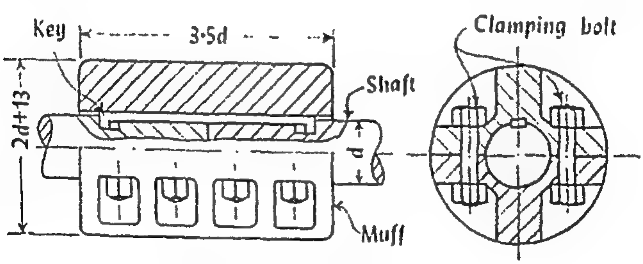
Outside diameter D of sleeve $= 2 \times$ diameter of shaft $+ 13$ mm

Length of the sleeve $= 3 \cdot 5 \times$ diameter of shaft

Length of the key is at least equal to the length of the sleeve.

7-16. Clamp or compression coupling:

This is the modification of the sleeve coupling. The sleeve is split in two halves which are connected by through bolts which are housed in the recesses formed in the sleeve (fig. 7-13). This is also of the rigid type. The coupling can be readily assembled and dissembled very easily. Generally, the coupling is placed over the ends of the shaft to be connected and key is inserted. In fig. 7-13, the key is first placed in position afterwards two halves of the clamp coupling are tightened. *The coupling is so bored that upon assembly of two halves of the coupling, it clamps tightly against the shaft.* The ribs add strength and serve as a protection to the workman against injury from the revolving bolt heads and nuts. The considerable frictional holding power is set up between the shafts and the coupling. This coupling may be used for the transmission of heavy torques. It is conveniently used for the line shaft service.



Clamp or compression coupling

FIG. 7-13

The following are the usual proportions for the sleeve of the compression coupling:

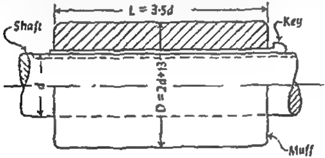
Diameter of the sleeve $= 2 \times d + 13$ mm

Length of the sleeve $= 3 \cdot 5d$.

In order to calculate the diameter of the holding down bolts, we assume that the entire torque of the shaft is transmitted by friction between the clamp and the shaft.

of a sleeve or hollow cylinder, generally made of cast iron, which is fitted over the ends of the two shafts to be connected and then keyed by means of a sunk key, whose material is generally the same as that of the shafts to be connected. The torque is transmitted from one shaft to the sleeve and then to the other shaft.

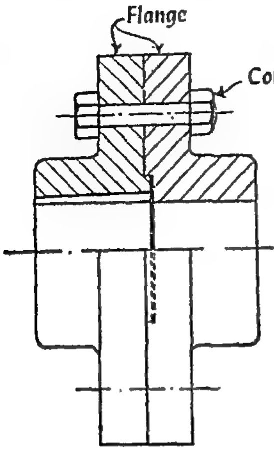
As it has no projecting parts, its exterior is perfectly smooth which is the clear advantage from safety point of view. Though the coupling is simple one, it requires very careful fitting. The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, many engineers make the key in two lengths and drive them both in from the same end one for each shaft or they may be driven from opposite ends. The following design procedure is suggested:



Sleeve or muff coupling
FIG. 7-12

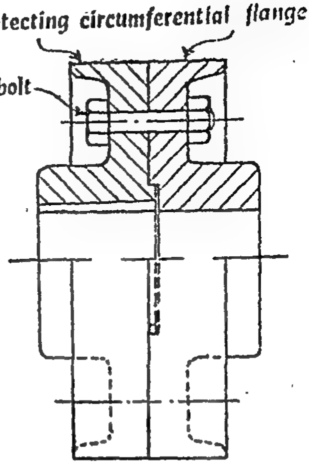
The sleeve is designed as a hollow shaft transmitting the entire torque of the shaft. As the sleeve is made of cast iron, the permissible value of the shear stress is taken to be 140 kg/sq cm. The width and thickness of the key are obtained from the proportions. The length of the key is calculated from the consideration that the moments of the shearing and crushing resistances of the key must each be equal to or greater than the torque of the shaft. The effective length of the key for the calculation purpose is that length which is in one of the two shafts. If we use one key, the total length of the key will be twice the calculated value.

7-17. Flange-coupling:



Flange coupling

Fig. 7-14



Flange coupling of protected type

Fig. 7-15

They are designed to withstand severe service. It is of the rigid type (as shown in fig. 7-14). This coupling consists of two flanges keyed to two shafts. These two flanges are coupled together by means of bolts *fitted in reamed holes* so that the bolts share the load equally. On important works flanges are forced on the shafts and keyed tightly in place. As a protection to workman the nut and bolt heads are covered by projecting flanges. To insure true alignment, one shaft may enter the coupling of the other shaft by 1 cm or a cylindrical projection is provided on one flange which fits into the corresponding recess in the other as shown in fig. 7-15. Coupling for marine or automotive propeller shafts demand great strength and reliability; for this service the flanges may be forged integral with shafts as shown in fig. 7-16, in which two bolts are shown with nuts.

The flanges of the coupling are usually constructed of cast iron but sometimes steel is also used. Coupling bolts are made of steel.

The flanges of the coupling are located very close to the bearings.

Let d_1 be the root diameter of the clamping bolts, n the number of clamping bolts, d the diameter of the shaft, μ the coefficient of friction between the shaft and the sleeve, f_t the allowable tensile stress in the bolt material and T the torque to be transmitted by the shaft.

Area of the cross section of the bolt $= \frac{\pi}{4}\, d_1^2$.

As the permissible tensile stress intensity is f_t, load per bolt will be $\frac{\pi}{4}\, d_1^2\, f_t$.

Clamping load on the shaft $= \frac{n}{2} \times \frac{\pi}{4}\, d_1^2\, f_t$.

Clamping pressure intensity between the shaft and sleeve

$$= \frac{\frac{n}{2} \times \frac{\pi}{4}\, d_1^2\, f_t}{d \times l/2} \text{ where } l \text{ is the length of the sleeve.}$$

If μ be the coefficient of friction between sleeve and shaft, then the tangential frictional force per sq cm of shaft periphery will be $\mu \times \frac{\pi}{4}\, \frac{d_1^2\, f_t\, n}{dl}$ kg.

The torque that can be transmitted by the sleeve coupling

= radius of the shaft $\times$ peripheral area of the shaft $\times$ tangential frictional force per sq cm of shaft periphery

$$\therefore\quad T = \frac{d}{2} \times \pi \frac{dl}{2} \times \mu \times \frac{\pi}{4}\, d_1^2\, \frac{f_t n}{dl}$$

$$= \frac{\pi^2}{16}\, d_1^2\, f_t\, \mu\, n\, d \qquad \ldots(\text{i})$$

From the above equation, the diameter of the coupling bolt can be calculated. If P be the load due to clamping in each bolt,

$$P = \frac{4\,T}{d\,n\,\pi\mu} \qquad \ldots(\text{ii})$$

The coefficient of friction between the sleeve and the shaft depends on the material, working of the bore and the pressure between the surfaces. The approximate value of the coefficient of friction is taken to be 0.3. The number of bolts should be multiple of 4.

Let d be the diameter of the shaft, d_1 the diameter of the bolt, B the diameter of the bolt circle, n the number of bolts, f_s allowable shear stress in bolts and T the torque transmitted by the coupling.

The bolts are subjected to direct shear.

The area resisting direct shear of each bolt $= \frac{\pi}{4} d_1^2$.

As the permissible shear stress intensity in bolt material is f_s, shear load on each bolt will be $\frac{\pi}{4} d_1^2 f_s$.

Torque that can be transmitted by each bolt $= \frac{\pi}{4} d_1^2 f_s \times \frac{B}{2}$.

As there are n bolts, the total torque that can be transmitted by the coupling will be $n \times \frac{\pi}{4} d_1^2 f_s \times \frac{B}{2}$.

$$\therefore \quad T = n \times \frac{\pi}{4} d_1^2 f_s \times \frac{B}{2}.$$

From the above equation the size of the bolt is determined when number of bolts, pitch circle diameter and allowable shear stress are known.

The following are the usual proportions for the cast iron flange coupling:
Outside diameter of the hub = 2 × diameter of the shaft
Pitch circle diameter of the bolt = 3 × diameter of the shaft
Thickness of the flange = $\frac{1}{2}$ × diameter of the shaft
Thickness of the protective circumferential flange = $\frac{1}{4}$ × diameter of the shaft.
Length of the hub = 1·5 × diameter of the shaft

n = 3 for shaft diameters upto 4 cm
= 4 for shaft diameters upto 10 cm
= 6 for shaft diameters upto 18 cm.

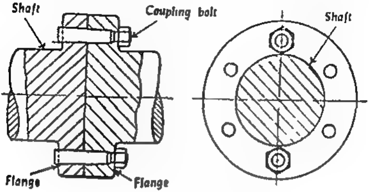
In ordinary work, the bolts fit the holes with clearance to allow for slight variations in the positions of the matching holes. These inaccuracies cause the bolts to share the load unequally. For this reason, in such cases, it is assumed that half the bolts carry the load.

Transverse loads induce bending stresses in the flange which reverse during a complete rotation as a result the fatigue failure is likely to occur. However the flanges of the standard coupling are such that the failure from fatigue is not expected.

7-18. Marine type of flange coupling:

Fig. 7-16 shows the coupling which is used in marine engineering. The flanges in this type of coupling are forged on the ends

The following design procedure is suggested for the flange coupling·



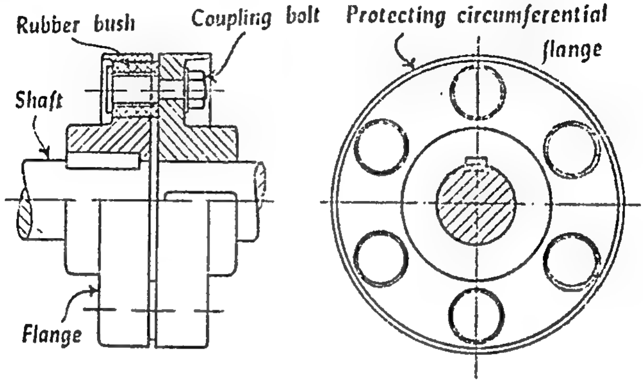
Marine type flange coupling

Fig 7-16

The outside diameter of the hub is calculated by considering the hub to be a hollow shaft transmitting the torque of the shaft, the inner diameter of the hub being equal to the diameter of the shaft. The hub must also provide the sufficient length for the key so that its resisting moment about the axis of the shaft will be at least equal to the *torsional* strength of the shaft. The thickness of the flange should be such that it does not shear off at the hub, at the same time the crushing stress on the bolt is not exceeded. To prevent the shearing of the flange at the place where it joins the hub, the moment of the shearing resistance of the flange must at least be equal to the torsional moment transmitted by the shaft. The number of bolts is fixed depending upon the diameter of the shaft. The pitch circle diameter is fixed such that enough space is provided between the hub and the lip so that the coupling bolts can be tightened by using a socket wrench. The size of the bolts is proportioned so that the bolts can transmit the full torsional strength of the shafts. The key for each flange is designed as explained in art. 7-9. The thickness of the circumferential safety flange should not be less than one-half that of the radial flange. The inside diameter of the circumferential flange is such that the bolt holes are located midway between the flange and the hub.

The following derivation is commonly used to determine the size of the coupling bolts:

meters are enlarged and covered with a flexible material like leather or rubber washers and the drive takes place through the medium of compressible leather or rubber washer.



Flexible flange coupling
FIG. 7-17

The following procedure for the design is adopted:

The usual proportions for the rigid type of coupling are determined and these will be modified to reduce the bearing pressure on the rubber bushes, which are the main feature of the design. For available thickness of rubber bushes, the designer should refer the catalogue from manufactures. The suitable value for the bearing pressure will be 3·5 kg/sq cm if longer life is desired although higher values than this are sometimes specified.

From the torque considerations, the load on each bolt can be calculated. In order to keep the dimensions of the rubber bushing reasonably low the pitch circle diameter will be increased in flexible coupling and the number of bolts will also be more.

Let l be the length of the bush and d its outside diameter. If p be the bearing pressure on the pin, then load on each pin will be equal to $p \times d \times l$, which we denote by F.

The torque transmitting capacity of the coupling is given by $T = n \times F \times R$

where n is the number of pins on pitch circle of radius R and T the torque to be transmitted by coupling.

Since the pin is not rigidly held in left hand face [see fig. 7-17] and the rubber is compressible, the tangential force F at the

of the shaft. The bolts are generally tapered which are made with or without heads. The bolt holes are always reamed after the flanges are placed together; thus insuring perfectly fitted bolts as a result each bolt will share the same load. The method of arriving at the diameter of the bolt is similar to that given for flange coupling. The thickness of the flange is obtained from shear stress consideration, and finally it should be checked for crushing stress consideration.

Generally, the following proportions in terms of the diameter of the shaft, d, are adopted for the coupling:-

Diameter of the bolt circle will be $1.6d$

Number of bolts $= \frac{d \text{ cm}}{7.5} + 2$

Number of bolts must be even

Thickness of the flange $= \frac{d}{3}$

Taper of bolt $= 1$ in 32

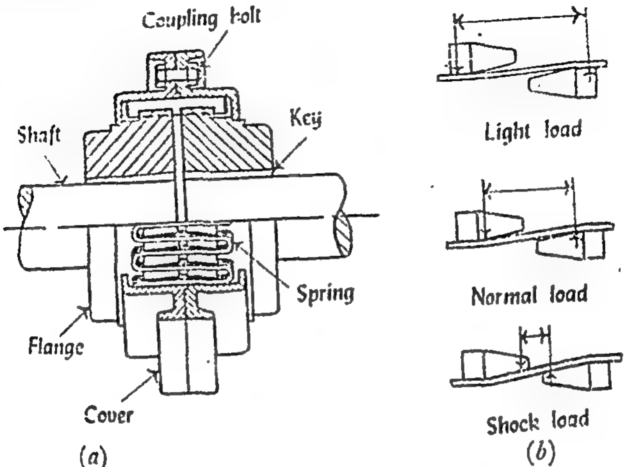
7-19. Flexible coupling:

Perfect alignment of two theoretically collinear shafts is practically impossible to attain, and still more difficult to maintain because of bearing wear and other causes. Misalignment of the shafts cause continuous stress reversal and excessive bearing wear. For these reasons flexible couplings are employed for moderate or heavy duty transmission service. The flexible couplings prevent transmission of shock from one shaft to other and eliminate stress reversals when either shaft is subjected to deflection at or near the coupling. Numerous forms of flexible couplings are manufactured by various companies. We shall not go into the details of all those couplings but shall consider some of them.

7-20. Bushed pin type of flexible coupling:

This type of coupling has a cushioning effect due to the flexible element inserted in one of the flanges. They are used to connect directly prime-mover and electric generator or an electric motor and a centrifugal pump or an electric motor and a reducing gear. Fig. 7-17 shows the coupling with its essential elements. From the figure it can be easily seen that it is the modification of the rigid type of flange coupling. The coupling bolts are known as pins having the special shape as shown in the figure. These pins are rigidly fastened by nuts to one of the flanges while their dia-

of flange coupling. The springs are made of steel having an elastic limit of 14,000 kg/sq cm when hardened and tempered. They are enclosed in grease tight cover.



Bibby type flexible coupling

Fig. 7-18

From fig. 7-18(*b*) it is easy to see that under light load, the coupling is very flexible and resilient to absorb vibration. Under normal load the span is decreased and the strength is increased. Under over load the span is smallest and the stiffness is maximum.

In large couplings to decrease the spring cross section the coils are laid in two or three rows.

Depending on the tooth contour these couplings may have a linear or non-linear characteristic.

7-22. Leather pad type flexible coupling:

Fig. 7-19 shows the leather pad flexible coupling. This type of coupling is used where the changes in the length of the shaft due to temperature are to be accommodated. The material of the coupling may be cast iron, cast steel or steel.

The following procedure is adopted for the design of the coupling:

(i) The torque transmitted by the coupling is calculated from the horse power and the speed of rotation.

pitch circle radius R will exert a bending action on the pin. Its effect on the smaller diameter portion will be reduced by the longitudinal resistance of the shoulder to bending but in estimating bending stresses, this resistance will be neglected.

We assume that load F is distributed on the bush uniformly.

The maximum bending moment on the pin $= \frac{Fl}{2}$.

If d' be the diameter of the pin,

$$\frac{Fl}{2} = \frac{\pi}{32} d'^3 f \quad \text{(i)}$$

After the bending stresses have been calculated, the pin should be checked for either principal stress or for shear stress. Since the applied shear stress will be very small compared with the direct stress, the maximum principal stress in the reduced section will be the design criterion, which may be taken as 420 kg/sq cm. With bolts of ordinary mild steel the value may be taken as 280 kg/sq cm.

Note: It can be easily seen that the two halves of the coupling will be dissimilar in construction as a result separate pattern should be made for both the flanges.

As this coupling is used for high speed service, it should be machined all over for perfect alignment and balance

7-21. Bibby type of flexible coupling (Fig. 7-18).

This type of coupling is used for high degree of resiliency, for accommodating loss of alignment and for damping out shocks and vibrations in the heavier drives. Some of the applications of these couplings are found in the following drives

To gear steam engine with rotary pumps, to drive electric generators from steam turbines, to connect oil engines with vertical pumps, to gear up propellers with air ship engines.

The construction consists of two serrated discs, which are keyed to the respective shafts. These serrated discs are connected by a grid spring. As shown in fig. 7-18(b), the grooves are flared so that many spring members have long flexible spans at normal loads but become supported by sides of the grooves when overloads occur.

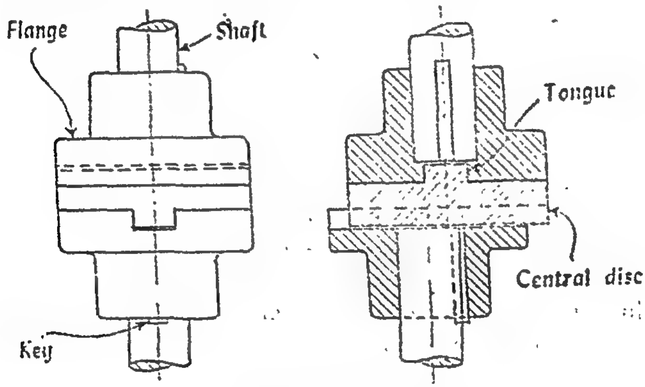
We assume that the torque is transmitted by the spring by shearing action similar to that of the coupling bolts in rigid type

force at the pitch circle radius will exert a bending action on the bolt and hence the bolts should be checked for bending as explained in article 7-20. The value of the bending stress is limited to 750 kg/sq cm.

Experience indicates that the peripheral velocity of C.I. coupling does not exceed 1,800 metre/min because at a higher frequency of load repetition the rubber pads become heated and disintegrate.

7-23. Oldham's coupling:

This coupling is a rigid coupling which is used for connecting two shafts whose axes are parallel and a short distance apart. Fig. 7-20 shows the constructional details of this coupling. It

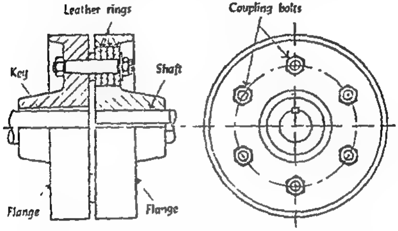


Oldham's coupling
FIG. 7-20

consists of two flanges each having a rectangular recess and each flange is keyed to the shaft to be connected. There is a central disc between the flanges. On the opposite faces of the central disc there are two rectangular projecting parts which are at right angles to each other and which fit into corresponding recesses in two flanges. With this coupling the rotary motion is transmitted by means of the sliding contact. The angular velocity of the driver and the driven is the same at any instant. This coupling is the inversion of the double slider crank chain.

This Oldham's coupling is, sometimes, used as a flexible coupling. In such cases the central member is made of fibre or has leather faced contact surfaces.

(ii) The diameter of the shaft is calculated from the torsion formula $T = \frac{\pi}{16}\, d^3 f_s$ or $d = 1{\cdot}72 \sqrt[3]{\frac{T}{f_s}}$. The permissible value for St 50 is limited to 200 kg/sq cm. In order to account for bending lower value of permissible shear stress is adopted.



Leather pad flexible coupling

Fig 7-19

(iii) The diameter of the hub of the coupling is taken from $2d$ to $2{\cdot}2d$ for cast iron and $1{\cdot}8d$ to $2d$ for cast steel and steel.

(iv) The length of the hub of the coupling is taken from $1{\cdot}5d$ to $2d$ for cast iron and d to $1{\cdot}3d$ for cast steel and steel.

(v) The number of bolts are located on a pitch circle radius, which is larger than that for the rigid coupling in order to reduce the shear load on the bolts necessary to transmit a given power.

(vi) The thickness of the rubber pad varies from 0·4 to 0·5 cm. The number of pads will depend upon the bearing load on each bolt. The bearing pressure varies from 10 to 20 kg/sq cm.

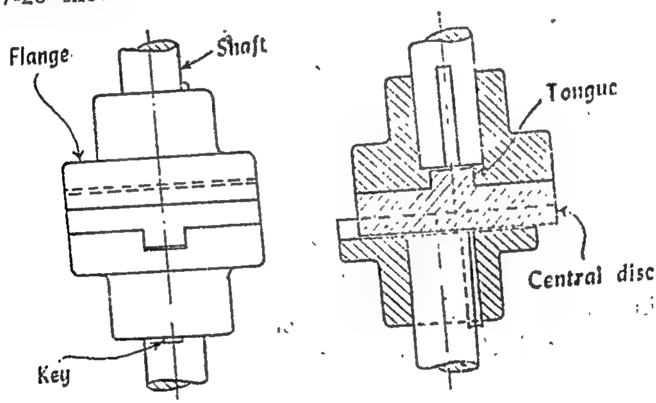
(vii) The size of the bolt is calculated from bearing considerations. Since the bolt is not rigidly held in the right hand face and the rubber is compressible the tangential

force at the pitch circle radius will exert a bending action on the bolt and hence the bolts should be checked for bending as explained in article 7-20. The value of the bending stress is limited to 750 kg/sq cm.

Experience indicates that the peripheral velocity of C.I. coupling does not exceed 1,800 metre/min because at a higher frequency of load repetition the rubber pads become heated and disintegrate.

7-23. Oldham's coupling:

This coupling is a rigid coupling which is used for connecting two shafts whose axes are parallel and a short distance apart. Fig. 7-20 shows the constructional details of this coupling. It



Oldham's coupling
FIG. 7-20

consists of two flanges each having a rectangular recess and each flange is keyed to the shaft to be connected. There is a central disc between the flanges. On the opposite faces of the central disc there are two rectangular projecting parts which are at right angles to each other and which fit into corresponding recesses in two flanges. With this coupling the rotary motion is transmitted by means of the sliding contact. The angular velocity of the driver and the driven is the same at any instant. This coupling is the inversion of the double slider crank chain.

This Oldham's coupling is, sometimes, used as a flexible coupling. In such cases the central member is made of fibre or has leather faced contact surfaces.

The following are the usual proportions for the cast iron Oldham's coupling.

Diameter of the shaft $= d$

Diameter of the bosses $= 2d$

Distance between centre lines of shafts $= e$

Diameter of disc or flanges $= D_1 = 3d + e$

Length of the boss $= 1{\cdot}75d$

Breadth of grooves $= \dfrac{D_1}{6} = w$

Thickness of the groove $= \dfrac{\text{breadth of groove}}{2}$

Thickness of the central disc $t = \dfrac{w}{2}$

Thickness of flange $= \frac{3}{4}d$

7·24. *Universal coupling:*

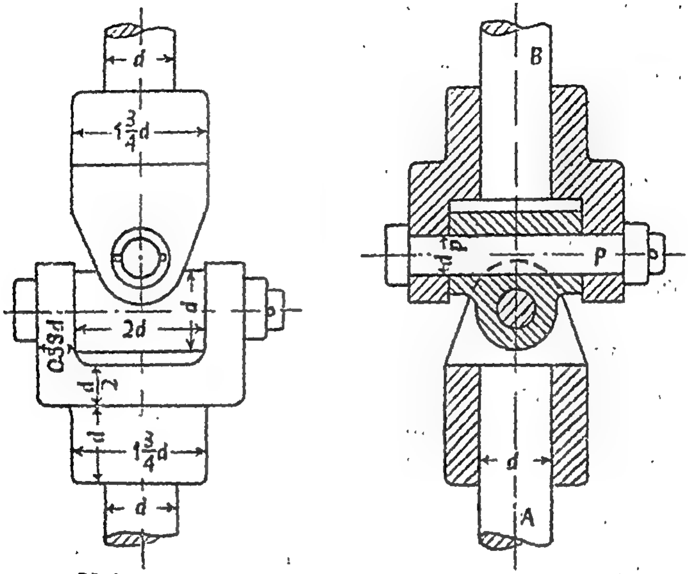
These couplings are rigid couplings that connect shafts whose axes will intersect if produced The coupling shown in fig. 7-21 consists of two forks which are keyed to the shafts. These two forks are pin jointed to a centre-block which has two arms at right angles to one another. The angle between the shaft axes may vary slightly during operation but these joints should not be used to compensate for excessive misalignment. The usual proportions for the joint when made of mild steel are shown in fig 7-21.

It can be shown from first principles that the speed of the driven shaft for any given angular position θ of the driving shaft is $\dfrac{\omega \cos \alpha}{1 - \cos^2\theta \sin^2\alpha}$ where θ = measured from the position in which the driving shaft fork lies in the plane of the shafts, ω the uniform velocity of the driver shaft and α the angle between two axes. The variability of the velocity ratio can be eliminated by using a double Hooke's joint with an intermediate shaft and the forks at the ends of the intermediate shaft lie in the same plane The intermediate shaft should make equal angles with the driver and driven shafts

The following procedure for the design of shafts and the pin is suggested. From the horse power requirement at a given speed, the mean torque on the shaft can be calculated and by knowing the permissible value of shear stress we get the diameter of the shafts. The pin is in double shear. If d_p be the diameter of the pin which is in double shear, we get

$$\frac{\pi}{4} d_p^2 \times f_s \times 2d = \text{torque to be transmitted.}$$

In the above equation, f_s denotes the allowable shear stress in the pin. The other dimensions are obtained from the proportions given in fig. 7-21.



Universal coupling or Hooke's coupling
FIG. 7-21

Examples:

1. Design and draw a sleeve or muff coupling which is used to connect two steel shafts transmitting 50 h.p. at 330 r.p.m.

The diameter of the shaft is calculated from the maximum torque transmitted. The dimensions of the cross section of the key (width and thickness) are found out from proportions. The dimensions of the sleeve are determined from the proportions and checked for the stresses. The proportions for the sleeve are as follows:

Diameter of the shaft $= d$
Diameter (outside) of the sleeve $= D = 2d + 1{\cdot}3$ cm
Length of the sleeve $= 3\frac{1}{2}d$.

The coupling may fail to transmit the full magnitude of the shaft torque due to the following causes:

(a) The coupling may twist off.
(b) The key may fail due to shear or crushing.

Moment of resistance for the coupling $= f_s \times$ section modulus

$$= f_s \times \frac{\pi}{16} \frac{(D^4 - d^4)}{D},$$

The following are the usual proportions for the cast iron Oldham's coupling.

Diameter of the shaft = d

Diameter of the bosses = $2d$

Distance between centre lines of shafts = e

Diameter of disc or flanges = $D_1 = 3d + e$

Length of the boss = $1{\cdot}75d$

Breadth of grooves = $\frac{D_1}{6} = w$

Thickness of the groove = $\frac{\text{breadth of groove}}{2}$

Thickness of the central disc $t = \frac{w}{2}$

Thickness of flange = $\frac{3}{4}d$.

7-24. Universal coupling:

These couplings are rigid couplings that connect shafts whose axes will intersect if produced. The coupling shown in fig. 7-21 consists of two forks which are keyed to the shafts. These two forks are pin jointed to a centre-block which has two arms at right angles to one another. The angle between the shaft axes may vary slightly during operation but these joints should not be used to compensate for excessive misalignment. The usual proportions for the joint when made of mild steel are shown in fig. 7-21.

It can be shown from first principles that the speed of the driven shaft for any given angular position θ of the driving shaft is $\frac{\omega \cos \theta}{1 - \cos^2\theta \sin^2\alpha}$ where θ is measured from the position in which the driving shaft fork lies in the plane of the shafts, ω the uniform velocity of the driver shaft and α the angle between two axes. The variability of the velocity ratio can be eliminated by using a double Hooke's joint with an intermediate shaft and the forks at the ends of the intermediate shaft lie in the same plane. The intermediate shaft should make equal angles with the driver and driven shafts.

The following procedure for the design of shafts and the pin is suggested. From the horse power requirement at a given speed, the mean torque on the shaft can be calculated and by knowing the permissible value of shear stress we get the diameter of the shafts. The pin is in double shear. If d_p be the diameter of the pin which is in double shear, we get

$$\frac{\pi}{4} d_p^2 \times f_s \times 2d = \text{torque to be transmitted.}$$

We adopt 4 cm. Crushing area provided is $4 \times 6 \cdot 5 = 26$ sq cm which will give reasonably low value of the stress.

5. *A mild steel shaft has to transmit 100 h.p. at 200 r.p.m. The allowable stress in the shaft is limited to 400 kg/sq cm and the angle of twist is not to exceed 1° in a length of 20 diameters. Calculate the suitable diameter for the shaft. Design and draw a cast iron flange coupling of protected type for this shaft. The safe shear stress in the coupling bolts is limited to 280 kg/sq cm.*

The diameter of the shaft is calculated from the maximum torque transmitted. Next, other dimensions are found from proportions and checked for the stresses. The proportions are as follows, the material being cast iron for the flange.

Diameter of the shaft $= d$

Outside diameter of the hub $= 2d$

Pitch circle diameter of bolts $= 3d$

Length of the hub $= 1 \cdot 5d$

Thickness of the flange $= \dfrac{d}{2}$

Thickness of the circumferential flange $= \dfrac{d}{4}$.

Number of bolts $= 3$, for shaft diameter upto 4 cm
$= 4$, for shaft diameter upto 10 cm
$= 6$, for shaft diameter upto 18 cm.

The coupling may fail to transmit the full magnitude of the shaft torque due to the following reasons:

(a) Bolts may fail by shearing.
(b) Bolts may fail by crushing.
(c) Flange may twist off at the hub.
(d) Flange may be broken by repeated bending.
(e) Key may fail due to crushing or shearing.

If d_1 be the diameter of the bolt and f_s the shearing stress intensity induced in the material of the bolt, then shear resistance of n bolts will be $n \dfrac{\pi}{4} d_1^2 f_s$. If T_{max} is the torque to be transmitted, then

$$T_{max} = n \times \frac{\pi}{4} d_1^2 f_s \times \text{pitch circle radius.}$$

Crushing load on each bolt $= t_1 \times d_1 \times f_b$ where t_1 is the thickness of the flange and f_b the crushing stress.

The bending of the flange due to deflection of the shaft may cause a failure by progressive fracture. By locating the bearing very near to coupling, this tendency may be decreased.

Shearing load on the key $= f_s \times$ width of the key $\times$ length of key.

Crushing load on the key $= f_c \times \dfrac{\text{thickness of the key} \times \text{length of key}}{2}$

If we design the coupling by proportions, then it will be strong for the shear of the flange and the hub.

$$\text{Torque transmitted} = \frac{71620 \times \text{h.p.}}{\text{speed}} = \frac{71620 \times 100}{200}$$
$$= 35{,}810 \text{ kg cm.}$$

Assuming the maximum torque to be 25% greater than the mean,

$$T_{max} = 1{\cdot}25\ T_{mean} = 1{\cdot}25 \times 35810 = 45{,}000 \text{ kg cm.}$$

If d cm be the diameter of the shaft, then

$$\frac{\pi}{16} d^3 \times 400 = 45000$$

or $$d = \sqrt[3]{\frac{45000}{400} \times \frac{16}{\pi}} = 8{\cdot}3 \text{ cm, we adopt 9 cm.}$$

According to rigidity view point, $\dfrac{T_{max}}{J} = \dfrac{f_s}{d/2} = \dfrac{G\theta}{l}$

$$\therefore\ f_s = \frac{0{\cdot}84 \times 10^6}{20d} \times \frac{\pi}{180} \times \frac{d}{2} = 370 \text{ kg/sq cm,}$$

which is less than 400 kg/sq cm. Therefore, the diameter of the shaft is to be designed from the rigidity view point

$$T_{max} = \frac{\pi}{16} \times d^3 \times 370 = 45{,}000$$

or $$d = \sqrt[3]{\frac{45000}{370} \times \frac{16}{\pi}} = 8{\cdot}56 \text{ cm, we adopt 9 cm.}$$

With usual proportions the dimensions of the key will be as under: width of key 25 mm; thickness of key 18 mm and length of the key 140 mm. The material of the key is the same as that of the shaft so $f_s = 400$ kg/sq cm and $f_c = 850$ kg/sq cm.

$$\text{Shearing force on the key} = \frac{45000 \times 2}{9} = 10{,}000 \text{ kg.}$$

$$\text{Shear stress induced in the key} = \frac{10000}{14 \times 2{\cdot}5} = 286 \text{ kg/sq cm}$$

which is less than the permissible, hence it is safe.

$$\text{Crushing stress induced in the key} = \frac{10000}{14 \times 0{\cdot}9} = 795 \text{ kg/sq cm}$$

which is less than the permissible value.

The outside diameter of the hub is taken as $2 \times 9 = 18$ cm.

Treating the hub as a hollow shaft, we determine the induced shear stress in the hub.

$$45000 = \frac{\pi}{16}\left[\frac{18^4 - 9^4}{18}\right] f_s$$

$$\therefore \quad f_s = \frac{45000 \times 16 \times 18}{\pi [18^4 - 9^4]} = 42 \text{ kg/sq cm}$$

which is well within limits.

Hub length is taken as length of the key which is 14 cm.

We adopt 4 bolts on pitch circle diameter of $3 \times 9 = 27$ cm.

$$\text{Force on each bolt} = \frac{45000}{\frac{27}{2} \times 4} = 835 \text{ kg.}$$

If d_1 cm be the diameter of the bolt, which is in single shear, we get

$$\frac{\pi}{4} d_1^2 \times 280 = 835$$

or

$$d_1 = \sqrt{\frac{835 \times 4}{280 \times \pi}} = 1{\cdot}95 \text{ cm; we adopt 2 cm.}$$

We take thickness of the flange as $\frac{d}{2} = \frac{9}{2} = 4{\cdot}5$ cm.

$$\text{Crushing stress intensity on bolt} = \frac{835}{2 \times 4{\cdot}5} \doteq 93 \text{ kg/sq cm}$$

which is well within limits.

The thickness of the protecting circumferential flange $= 2{\cdot}3$ cm.

6. *Design a bushed pin type of flexible coupling for connecting the motor and centrifugal pump shafts for the following duty:*

H.P. to be transmitted 20

Speed in r.p.m. 1,000.

The diameters of the motor and pump shafts are 5 cm and 4 cm respectively. Bearing pressure on rubber bush is 3 kg/sq cm; allowable shear stress in pins 200 kg/sq cm.

Proportions of the rigid type of flanged coupling are more or less standardised by manufacturers and by usage. For flexible coupling, the above proportions will be modified chiefly to reduce the bearing pressure on the rubber bushes which are the main features of the design.

The outer diameters of the hubs are each twice the shaft diameter and the length corresponds to that for the standard rigid type. The overall diameter and width of flanges are fixed by the dimensions of the bushes and the pitch circle diameter of pins.

For rigid coupling for this duty we shall require 4 bolts. Here we adopt 6 bolts. The diameter of the pin in terms of the diameter of the shaft and the number of pins is given by the relation pin

If we design the coupling by proportions, then it will be strong for the shear of the flange and the hub.

$$\text{Torque transmitted} = \frac{71620 \times \text{h.p.}}{\text{speed}} = \frac{71620 \times 100}{200}$$
$$= 35{,}810 \text{ kg cm.}$$

Assuming the maximum torque to be 25% greater than the mean,

$$T_{max} = 1{\cdot}25\ T_{mean} = 1{\cdot}25 \times 35810 = 45{,}000 \text{ kg cm.}$$

If d cm be the diameter of the shaft, then

$$\frac{\pi}{16} d^3 \times 400 = 45000$$

or $$d = \sqrt[3]{\frac{45000}{400} \times \frac{16}{\pi}} = 8{\cdot}3 \text{ cm; we adopt 9 cm.}$$

According to rigidity view point, $\dfrac{T_{max}}{J} = \dfrac{f_s}{d/2} = \dfrac{G\theta}{l}$

$$\therefore\ f_s = \frac{0{\cdot}84 \times 10^6}{20d} \times \frac{\pi}{180} \times \frac{d}{2} = 370 \text{ kg/sq cm,}$$

which is less than 400 kg/sq cm. Therefore, the diameter of the shaft is to be designed from the rigidity view point.

$$T_{max} = \frac{\pi}{16} \times d^3 \times 370 = 45{,}000$$

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With usual proportions the dimensions of the key will be as under: width of key 25 mm, thickness of key 18 mm and length of the key 140 mm. The material of the key is the same as that of the shaft so $f_s = 400$ kg/sq cm and $f_c = 850$ kg/sq cm.

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$$\text{Crushing stress induced in the key} = \frac{10000}{14 \times 0{\cdot}9} = 795 \text{ kg/sq cm}$$

which is less than the permissible value.

The outside diameter of the hub is taken as $2 \times 9 = 18$ cm.

Treating the hub as a hollow shaft, we determine the induced shear stress in the hub.

$$\text{Direct shear stress} = \frac{30}{\frac{\pi}{4} \times 1{\cdot}4^2} = 19{\cdot}6 \text{ kg/sq cm.}$$

Since the pin is not rigidly held in the left hand face and the rubber is compressible the force of 30 kg at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load along a bush, the arm of the bending moment at the reduced section (at 14 mm diameter) will be $2 + 0{\cdot}5 = 2{\cdot}5$ cm. Bending moment on the section will be $30 \times 2{\cdot}5 = 75$ kg cm. The bending stress on the section will be $\dfrac{75}{\frac{\pi}{32} \times 1{\cdot}4^3} = 280$ kg/sq cm.

Maximum shear stress on pin $= \frac{1}{2}\sqrt{280^2 + 4 \times 19{\cdot}6^2} = 144$ kg/sq cm which is less than the permissible value of the shear stress which is 200 kg/sq cm.

7. *A coupling of the Faulk type (Bibby type) is keyed to two 25 mm transmission shafts. The shafts rotate at 950 r.p.m. If the connecting strip is 4 cm from the axis of the shaft, how many folds of $0{\cdot}25 \times 3$ mm steel are required? Permissible shear stress for the shaft material is 400 kg/sq cm. Permissible shear stress for the steel fold may be taken as 2,000 kg/sq cm. Under what conditions could you rate this coupling at 12·5 h.p.?*

$$\text{Torque that can be transmitted by shaft} = \frac{\pi}{16}\, d^3 f_s$$

$$= \frac{\pi}{16} \times 2{\cdot}5^3 \times 400 = 1{,}220 \text{ kg cm.}$$

We assume that the folds are subjected to shear only. Let us consider the torque transmitting capacity of one fold.

$$\text{Shear load that can be resisted by each fold} = \frac{0{\cdot}25 \times 3 \times 2000}{100}$$

$$= 15 \text{ kg.}$$

Torque that can be resisted by each fold $= 15 \times 4 = 60$ kg cm.

$$\therefore \text{ Number of folds} = \frac{1220}{60} = 20{\cdot}4; \text{ we adopt 21 folds.}$$

Torque capacity of the coupling = 1,220 kg cm.

$$\text{Metric h.p.} = \frac{1220 \times 950}{71620} = 16{\cdot}2.$$

$$\text{diameter} = \frac{0{\cdot}5d}{\sqrt{n}} = \frac{0{\cdot}5 \times 5}{\sqrt{6}} = 1{\cdot}3 \text{ cm; we adopt } 1{\cdot}4 \text{ cm.}$$

From fig. 7-17 we see that the portion of the pin of least diameter (1·4 cm) is threaded and secured in the right hand half coupling (at the pump end) by the standard nut and washer and the shoulder of the enlarged portion of the diameter is 22 mm. On the enlarged portion of the rod, a thin brass sleeve of 2 mm thick is slid. Let us assume that the thickness of the rubber bush is 6 mm. The overall diameter of the rubber bush will be

$$22 + [\text{illegible}] \times [\text{illegible}] + [\text{illegible}] \times 6 = 38 \text{ mm.}$$

Let l be the necessary length of the bush; then its projected area or bearing area is $3{\cdot}8 \times l$ sq cm. If F be the force to be applied at each pin to produce the torque to transmit the given power, we have $n \times F \times R = T$

where n is the number of pins on pitch circle radius R and T is the torque transmitted

$$\text{Torque transmitted by coupling} = \frac{71620 \times 20}{1000} = 1{,}432 \text{ kg cm.}$$

Let us assume the pitch circle radius to be 8 cm. The number of bolts is 6.

$\therefore\ 8 \times 6 \times F = 1432.$

$\therefore\ F = \dfrac{1432}{8 \times 6} = 30$ kg. Bearing pressure intensity is limited to 3 kg/sq cm. Therefore, minimum projected area will be $\dfrac{30}{3} = 10$ sq cm.

$\therefore\ 3{\cdot}8 \times l = 10$ or $l = \dfrac{10}{3{\cdot}8} = 2{\cdot}64$ cm.

We adopt the length of the rubber sleeve as 4 cm. Clearance of 5 mm is left between the faces of the two halves of the coupling. There is no rigid connection between two halves of the couplings. The drive takes place through the medium of the compressible rubber bushes.

14 mm diameter portion of the pin should be a tapping fit in the coupling disc to avoid bending stresses. The threaded portion should be as short as possible so that the shear can be taken up by the unthreaded neck.

pins if the permissible shear stress in the pin material is not to exceed 300 kg/sq cm. *Ans.* *13 mm dia. pins.*

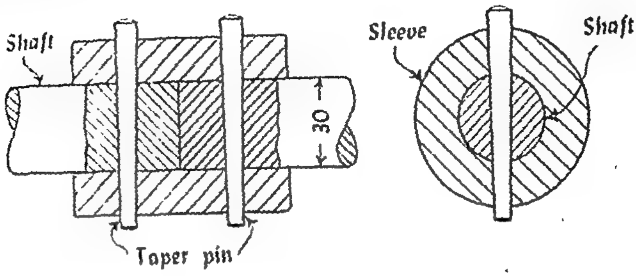


FIG. 7-22

4. *A flange coupling transmits 35 h.p. at 1,400 r.p.m. The pitch circle diameter of the coupling bolts is 13 cm. Calculate the size of the bolts if the permissible shear stress intensity is 250 kg/sq cm. Assume 4 bolts.* *Ans.* *M14.*

5. *The shaft of a steamship transmits 18,000 h.p. at 80 r.p.m. Assuming that the shaft is subjected to pure torsion, determine the diameter of the shaft if the permissible shear stress in the shaft material is limited to 350 kg/sq cm. Assuming the pitch circle diameter to be 1·5 times the diameter of the solid shaft, determine the diameter of the coupling bolt if the permissible value of the shear stress in the bolt is limited to 280 kg/sq cm. Assume 12 bolts.*

Ans. *380 mm; 15 mm.*

6. *A propeller shaft is made up by joining together number of solid shafts. The joint is made by forging the ends of the shaft in the form of a flange and bolting the flanges together by means of 8 bolts. If the shaft transmits 90 h.p. at 100 r.p.m., determine the size of the shaft, the diameter and the thickness of the flange and the diameter and pitch circle diameter of bolts. Permissible stresses are* $f_s = 630$ *kg/sq cm;* $f_b = 350$ *kg/sq cm.* *Ans.* *Shaft dia. 80 mm.*

7. *Each half of a rigid flange coupling with fitted bolts for a shaft of diameter D is to be installed on the shaft with a square key 1·25D long. Five bolts are to be used on a bolt circle of diameter 5D,. Assume coupling, bolts, shaft, and key to be of the same material.*

(a) *What should be the diameter d of the bolts in terms of D to make the coupling as strong as the shaft in torsion?*

Hence we could rate the coupling at 12·5 h.p. with

$$\left\{\frac{16\cdot 2}{12\cdot 5} - 1\right\} \times 100 = 30\% \text{ possibility of overload.}$$

8. In a Hooke's joint a driving shaft transmits a mean twisting moment of 55,000 kg cm to the driven shaft. Suggest the suitable diameter for the shafts and the pins, assuming the safe shearing stresses to be ~~650~~ 560 *kg/sq cm and 280 kg/sq cm for the shafts and pins respectively. The shafts are assumed to be subjected to torsion only.*

If d cm be the diameter of the shaft, then

$$\frac{\pi}{16}\, d^3 \times 560 = 55000.$$

$$\therefore \qquad d = \sqrt[3]{\frac{55000 \times 16}{560 \times \pi}} = 7\cdot 93 \text{ cm, we adopt 8 cm.}$$

The pin is in double shear. The planes across which shear takes place are at a distance of twice the diameter of the shaft.

Let d_p be the diameter of pin, then resisting moment of the pin is equal to the torque transmitted. The resisting moment of the pin $= 2 \times \frac{\pi}{4}\, d_p^2 \times 280 \times 8$ kg cm. and the torque transmitted is 55,000 kg cm.

By equating two, we get

$$d_p = \sqrt{\frac{55000 \times 4}{2 \times \pi \times 280 \times 8}} = 3\cdot 96 \text{ cm, we adopt 4 cm}$$

diameter pin.

Exercises:

1. The split muff coupling transmits 90 h.p. at 70 r.p.m. The diameter of the shaft is 20 cm. Assuming that the two halves of the coupling are connected by 8 bolts, determine the diameter of each bolt if the permissible tensile stress intensity in the bolt material is limited to 800 kg/sq cm. The coefficient of friction between the shaft and the muff may be taken as 0·15.
Ans. M 42.

2. Design and draw a neat sketch of a muff coupling to transmit 50 h.p. at 120 r.p.m. Choose your own material and suitable values for the stresses.

3. Fig. 7-22 shows the muff coupling for 30 mm diameter shafts transmitting 4 h.p. at 240 r.p.m. Determine the diameter of the taper

13. Two shafts inclined at an angle of 30° are connected by Hooke's joint. 4,000 kg cm torque is to be transmitted by the coupling. The distance between the journal bearings on each fork is 10 cm. The overhungs of the driving and the driven shafts are 20 and 50 cm respectively. Calculate the bearing reactions for each shaft.

Ans. 115 kg; 40 kg.

14. The split muff coupling transmits 20 h.p. at 100 r.p.m. The shaft diameter is 6·25 cm. Assuming that the two halves of the coupling are connected by 8 bolts, determine the diameter of each bolt if the permissible tensile stress for the bolt is 700 kg/sq cm. The coefficient of friction between the shaft and the muff may be taken as 0·2.

(Sardar Patel University, 1967)

15. A mild steel shaft has to transmit 200 h.p. at 400 r.p.m. The allowable stress is limited to 400 kg/sq cm and angle of twist is not to exceed 1° in a length of 20 diameters. Calculate the suitable diameter for the shaft. Draw a cast iron flange coupling for the shaft after designing the coupling bolts, keys and flange width.

(Bombay University, 1969)

EXAMPLES VII

1. During the design of a certain steam ship, the propeller shaft is required to transmit 2,500 h.p. at 125 r.p.m. The designer has the choice of three steels *A*, *B*, and *C* having the prices and permissible values of the shear stresses shown in the table. The prices are given for the completed solid shaft. The prices for the hollow shaft are to be increased 43 paise per kg.

Steel	Price per kg	Permissible shear stress kg/sq cm
A	Rs. 2·75	350
B	Rs. 4·00	450
C	Rs. 4·65	650

The angular twist is limited to 1° in a length of 20 diameters. (a) If the weight is the primary consideration what material and what shaft diameter would you use for a solid shaft and a hollow shaft whose inside diameter is 0·6 of the outside diameter?

(b) If the prices were to be the primary consideration, what diameter shaft would you suggest for a solid shaft and a hollow shaft whose inside diameter is 0·6 of the outside diameter?

(b) What should be the dimension of the key in terms of D if the key is to transmit the full capacity of the shaft in torsion?

Ans. (a) *Based on shear being uniform,* $d = 0{\cdot}122D$.
Based on maximum shear $= 4/3$ *average,*
$d = 0{\cdot}101D$.
(b) *Width* $=$ *thickness* $= 0{\cdot}236D$.

8. Design a shaft coupling to transmit 10 horse power at 1,000 r.p.m. The permissible stresses for the coupling and key 650, 350 and 1,000 kg/sq cm in tension, shear and compression respectively.
Draw the freehand sketch showing important dimensions.

9. Design and draw a protected type of C.I. flange coupling for a steel shaft transmitting 20 h.p. at 200 r.p.m. and having an allowable shear stress of 400 kg/sq cm. The working stress in both should not exceed 300 kg/sq cm. Assume that same material is used for shaft and key and that the compressive stress is twice the value of its shear stress and the maximum torque is 25% greater than the full-load torque.

Take shear stress of cast iron $= 40$ *kg/sq cm.*

10. A mild steel shaft has to transmit 10 h.p. at 200 r.p.m. The allowable stress in the shaft is limited to 420 kg/sq cm and the angle of twist is not to exceed $1°$ *in a length of 20 diameters. Calculate the suitable diameter for the shaft and design a cast iron flange coupling of protected type for this shaft. The safe stress in the coupling bolts is limited to 280 kg/sq cm. Take maximum torque to be 1·25 times the mean torque.*
$G = 8{\cdot}4 \times 10^5$ *kg/sq cm.*

11. A simple flange coupling of C.I. is used for connecting two shafts of 50 mm diameter. The flanges are fitted with 6 bolts on a pitch circle diameter of 15 cm. The shafts transmit a torque which has an average value of 9,225 kg cm, maximum value being 30% more than the average value. The running speed of the shaft is 400 r.p.m. Assume suitable stresses, design the coupling along with the key. Make a neat sketch of the coupling.

12. A propeller shaft is assembled together by joining a number of shafts, which have flanges fitted with keys at both ends. The flanges at the end of two shafts are bolted together by 8 bolts at each joint. The shaft transmits 100 h.p. at 120 r.p.m. Design and make a fully dimensioned sketch of the flanged joint connecting the two shafts together. Show clearly how the shafts are aligned properly.

Both steels have a shear modulus of 8×10^5 kg/sq cm.

Comment on the relative merits of each design and state with reasons which you would choose to adopt.

Ans. (a) 70 mm
(b) 75 mm (outside); dia ratio 0·6 suitable.

5. A gear wheel is keyed to a shaft which is supported by bearings at 75 cm centres. The bearings are equally spaced on each side of the wheel. The wheel, which weighs 225 kg is driven by a pinion and transmits 120 h.p. when the speed of the revolution is 80 r.p.m. The teeth are machine cut of pressure angle 20°. The pitch circle diameter of the gear is 120 cm. If the allowable shear stress in the shaft is 350 kg/sq cm, determine the diameter of the wheel shaft.

Ans. 125 mm.

6. A 150 cm diameter pulley transmits 200 h.p. at a speed of 500 r.p.m. to a 60 cm diameter pulley [fitted on a shaft to which a flange coupling is to be attached.

(i) Assuming suitable number of bolts and the bolt circle diameter, determine the required size of the bolts, using a design shear stress for bolts of 200 kg/sq cm.

(ii) Determine the hub length of each part of the coupling, if a 5 mm × 10 mm flat key is to be used. Assume design stress values of 560 kg/sq cm in shear and 1,400 kg/sq cm in bearing. The shaft is 50 mm in diameter.

Make a neat dimensioned sketch of the coupling showing clearly the provision made for the correct alignment of the two halves of the coupling.

7. The speed of an engine shaft is 300 r.p.m. The flywheel on the shaft is 150 cm external diameter; the rim is rectangular in section and measures 15 cm wide and 10 cm deep. Determine the kinetic energy which the flywheel will have at this speed. The running conditions are such that the flywheel varies in speed four times during each revolutions setting up harmonic oscillation and twisting the shaft through an angle of 0·2° for each oscillation. Determine the torque involved in this action.

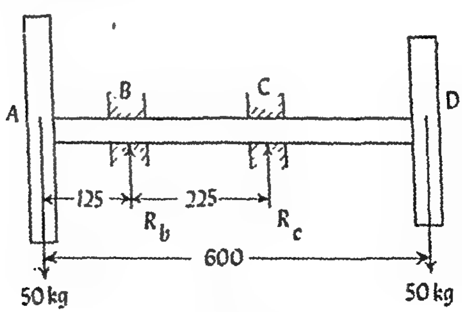


FIG. 7-24

8. A steel shaft has two pulleys, *A* and *D*, fixed to the ends as shown in fig. 7-24 and it is supported by two bearings at *B* and *C*. Vertically downward

2. A machine which runs at 200 r.p.m. requires a maximum of 10 h.p. It is to be driven from an overhead line shaft by a belt drive inclined at 55° to the horizontal.

The pulley on the machine is of 90 cm diameter, weighs 100 kg and has its centre 120 cm above ground level. The line shaft is 240 cm above ground level and runs at 300 r.p.m.

Determine a suitable width of flat belt for the drive, making provision for a starting torque 20 per cent greater than the maximum running torque. The *belting to be used is* [illegible] *mm thick and has a working tensile stress 25 kg/sq cm.* The coefficient of friction between belt and pulleys is 0.3. Centrifugal tension may be neglected. (The ratio of the belt tensions is given by $\frac{T_1}{T_2} = e^{\mu\theta}$.)

The machine pulley is mounted on a solid steel shaft of 65 mm diameter and overhangs the bearing by 23 cm. Check that the shaft is not overstressed where it enters the bearing. Assume a working shear stress of [illegible] kg/sq mm.

Ans. [illegible] cm.

3. The traction sheave of a goods hoist is to be of 95 cm diameter and may be assumed to weigh 250 kg with a radius of gyration of 45 cm. It is mounted centrally on a solid shaft supported in two bearings 70 cm apart.

The cage weighs 1,000 kg (including an allowance for the ropes) and the greatest load to be carried weighs 800 kg. The counterbalance weighs 1,150 kg. The arrangement is shown diagrammatically in fig. 7-23. The maximum lifting velocity is to be 3.6 metre/sec and is to be attained after a rise of 8 metre from rest with uniform acceleration.

Determine the required number of hoisting ropes of 15 mm diameter each *having a permissible working load of* 350 kg.

Estimate the maximum horse power required and determine a suitable shaft size, using a working shear stress of 5 kg/sq mm.

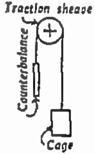


Fig. 7-23

Give a dimensioned sketch of a suitable sheave. Specify the size of the key to secure it to the shaft, assuming working stresses of 6 kg/sq mm in shear and 15 kg/sq mm in bearing.

Ans. 6 ropes; 43 h.p., 100 mm.

4. A shaft is to transmit 150 h.p. at 500 r.p.m. with provision for a 20% overload. The angle of twist must not exceed 1° in a length of fifteen diameters. For practical reasons the external diameter must not be more than 85 mm or less than 50 mm. Keyway about 5 mm deep are to be cut in places.

Design the shaft in each of the following alternative materials:

(a) Solid circular section mild steel with a working shear stress of 600 kg/sq cm.

(b) Hollow circular section alloy steel with a working shear stress of 1,000 kg/sq cm.

Both steels have a shear modulus of 8×10^5 kg/sq cm.

Comment on the relative merits of each design and state with reasons which you would choose to adopt.

Ans. (a) 70 mm
(b) 75 mm (outside); dia ratio 0·6 suitable.

5. A gear wheel is keyed to a shaft which is supported by bearings at 75 cm centres. The bearings are equally spaced on each side of the wheel. The wheel, which weighs 225 kg is driven by a pinion and transmits 120 h.p. when the speed of the revolution is 80 r.p.m. The teeth are machine cut of pressure angle 20°. The pitch circle diameter of the gear is 120 cm. If the allowable shear stress in the shaft is 350 kg/sq cm, determine the diameter of the wheel shaft.

Ans. 125 mm.

6. A 150 cm diameter pulley transmits 200 h.p. at a speed of 500 r.p.m. to a 60 cm diameter pulley [fitted on a shaft to which a flange coupling is to be attached.

(i) Assuming suitable number of bolts and the bolt circle diameter, determine the required size of the bolts, using a design shear stress for bolts of 200 kg/sq cm.

(ii) Determine the hub length of each part of the coupling, if a 5 mm × 10 mm flat key is to be used. Assume design stress values of 560 kg/sq cm in shear and 1,400 kg/sq cm in bearing. The shaft is 50 mm in diameter.

Make a neat dimensioned sketch of the coupling showing clearly the provision made for the correct alignment of the two halves of the coupling.

7. The speed of an engine shaft is 300 r.p.m. The flywheel on the shaft is 150 cm external diameter; the rim is rectangular in section and measures 15 cm wide and 10 cm deep. Determine the kinetic energy which the flywheel will have at this speed. The running conditions are such that the flywheel varies in speed four times during each revolutions setting up harmonic oscillation and twisting the shaft through an angle of 0·2° for each oscillation. Determine the torque involved in this action.

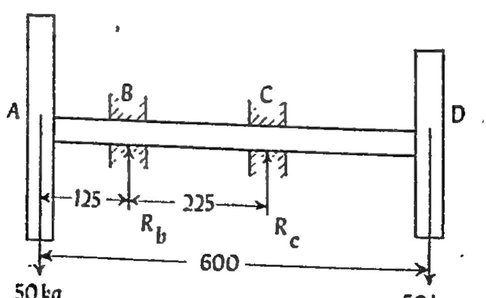


Fig. 7-24

8. A steel shaft has two pulleys, *A* and *D*, fixed to the ends as shown in fig. 7-24 and it is supported by two bearings at *B* and *C*. Vertically downward

loads of 50 kg are transmitted to the shaft by each pulley. If the shaft transmits 80 h.p at 900 r.p.m., calculate the suitable uniform shaft diameter using the maximum shear stress theory and satisfying the folllowing conditions. Maximum allowable shear stress in shaft, 450 kg/sq cm; maximum allowable twist between pulleys 0.20 degree.

Discuss reasons for and against using a stepped shaft in this case.

Take the modulus of rigidity for the shaft, $G = 8.4 \times 10^5$ kg/sq cm.

Ans 75 mm

9. A locomotive brake system operates through a solid steel shaft supported in bearings 100 cm apart. A horizontal crank arm 35 cm long and placed centrally between the bearings is linked to the piston of a vacuum cylinder of 75 cm diameter. Two vertical cranks each 20 cm long are spaced at a distance of 35 cm on either side of the central crank and are connected to horizontal pull rods. If the maximum vacuum in the cylinder is 1 kg/sq cm and the maximum allowable shear stress in the shaft = 420 kg/sq cm, calculate a suitable shaft diameter.

Ans 150 mm.

10. Design a hollow steel shaft to transmit 350 h.p at 400 r.p.m. with provision for 25% overloa [illegible] exceed 1° in a length of 2 [illegible] two lengths of this shaft. [illegible] stress of 580 kg/sq cm and a working crushing stress of 1,400 kg/sq cm with $G = 0.84 \times 10^6$ kg/sq cm.

The diameter of the shaft and the coupling bolts, and the key sizes are to be computed. Other dimensions may be decided by judgement. Make the outside diameter of the shaft twice its inside diameter. Give a fully dimensioned sketch of the coupling.

Describe briefly with the aid of an outline sketch a form of coupling which would be suitable if the two lengths of shaft were slightly misaligned.

Ans. 12 cm outside diameter, 6 bolts, on 16 mm diameter on 28 cm diameter pitch circle

11. A shaft is to transmit 270 h.p at 120 r.p.m. Determine suitable cross sectional dimensions for a hollow steel shaft. Assume a reasonable diameter ratio and adopt a working shear stress of 6 kg/sq mm. Estimate the saving in weight per metre length compared with a solid shaft. The density of the steel is 7·8 gm/cu cm. Design a coupling to connect two lengths of this shaft which are in line. Compute as many parts as practicable using reasonable working stresses. Other parts are to be decided by judgement.

Give a fully dimensioned working sketch of the coupling.

Ans. 12 cm outside and 6·5 cm inside diameter would suit.

12. Design and draw an Oldham's coupling to transmit 100 h.p at 250 r.p.m. Take f_s for shaft and key 8,000 psi (560 kg/sq cm), f_b for shaft and key 10,000 psi (700 kg/sq cm). Diameter of coupling = $3d$ where d = diameter of shaft. Width of tongue = $0{\cdot}45d$. Assume any data and stresses which are not given.

(Gujarat University, 1959)

Shaft Diameter mm	Key size Width × height mm × mm
12 – 17	5 × 5
18 – 22	6 × 6
23 – 30	8 × 7
31 – 38	10 × 8
39 – 44	12 × 8

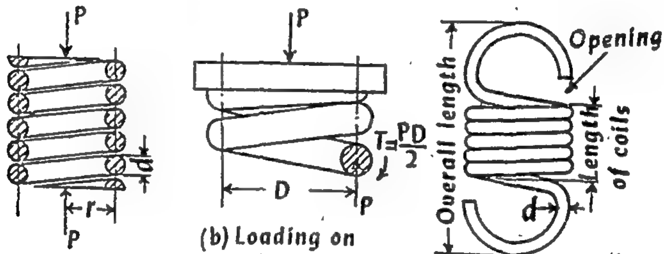
(University of Bombay, 1969)

23. A cast iron flange coupling has to transmit 50 h.p. at 150 r.p.m. Provide six bolts in the coupling and space them on a pitch circle of diameter 3·5 times the diameter of the shaft approximately. The material of bolts and shafts is steel of ultimate shearing strength 2,100 kg/sq cm. Design the coupling and make a fully dimensioned working sketch [illegible] y one bolt in position.

(v) To change the vibration characteristics of a member as in flexible mounting of motors.

8-2. Closed coiled helical spring subjected to axial loading — circular wire:

A helical spring is considered close coiled if the plane containing each coil is nearly perpendicular to the axis of the helix. This will be the case when the helix angle is small and generally a spring is taken as close coiled if the angle be less than 10°. The effect of an axial load on close coiled spring may be investigated by cutting the wire and considering the equilibrium of the upper part as in fig. 8-1(*b*).



(a) Compression spring (b) Loading on helical spring (c) Tension spring

Forces acting on springs

FIG. 8-1

Let P = axial load

D = mean diameter of coils

d = diameter of spring wire

δ = deflection of the spring

n = number of active coils

C = spring index $= \frac{D}{d}$ for circular wires

G = modulus of rigidity

f_s = shear stress induced in the wire of the spring.

l = length of the spring wire (active).

It will be observed that for equilibrium the material of the wire must provide the resisting torsional moment $T = \frac{PD}{2}$ as shown in fig. 8-1(*b*).

CHAPTER **VIII**

SPRINGS

8-1. Introduction:

A spring may be defined as an elastic body or resilient member whose primary function is to deflect or distort under load; it recovers its original shape when load is released.

The important types of springs are:

(i) Helical compression or tension springs, in which the major stress is shear due to twisting. They are made of wire coiled into a helical form, the load being applied along the axis of the helix.

(ii) Helical torsion springs in which the major stresses are tensile and compressive due to bending. They are similar in form to helical compression springs, the torque being applied about the axis of the helix.

(iii) Spiral springs in which the major stresses are tensile and compressive due to bending. They consist of flat strip wound in the form of a spiral and loaded in torsion.

(iv) Leaf springs in which the major stresses are tensile and compressive. They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency and resilience. Leaf springs may be full elliptic, semi-elliptic or cantilever.

(v) Belliville springs, in which the major stresses are tensile and compressive, are composed of coned discs which may be stacked up to give a variety of spring load-deflection characteristics.

The important functions of springs are:

(i) To apply forces and to control motions as in brakes and clutches

(ii) To measure forces as in a spring balance

(iii) To store energy as in clock springs

(iv) To cushion or reduce the effect of shock or impact loading as in carriage springs

The load required to produce unit deflection is known as the stiffness of the spring or the spring rate.

$$\therefore \quad \frac{P}{\delta} = k = \frac{P}{\dfrac{8PD^3n}{Gd^4}} = \frac{Gd^4}{8D^3n} = \frac{Gd}{8C^3n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

where C is the spring index as defined earlier. *The stiffness of the spring is the function of the geometrical dimensions of the springs, and the material of the spring.*

While choosing the pitch of the coils, the following points should be noted:

(i) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress will not exceed the yield point in torsion.

(ii) The spring should not close up before the maximum service load is reached.

The design of a helical compression spring is frequently completed by specifying the compressed length of the spring. The solid length of a spring $= d \times n'$ where n' is the number of coils and d is the diameter of wire of the spring.

$$\text{Free length} = l_p + \delta_p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

where

l_p = length of spring compressed under a load P

δ_p = deflection under a load P.

The pitch p of the coils is given by

$$p = \frac{\text{free length} - \text{solid length}}{n'} + d. \quad \ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

8-3. Helical springs of non-circular wire:

In order to provide greater resilience in a given space and to provide for pre-determined altering of the stiffness of the spring, helical springs with square or rectangular wires are used.

The maximum shearing stress in a helical tension or compression spring made of rectangular wire is equal to

$$f_s = \frac{KPD\,(3b + 1{\cdot}8t)}{2b^2\,t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

where t = short dimension of rectangular cross section of the wire

b = long dimension of rectangular cross section of the wire, and this dimension is parallel to the axis of the spring.

The torsional shear stresses are set up within the material of the wire and the maximum value of the shear stress induced in the material may be obtained by the equation

$$T = \frac{PD}{2} = \frac{\pi}{16}\, d^3 f_s \text{ .... (i)}$$

It should be remembered that there will be direct shear stress equal to $\frac{P}{\frac{\pi}{4} d^2}$, but this will be small in comparison with the torsional shear stress and may be ignored. Likewise bending stresses due to the obliquity of coils are neglected.

In order to account for the effects of direct shear and wire curvature a stress factor as defined by A. M. Wahl has been introduced. The maximum resisting torque is given by the equation

$$T = \frac{PD}{2} = \frac{f_s}{K} \cdot \frac{\pi}{16} d^3 \text{ .... (ii)}$$

where $K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ where C is a spring index.

The size of the wire should be selected from IS· 1137-1959. Appendix XII gives the sizes of wire to be used for the design of springs.

For circular wires $C = \frac{D}{d}$, while for square wire and rectangular wire springs, the spring index has the value D divided by the radial thickness of the wire.

The Wahl stress factor has been adoped generally for the design of helical springs.

It can be shown that the deflection of a helical spring may be calculated with sufficient accuracy from the formula

$$\delta = \frac{8\, PD^3 n}{Gd^4} \text{ .... (iii)}$$

In the above formula, the value of n is the active turns. The ends of the coil which are in contact with the seat are inactive. For tension springs ending in hooks, the deflection of the hooks may be added to the deflection derived from the actual number of turns. In order to make allowance for this deflection, the number of active turns can be taken slightly greater than the actual turns.

$$\therefore \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} \quad \ldots\ldots\ldots\ldots \quad \text{(iii)}$$

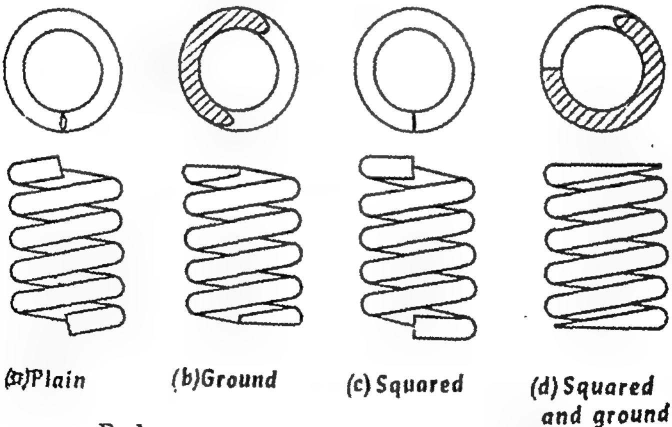
Thus, with use of round wire, with the same grade of material, the same stresses will be induced in the coils if the same spring index is provided.

8-5. General considerations in design of compression and tension springs:

Springs are usually made from high carbon steels (0·7 to 1·0%) or from medium carbon alloy steels. 18/8 stainless steel, spring brass, phosphor bronze, monel and other metal alloys are used for corrosion resistant springs. Small size springs are usually wound cold and are given heat treatment for desired properties. The large diameter wires are wound hot and heat treated after winding. Brass and bronze cannot be hardened by heat treatment and these springs are not treated after winding.

The allowable strength of a spring wire decreases with the increase in diameter of the wire. Therefore, higher working stresses are allowable for small wires than large wires. The stress factor depends on the spring index which is assumed for the design of the spring. For general industrial uses the spring index should be 8 to 10; for clutch and valve springs 5 is common. If no data are available, spring index of 8 should be adopted.

Steel wire sizes are available in fractional dimensions and should be selected from either SWG or W and M gauge numbers. The non-ferrous wire sizes are usually given by B and S gauge numbers. Diameters of wire are recommended by IS: 1137-1959.



End connections for compression springs
FIG. 8-2

Fig. 8-2 shows the end connections for compression springs. The plain ends are the most economical to manufacture. The closest approach to the

For square wire, $b = t = a$ the side of the square, then the equation for stress will be

$$f_s = K\frac{2\cdot4PD}{a^3} \quad \text{(ii)}$$

The deflection of a coil spring with rectangular wire is

$$\delta = \frac{2\cdot45\,PD^3n}{Gt^3\,(b - 0\cdot56t)} \quad \text{(iii)}$$

By putting $b = t = a$ in equation (iii), we get the expression for the deflection of a helical coil spring of square wire as

$$\delta = \frac{5\cdot58\,PD^3n}{Ga^4} = \frac{5\cdot58\,PC^3n}{Ga} \quad \text{(iv)}$$

The expression for the stiffness of the spring of square and rectangular wires can be obtained.

8-4. Concentric helical springs:

Springs used on railway trucks and in many automobile clutches consist of two concentric helical coils. These two helical coils are wound right and left hand so as to avoid binding of adjacent coils. Generally, the free length and solid lengths of these springs are equal, as a result both these springs are deflected by equal amounts. These springs may be of round, rectangular or square wire. Generally these springs are both made of the same grade of material

In the design of concentric springs the same shear stress is aimed at in both the coils.

The deflection of a spring in term of stress induced is given by the expression

$$\delta = \frac{\pi n D^2 f_s}{dG} \quad \text{(i)}$$

$$\therefore \quad \frac{\delta}{f_s} = \frac{\pi n D^2}{d\,G} = \frac{\pi\, nd\, D^2}{G\,d^2} = \frac{\pi}{G}\, nd\left(\frac{D}{d}\right)^2 \quad \text{(ii)}$$

As the solid height of both the springs is the same, by adopting the subscripts 1 and 2 to the inner and outer coils respectively, we have $n_1d_1 = n_2d_2$.

As the material and deflection are the same, for equal stresses in both the springs, we have

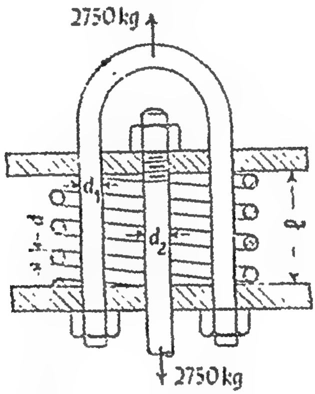
$$\frac{\pi\, n_1d_1\left(\dfrac{D_1}{d_1}\right)^2}{G} = \frac{\pi\, n_2d_2\left(\dfrac{D_2}{d_2}\right)^2}{G}.$$

Fig. 8-3 shows the types of spring ends for tension loading. Some of these designs may result in a considerably greater stress in the spring than that calculated on the basis of an axial load.

Often tension springs are made with plain ends and spring ends are attached to this, as shown in fig. 8-4.

Fig. 8-5 shows the arrangement where compression spring is used to carry a tension load.

To provide for local stress concentration in tension springs, the design stress for tension springs should not exceed 70% of that used with compression springs.



Compression spring carrying axial tensile load
FIG. 8-5

The majority of tension springs that are manufactured are wound with sufficient initial tension (or pre-load) to keep the springs closed up when the spring is unloaded. The amount of initial tension put into spring when it is wound varies in general as an inverse function with the spring index. Tension springs coiled with initial tension can be wound with greater precision particularly with respect to free length requirements of the spring.

Springs are often grouped into classes according to service conditions. Light service consists of operation under static conditions. Examples of such service are safety valves, spring couplings, etc.

Average service includes springs in which the load is not static but the load changes are not frequent. Examples are governor springs, automobile suspension springs, etc.

In severe service conditions, the load changes are cyclic as in I.C. engine valve springs.

For the same size of the wire, the allowable stresses are maximum for light service springs and minimum for severe service springs.

Compression springs are likely to buckle if it is too long compared to its mean diameter. As far as possible this condition should be avoided.

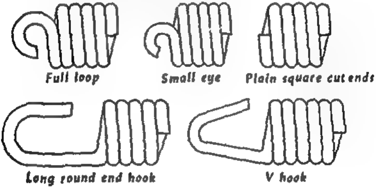
axial loading is given by squared and ground ends. In all springs there is eccentricity of loading introduced by the end connections. This eccentricity can be neglected in compression springs having squared and ground ends, having large spring indices, and having six or more active coils. It should be remembered that whatever portion of the spring is in contact with the seat, is inactive.

The following gives the value of inactive turns:

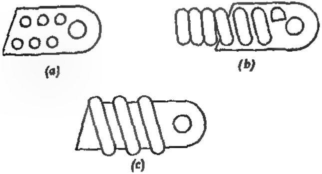
Plain ends	½ turn	Squared ends 2
Ground ends	1 ,,	Squared and ground ends 2.

The design of tension springs differs, in the main, from the design of compression springs because of:

(i) Type of spring ends
(ii) Bending stress in tension springs
(iii) Initial tension in extension springs
(iv) Lack of inherent restraint in case of spring rupture



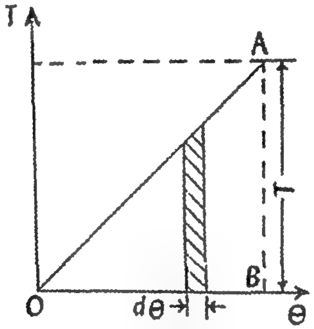
Spring ends for tension loading
Fig. 8-3



Separate spring ends
Fig. 8-4

2. *Derive a formula for the strain energy stored in a circular cross section rod loaded by a pure torque. Hence, calculate the weight of the helical spring made of circular section wire which is required to bring to rest a mass of 20 kg moving with a velocity of 3 metre/sec. The maximum allowable stress in the spring is to be 5,600 kg/sq cm and the modulus of rigidity is $0{\cdot}84 \times 10^6$ kg/sq cm.*

If the spring is made from 2 SWG wire of diameter 7·010 mm with a mean coil diameter of 6 cm, calculate the stiffness of the spring and the number of coils required.



Strain energy diagram for a spring

FIG. 8-6

The strain energy of a circular bar loaded in torsion may be calculated from a diagram of torsion (fig. 8-6) in which torque is represented by the ordinates and the angle of twist by the abscissas. Within elastic limit the angle of twist is proportional to twisting moment. The small area shaded in figure represents the work done by the torque during an increment $d\theta$ in the angle of twist θ.

The area $OAB = \dfrac{T\theta}{2}$ represents the total energy stored in a bar during twist.

Remembering $\theta = \dfrac{Tl}{GJ}$, we get

strain energy $= \dfrac{T^2 l}{2GJ}$.

On substitution of values $T = \dfrac{\pi}{16}\, d^3 f_s$ and $J = \dfrac{\pi}{32}\, d^4$,

we get

strain energy $= \dfrac{f_s^2}{4G} \times \dfrac{\pi}{4} d^2 l = \dfrac{f_s^2}{4G} \times$ volume of the spring.

Kinetic energy to be stored in the spring $= \frac{1}{2}\dfrac{W}{g} v^2$

$$= \tfrac{1}{2} \times \frac{20}{9{\cdot}81} \times 3^2 = 9{\cdot}17 \text{ kg metre} = 917 \text{ kg cm.}$$

When helical springs are subject to rapid alteration of load, surging of the coil may occur due to dynamical effects The spring failure may occur due to surging. According to W.M. Griffith the critical frequency of a spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto the twentieth order

Generally, it is customary to state the free length of the spring which means the free height of the spring under no load.

Due to the following reasons the tension springs are used sparingly.

(i) They are costlier.
(ii) They require more elaborate arrangements for end connections.
(iii) They are likely to be stressed beyond elastic limit.
(iv) If the spring breaks, the control is lost.

Examples:

1. A closed coiled helical spring is required to have a sliding fit over a rod of 25 mm in diameter. The spring is to carry a maximum axial load of 12 kg and deflection at this load is to be 2 cm. The shearing stress must not exceed 3,000 kg/sq cm. Suggest the suitable dimensions for the spring. Take G = 840,000 kg/sq cm.

As the spring is to slide over a 25 mm diameter rod, we assume 30 mm as the mean diameter of the coil. The maximum load on the spring is 12 kg Maximum torque on the section of the coil $= \frac{12 \times 3}{2} = 18$ kg cm If d_w be the diameter of the wire, we have

$$\frac{\pi}{16} d_w^3 \times 3000 = 18$$

or $$d_w = \sqrt[3]{\frac{18 \times 16}{3000 \cdot \pi}} = 0.313 \text{ cm.}$$

We assume a 10 SWG wire whose diameter is 0.3251 cm. (According to IS: 1137 — 1959, the size will be 0.335 cm.) The stiffness of the spring $\frac{12}{2} = 6$ kg/cm.

Spring index $= \frac{3}{0.3251} = 9.23$.

$$\text{Stiffness} = \frac{Gd_w}{8C^3n}$$

$$6 = \frac{840000 \times 0.3251}{8 \times 9.23^3 \times n}$$

or $$n = \frac{840000 \times 0.3251}{6 \times 8 \times 9.23^3} = 7.2 \text{ active turns.}$$

Assuming squared and ground ends, we have 9.2 actual turns.

The spring index is 6. The stress factor, as defined by A.M. Wahl, is $K = \frac{4C-1}{4C-4} + \frac{0.616}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.252.$

If a be the side of the square, the mean coil diameter will be $6a$. With usual notations, the maximum shearing stress in a helical compression or tension spring made of square wire is given by

$$f_s = K \frac{2.4\, PD}{a^3}.$$

$$\therefore \quad 3600 = 1.252 \times \frac{2.4 \times 646 \times 6a}{a^3}$$

$$\therefore \quad a = \sqrt{\frac{1.252 \times 2.4 \times 646 \times 6}{3600}} = 1.8 \text{ cm.}$$

We adopt 1·8 cm as the side of a square section.

The mean diameter of the coil will be $6 \times 1.8 = 10.8$ cm.

The spring rate of a close-coiled helical spring of square section is given by $\frac{Ga^4}{5.58\, n\, D^3}$.

$$\therefore \quad 32.3 = \frac{840000 \times 1.8^4}{5.58 \times n \times 10.8^3}.$$

$$\text{or} \quad n = \frac{840000 \times 1.8^4}{5.58 \times 32.3 \times 10.8^3} = 38.8 \text{ turns (active).}$$

4. A motor vehicle single plate clutch is to have both sides of the plate effective. It is to transmit 30 horse power at a speed of 2,800 r.p.m. and a 20 per cent overload is to be allowed for. The pressure intensity on the friction surface is not to exceed 0·85 kg/sq cm and the surface speed at the mean radius must not be greater than 2,100 metre/minute. The co-efficient of friction for the surfaces is 0·3. The outside diameter of the surfaces is to be 1·3 times the inside diameter.

The axial thrust is to be provided by six springs of about 2·5 cm coil diameter. Their safe shear stress is limited to 4,200 kg/sq cm and the modulus of rigidity G is 0·84 × 10⁶ kg/sq cm.

Design these springs selecting wire from the following gauges:

SWG	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Dia. (mm)	*5·893*	*5·385*	*4·877*	*4·470*	*4·064*	*3·658*	*3·251*	*2·946*	*2·642*

The clutch is to transmit 30 horse power at a speed of 2,800 r.p.m.

$\therefore \quad 917 = \dfrac{5600^2}{4 \times 0{\cdot}84 \times 10^6} \times$ volume of the spring.

$\therefore \quad V = 97{\cdot}6$ cu cm.

If n be the number of active coils in the spring, then

$$97{\cdot}6 = \pi \times 6 \times n \times \frac{\pi}{4} \times 0{\cdot}701^2$$

or
$$n = \frac{97{\cdot}6 \times 4}{\pi \times 6 \times \pi \times 0{\cdot}701^2} = 13{\cdot}5.$$

Spring index $= \dfrac{D}{d} = \dfrac{6}{0{\cdot}701} = 8{\cdot}57.$

Stiffness of the spring $= \dfrac{Gd}{8C^3n} = \dfrac{0{\cdot}84 \times 10^6 \times 0{\cdot}701}{8 \times 8{\cdot}57^3 \times 13{\cdot}5}$

$= 8{\cdot}65$ kg/cm.

Weight of the spring $= 7{\cdot}25 \times 97{\cdot}6$

$= 710$ gm or $0{\cdot}71$ kg.

3. A loaded narrow gauge car weighing 1,500 kg and moving at a velocity of 1·30 metre/second is brought to rest by a bumper consisting of two helical steel springs of square section in which the mean diameter of the coil is six times the side of the square section. In bringing the car to rest the springs are to be compressed 20 cm. The value of the shear stress is not to exceed 3,600 kg/sq cm. Determine the following: (a) maximum load on each spring; (b) side of the square section of wire; (c) mean diameter of coils; (d) number of active coils. Assume G = 840,000 kg/sq cm.

Helical springs are made of square or rectangular section wound flatwise or edgewise in order to provide greater resilience in a given space and to provide pre-determined altering of spring rate by grinding off the outside of the coils

The kinetic energy of the moving car is to be stored in the spring when it is compressed 20 cm. There are two springs.

The kinetic energy of the moving car $= \frac{1}{2} \times \dfrac{W}{g} v^2$

$= \frac{1}{2} \times \dfrac{1500}{9{\cdot}81} \times (1{\cdot}3)^2 = 129{\cdot}2$ kg metre $= 12{,}920$ kg cm.

If P be the maximum force on each spring, then

$$\frac{P}{2} \times 2 \times 20 = 12920$$

or
$$P = \frac{12920}{20} = 646 \text{ kg}.$$

The spring rate $= \dfrac{646}{20} = 32{\cdot}3$ kg/cm.

or $d_w = \sqrt[3]{\dfrac{33{\cdot}4 \times 16}{4200 \times \pi}} = 0{\cdot}342$ cm.

From the table, we use 9 SWG wire whose diameter is 0·3658 cm.

If we assume eight free coils then the total length of wire will correspond to ten coils allowing for the ends.

Length of wire required will be $\pi \times 2{\cdot}5 \times 10 = 78{\cdot}5$ cm.

To give the axial thrust of 26·8 kg required per spring, the initial compression will be given by the formula

$\delta = \dfrac{8PD^3n}{G\,d^4}$ with usual notations.

On substitution of values, we have

$$\delta = \frac{8 \times 26{\cdot}8 \times 2{\cdot}5^3 \times 8}{0{\cdot}84 \times 10^6 \times 0{\cdot}3658^4} = 1{\cdot}77 \text{ cm.}$$

$\therefore$ The minimum free length of the spring $= 8 \times 0{\cdot}3658 + 1{\cdot}77$
$= 4{\cdot}7$ cm.

But due to wear, slipping, etc. the compression may have to be increased when the clutch is in service. If, therefore, the total free length be made 5 cm, sufficient margin is provided. The total thrust available by complete compression of the spring will be

$$\frac{(5 - 8 \times 0{\cdot}3658)}{1{\cdot}77} \times 26{\cdot}8 = 46{\cdot}6 \text{ kg.}$$

Note: When the engine is running on the test bed, the springs would be adjusted by trial until the designed horse power is obtained at the rated speed.

5. *The table below gives particulars, of concentric helical springs. If the spring is subjected to an axial load of 40 kg, determine for each spring (a) the change in length, (b) the amount of load carried and (c) the shear stress induced in the wire. $G = 0{\cdot}84 \times 10^6$ kg/sq cm.*

	Mean coil diameter	*Size of wire*	*Diameter of wire mm*	*No. of turns*	*Free length*
Inner spring	*3 cm*	*8 SWG*	*4·064*	*8*	*7·5 cm*
Outer spring	*4 cm*	*6 SWG*	*4·877*	*10*	*9 cm*

The outer spring takes the whole load until its compression is 1·5 cm, after which both springs are compressed together.

The load required to deflect the outer spring 1·5 cm is given by the equation

$$\delta = \frac{8\,P\,D^3\,n}{G\,d^4}.$$

Torque to be transmitted $= \dfrac{71620 \times \text{h.p.}}{\text{speed}} = \dfrac{71620 \times 30}{2800}$

$= 770$ kg cm.

As 20 per cent over load is allowed, the clutch is to be designed to transmit $1{\cdot}2 \times 770 = 925$ kg cm torque.

Let r be the inner radius of the friction lining; then $1{\cdot}3r$ will be the outer radius of the friction lining. The pressure intensity is not to exceed 0·85 kg/sq cm. We assume that wear is uniform. The maximum intensity will occur at the inner radius and we have, from the mechanics of machine, the relation $0{\cdot}85 \times r =$ constant $= C$ for the assumption of uniform wear.

The torque transmitted in kg cm $= \pi\mu C\,[(1{\cdot}3r)^2 - r^2)]$

$= \pi \times \frac{3}{10} \times 0{\cdot}85r \times 0{\cdot}69r^2 = 0{\cdot}55r^3.$

As both the friction surfaces are effective, torque transmitted by each friction surface $= \dfrac{925}{2} = 462{\cdot}5$ kg cm.

$\therefore\quad 462{\cdot}5 = 0{\cdot}55r^3$

or $\qquad r = \sqrt[3]{\dfrac{462{\cdot}5}{0{\cdot}55}} = 9{\cdot}42$ cm, we adopt 10 cm.

Outer radius $= 1{\cdot}3 \times 10 = 13$ cm.

The mean radius $= \dfrac{10 + 13}{2} = 11{\cdot}5$ cm.

$\therefore$ Surface speed at mean radius $= \pi \times 2 \times 0{\cdot}115 \times 2800$

$= 2{,}020$ metre/minute which is less than the allowed value.

The constant $C = 0{\cdot}85 \times 10 = 8{\cdot}5$ kg/cm.

The axial load to be applied $= 2\pi C\,(R - r)$

$= 2\pi \times 8{\cdot}5\,(13 - 10) = 160$ kg.

The axial thrust is to be provided by 6 springs

$\therefore$ Maximum load on each spring $= \dfrac{160}{6} = 26{\cdot}7$ kg

Let us assume the mean diameter of coil as 2·5 cm.

$\therefore$ Maximum torque $= \dfrac{26{\cdot}7 \times 2{\cdot}5}{2} = 33{\cdot}4$ kg cm.

If d_w be the diameter of the wire, then

$\dfrac{\pi}{16}\, d_w^3 \times 4200 = 33{\cdot}4$

it is not too long nor the coils so few that the deflection of each coil is excessive.

d cm	$R = 8{\cdot}25d^3$ cm	$n = \dfrac{0{\cdot}585}{d^5}$
0·5	1·029	18·4
0·53	1·227	13·75
0·56	1·450	10·40
0·60	1·778	7·35
0·63	**2·058**	**5·80**
0·67	2·478	3·90
0·71	2·947	3·17
0·75	3·474	2·43
0·80	4·216	1·75

From the above table, we adopt 0·63 cm as the diameter of wire having 5·80 as active turns.

7. *The spring loaded safety valve for a boiler is required to blow off at a pressure of 10 kg/sq cm. The diameter of the valve is 6 cm, and the maximum lift of the valve is 1·5 cm.*

Design the suitable compression spring for the safety valve assuming the spring index to be 6 and providing initial compression of 3 cm.

The maximum shear stress in the material of the wire is limited to 4,500 kg/sq cm. $G = 0{\cdot}84 \times 10^6$ kg/sq cm.

Load on the valve when it just begins to lift $= \dfrac{\pi}{4} \times 6^2 \times 10$

$= 282$ kg.

The valve is kept tight on its seat against steam load of 282 kg by providing initial compression of 3 cm. Therefore, the stiffness of the spring $= \dfrac{282}{3} = 94$ kg/cm.

The lift of the valve is 1·5 cm. Therefore, the maximum compression of the spring is $3 + 1{\cdot}5 = 4{\cdot}5$ cm.

Maximum load on the spring when the valve is in full open position $= 4{\cdot}5 \times 94 = 423$ kg.

The assumed spring index is 6. The stress factor, as defined by A.M. Wahl, is $K = \dfrac{4C-1}{4C-4} + \dfrac{0{\cdot}615}{C} = \dfrac{4\times 6-1}{4\times 6-4} + \dfrac{0{\cdot}615}{6}$

$= 1{\cdot}252.$

$\therefore \quad 1{\cdot}5 = \dfrac{8P \times 4^3 \times 10}{0{\cdot}84 \times 10^6 \times 0{\cdot}4877^4}$

or $\quad P = 13{\cdot}9$ kg.

After this load each spring will take a proportional of the additional load of $40 - 13{\cdot}9 = 26{\cdot}1$ kg..

Let x be the further compression of the outer spring and total deflection of the inner spring.

Additional load taken by outer spring $= \dfrac{13{\cdot}9 \times x}{1{\cdot}5} = 9{\cdot}26x.$

Load taken by inner spring = stiffness of inner spring $\times x$

$$= \frac{0{\cdot}4064^4 \times 0{\cdot}84 \times 10^6}{8 \times 3^3 \times 8} \times x = 13{\cdot}2x.$$

$\therefore \quad 13{\cdot}2x + 9{\cdot}26x = 26{\cdot}1$

or $\quad x = \dfrac{26{\cdot}1}{22{\cdot}46} = 1{\cdot}16$ cm.

$\therefore$ Change in length of outer spring $= 1{\cdot}5 + 1{\cdot}16 = 2{\cdot}66$ cm.

Change in length of inner spring $= 1{\cdot}16$ cm.

Load shared by inner spring $= 13{\cdot}2 \times 1{\cdot}16 = 15{\cdot}3$ kg.

Load shared by outer spring $= 40 - 15{\cdot}3 = 24{\cdot}7$ kg.

Stress induced in outer spring $= \dfrac{16 \times 24{\cdot}7 \times 2}{\pi \times 0{\cdot}4877^3} = 2{,}170$ kg/sq cm.

Stress induced in inner spring $= \dfrac{16 \times 15{\cdot}3 \times 1{\cdot}5}{\pi \times 0{\cdot}4064^3} = 1{,}750$ kg/sq cm.

The maximum shear stress is induced in the outer spring.

6. *Design a spring, having a stiffness of 40 kg/cm, to sustain a maximum load of 100 kg. The maximum shear stress is not to exceed 4,200 kg/sq cm. Take modulus of rigidity as 840,000 kg/sq cm.*

Let R be the mean radius of the coil. Maximum torque will be $100R$ kg cm. If d cm be the diameter of the wire, then

$\dfrac{\pi}{16} d^3 \times 4200 = 100R$

or $\quad R = 8{\cdot}25\ d^3.$

$$\text{Stiffness} = \frac{Gd}{8C^3n} = \frac{Gd^4}{64R^3n}$$

$$40 = \frac{840000 \times d^4}{64\,(8{\cdot}25d^3)^3 n} \quad \text{or } n = \frac{0{\cdot}585}{d^5}.$$

Now we calculate the value of R and n for different values of d until spring with a suitable number of coils is found such that

Torque $= 423 \times 3\, d_w$ where d_w is the diameter of the spring wire.

$$423 \times 3d_w = \frac{\pi}{16} d_w^3 \times \frac{4500}{1 \cdot 252}$$

or $$d_w = \sqrt{\frac{423 \times 3 \times 16 \times 1 \cdot 252}{4500 \times \pi}} = 1 \cdot 31 \text{ cm.}$$

From IS: 1137—1959, we adopt 14 mm; i.e. 1·4 cm.

Mean diameter of the coil $= 1 \cdot 4 \times 6 = 8 \cdot 4$ cm. If n be the number of active turns in the spring, then

$$94 = \frac{0 \cdot 84 \times 10^6 \times 1 \cdot 4^4}{8 \times 8 \cdot 4^3 \times n}$$

or $$n = \frac{0 \cdot 84 \times 10^6 \times 1 \cdot 4^4}{8 \times 8 \cdot 4^3 \times 94} = 7 \cdot 4 \text{ turns}$$

Assuming squared and ground ends we have 9 actual turns. Free length of the spring = solid length + maximum compression + clearance between adjacent coils (1 mm between adjacent coils)

$$= 9 \times 1 \cdot 4 + 4 \cdot 5 + 0 \cdot 1 \times 8$$
$$= 17 \cdot 9 \text{ cm say } 18 \text{ cm}$$

8. *A helical spring whose mean diameter of coils is 8 times that of the wire is to absorb 4,000 kg cm of energy. The initial compression of the spring is 5 cm and compresses by additional 7 cm while absorbing the shock. The maximum allowable stress is 4,000 kg/sq cm and $G = 0 \cdot 84 \times 10^6$ kg/sq cm. Determine the diameter of the wire and the number of active turns. Neglect the effect of stress concentration*

Let k be the stiffness of the spring in kg/cm. The initial load on the spring is $5\,k$ kg. As the spring further compresses by 7 cm, the maximum load on the spring is $(5+7)k = 12k$ kg

Mean spring force during compression $= \dfrac{12k + 5k}{2} = 8 \cdot 5k$ kg

The energy absorbed during shock $= 8 \cdot 5k \times 7 = 59 \cdot 5k$ kg cm.

$\therefore\ 4000 = 59 \cdot 5\, k$

or $$k = \frac{4000}{59 \cdot 5} = 67 \cdot 3 \text{ kg/cm.}$$

Maximum spring force $= 67 \cdot 3 \times 12 = 807$ kg.

Spring index is 8. If d_w be the diameter of the spring wire, then $807 \times \dfrac{8\, d_w}{2} = \dfrac{\pi}{16} d_w^3 \times 4000$

or $$d_w = \sqrt{\frac{807 \times 4 \times 16}{4000 \times \pi}} = 2 \cdot 02 \text{ cm.}$$

compressed by 8 cm. The outside diameter of the spring should not exceed 18 cm. What is the wire diameter, coil diameter and number of coils? The value of the shear stress induced is not to exceed 2,400 kg/sq cm. Take $G = 0{\cdot}84 \times 10^6$ kg/sq cm.

Ans. 2·65 mm; 15 cm; 11·4 active turns.

10. *Two helical springs of the same axial length but of different coil diameters are placed co-axially, one inside the other, so as to share a compressive load. If they are made from the same diameter of wire, show that the maximum shear stress will be the same in each, provided the number of coils is made inversely proportional to the square of coil diameters.*

11. *Two helical springs of the same axial length but different diameters of coils are placed co-axially, one inside the other, so as to share an axial load of 320 kg with a deflection of 2·5 cm under load. The maximum shear stress permissible in the material is not to exceed 1,500 kg/sq cm. Neglect the effect of stress concentration factor. The values of the spring index may be taken as 5 and 9. The diameter of the wire may be taken to be the same. Modulus of rigidity = $0{\cdot}84 \times 10^6$ kg/sq cm.*

Ans. Ratio of turns = 3·24.

12. *A car engine rated at 12 h.p. gives a maximum torque of 800 kg cm which is being transmitted by a clutch of single plate type whose both sides are effective. The maximum value of the axial thrust which is provided by 8 close coiled helical springs of spring index 8 is 200 kg. Determine the diameter of the spring wire if the permissible stress is limited to 5,800 kg/sq cm. If the compression of each spring be 2·5 cm, determine the number of active turns if the modulus of rigidity be $0{\cdot}84 \times 10^6$ kg/sq cm.*

Ans. 10 SWG; 6·7 active turns.

13. *Four springs are used in a centrifugal clutch coupling transmitting 36 h.p. at 1,000 r.p.m. The maximum centrifugal force when the spring extends by 0·6 cm is 90 kg. The available space does not permit the outside diameter of the coil to exceed 3·5 cm. Design the suitable spring dimensions if the permissible shear stress is limited to 6,000 kg/sq cm. Modulus of rigidity = $0{\cdot}84 \times 10^6$ kg/sq cm.*

Ans. 6 SWG wire; mean diameter of coil 2·5 cm; 1·63 active turns.

14. *The valve opens against a spring load provided by two concentric helical springs which are close coiled. Both springs are of the same material. The free length of outer spring is 13 mm smaller than that of the inner spring. The outer spring has 12 coils of mean diameter 25 mm, the wire diameter being 3 mm and an initial compression of 5 mm when the valve is closed.*

The greatest force required to open the valve 10 mm is 15 kg. Determine the stiffness of the inner spring. If the radial clearance between the springs is 0·1 mm, determine the diameter of the wire of the inner spring if it has 10 coils. $G = 8{\cdot}4 \times 10^5$ kg/sq cm.

Ans. 2·9 kg/cm; 2·3 mm.

15. *Two close coiled helical springs are arranged concentrically one inside the other. Both springs have the same number of effective coils and same overall length, but the mean coil diameter of the outer spring is twice that of the inner spring which is made of bronze. The outer spring is made of steel. The springs are designed to act together when a force is applied, so that both suffer the same change in length and each carries half the force. Determine the ratio of the wire diameters and the ratio of stresses produced in wires, if the modulus of rigidity of steel is twice that of bronze.*

$$\textit{Ans.}\quad \frac{\text{dia of brass}}{\text{dia of steel}} = \frac{1}{\sqrt{2}}\,;\quad \frac{\text{stress in brass}}{\text{stress in steel}} = \frac{\sqrt{2}}{1}\,.$$

16. *An engine indicator has a plunger diameter of 19 mm. The indicator spring attached to this plunger is to be compressed 13 mm when the steam pressure is 7 kg/sq cm. Assuming the spring index to be 6, design the spring. $G = 8 \times 10^5$ kg/sq cm; $f_s = 5{,}600$ kg/sq cm*

Ans. 8 coils of wire diameter 1·6 mm.

17. *A carbon steel spring is to be subjected to a load that varies from 200 kg to 500 kg. The outside diameter should be between 9 cm to 10 cm, the spring index between 5 and 10, approximate stiffness 90 kg/cm. Choose a steel, and for a design factor of 1·4 by the Wahl line, find the wire diameter. Also determine the number of active coils and the free length for squared and ground ends.*

8-6. Torsion helical springs:

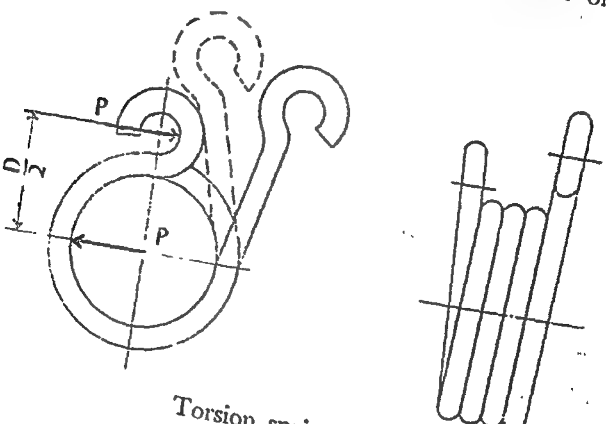
These springs are similar to helical compression springs in form and are loaded by a torque about the axis of the helix. The primary stress in helical torsion springs (fig. 8-8) is flexural in contrast to the helical compression or tension springs where the stresses are torsional shear stresses. Torsion springs are used for transmission of small torques and are used as cushions on flexible drives transmitting rotary motions or torque. Further examples in torsion spring applications include door hinge springs, spring for starters in automobiles and springs for brush holders in electric

motors. They are made primarily for the purposes of transmitting external torque to the spring.

When a helical spring is subjected to a twisting moment the wire is under bending action due to change in curvature of the coils and flexural stresses are induced. The total angle θ of the twist in radians is given by

$$\theta = \frac{Tl}{EI} = \frac{64\,T D n}{E d^4} \quad \text{......(i)}$$

where T is the twisting moment, l the length of the spring wire, E the modulus of elasticity, I the rectangular moment of inertia of wire, d the diameter of wire, D the mean diameter of coil and n the number of active coils.



Torsion spring

FIG. 8-8

For the bending stress in wire, we have

$$f_b = \frac{32\,T}{\pi d^3} \quad \text{......(ii)}$$

For a spring made of square wire of side a

$$\theta = \frac{12\pi\,T D n}{E a^4} \quad \text{......(iii)}$$

$$f_b = \frac{6T}{a^3} \quad \text{......(iv)}$$

The energy stored in torsion spring of round

The greatest force required to open the valve 16 mm is 15 kg. Determine the stiffness of the inner spring. If the radial clearance between the springs is 0.1 mm, determine the diameter of the wire of the inner spring if it has 10 coils. $G = 8{\cdot}4 \times 10^5$ *kg/sq cm.*

Ans. $2{\cdot}9$ *kg/cm*; $2{\cdot}3$ *mm.*

15. Two close coiled helical springs are arranged concentrically one inside the other. Both springs have the same number of effective coils and same overall length, but the mean coil diameter of the outer spring is twice that of the inner spring which is made of bronze. The outer spring is made of steel. The springs are designed to act together when a force is applied, so that both suffer the same change in length and each carries half the force. Determine the ratio of the wire diameters and the ratio of stresses produced in wires, if the modulus of rigidity of steel is twice that of bronze.

$$\text{Ans. } \frac{\text{dia of brass}}{\text{dia of steel}} = \frac{1}{\sqrt{2}}, \quad \frac{\text{stress in brass}}{\text{stress in steel}} = \frac{\sqrt{2}}{1}.$$

16. An engine indicator has a plunger diameter of 19 mm. The indicator spring attached to this plunger is to be compressed 13 mm when the steam pressure is 7 kg/sq cm. Assuming the spring index to be 6, design the spring. $G = 8 \times 10^5$ *kg/sq cm;* $f_s = 5{,}600$ *kg/sq cm*

Ans. 8 coils of wire diameter 1·6 mm.

17. A carbon steel spring is to be subjected to a load that varies from 200 kg to 500 kg. The outside diameter should be between 9 cm to 10 cm, the spring index between 5 and 10, approximate stiffness 90 kg/cm. Choose a steel, and for a design factor of 1·4 by the Wahl line, find the wire diameter. Also determine the number of active coils and the free length for squared and ground ends.

8-6. Torsion helical springs:

These springs are similar to helical compression springs in form and are loaded by a torque about the axis of the helix. The primary stress in helical torsion springs (fig. 8-8) is flexural in contrast to the helical compression or tension springs where the stresses are torsional shear stresses. Torsion springs are used for transmission of small torques and are used as cushions on flexible drives transmitting rotary motions or torque. Further examples in torsion spring applications include door hinge springs, spring for starters in automobiles and springs for brush holders in electric

Due to application of external force at a radius r, twisting moment $T = Pr$ is set up, which is balanced by the resisting moment of the spring.

Assuming both ends of springs are clamped, we have

$$\theta = \frac{Prl}{EI} = \frac{Tl}{EI} \quad \ldots\ldots (i)$$

$$\text{max } f = \frac{12\,Pr}{wt^2} = \frac{12T}{wt^2} \quad \ldots\ldots (ii)$$

$$\delta = \theta r = \frac{Pr^2 l}{EI} = \frac{flr}{Et} \quad \ldots\ldots (iii)$$

$$\text{Strain energy of the spring} = \frac{f^2}{24E} \times \text{volume of the spring} \quad (iv)$$

Examples:

1. A flexible shaft transmits 25 kg cm torque. It consists of a torsion spring in which the permissible stress is limited to 4,500 kg/sq cm. What must be the diameter of the wire?

In case of helical springs in torsion, the maximum bending stress in the wire is given by

$$f = \frac{32T}{\pi d^3}.$$

$$\therefore \quad 4500 = \frac{32 \times 25}{\pi d^3}$$

$$\text{or} \quad d = \sqrt[3]{\frac{32 \times 25}{\pi \times 4500}} = 0{\cdot}385 \text{ cm.}$$

We adopt 8 SWG wire or 0·385 cm diameter wire.

2. A flat spiral steel spring is to give a maximum torque of 14 kg cm for a maximum stress of 7,000 kg/sq cm. What thickness and length are necessary to give three complete turns of motion when stress decreases from 7,000 to zero? Space considerations limit the width of wire to 15 mm.

$$\text{Maximum stress} = \frac{12T}{wt^2}$$

$$\therefore \quad 7000 = \frac{12 \times 14}{1{\cdot}5t^2}$$

$$\text{or} \quad t = \sqrt{\frac{12 \times 14}{7000 \times 1{\cdot}5}} = 0{\cdot}127 \text{ cm.}$$

From IS: 1137 — 1959, we adopt 1·32 mm thickness.

Three complete turns are equal to $3 \times 2\pi = 6\pi$ radians.

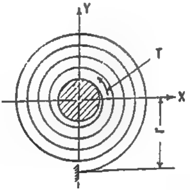
$\frac{fb^2}{8E} \times$ volume of the spring. Similar expression for the torsion spring of square wire will be

$$\frac{fb^2}{6E} \times \text{volume of the spring.}$$

All torsion springs should be installed such that the application of load winds up the wire as a result the diameter will be reduced. So clearance should be provided if the spring wire is to be wound around a mandrel Similarly in order to prevent sliding friction between the adjacent coils, there must be certain clearance between adjacent coils

8-7. Spiral springs:

This spring consists of a flat strip of rectangular section to form a spiral shape as in fig. 8-9. Usually the inner end is clamped to an arbor while the outer end may be pinned or clamped. The stresses induced in the spring material are flexural. Common applications of such springs are spring for clocks and motor brush holder springs.



Spiral spring
FIG 8-9

Let P = force acting at a radius r
l = length of the wire froming the spring
θ = angular deflection in radians
w = width of the wire
t = thickness of the wire
f = tensile or compressive stress in the wire
E = modulus of elasticity of the material of the spring.

Due to application of external force at a radius r, twisting moment $T = Pr$ is set up, which is balanced by the resisting moment of the spring.

Assuming both ends of springs are clamped, we have

$$\theta = \frac{Prl}{EI} = \frac{Tl}{EI} \quad \text{.......... (i)}$$

max $$f = \frac{12\,Pr}{wt^2} = \frac{12T}{wt^2} \quad \text{.......... (ii)}$$

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From IS: 1137 – 1959, we adopt 1·32 mm thickness.

Three complete turns are equal to $3 \times 2\pi = 6\pi$ radians.

We have $\theta = \dfrac{Tl}{EI}$ radian.

$$\therefore \quad l = \frac{\theta EI}{T} = \frac{6\pi \times 2\cdot1 \times 10^6 \times \frac{1}{12} \times 1\cdot5 \times 0\cdot132^3}{14}$$

$= 812$ cm.

3. *A pivoted roller follower is held in contact with the cam by a torsion spring. The moment exerted by the spring varies from 20 kg cm to 50 kg cm as the follower oscillates through 30°. Design the suitable spring with a factor of safety 2 based on the Soderberg line.*

Ultimate strength of steel 13,000 kg/sq cm and yield strength 11,000 kg/sq cm. Assume the stress concentration factor to be 1·08.

Mean torque $= \dfrac{20+50}{2} = 35$ kg cm.

Variable torque component $= \dfrac{50-20}{2} = 15$ kg cm.

Endurance limit $= 13000 \times 0\cdot5$
$= 6{,}500$ kg/sq cm.

Let d_w be the diameter of the spring wire. Then according to Soderberg criterion we get

$$\frac{1}{2} = \frac{35 \times 16}{11000 \times \pi d_w^3} + \frac{1\cdot08 \times 15 \times 16}{6500 \times \pi d_w^3}$$

$$\text{or} \quad d_w = \sqrt[3]{2\left[\frac{35 \times 5\cdot1}{11000} + \frac{1\cdot08 \times 15 \times 5\cdot1}{6500}\right]}$$

$= 0\cdot386$ cm.

We adopt 8 *SWG* wire having 0·4064 cm as the diameter of the wire. We adopt 5 cm as the mean diameter of the coil.

The angular deflection θ of a torsion spring subjected to opposing moments M at the ends is

$$\theta = \frac{Ml}{EI} = \frac{64 M D n}{E d_w^4}$$

where n is number of active coils, D is the mean diameter of the coil, d_w is the diameter of the wire and E is the modulus of elasticity.

$$\therefore \quad \frac{\pi}{6} = \frac{64 \times 30 \times 5 \times n}{2\cdot1 \times 10^6 \times (0\cdot4064)^4}$$

$$\text{or} \quad n = \frac{\pi}{6} \times \frac{2\cdot1 \times 10^6 \times (0\cdot4064)^4}{64 \times 30 \times 5}$$

$= 3\cdot56.$

Exercises:

1. *A flexible coupling transmits 10 h.p. at 500 r.p.m. through a torsional helical spring made of 16 mm square steel wire. Find the pitch diameter of the coil taking the permissible stress as 4,200 kg/sq cm. Also determine the number of active coils if the torsional deflection should not exceed 0·1 radian.*

Ans. 7·5 cm; 3·2.

2. *A flat spiral spring for holding an electric motor brush against the commutator is to be made of brass ribbon of 13 mm width. It is to exert a pressure of 0·5 kg against the brush. The line of action of the thrust against the brush passes 6 cm from the axis of the spring. The length of the ribbon used is 50 cm. Assume the maximum allowable stress to be 1,000 kg/sq cm and the modulus of elasticity of the brass to be 910,000 kg/sq cm. Determine the thickness of the ribbon and the number of degrees through which the axis must be turned to produce the force required.*

Ans. 0·170 cm; 18.7°.

3. *Derive an equation for the energy absorbed by a round wire torsion spring. Express the answer in terms of the volume of metal in coils.*

Ans. $\frac{f^2}{8E}$ × volume of the metal.

8-8. Leaf spring:

It is a modification of the beam of constant depth and constant strength, shown in fig. 8-10. Fig. 8-11 shows the leaf spring that will result from a spring (flat) of fig. 8-10.

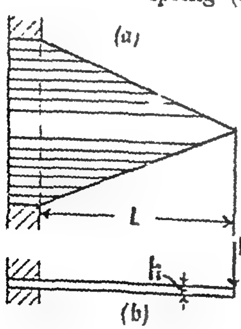
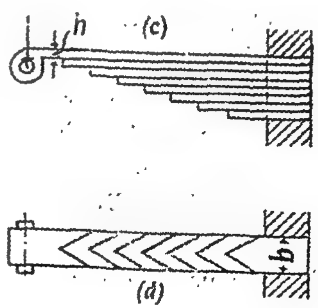


Plate spring
FIG. 8-10



Leaf spring
FIG. 8-11

In order to secure compactness flat spring is cut into strips, the strips are piled one above the other and we get a familiar cantilever spring. The ends of

Exercises:

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Ans. 7·5 cm; 3·2.

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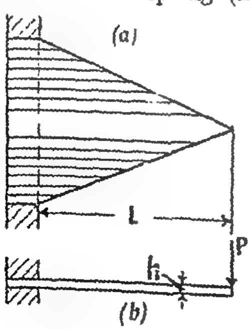
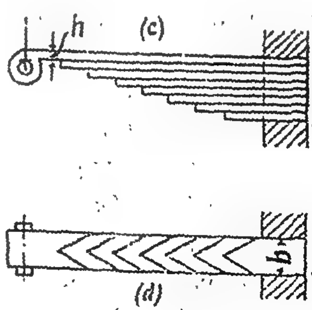


Plate spring
FIG. 8-10



Leaf spring
FIG. 8-11

In order to secure compactness flat spring is cut into strips, the strips are piled one above the other and we get a familiar cantilever spring. The ends of

individual leaves are usually modified by being made wider and thinner in order to make a neater appearance. The beam that is composed of by piling separate leaves, is just as strong as the beam with the leaves beside each other. In the former case the load is transmitted from one leaf to the other by bearing while in the latter by shear

The free end of the spring is modified for the attachment of the load. Leaf springs are initially curved and straighten under load. The semi-elliptical spring may be considered as two cantilevers and the elliptical spring as four cantilevers

Let

h = thickness of leaves,
b = width of each leaf
n = number of leaves
f = maximum stress
P = maximum load
l = length of a cantilever spring

$$f = \frac{6Pl}{bnh^2} \quad \ldots \quad \text{(i)}$$

Deflection δ is given by

$$\delta = \frac{6Pl^3}{b\,n E\,h^3} \quad \ldots \quad \text{(ii)}$$

The strain energy of a cantilever leaf spring is given by

$$U = \frac{f^2}{6E} \times \text{volume of the spring} \quad \text{(iii)}$$

The volume of the spring is given by

$$V = \frac{nblh}{2} \quad \ldots \quad \text{(iv)}$$

Laminated springs permit saving in material and greater deflection than a spring of constant depth, that is they have greater resilience and shock absorbing capacity.

The bundle of leaves is held together either by passing a bolt through the centre as shown in fig. 8-12 or by a shrunk band around them. The diameter of the bolt hole must be subtracted from the width when making calculations for strength. When band is used, it exerts a stiffening effect and the width of the band should be subtracted from the overall length of the spring to obtain the effective length. Shrunk bands are superior construction but are used only on heavy springs.

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A

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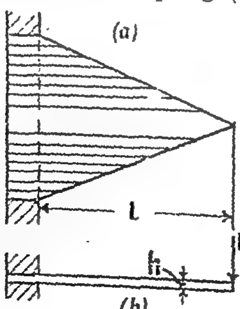
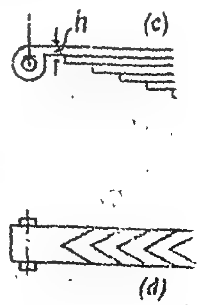


Plate spring
FIG. 8-10



Leaf spr
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The free end of the spring is modified for the attachment of the load. Leaf springs are initially curved and straighten under load. The semi-elliptical spring may be considered as two cantilevers and the elliptical spring as four cantilevers.

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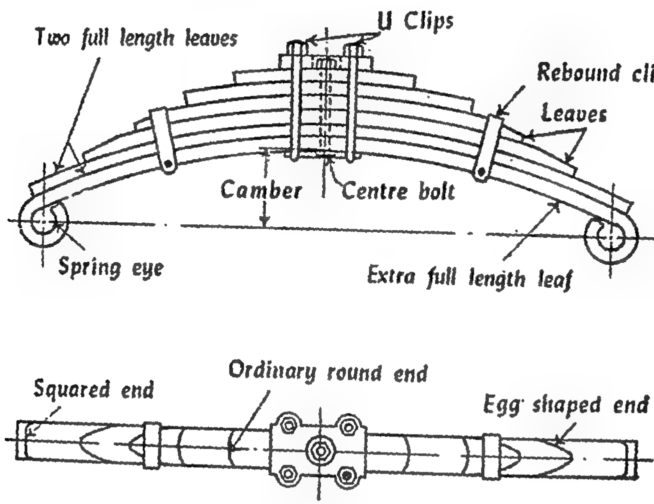
The bundle of leaves is held together either by passing a bolt through the centre as shown in fig. 8-12 or by a shrunk band around them. The diameter of the bolt hole must be subtracted from the width when making calculations for strength. When band is used, it exerts a stiffening effect and the width of the band should be subtracted from the overall length of the spring to obtain the effective length. Shrunk bands are superior construction but are used only on heavy springs.

As the load is applied to the spring, the curvature and the effective length change as a result the deflection rate is altered. By properly choosing the camber the spring may be made to soften or stiffen as the load is increased.

If all the leaves were to be given the same curvature before assembly, the leaves will separate during the rebound and dirt and grit may enter between them. This condition may be prevented by rebound clips as shown in fig. 8-12. The other way is to *nip* the spring. The nipping of a spring means the shorter leaves are given a slightly greater initial curvature as a result contact is maintained at all times.



Carriage spring

FIG. 8-12

On leaf springs for use on vehicles where there is a possibility of rebound, as on automobiles or carriages driven over a rough ground, clips are used to fasten together two or more of the leaves to strengthen them against the rebound. The long leaf fastened to the supports is called the main leaf or the master leaf, whose ends are bent to form an eye. The spring is supported by means of hinges. This arrangement produces longitudinal loads and additional stresses. To these may be added stresses resulting from twisting and transverse forces. Therefore, the master leaf is made sometimes of stronger material than the rest of the leaves. In some leaf springs the master leaf is made thicker than the others. The only advantage in having the thicker leaf is that on rebound the single leaf to the first restraining clip is stronger.

Laminated springs for heavy loads have under the master leaf additional full length leaves. With such an arrangement, the spring is no longer a spring of uniform strength.

The following formulas pertaining to semi-elliptic springs for extra full length leaves were developed by E. R. Morrison.

$$P = \frac{2n\,b\,h^2 f}{3L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(v)$$

Without extra full length leaves:

$$\delta = \frac{3\,PL^3}{8Enbh^3} = \frac{L^2 f}{4\,Eh} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(vi)$$

With extra full length leaves.

$$\delta = \frac{3\,P\,L^3}{4\,(2+r)\,En\,bh^3} = \frac{L^2 f}{2\,(2+r)\,Eh} \qquad \ldots\ldots(vii)$$

where

L = total length of the spring

$r = \dfrac{n'}{n}$

n' = number of *extra* full length leaves

n = total number of leaves.

If there are three full length leaves, $n' = 2$.

The suitable material for the manufacture of such (leaf) springs will be highly hardened plain carbon steels, silico manganese steels and vanadium steels Silico manganese steels are used in automobile industry while carbon steels are used mainly by railways

Spring plates are made in accordance with Birmingham wire gauge

The suitable thicknesses can also be selected from IS. 1137-1959

Examples:

1. A cantilever spring 60 cm long is composed of twelve leaves each 6 cm wide. If the allowable flexural stress is 10,000 kg/sq cm, determine the thickness of each leaf if the maximum load at free end is 1,000 kg.

The maximum stress is given by the formula

$$f = \frac{6Pl}{b\,n\,h^2}.$$

On substitution of values, we get

$$10000 = \frac{6 \times 1000 \times 60}{6 \times 12 \times h^2}$$

or

$$h = \sqrt{\frac{6 \times 1000 \times 60}{6 \times 12 \times 10000}} = 0{\cdot}707 \text{ cm.}$$

From IS: 1137-1959, we adopt 0·71 cm.

2. *A flat spring 140 cm long is to be made of ten leaves 6 cm wide two of which extends the full length of the spring. It is to have a deflection of 10 cm when subjected to a load of 350 kg. The leaves are held together at the centre by a band 10 cm wide. Determine the thickness for the leaves and the maximum stress induced. $E = 2{\cdot}1 \times 10^6$ kg/sq cm.*

Due to the stiffening effect of the band, the equivalent length will be $140 - 10 = 130$ cm.

Extra full length leaf is 1.

$\therefore \quad r = \frac{1}{10} = 0{\cdot}1.$

$$\text{Deflection} = \frac{3PL^3}{4(2+r)\,Enbh^3}.$$

On substitution of values, we get

$$10 = \frac{3 \times 350 \times 130^3}{4\,(2+0{\cdot}1)\;2{\cdot}1 \times 10^6 \times 10 \times 6 \times h^3}$$

or $\quad h = 1{\cdot}3$ cm.

From IS: 1137 of 1959 we adopt 1·32 cm thickness.

$$\text{Maximum stress} = \frac{3LP}{2nbh^2} = \frac{3 \times 130 \times 350}{2 \times 10 \times 6 \times 1{\cdot}32^2}$$
$$= 655 \text{ kg/sq cm.}$$

3. *Design a cantilever leaf spring to absorb 8,000 kg cm of energy without exceeding a deflection of 15 cm and the permissible stress 8,750 kg/sq cm. The length of the spring is 60 cm and the modulus of elasticity is $2{\cdot}1 \times 10^6$ kg/sq cm.*

Let P be the maximum load on the spring when the deflection is 15 cm. By principle of work we get

$$P \times \frac{15}{2} = 8000$$

or $\quad P = \frac{8000 \times 2}{15} = 1{,}070$ kg.

The maximum stress is given by the equation, with usual notations,

$$f = \frac{6Pl}{bnh^2},$$

or $\quad bnh^2 = \frac{6Pl}{f}$

$$= \frac{6 \times 1070 \times 60}{8750} = 44{\cdot}2 \text{ cm}^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$$\text{Volume of the spring} = \frac{bnhl}{2}.$$

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3. *Design a cantilever leaf spring to absorb 8,000 kg cm of energy without exceeding a deflection of 15 cm and the permissible stress 8,750 kg/sq cm. The length of the spring is 60 cm and the modulus of elasticity is* $2{\cdot}1 \times 10^6$ *kg/sq cm.*

Let P be the maximum load on the spring when the deflection is 15 cm. By principle of work we get

$$P \times \frac{15}{2} = 8000$$

or $$P = \frac{8000 \times 2}{15} = 1{,}070 \text{ kg.}$$

The maximum stress is given by the equation, with usual notations,

$$f = \frac{6Pl}{bnh^2}$$

or $$bnh^2 = \frac{6Pl}{f}$$

$$= \frac{6 \times 1070 \times 60}{8750} = 44{\cdot}2 \text{ cm}^3 \quad \dots\dots\dots\dots\dots\text{(i)}$$

$$\text{Volume of the spring} = \frac{bnhl}{2}.$$

As the energy of a cantilever spring is equal to $\frac{f^2}{6E}$ × volume of the spring, the volume of the spring is given by

$$\frac{bnhl}{2} = \frac{8000 \times 6 \times 2{\cdot}1 \times 10^6}{8{\cdot}75^2 \times 10^6} = 1{,}330 \text{ cu cm.} \quad \ldots\ldots\ldots\text{(ii)}$$

Dividing equation (i) by (ii) we get

$$\frac{2h}{l} = \frac{44{\cdot}2}{1330}$$

or $$h = \frac{44{\cdot}2 \times 60}{2 \times 1330}$$

$$= 1 \text{ cm.}$$

∴ $$bn = 44{\cdot}2$$

We adopt 8 leaves of 6 cm width.

4. *The rear spring of an automobile has 9 leaves, each with an average thickness of 6 mm and a width of 50 mm. The length of the spring is 140 cm and the total weight on the spring is 600 kg. The spring is semi-elliptical one.*

Determine the rate of the spring, and the maximum stress caused by the dead weight. What approximate repeated maximum force (0 to F_{max}) would cause impending fatigue in 10^5 cycles, the number of applications of the maximum load expected during the ordinary life of a car?

Ultimate strength = 14,000 kg/sq cm

Yield strength = 12,000 kg/sq cm

$$\text{Stiffness of the spring} = \frac{8Enbh^3}{3l^3}$$

$$= \frac{8 \times 2{\cdot}1 \times 10^6 \times 9 \times 5 \times 0{\cdot}6^3}{3 \times 140^3}$$

$$= 19{\cdot}8 \text{ kg/cm.}$$

$$\text{Maximum stress} = \frac{3Pl}{2nbh^2}$$

$$= \frac{3 \times 600 \times 140}{2 \times 9 \times 5 \times 0{\cdot}36} = 7{,}800 \text{ kg/sq cm.}$$

When the load varies from zero to maximum, the mean load is equal to the variable load component is equal to $0{\cdot}5F_{max}$.

∴ $$f_a = f_m = \frac{3Pl}{2nbh^2} = \frac{3 \times 0{\cdot}5\,F_{max} \times 140}{2 \times 9 \times 5 \times 0{\cdot}36}$$

$$= 6{\cdot}5\ F_{max} \text{ kg/sq cm.}$$

$$\text{Endurance limit} = 0{\cdot}5 \times 14000$$

$$= 7{,}000 \text{ kg/sq cm.}$$

As we are designing for finite life of 10^5 cycles, endurance strength will be $7000 \left[\frac{10^6}{10^5}\right]^{0.09} = 8{,}600$ kg/sq cm.

For impending fatigue, the design factor N will be equal to 1. We take stress concentration factor as 1·4.

$$1 = \frac{6.5\,F_{max}}{12000} + \frac{6.5\,F_{max} \times 1.4}{8600}$$
$$= \frac{6.5\,F_{max}}{1000}\left[\frac{1}{12} + \frac{1}{6.12}\right]$$

From the above equation we get

$$F_{max} = 625 \text{ kg.}$$

Exercises:

1. *Design a semi elliptical carriage spring of 120 cm long to carry a dynamic load of 5,500 kg with a maximum deflection of 9 cm. $E = 2 \times 10^6$ kg/sq cm and $f = 6{,}500$ kg/sq cm.*

Ans. 10 leaves of 9 cm width and 1·32 cm thick.

2. *Design the cantilever leaf spring to absorb 6,200 kg cm energy without exceeding a deflection of 15 cm and the permissible stress 8,750 kg/sq cm. The length of the spring is 60 cm. The modulus of elasticity is 2.1×10^6 kg/sq cm.*

Ans. 6 leaves 6 cm wide and thickness of each leaf 1 cm.

3. *Design a cantilever plate spring to absorb 375 kg cm without exceeding a deflection of 2·5 cm and the permissible stress 10,000 kg/sq cm. The length of the spring is 16·5 cm. The modulus of elasticity is 2×10^6 kg/sq cm.* *Ans. Width 9·5 cm, thickness 0·56 cm.*

4. *A semi-elliptic automobile spring 145 cm long carries a total load of 900 kg. The spring is composed of 10 leaves two of which are full length, each 6 cm wide. Determine the necessary thickness and the resultant stress to give a deflection of 6·3 cm. $E = 2.1 \times 10^6$ kg/sq cm.*

Ans. 1·18 cm; 2,320 kg/sq cm.

5. *A car weighing 1,450 kg is supported by four semi-elliptic springs with the load equally distributed on the front and rear axles. Considering the available space and the proportions as commonly used in automobile practice, it is decided to make the spring 137 cm long and 5 cm wide. Determine the leaf thickness and the number of leaves, if the deflection at rest is assumed to be 10 cm and the allowable stress 4,200 kg/sq cm.*

Ans. 0·95 mm; 5.

6. *A semi-elliptic spring with a total effective length of 80 cm has ten leaves, two of which are full length, which are 6·7 mm thick and 50 mm wide. It is desired to replace this spring by a helical spring of 100 mm mean diameter and of such proportions that for any load it will have the same value of the induced stress and the same deflection as the leaf spring. Suggest the suitable diameter of the wire and the number of turns.*

Ans. 17 *mm*; 6·C.

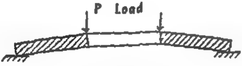
7. *Derive an equation for the energy absorbed by a cantilever spring of uniform strength. Express the answer in terms of metal volume and compare it with the energy absorbed by a beam of constant width and hence show that laminated springs permit saving in material.*

Ans. $\frac{f^2}{6E} \times$ *volume*, $\frac{f^2}{18E} \times$ *volume*.

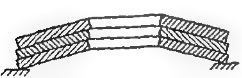
✓8. *Design a cantilever plate spring to absorb 375 kg cm without exceeding a deflection of 2·5 cm and the permissible stress 10,000 kg/sq cm. The length of the spring is 16·5 cm.* $E = 2 \times 10^3$ *kg/sq cm.*

Ans. Thickness 5 4 mm; width 10 cm.

8·9. Belleville springs:



(a) Belleville spring



(b) Belleville springs stacked in parallel

Belleville springs
Fig. 8-13(*a*) and (*b*)

These have high capacity and a relatively small space requirement, in direction of load application but this is obtained at the expense of relatively non-uniform stress distribution across the section.

Both the free length and the diameter of the spring must not exceed 3·3 cm. Assume the springs to be of steel having a working shear stress of 5,600 kg/sq cm with $G = 8{\cdot}4 \times 10^5$ kg/sq cm.

Calculate the speed at which the clutch will begin to engage. Suggest a suitable material for the friction linings and describe a method of fixing them to the shoes. Ans. 71 sq cm; $4\frac{1}{2}$ coil 25 mm mean diameter 0·53 mm diameter wire would suit.

6. A locomotive engine weighing 110 tonnes and moving at 6 km/hour strikes a buffer and is stopped in a distance of 30 cm. The buffer contains four carbon steel helical springs mounted in parallel with a friction element that requires a constant force of 15 tonnes to compress it. Assuming that the draft gear of the engine has 10 cm travel when the engine strikes the buffer and absorbs one half the energy, design the buffer springs for a spring index of 5 and design stress $f_s = 5{,}000$ kg/sq cm and $G = 0{\cdot}84 \times 10^6$ kg/sq cm.

7. Fig. 8-5 shows a compression spring for tension rods. Calculate the values of d, d_1, d_2 and l when the spring is free. Also, calculate the pitch and total number of coils of the spring. Assume:

Maximum tension in the rod 2,750 kg

Allowable stress in the rod 700 kg/sq cm

Deflection of the spring when the load changes from 2,500 kg to 2,750 kg is 1·5 cm

Spring index 6

Allowable stress in spring wire 5,600 kg/sq cm

Modulus of rigidity $8{\cdot}5 \times 10^5$ kg/sq cm.

Draw a fully dimensioned sketch of the spring in its free state.

8. A pair of buffers is required to stop a vehicle, which has no buffer springs, in a distance of 25 cm. The carriage weighs 2 tonnes and is moving at a speed of 6 km/hour. Design a spring of square section steel in which the mean diameter of the coil is 6 times the side of the square section, stress f_s not to exceed 3,500 kg/sq cm.

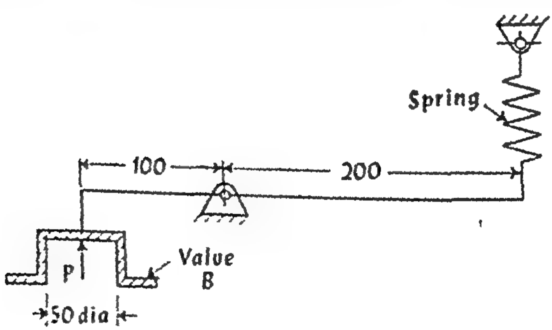


FIG. 8-15

9. Fig. 8-15 shows a safety device operated by a lever and tension spring. During the normal closed position of the valve B the pressure on the valve is

4.2 kg/sq cm. The maximum lift of the valve is 8 mm under a pressure of 5.6 kg/sq cm. The diameter of the valve is 5 cm. Design the spring making allowance for direct shear, bending, etc. Take allowable shear stress 4,200 kg/sq cm and $G = 8 \times 10^5$ kg/sq cm and spring index 7. Draw the spring which you have designed showing the end connections for adjusting the initial tension.

Ans. 0.53 cm, 19 turns.

10. In fig. 8-16, S and S' are two portions of a shaft. Arms A and A' are rigidly attached to the adjacent ends of the shaft as shown. The ends of the arms are furnished with hooks which are connected by two like coil springs as shown. Thus, it is possible to transmit energy from one shaft to other. Length of each arm $CB = C'B' = 24''$ (60) cm; the natural length of each spring is $= 8''$ (20) cm. When the shaft is rotating at 200 r.p.m., the angle between the arms is $60°$ and the h.p. transmitted is 15.

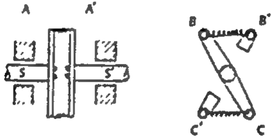


FIG. 8-16

Calculate the diameter of the spring wire, mean diameter of the coil and the number of coils assuming $G = 12 \times 10^6$ psi (8.4×10^5 kg/sq cm), $f_s = 60{,}000$ psi (4,200 kg/sq cm) and spring index 6. M. S. University of Baroda, 1959

11. A helical spring is wound from a commercial wire which is found to be 3% larger than the specified diameter d. To what mean radius in terms of the specified radius R must its coils be wound so that its deflection under the design load may remain unaltered? How will the shear stress in this spring then compare with that in the specified spring?

12. A car weighing 900 kg when loaded has a wheel base of 270 cm and its centre of gravity is 105 cm behind the front axle centre. The car is to be sprung on four similar longitudinal semi-elliptic carriage springs each 84 cm between the shackle pins. Design a suitable spring using a factor of safety of 1.5 on the proof stress of 8,400 kg/sq cm. The static loads are to be multiplied by a load factor of 2.5 to allow for impact loads. Young's modulus is 2.1×10^6 kg/sq cm.

Give a well proportioned and fully dimensioned sketch of the spring and show the method of securing a rear spring to the back axle.

Ans. 10 leaves 40 mm by 6.3 mm, radius 92 cm would be suitable

13. A semi-elliptical laminated spring 100 cm long between the supports carries a central load of 450 kg and is made of spring steel having a safe maximum stress limit of 4,200 kg/sq cm. The spring whose maximum deflection is

under the load is to be 13 cm becomes straight under this load. Calculate the various dimensions of the spring.

Draw a neat sketch of the longest leaf showing the various dimensions and the arrangement provided for attaching the spring to the shackles. Give also the lengths of the other plates.

14. Design the following springs:

(a) A semi-elliptic carriage spring 100 cm long to absorb 1,650 kg cm of energy, under the proof load of 450 kg (the proof load being the load to straighten the spring). Assume a working stress of 3,500 kg/sq cm and take the modulus of elasticity to be $2{\cdot}1 \times 10^6$ kg/sq cm.

Specify the initial radius of curvature of the plates.

(b) A helical valve spring of circular section steel wire to fulfil the following requirements:

Spring force holding valve closed — 22·5 kg.

Maximum spring force, occuring when valve opens 6·3 mm — 54·5 kg.

Spring to fit snugly into a recess of 7 cm diameter.

Assume a working shear stress of 2,100 kg/sq cm and take the modulus of rigidity to be 8×10^5 kg/sq cm. Make a dimensioned sketch of each spring.

Ans. (a) 8 leaves, 6·5 cm wide by 6·3 mm thick would suit. Radius of curvature 180 cm.

(b) 4 free coils of 8 mm dia wire, mean coil diameter 60 mm would suit.

15. A spring is required to support a maximum load of 270 kg and to have a stiffness of about 72 kg/cm. Two alternative types are to be considered as follows:

(a) a semi-elliptic leaf spring of 70 cm span, using leaves 5 cm by 0·475 cm section to be loaded centrally.

(b) a helical compression spring of outside diameter not exceeding 125 mm.

Compute suitable sizes for each type of spring, give a dimensioned working sketch of each and compare their weights assuming the density of the steel used 7·8 gm/cu cm. Other particulars of the steels are:

Leaf spring: working stress 3,150 kg/sq cm

$E = 2{\cdot}11 \times 10^6$ kg/sq cm

Helical spring: working shear stress 2,360 kg/sq cm

$G = 8{\cdot}35 \times 10^5$ kg/sq cm.

Ans. (a) 8 leaves; weight about 5·9 kg.

(b) 6 coils of 15 mm dia. wire; weight about 1·4 kg.

16. A single plate clutch is to transmit 60 metric H.P. at 1,500 r.p.m. The faces of the clutch plate are lined to have coefficient of friction 0·32 and permissible bearing pressure of 2·1 kg/sq cm. The clutch is engaged by 12 springs of 3 cm mean diameter. These springs are arranged symmetrically. The springs compress by 2 mm for disengagement with an increase of pressure of 10%.

4·2 kg/sq cm. The maximum lift of the valve is [illegible] mm under a pressure of 5·6 kg/sq cm. The diameter of the valve is 5 cm. Design the spring making allowance for direct shear, bending, etc. Take allowable shear stress 4,200 kg/sq cm and $G = $ [illegible] $\times 10^5$ kg/sq cm and spring index 7. Draw the spring which you have designed showing the end connections for adjusting the initial tension.

Ans. 0·53 cm, [illegible] turns

[illegible] In fig 8-16, S and S' are two portions of a shaft. Arms A and A' are rigidly attached to the adjacent ends of the shaft as shown. The ends of the arms are furnished with hooks which are connected by two like coil springs as shown. Thus, it is possible to transmit energy from one shaft to other. Length of each arm $CB = C'B' = 24''$ (60 cm), the natural length of each spring is $= 8''$ (20 cm). When the shaft [illegible] rotating at 200 r.p.m., the angle between the arms is 60° and the h.p. transmitted is 15.

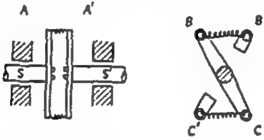


Fig. 8-16

Calculate the diameter of the spring wire, mean diameter of the coil and the number of coils assuming $G = 12 \times 10^6$ psi ($8·4 \times 10^5$ kg/sq cm), $f_s = 60{,}000$ psi (4,200 kg/sq cm) and spring index 8. (M. S. University of Baroda, 1959)

11. A helical spring is wound from a commercial wire which is found to be X% larger than the specified diameter 'd'. To what mean radius in terms of the specified radius 'R' must its coils be wound so that its deflection under the design load may remain unaltered? How will the shear stress in this spring then compare with that in the specified spring?

12. A car weighing 900 kg when loaded has a wheel base of 270 cm and its centre of gravity [illegible] 145 cm behind the front axle centre. The car [illegible] to be sprung on four similar longitudinal semi-elliptic carriage springs, each 81 cm between the shackle points. Design a suitable spring using a factor of safety of 1·5 on the proof stress of 8,400 kg/sq cm. The static loads are to be multiplied by a load *factor of 2·5 to allow for impact loads.* Young's modulus is $2·1 \times 10^6$ kg/sq cm.

Give a well proportioned and fully dimensioned sketch of the spring and show the method of securing a rear spring to the back axle.

Ans. 10 leaves 40 mm by 6·3 mm, radius 92 cm would be suitable.

13. A semi-elliptical laminated spring 100 cm long between the supports carries a central load of 450 kg and is made of spring steel having a safe maximum stress limit of 4,200 kg/sq cm. The spring whose maximum deflection

Draw a neat sketch of the spring showing the method of obtaining lengths of individual leaves.

(Bombay University, 1969)

21. A solenoid brake is to be actuated by a helical compression spring of free length 35 cm and is to exert a maximum force of 1,000 kg when compressed by 8 cm. The outside diameter of the spring should not exceed 18 cm. What is the wire diameter, coil diameter and number of coils? The value of the shear stress induced is not to exceed, 2,400 kg/sq cm. Take $G = 0{\cdot}84 \times 10^6$ kg/sq cm.

(Sardar Patel University, 1969)

22. Design a cantilever leaf spring to absorb 8,000 kg cm of energy without exceeding a deflection of 15 cm and the permissible stress of 8,750 kg/sq cm. The length of the spring is 60 cm and modulus of elasticity $2{\cdot}1 \times 10^6$ kg/sq cm.

(Sardar Patel University, 1970)

If the pressure is parallel to the axis of the shaft, the bearing is called a thrust bearing. In a thrust bearing if the shaft terminates at the bearing surface the bearing is a pivot bearing. If the shaft extends through and beyond the bearing, the bearing is known as a collar bearing. The common examples of collar thrust bearing are bearings of propeller shafts, shafts carrying worm and bevel gears and spindles of drill presses.

In rolling bearing, the surfaces are in rolling contact in contrast to sliding contacts as in sliding bearings. If the supported member runs on cylindrical or conical rollers, the bearing is known as roller bearing. If hardened steel balls are used in place of rollers, the bearing is termed as ball bearing.

9-2. Bearing area:

The area of a surface of a bearing perpendicular to the direction of the pressure is called the projected area. For a cylindrical journal bearing of diameter d and length l the projected area is $d \times l$. For a pivot bearing of diameter d the projected area is $\frac{\pi}{4} d^2$. For a collar bearing having n collars of inside diameter d and outside diameter d_1, the projected area is $\frac{\pi}{4} n\,(d_1^2 - d^2)$.

In all calculations on the area of the surface of a bearing, the projected area only is considered.

If P is the total load on any bearing and p the mean intensity of pressure, then

$P = p \times$ projected area of the bearing..................(i)

The permissible value of p varies greatly in different cases in practice. It is generally smaller when the speed is greater. It also depends upon the nature of the load and the method of lubrication. At a later stage this point will be considered in more details.

9-3. Sliding bearings: Solid Journal bearings:

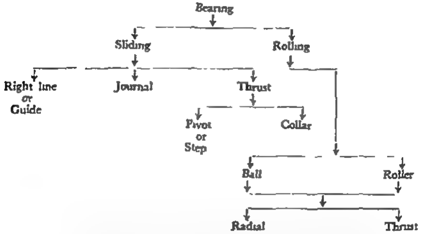
The simplest form of bearing consists of a cylindrical piece of metal with a hole to receive a pin or shaft which makes a running fit. Such a bearing known as a solid journal bearing is used when the load is small and the wear is immaterial. Solid journal bearing can be made integral with a frame of the machine as shown in fig. 9-1. When this bearing is manufactured as a separate piece, it is provided with extensions to receive bolts for fastening

CHAPTER **IX**

BEARINGS

9-1. Classification:

When there is relative motion between two machine parts, one of which supports the other, the supporting member is called a bearing.



The right line bearing is one in which the relative motion is parallel to the elements of the sliding surfaces, which may be flat as the guides on engine crossheads, the ways of milling machines and small planers or they may be circular as in case of spindles of boring and drilling machines.

*If the relative motion between two machine parts is of rota*tion and the pressure on the bearing is perpendicular to the axis of the shaft (along the radius of the shaft), the bearing is known as a journal bearing. The part which is enclosed by and rubs against the other is called the journal and the part which encloses the journal is called the box or bearing. Mostly the journal rotates in the fixed bearing but in a few cases both the journal and bearing are in motion, for example, a crank pin and its bearing in the connecting rod. In some cases the journal is fixed and the bearing rotates as in a hoisting drum or a loose pulley.

If the pressure is parallel to the axis of the shaft, the bearing is called a thrust bearing. In a thrust bearing if the shaft terminates at the bearing surface the bearing is a pivot bearing. If the shaft extends through and beyond the bearing, the bearing is known as a collar bearing. The common examples of collar thrust bearing are bearings of propeller shafts, shafts carrying worm and bevel gears and spindles of drill presses.

In rolling bearing, the surfaces are in rolling contact in contrast to sliding contacts as in sliding bearings. If the supported member runs on cylindrical or conical rollers, the bearing is known as roller bearing. If hardened steel balls are used in place of rollers, the bearing is termed as ball bearing.

9-2. Bearing area:

The area of a surface of a bearing perpendicular to the direction of the pressure is called the projected area. For a cylindrical journal bearing of diameter d and length l the projected area is $d \times l$. For a pivot bearing of diameter d the projected area is $\frac{\pi}{4} d^2$. For a collar bearing having n collars of inside diameter d and outside diameter d_1, the projected area is $\frac{\pi}{4} n (d_1^2 - d^2)$. In all calculations on the area of the surface of a bearing, the projected area only is considered.

If P is the total load on any bearing and p the mean intensity of pressure, then

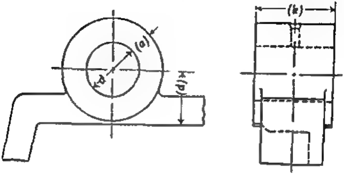
$$P = p \times \text{projected area of the bearing} \quad \text{.................(i)}$$

The permissible value of p varies greatly in different cases in practice. It is generally smaller when the speed is greater. It also depends upon the nature of the load and the method of lubrication. At a later stage this point will be considered in more details.

9-3. Sliding bearings: Solid Journal bearings:

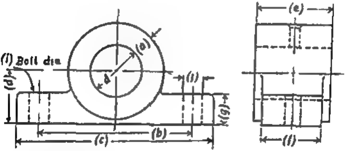
The simplest form of bearing consists of a cylindrical piece of metal with a hole to receive a pin or shaft which makes a running fit. Such a bearing known as a solid journal bearing is used when the load is small and the wear is immaterial. Solid journal bearing can be made integral with a frame of the machine as shown in fig. 9-1. When this bearing is manufactured as a separate piece, it is provided with extensions to receive bolts for fastening

it to the frame as shown in fig. 9-2. To give the necessary length of a bearing when it is made integral with the frame, the frame is increased in thickness by casting bosses on it.



(a) 0 4d (d) d (k) 1 5d+0 75"

Solid journal bearing integral with the frame of the machine
Fig 9-1



(a) 0.4d (d) d (g) 0.5d+0.25"

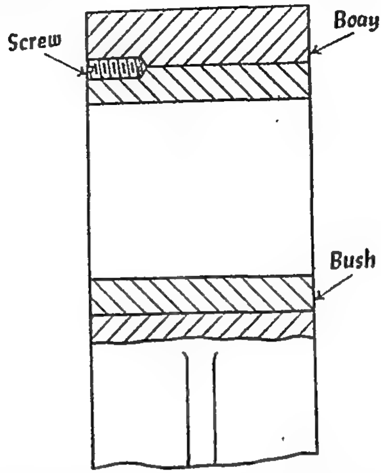
(b) 2 5d+1.25" (e) 1.5D+0 75" (i) 0.25d+0 125"

(c) 3 5d+1 75" (f) 1 25d+0 625" (l) 0 4d

Solid journal bearing
Fig. 9-2

The main drawback of this form of bearing is the absence of provision for adjustment in case of wear The first refinement of this bearing is to bore the hole larger than the shaft and to fit a solid bush of softer metal which takes up the wear. The bush may be a tight driving fit in the outer support or it may be kept in place by a screw as shown in fig. 9-3. To re-new the bearing it is only necessary to re-new the bush.

The solid bearing can only be used when there are no enlargements on the shaft which would prevent it passing into the bearing.



The arrangement for keeping the bush in position

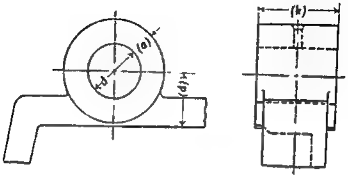
FIG. 9-3

9-4. Divided journal bearing: Plummer block:

When it is not desirable to place a shaft in its bearing by introducing it endwise, the bearing is divided and the parts are held together by bolts or other fastenings. This arrangement also provides for wear. *The bearing is usually split into two parts by a plane, passing through the axis of the shaft, normal to the direction of the load.* The top part of the divided bearing is called the cap, which is fastened to the lower part called base, frame or pedestal by bolts and nuts as shown in fig. 9-4. *Separable bearings are made for shafts with $d = 50$ to 500 mm and their caps are secured either with two bolts (light bearings $d < 80$ mm or with four bolts (heavy bearings, $d > 80$ mm).* The wear is taken up by removing some of the metal on the dividing surface or by thinning the shims. To insure proper alignment of the two parts, an off-set is made on the line of division.

The plummer block (fig. 9-4) is an independent shaft bearing adopted for bolting to a support on a masonry wall, a steel girder or on a large machine base. The usual design of the plummer block provides for ample rigidity, for replaceable and adjustable brasses and for definite lubrication.

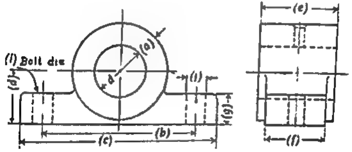
it to the frame as shown in fig. 9-2. To give the necessary length of a bearing when it is made integral with the frame, the frame is increased in thickness by casting bosses on it.



(a) 0.4d (d) d (k) 1.5d+0.75"

Solid journal bearing integral with the frame of the machine

Fig. 9-1



(a) 0.4d (d) d (g) 0.5d+0.25"

(b) 2.5d+1.25" (e) 1.5D+0.75" (i) 0.25d+0.125"

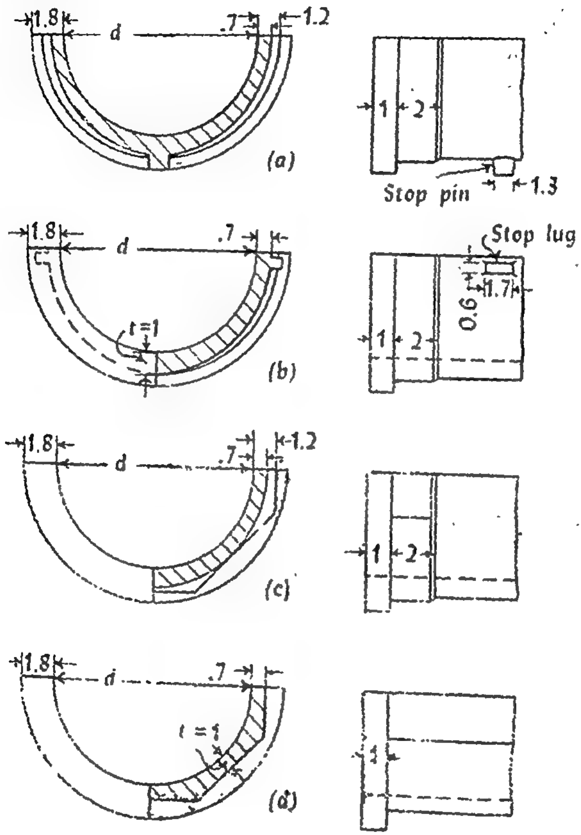
(c) 3.5d+1.75" (f) 1.25d+0.625" (l) 0.4d

Solid journal bearing

Fig. 9-2

The main drawback of this form of bearing is the absence of provision for adjustment in case of wear. The first refinement of this bearing is to bore the hole larger than the shaft and to fit a solid bush of softer metal which takes up the wear. The bush may be a tight driving fit in the outer support or it may be kept in place by a screw as shown in fig. 9-3. To re-new the bearing it is only necessary to re-new the bush.

The unit for proportion is usually t, the thickness of that part of the brass which supports the load and this may be taken as $t = 0{\cdot}08d + 0{\cdot}1''$ (3 mm).



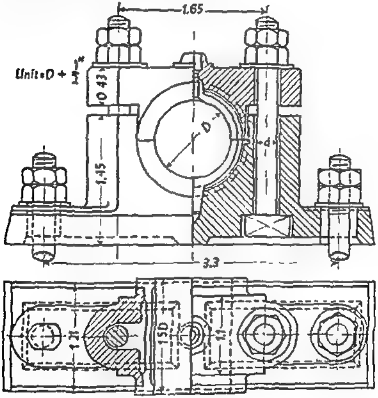
Brasses

FIG. 9-5

9-5. Lubrication methods:

For the most efficient operation of the bearings, they must be properly lubricated. Under ideal conditions the rubbing surfaces actually become separated and the upper surface floats on the lubricant. This is known as fluid film lubrication. In such cases the friction is very low and the wear is practically eliminated.

The halves of a split bush are known as brasses or steps. They are made of special material known as bearing material. A good bearing material is softer than the journal, has a low coefficient of friction, has sufficient compressive strength, good conductivity for heat, small amount of wear and low cost. The commonly adopted bearing materials are brass, bronze, white metal and special antifriction alloys.



Plummer block

Fig. 9-4

Brasses must be of such form that they cannot move axially or rotate in the housing. The axial movement of the brasses is prevented by means of the flanges on the ends and the rotation is prevented by various means, some of which are shown in fig. 9-5. In all forms of brasses the amount of machining should be reduced to a minimum and for this reason portions preventing rotation usually extend only a short portion of the length of the brass.

A towards C and the fluid pressure in the oil rises. When the speed is higher, the pressure is sufficient to carry the weight of the journal; there is then no metal contact, but the shaft is supported by the oil, being nearest to the bearing surface at a point P_3 on the off side; this will be the point of maximum fluid pressure and the position of the shaft will be as shown in fig. 9-7. Under such conditions the fluid friction in the lubricant is substituted for sliding friction between the journal and the bearing. Fig. 9-7 shows the ideal variation of pressure in the converging film in radial and axial directions.

The coefficient of friction for a complete film lubrication can be expressed as

$$\mu = \phi \left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right) \quad \ldots\ldots (i)$$

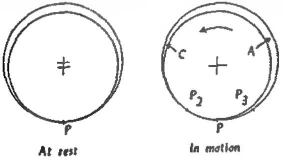
where μ = coefficient of friction
ϕ = a functional relationship
Z = absolute viscosity of a lubricant in centipoises
N = speed of journal in r.p.m.
p = permissible bearing pressure in kg/sq cm on projected area
d = diameter of journal
c = difference between diameter of bushing and diameter of journal
l = length of the bearing.

The quantity $\frac{ZN}{p}$ is a dimensionless number and is known as the bearing characteristic number. Fig. 9-8 shows the variation of the coefficient of friction with operating values of $\frac{ZN}{p}$. Bearing factor $\frac{ZN}{p}$ helps to predict the performance of a bearing. It can be seen from the curve that there is a minimum value of the coefficient of friction for a particular value of the bearing characteristic number, which we denote by the letter α. Values of $\frac{ZN}{p}$ greater than α indicate that the bearing may operate with complete film lubrication. At values less than α the rapid rise in the coefficient of friction indicates that the oil film has ruptured and there is metal to metal contact. The value

It can be proved by the theory of fluid lubrication that a positive pressure can be built up and that the load is supported by the fluid acting as a converging film. For a quantitative determination of the pressures in a bearing, corresponding to given operating conditions, it is necessary to resort to hydrodynamic theory.

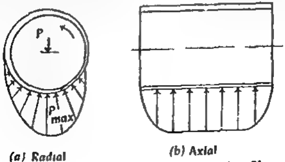
The action in a lubrication of a plain journal bearing is briefly described as under·

When the journal is at rest, it makes contact with the bearing at its lowest point P leaving a crescent-shaped space above, which is filled with lubricant, shown to an exaggerated scale in



Wedge action of a film in a bearing
FIG 9-6

fig. 9-6. When the shaft rotates in counter clockwise direction as shown, it tends to climb slightly until slipping occurs when contact is made at P_2 slightly to one side of P. As the speed of the shaft increases, it carries the oil with it from the wide space



Variation of pressure in a converging film
FIG. 9-7

The absolute viscosity Z of a lubricant in centipoise is given by

$$Z = \rho \left(0{\cdot}22\, S - \frac{180}{S}\right) \quad \text{.....(iii)}$$

where ρ = specific gravity of liquid

S = Saybolt reading in seconds.

The absolute viscosity of any oil varies with its specific gravity which also changes with temperature.

The specific gravity of the liquid is given by

$$\rho = \rho_{60} - 0{\cdot}000365\,(t - 60) \quad \text{.....(iv)}$$

where ρ_{60} is the specific gravity at 60°F and t the temperature of the oil film.

The specific gravity of ordinary lubricating oils at running temperatures may vary from about 0·8 to somewhat over 0·9 with 0·9 as an average satisfactory for approximate computations.

Small clearance means more heat, but greater load capacity; large clearance means less heat but is mechanically undesirable. Hence the designer must select the best compromise.

Rule of thumb: 0·0025 cm diametral clearance for shafts 25 mm diameter or less; on larger shafts, 0·0025 cm plus 0·0025 cm for each cm of shaft diameter, upto 0·015 cm. The ratio $\frac{c}{d}$ is known as the clearance ratio.

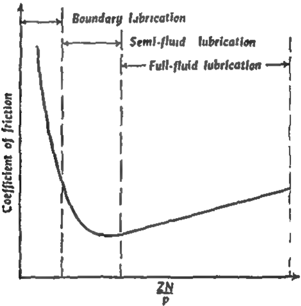
From the stand point of side leakage, a bearing with a large $\frac{l}{d}$ ratio is preferable. However space requirements, manufacturing tolerances and shaft deflections are better met by a bearing of short length. The usual value of $\frac{l}{d}$ ratio for general industrial machinery lies between 1 and 2. It is less than 1 for highly loaded engine bearings.

The average oil film thickness is $\frac{c}{4}$.

The pressure at which the oil film breaks down so that metal to metal contact begins is known as the critical pressure, which depends on the materials of the bearing and on the degree of smoothness of the contact surfaces. The maximum permissible unit pressure is given by

$$p = \frac{ZN}{475 \times 10^6} \left(\frac{d}{c}\right)^2 \frac{l}{d + l} \quad \text{.....(v)}$$

of $\frac{ZN}{p}$ when the oil film ruptures is known as the bearing modulus. When a bearing operates near this value, slight increase in pressure or decrease in speed may be accompanied by increase in friction, wear and heating. To prevent such conditions the bearing should operate at values of $\frac{ZN}{p}$ at least three times the minimum value α and if the bearing is subjected to large fluctuations of load and heavy impacts, values as high as 15α may be used.



Variation of coefficient of friction with bearing characteristic number

Fig. 9-8

Experimental data on small journal bearings by McKee established the following approximate equation for the coefficient of friction, which is the equation of the straight line portion of the curve shown in fig. 9-8

$$\mu = \frac{33}{10^{10}}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

where k depends on the ratio of the bearing length to the diameter. When $\frac{l}{d}$ ratio is less than 2·8 and more than 0·75, the value of the correction factor k may be taken as 0·002.

In case of comparatively long journals $\frac{l}{d} > 1{\cdot}5$, the load is distributed nonuniformly along the length of the stationary shell which brings about heavy wear of the ends of the shell as the bearing shell can not follow the angular deviations of the journal axis caused by the deformations of the shaft under load. Thus for *such type of bearings use should be made of bearings with self aligning shells.* Such shells have spherical lugs in the middle resting on spherical supporting surfaces in the housing and can turn on these surfaces to suit the inclination of the shaft. Such types of bearings are provided with ring lubrication.

9·6. Oil Grooving:

The purpose of the oil groove is to assist in the distribution of the oil between the rubbing surfaces. The maximum intensity of pressure in the oil film of bearings with ordinary clearances varies from two and one-half to four times the bearing pressure in kg/sq cm of projected area. At the groove the pressure of the oil is the supply pressure which is considerably less than the hydrodynamic pressure required to support the bearing load. Thus, a bearing with a central circumferential oil groove will in effect be equivalent to two narrow bearings and the presence of a groove in the region of positive pressure causes the reduction in load carrying capacity of the bearing. The best place to put the grooves is at right angles to the line of action of the load. Under no circumstances should the grooves run from regions of high pressure to regions of low pressure, for they would then act as drains running the oil from the bearing. The diagonal grooving should be avoided. The grooves should be cut shallow, with the bevelled edges to permit the free entrance of oil between the journal and bearing.

9·7. Heating of bearings:

The power lost in friction in the bearing is converted to heat and must be radiated from the housing without producing excessive temperatures. If the temperature of the bearing increases, the viscosity of the oil decreases as a result the oil squeezes out and the bearing may seize.

The heat generated at the bearing is given by

$$H_1 = \mu p d l V \text{ kg metre/minute} \quad \ldots\ldots (i)$$

where V is rubbing velocity in metre/minute.

For design purposes, the bearing characteristic number and the clearance ratio are related by the equation.

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 1{\cdot}43 \times 10^9 \quad \text{.........................(vi)}$$

This equation is used in design to determine the permissible bearing pressure.

Copious splash, forced feed lubrication, submersion or flooding of the sliding surfaces are methods which give perfect lubrication. Ring oiling, collar oiling or chain oiling in journal bearings furnish the conditions for semi-perfect lubrication while grease cup, wick oiler, drop oiler and capillary oiler give imperfect lubrication.

The following table gives the values commonly adopted for the journal bearing design practices:

Machinery	Bearing	$\frac{l}{d}$ ratio	Maximum p kg/sq cm	Operating Z centi-poise	$\frac{ZN}{p}$
Stationary slow speed steam engines	Main	1·0—2·0	28	60	280
	Crank pin	0·9—1·3	105	80	85
	Wrist pin	1·2—1·5	125	60	70
Stationary high speed steam engines	Main	1·5—3·0	30	15	65
	Crank pin	0·9—1·5	42	30	85
	Wrist pin	1·3—1·7	125	25	70
Gas and oil engines four stroke	Main	0·6—2·0	50—85	20	280
	Crank pin	0·6—1·5	100—125	to	140
	Wrist pin	1·5—2·0	125—135	65	70
Gas and oil engines two stroke	Main	0·6—2·0	35—125	20	350
	Crank pin	0·6—1·5	70—105	to	170
	Wrist pin	1·5—2·0	85—125	65	140
Aircraft engines and automobile engines	Main	0·8—1·8	55—120		210
	Crank pin	0·7—1·4	105—225	8	140
	Wrist pin	1·5—2·2	160—350		115
Reciprocating compressors and pumps	Main	1·0—2·2	20	30	420
	Crank pin	0·9—1·7	45	to	280
	Wrist pin	1·5—2·0	70	80	140
Machine tools	Main	1·0—4·0	20	40	15
Steam turbines	Main	1·0—2·0	7—18	2—16	1,400
Railway cars	Axle	1·9	35	100	700
Centrifugal pumps, motors and generators	Rotor	1·0 to 2·0	7—14	25	2,800

tions should be within permissible limits. This is very important in cases where the shafts carry gears whose teeth must be kept accurately in mesh. This is especially true for the mountings of bevel gear trains.

9.9. Bearing materials:

The following table gives a quick round up of materials and their characteristics, limitations and applications:

Materials	Characteristics	Limits	Applications
Wood (maple lignum vitai)	Self lubricating, low cost, long life	Light loads at high speed, under 65°C	Conveyors
Cast iron	Low friction, low cost develops high graze	Not over 35 kg/sq cm and 40 metre/min	Cam shafts, light transmission
Steel	Low cost	Light loads, 45 metre/min	Guides
Bronze bushing	Low cost, simple construction	Loads upto 200 kg/sq cm, shaft speeds upto 270 metre/min	All equipment
Heavy babbit liner on steel or cast iron	Long life, low friction, must have good lubrication	Steady loads under 70 kg/sq cm	Motor, turbine shafting
Light liner on steel or bronze backing	Heavy duty, general purpose, good for dynamic loads	About 350 kg/sq cm, and 1,200 metre/min	Gas and Diesel engines, compressors
Rubber	Low friction, resists abrasion, shock absorbent, long life	About 6 kg/sq cm, needs water lubrication	Marine propellers, pumps, turbines
Carbon graphite	No lubrication needed, light duty applications	Under 450°C 42 kg/sq cm, at low speeds	Electric motors, metres, conveyors
Moulded plastic laminate	Low friction, stronger than babbit when water lubricated	About 120 metre per min and 175 kg/sq cm; must be well cooled	Pumps, propellers
Moulded plastic	Low friction, clean	Low loads if used at high speeds	Dairy, textile and food machinery

The heat radiated by the bearing depends on the form of the radiating surface, on the temperature difference, on the mass of the bearing and on the amount of air flowing. However, for the convenience in bearing design the heat radiating capacity of the bearing is expressed in terms of the projected area of the journal. Thus the equation for the radiated heat becomes

$$H_2 = C\,l\,d\,(t_b - t_a) \text{ kg metre/min} \quad \text{(ii)}$$

where t_b = temperature of bearing surface, deg C

t_a = temperature of surrounding air, deg C

C = heat-dissipation coefficient in kg metre/min/sq cm of projected bearing area per deg C.

The values of C have been determined experimentally by O. Lasche. The values depend on temperature difference, ventilation of the bearing and the type of the bearing.

If the bearing conditions are such that the work of friction is high, artificial cooling should be provided. The two common ways of artificial cooling of the bearings are, first, by circulating an excess of lubricating oil to the bearing and second by water cooling of the bearing shell.

The maximum temperature allowable for a bearing depends on the lubricant used but should never be allowed to exceed 60°C.

9-8. Design procedure for Journal bearing:

The following procedure is adopted for the design of journal bearing:

(i) The diameter of the journal is determined either from considerations of strength or stiffness.

(ii) The bearing length is determined by choosing a ratio from the table of article 9-5.

(iii) The bearing pressure is checked for the probable satisfactory value.

(iv) By equating the heat generated at the bearing to the heat dissipated from it, it is determined whether artificial cooling is necessary or not.

Sometimes the dimensions of the bearing are determined directly from the load, allowable bearing pressure and $\frac{l}{d}$ ratio.

The strength and stiffness of the bearing, bearing cap and housing should also be considered. It is important that deflec-

On substitution of values, we have

$$60\left(\frac{7\cdot5}{0\cdot02}\right)^2 = Z_2\left(\frac{7\cdot5}{0\cdot015}\right)^2$$

or $$Z_2 = 60\left(\frac{0\cdot015}{0\cdot020}\right)^2 = 33\cdot75 \text{ centipoise.}$$

2. *A steam turbine shaft 200 mm diameter turns at 1,800 r.p.m. and is supported in a journal bearing on which the total load is 9,000 kg. The room temperature is 30°C. If the bearing temperature is 60°C, and allowable bearing pressure 15 kg/sq cm, determine the length of the bearing and the amount of heat to be removed by the lubricant per minute. Viscosity of the oil at 60°C is 21 centipoise.*

Minimum projected area of the bearing necessary $= \dfrac{9000}{15}$

$= 600$ sq cm.

Length of the bearing $= \dfrac{600}{20} = 30$ cm.

Temperature of the bearing = 60°C.

The coefficient of friction $= \dfrac{33}{10^{10}}\left(\dfrac{ZN}{p}\right)\left(\dfrac{d}{c}\right) + 0\cdot002.$

We assume $\dfrac{c}{d}$ ratio as 0·001.

$$\therefore \quad \mu = \frac{33}{10^{10}}\left[\frac{21 \times 1800}{15}\right]10^3 + 0\cdot002 = 0\cdot0103$$

$V = \pi dN = \pi \times 0\cdot2 \times 1800 = 1{,}130$ metre/min.

Heat generated $= H_1 = \mu PV = 0\cdot0103 \times 9000 \times 1130$

$= 105{,}000$ kg metre/minute.

3. *A shaft 20 cm diameter has a speed of 2,600 r.p.m. and runs in a bearing which has a length 1·2 times the diameter. The bearing pressure is 8 kg/sq cm and the coefficient of friction at the bearing surface is 0·006. Calculate the frictional loss in horse power units. The temperature of the bearing is controlled by the flow of oil through the bearing. If the difference between the outlet temperature and that at inlet is 20°C, obtain the quantity of oil required if the specific heat is 0·46.*

Length of the bearing $= 1\cdot2 \times 20 = 24$ cm.

Load on the bearing $= 24 \times 20 \times 8 = 3{,}840$ kg.

$V = \pi \times 0\cdot20 \times 2600 = 1{,}633$ metre/min.

Sliding bearings have a number of disadvantages including comparatively high friction losses, great length of the bearing shell, [illegible] number [illegible] necessity for constant supervision These disadvantages are nonexistent in the rolling contact bearings where sliding friction (friction of the first order) is replaced by rolling friction (friction of the second order).

Examples:

1. A bearing 7·5 cm diameter has a shaft speed of 300 r.p.m. and a lubricating oil of absolute viscosity, Z, of 60 is used. With a bearing clearance of 0·02 cm and a bearing pressure of 14 kg/sq cm, this oil is satisfactory in operation. If it is necessary to change the speed to 400 r.p.m. determine the pressure at which the bearing should operate.

If when designed for a speed of 300 r p m and pressure of 14 kg/sq cm, the clearance had been made 0·015 cm, what change should be made in the oil?

For a given clearance ratio, for the satisfactory operation of the bearing, the bearing characteristic number $\frac{ZN}{p}$ should remain the same for any change in the operating variables.

$$\therefore \quad \frac{Z_1N_1}{p_1} = \frac{Z_2N_2}{p_2}.$$

As the same oil is used, Z_1 is equal to Z_2 Hence $\frac{N_1}{p_1} = \frac{N_2}{p_2}$.

On substitution of values, we have $\frac{300}{14} = \frac{400}{p_2}$.

$$\therefore \quad p_2 = 14 \times \frac{400}{300} = 18{\cdot}7 \text{ kg/sq cm}$$

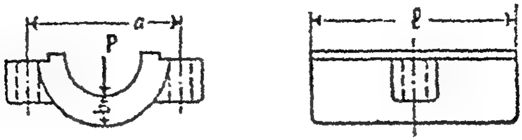
When the clearance ratio changes, for the satisfactory operation of the bearing, $\frac{ZN}{p}\left[\frac{d}{c}\right]^2$ should remain constant.

$$\therefore \quad \frac{Z_1N_1}{p_1}\left[\frac{d_1}{c_1}\right]^2 = \frac{Z_2N_2}{p_2}\left[\frac{d_2}{c_2}\right]^2.$$

As the speed and the bearing pressure remain the same,

$$Z_1\left[\frac{d_1}{c_1}\right]^2 = Z_2\left[\frac{d_2}{c_2}\right]^2.$$

a considerable pressure comes on the top. The cap is generally regarded as a beam supported by the holding down bolts or screws and loaded at the centre (fig. 9-9).



Forces acting on a bearing cap
FIG. 9-9

The maximum bending moment at the centre $= \frac{Pa}{4}$.

The moment of resistance $= \frac{1}{6}\, l\, b^2\, f$.

By equating the bending moment to the moment of resistance, we get

$$b = \sqrt{\frac{3}{2} \times \frac{Pa}{fl}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

When oil holes are provided in the cap, the length will be the bearing cap length less the diameter of the oil hole.

The cap should also be investigated for the stiffness. For the cap of a common end journal, good engineering practice limits the deflection to 0·025 mm. The expression for the thickness of a cap based upon the formula for the deflection δ of a simple beam loaded at the centre is given by

$$b = 0{\cdot}63\, a \sqrt[3]{\frac{P}{l\,\delta\,E}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

The bolts, screws or studs, used for holding down the cap, are generally assumed to be subjected to a simple tension and as a rule each bolt is designed for 33% overload, i.e. for a load equivalent to $\frac{4P}{3n}$ where n equals the number of bolts used for holding down the cap.

Examples:

1. Each of the main bearings of a triplex pump is 5 cm in diameter and 10 cm long and sustains a load of 900 kg. The load is directed against the cap. The distance between the centre lines of the bolts on either side of the cap is 10 cm. The cap is to be made of cast iron having an ultimate

Heat generated due to friction in horse power units will be equal to $\frac{\mu PV}{4500} = \frac{0.006 \times 3840 \times 1633}{4500} = 8.4$ h.p.

Heat equivalent of f.h.p. at the bearing $= \frac{8.4 \times 4500}{427}$

$= 88.5$ kcal/min.

Heat generated at the bearing is taken away by the lubricating oil.

If m be the mass of the lubricating oil flowing through the bearing per minute, then

$m \times 0.46 \times 20 = 88.5$

or $\quad m = \frac{88.5}{0.46 \times 20} = 9.65$ kg/min.

Exercises.

1. *A bearing for a centrifugal pump has a diameter of 75 mm and a length of 120 mm. The journal is machined so as to give a radial clearance of 0·0015 cm per cm radius. The journal rotates at 1,440 r.p.m. and carries a total load of 1,000 kg. Oil is supplied with a viscosity of 30 centipoise at the operating temperature. Determine the coefficient of friction, the actual value of the bearing characteristic number and the heat generated in kcal per minute.* *Ans.* 0·0031; 3,900; 4·9 kcal/minute.

2. *A steam turbine rotor is to run at 4,000 r.p.m. and is supported by bearings having a diameter of 20 cm and a length of 50 cm. The bearing pressure is limited to 5 kg/sq cm of projected area. Assume the coefficient of friction as 0 008. Determine the amount of the bearing cooling water required per minute if the rise in temperature of circulating water is limited to 22°C.* *Ans.* 215 kg.

3. *The flow of oil through a bearing is used to control the rise in temperature. The flow is regulated so that the difference in temperature between inlet and outlet is 20°C. The bearing diameter is 125 mm and ratio of length of bearing to diameter is 1·25. The shaft runs at a speed of 2,500 r.p.m. and bearing pressure is 15 kg/sq cm. Coefficient of friction between the rubbing surfaces is 0 006 If the specific gravity of the oil is 0 88, obtain the quantity of oil required.*

9-10. Design of bearing caps and bolts:

The cap of bearings should not be subjected to heavy loads; however, there are cases in which the circumstances are such that

We adopt the following materials for the various parts: cap — malleable cast iron; stud — mild steel; bolt — mild steel. The permissible stresses for cap, stud and bolt materials, will be taken as 350 kg/sq cm.

The dividing plane of the journal bearing is such that the plane is normal to the direction of the load. The resultant load is $\sqrt{1000^2 + 800^2} = 1,280$ kg and is inclined at an angle of $\tan^{-1} \dfrac{800}{1000} = 38°\text{-}48'$ to the direction of the horizontal component of the load which is 1,000 kg.

Let us assume the bearing pressure as 15 kg/sq cm.

Minimum bearing area required $= \dfrac{1280}{15} = 85{\cdot}5$ sq cm.

$\therefore$ Length of the bearing $= \dfrac{85{\cdot}5}{7{\cdot}5} = 11{\cdot}35$ cm; we adopt 12 cm.

We design the cap as a beam simply supported at the stud centre lines and loaded at the centre by a concentrated load of 1,280 kg. *Another assumption regarding the design of the bearing cap is that the load is taken as uniformly distributed over the length of the beam, equal to shaft diameter at the middle portion of the beam.* The former consideration gives a stronger cap and hence we adopt it.

The diameter of the shaft is 7·5 cm. We adopt the distance between two studs as 14 cm. Maximum bending moment on the cap $= \dfrac{1280 \times 14}{4} = 4,470$ kg cm.

The axial length of the cap = 12 cm.

If t be the thickness of the cap, then

$$4470 = \tfrac{1}{6} \times 12t^2 \times 350$$

or $$t = \sqrt{\frac{4470 \times 6}{12 \times 350}} = 2{\cdot}52 \text{ cm; we adopt } 2{\cdot}7 \text{ cm.}$$

The studs are usually designed for 33% overload.

There are 2 studs.

Design load on each stud $= \dfrac{4}{3} \times \dfrac{1280}{2} = 855$ kg. We have assumed low value of the working stress to account for the initial stresses induced in them due to tightening.

strength of 1,400 kg/sq cm in tension and 7,000 kg/sq cm in compression. Taking the factor of safety to be 5, determine the thickness for the cap.

The cap is subjected to bending stresses which are tensile as well as compressive. As the ultimate strength in tension is less than that in compression, the tensile strength will be the design criterion.

Allowable stress $= \frac{1400}{5} = 280$ kg/sq cm.

With usual notations, the thickness of the cap is given by

$$b = \sqrt{\frac{3}{2}\frac{Pa}{fl}}$$

On substitution of values, we have

$$b = \sqrt{\frac{3}{2} \times \frac{900}{280} \times \frac{10}{10}} = 2{\cdot}19 \text{ cm, we adopt } 2{\cdot}2 \text{ cm.}$$

2. *A journal bearing for a 75 mm diameter shaft is shown in fig. 9-10. The horizontal and vertical reactions at the bearings are $H = 1,000$ kg and $V = 800$ kg. The cap C is fixed by two studs A to the main body. The bearing is fixed to the vertical R.S.J. by four bolts B.*

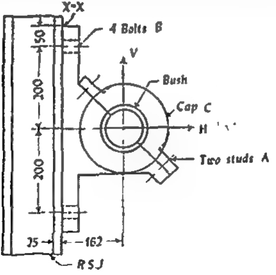


Fig. 9-10

Calculate the thickness of the cap C and sizes of the studs A and bolts B. Also, give the inclination of the dividing plane of the journal bearing.

The area of cross section at the shank of the bolt will be equal to $\frac{\pi}{4} \times 1{\cdot}6^2 = 2$ sq cm.

Stress induced at the section where there is a shear load of 200 kg, in addition to tensile load of 392 kg $= \frac{476}{2} = 238$ kg/sq cm which is less than 350 kg/sq cm. Therefore, the design is safe.

We adopt 4 — M 16 diameter bolts for connecting the bearing to R.S.J.

Exercises:

1. *The crank-pin of an automobile crank shaft is 5 cm in diameter and 5 cm long. The connecting rod cap is to be made of a steel drop forging having an ultimate tensile strength of 5,250 kg/sq cm. The distance between the centre lines of the holding down bolts is 7·5 cm. Taking the factor of safety 4, determine the thickness for the cap necessary to carry a load of 400 kg.* *Ans. 1 cm.*

2. *A connecting rod cap is fixed by two studs. The maximum tensile load on the connecting rod is 1,500 lb (680 kg). The studs are made of nickel steel having an endurance limit of 45,000 psi (3,150 kg/sq cm). Design and prepare a dimensioned free hand sketch of the stud with a suitable nut.* *(Gujarat University, 1953)*

3. *A bearing cap is to be made of malleable cast iron having an ultimate tensile strength of 45,000 psi (3,150 kg/sq cm). The journal is 3″ (7·5 cm) in diameter and the bearing is 4½″ (11·5 cm) long. The load on the cap is 1,500 lb (680 kg) and the bolts holding the cap in place are spaced 4″ (10 cm) from centre to centre measured at right angles to the shaft. Assuming a factor of safety of 6, determine the thickness of the cap.* *(Sardar Vallabhbhai Vidyapeeth, 1958)*

4. *Design the big end of a marine type of connecting rod. The load acting along the connecting rod is 11,000 lb (5,000 kg). Allowable bearing pressure on the crank pin is not to exceed 460 psi (30 kg/sq cm). Diameter of the crank pin is 4 in. (10 cm). Bending stress in the keep plate is not to exceed 5,000 psi (350 kg/sq cm). Width of keep plate is 4 in. (10 cm). Tensile stress in holding down bolts is not to exceed 5000 psi (350 kg/sq cm) and the distance between the centre lines of the bolts is to be kept 6 in. (15 cm).*

Give dimensioned sketches (elevation and plan) of the big end. *(Bombay University, 1955)*

The minimum cross sectional area at the bottom of the thread will be equal to $\frac{855}{350} = 2{\cdot}45$ sq cm.

From table, we adopt M22 stud whose area at the bottom of the thread is 3·03 sq cm.

Design of bolts B:

Due to the horizontal component of 1,000 kg all the bolts are subjected to the direct tensile load of $\frac{1000}{4} = 250$ kg. Due to the vertical component of 800 kg, the bearing will tend to turn about the line *XX* where it is connected to R.S.J. and due to this the bolts will be subjected to tensile loading. The maximum load will be in bolts situated away from the tilting edge. Let us assume that a bolt at a unit distance from the edge *XX* shares the load *W*. Equating the tilting moment of the load due to vertical component to the resisting moment of the bolt, we get

$$800 \times 16{\cdot}2 = 2\ [5W \times 5 + 45W \times 45]$$

or $$W = \frac{800 \times 16{\cdot}2}{2\ [25 + 2025]} = 3{\cdot}16 \text{ kg.}$$

Maximum load due to tilting on a bolt $= 45 \times 3{\cdot}16 = 142$kg.

$\therefore$ Maximum total tensile load on a bolt $= 250 + 142$

$= 392$ kg.

Bolts are also subjected to vertical shear load of 800 kg which is shared equally by each bolt.

Shear load on each bolt $= \frac{800}{4} = 200$ kg.

Principal load on the lower bolt $= \dfrac{392 + \sqrt{392^2 + 4 \times 200^2}}{2}$

$= 476$ kg.

The parallel part of the bolt is subjected to the principal load, while the threaded part is subjected to the tensile load only.

Minimum area at the bottom of the thread necessary will be equal to $= \frac{392}{350} = 1{\cdot}12$ sq cm

We adopt M 16 bolt whose area, at the bottom of the thread, is 1·57 sq cm.

3·5 to 17·5 kg/sq cm for rubbing speeds upto 60 metre/minute, the higher pressures being allowed for lower speeds. For speeds over 60 metre/minute the pressure should not exceed 7 kg/sq cm.

For slow speeds and intermittent service the bearing pressures may be taken upto 105 kg/sq cm. The coefficient of friction may be taken as 0·015.

The diameter d of the shaft is obtained from bearing stress considerations by the equation

$$d = \sqrt{\frac{4P}{\pi p}} \quad \ldots\ldots (i)$$

The horse power lost in friction at the bearing can be calculated. The work of friction appears as heat at the bearing. While calculating the power lost in friction *we assume that there is an uniform pressure intensity between the bearing surfaces.*

$$T = \frac{2}{3}\, \mu P r \quad \ldots\ldots (i)$$

where r is the radius of the shaft.

For collar bearing

$$T = \frac{2}{3}\, \mu P \left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] \quad \ldots\ldots (iii)$$

where r_2 is the outside radius of the collar and r_1 is the radius of the shaft.

The heat generated at the bearing is given by

$$H_1 = \tfrac{2}{3}\pi\mu P d N \text{ kg metre/minute} \quad \ldots\ldots (iv)$$

The above equations are modified when a shaft is bored with a shallow hole at the end.

The step bearing is difficult to lubricate as the oil is being thrown outward from the centre by a centrifugal force.

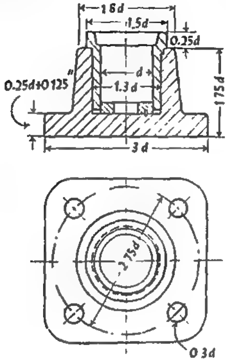
9-12. Collar bearings:

Such a bearing is used when it is not practicable to use the step bearing. The collars are placed, if possible, very close to the source of the axial thrust in order to relieve the shaft of any buckling action.

The bearing pressures for multi-collar thrust bearing are lower than that for pivot bearings. Usually the values of permissible bearing stress for pivot bearings are double than that for thrust

9-11. Foot step or pivot bearings:

Pivot bearings are used to support the lower ends of vertical shafts. The simple type of foot step bearing suitable for a slow running lightly loaded shaft is shown in fig. 9-11. The greatest



Foot step bearing
Fig. 9-11

wear on step bearing occurs at the outer radius where the velocity of rubbing is maximum. This trouble is partially eliminated by providing the thrust disc with a hole in the centre. Occasionally wear is distributed by several discs and each disc rotates at a fraction of the speed of the shaft.

If the shaft is not of steel, its end must be fitted with a steel face. This end is rounded and is supported on a cup shaped disc fitting in the foot step and prevented from rotating by a steel pin. The shaft is guided in a gunmetal bush and pressed into a pedestal. The bush is prevented from rotating by another pin. The allowable bearing pressures for the step bearings may be from

2. *A 8 cm shaft has on it an axial load of 3,500 kg which is taken by collar thrist bearing made up of 7 collars, each with an outside diameter of 13 cm. The shaft runs at 150 r.p.m. (a) What is the average bearing pressure? (b) What is the approximate work of friction?*

$$\text{Load per collar} = \frac{3500}{7} = 500 \text{ kg.}$$

$$\text{Area per collar} = \frac{\pi}{4}(13^2 - 8^2) = 82{\cdot}50 \text{ sq cm.}$$

$$\text{Bearing pressure} = \frac{500}{82{\cdot}5} = 6{\cdot}05 \text{ kg/sq cm.}$$

We assume the coefficient of friction to be 0·04.

Torque required to overcome friction at the collar bearing

$$= \frac{2}{3}\mu P\left[\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\right] = \frac{2}{3} \times \frac{4}{100} \times 3500\left[\frac{6{\cdot}5^3 - 4^3}{6{\cdot}5^2 - 4^2}\right] = 745 \text{ kg cm.}$$

$$\text{H.P.} = \frac{745 \times 150}{71620} = 1{\cdot}56.$$

Exercises:

1. *A rotary pump has a shaft of 6 cm and is required to sustain a load of 500 kg. Taking outside diameter of the collar as 1·5 times the diameter of the shaft, determine the number of collars required if the permissible bearing pressure is 3 kg/sq cm. If the maximum speed of the pump is 1,000 r.p.m., determine the horse power lost in friction. Take $\mu = 0{\cdot}03$. How will you determine the thickness of the collar?*

Ans. 5 collars; 0·745 H.P.; shear consideration.

2. *A twin screw ship driven by the turbine has a combined shaft horse power of 10,000 h.p. Each of the bearing has 12 shoes in contact with an equal number of collars on the shaft. Calculate the collar dimensions assuming that 35% of the bearing surface is lost due to opening in the shoe. Speed of the ship may be taken as 35 km/hour. The propelling efficiency is 80%. Outside diameter of the collar may be taken as 1·5 times inside diameter. The bearing pressure is not to exceed 3 kg/sq cm.*

Ans. 66 cm; 44 cm.

3. *Discuss the choice of lubricants and lubrication system for journal bearings.*

A foot step collar bearing 6" (15 cm) outside diameter and 4" (10 cm) inside diameter carries a load of 2,000 lb (910 kg). If shaft runs at 150 r.p.m., calculate the power lost in friction assuming uniform pressure. $\mu = 0{\cdot}04$.

bearings that are not water cooled. Kingsbury pivoted thrust bearings operate satisfactorily with pressures from 21 to 70 kg/sq cm.

The outer diameter of the collar may be made from 1·4 to 1·8 times the diameter of the shaft. The thickness of the collar should be checked for the shearing action. The collars are either integral parts of the shaft or rigidly fastened to it.

The number of collars are obtained from the equation

$$n = \frac{4P}{p\,\pi\,(d_2^2 - d_1^2)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

where d_2 = outside diameter of the collars

d_1 = diameter of the shaft.

It is assumed that load is evenly distributed over all collars. The heat generated at the bearing is given by the equation

$$H_1 = \tfrac{2}{3}\,\pi\mu P N \left(\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}\right) \text{ kg metre/minute.}$$

The coefficient of friction of collar bearings may be taken as 0·04, while due to the presence of a perfect oil film for Kingsbury ✓ bearings, it is $\frac{1}{10}$ the friction coefficient in a collar thrust bearing.

Examples :

1. A 10 cm shaft running at 200 r.p.m. is supported on a step bearing. The bearing area is annular with a 10 cm outside diameter and 4 cm inside diameter. The allowable average bearing pressure is to be taken as 10 kg/sq cm.

(a) What axial load may be supported?

(b) Determine the heat generated at the bearing.

$$P = \frac{\pi}{4}\,(10^2 - 4^2)\ 10 = 660 \text{ kg}$$

$$\text{Heat generated at the bearing} = \tfrac{2}{3}\pi\mu P N \left[\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}\right] \text{ kg metre/minute.}$$

We assume $\mu = 0{\cdot}015$. On substitution of values, we get

$$H_1 = \tfrac{2}{3} \times \frac{\pi \times 0{\cdot}015 \times 660 \times 200}{100}\left[\frac{10^3 - 4^3}{10^2 - 4^2}\right] = 462 \text{ kg metre/minute.}$$

$$= \frac{462}{427} = 1{\cdot}08 \text{ kcal/minute.}$$

(c) balls
(d) the cage or retainer which keeps the balls properly spaced about the periphery.

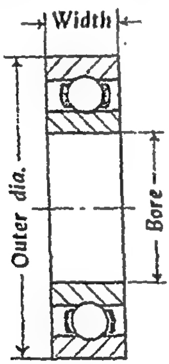
The contour of the raceway is grooved as shown in the fig. 9-12 a flat contour is also possible but the grooved raceway has the following advantages:

(a) The balls are retained in the bearing and the race is not likely to be distorted by the applied load.

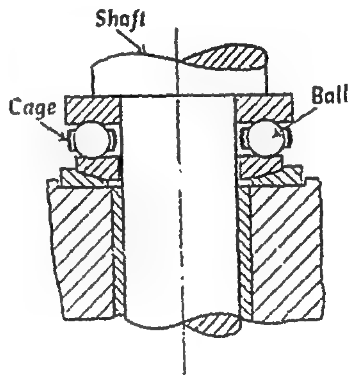
(b) Higher loads can be sustained.

(c) Considerable end thrust can be carried.

Though the single row bearing of fig. 9-12 is primarily intended for radial loads, it has a thrust capacity equivalent to 75% of its rated radial loads. The thrust capacity can be increased by employing bearings with deeper and closely fitting grooves.



Single row radial ball bearing
FIG. 9-12



Self aligning, spherical seated ball thrust bearing
FIG. 9-13

Double row ball bearings have two independent rows of balls in single inner and outer rows and have approximately twice the load capacity of the single row ball bearing. It can also sustain two directional thrust loads.

To compensate for a small degree of misalignment in erection and in installation and to permit some degree of deflection of the inner race, self aligning ball bearings are used. In such bearings the inner surface of the outer race is spherical.

In order to maintain accurate alignment of the shaft, pre-loaded ball bearings are used, which are placed under an initial load that is independent of the working load. Pre-loading tends to reduce the axial deflection under working loads. Pre-loaded ball bearings are used in precision equipments and for machine tool spindles.

Radial ball bearings are made in three series—light, medium and heavy. Light series ball bearings permit the smallest bearing width and outer diameter

Sketch a suitable bearing. Sketch an alternative arrangement using ball bearings. *(A.M.I.C.E.)*

Ans. 0·96 h.p. (0·97 metric h.p.)

4. *The propeller shaft of a steamer rotating at 100 r.p.m. is subjected to a torque of 200,000 lb in. (230,000 kg cm) and a thrust of 15,000 lb (6,800 kg). Design and prepare a working sketch of a suitable collar thrust bearing. Take the outside diameter of the thrust collars 1·5 times the diameter of the shaft, allowable bearing pressure, 50 psi (3·5 kg/sq cm), allowable stress for the material of the shaft and collars 5,000 psi (350 kg/sq cm) tensile, compressive and shear. Taking the coefficient of friction as 0·01, calculate also the h.p. lost in bearing friction.*

(Bombay University, 1953)

5. *Make a dimensioned sketch of a footstep bearing of a simple design proportioning the bearing surface so that the vertical shaft in the foot step may carry a load of 1,800 lb (820 kg) while running at a speed of 100 r.p.m. Allow a maximum bearing pressure of 250 psi (17·5 kg/sq cm). If the coefficient of friction be 0·05, calculate the power lost in friction at the bearing. Indicate clearly on your sketch the provisions made for lubrication and state what lubricants may be used for the purpose.*

(Sardar Vallabhbhai Vidyapeeth, 1957)

9-13. Anti-friction bearings:

Ball and roller bearings are known as anti-friction bearings and possess certain advantages over journal bearings. The actual bearing friction is less than that in sliding bearings, and, since it is principally rolling friction, results in little danger of abrasion to machines that are frequently started and stopped under load. Rolling bearings will maintain accurate alignment of parts for a long period, can carry heavy overload for a short time without failure and requires no attendance. Ball bearings occupy very little axial space. The disadvantages of these bearings are relatively high initial cost, large diameter and extreme care in manufacture.

9-14. Radial ball bearings:

Fig. 9-12 shows a single-row radial ball bearing. The main constituents of a ball bearing are:

(a) the outer race which fits in the frame of the machine

(b) the inner race which fits on the shaft

The SKF company give the hours of life for its bearing as

$$\text{hours of life} = \frac{\text{constant}}{\text{speed} \times \text{load}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

The constant of the equation depends on the size and type of bearing.

For Timken roller bearings the equation is

$$\text{hours of life} = \frac{\text{constant}}{\text{speed} \times \text{load}^{3\cdot 2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

The manufacturers of rolling bearings rate the bearings on the basis of the radial load the bearings will carry for a certain number of hours without more than 10% of the bearings failing. The capacities of SKF bearings are given for 500 hours while those of Timken bearings are given for 3,000 hours.

In order to select a proper bearing from the manufacturers' catalogue, first of all the actual radial and thrust loads are calculated. We determine the equivalent of the actual thrust and radial load at the given speed and desired hours of the life to the load at a speed of 500 r.p.m. and 500 hours of life for SKF bearings and 3,000 hours of life for Timken roller bearings. After the equivalent load has been calculated, the bearing which has at least this rated capacity is selected from the table.

If the load varies in magnitude then the allowance should be made for it. If the load is uni-directional and changes between limits P_{min} and P_{max}, the equivalent steady load is given by

$$P = \frac{2\,P_{max} + P_{min}}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

The SKF company state that the equivalent radial load for their ball bearing for 500 hours' life and 500 r.p.m. is given by the equation

$$W = \frac{x\,(R + y\,A)\,S}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

where W = calculated radial load for 500 hours' life and 500 r.p.m.

x = 1, for self aligning bearings, or other bearings with inner ring rotating, and

x = 1·33 for non-self aligning bearings with outer ring rotating

for a given bore size and they are used for moderate loads. Medium series bearings have load carrying capacity upto 40% greater than light series bearings of the same bore, but occupy more axial and radial space on the shaft and in the frame. Heavy series bearing have a load carrying capacity of from 20 to 30% greater than medium series bearings. Wherever possible single row ball bearings are preferred as it is difficult to secure an even distribution of load.

Rolling bearings should be protected from rust and dust. Rust may be prevented by packing the bearing in grease or submerging it in oil. Dust may be kept out by placing felt washers where the shaft passes through the housing

9-15. Roller bearings·

These bearings develop more friction but have a greater load capacity. The cylindrical roller bearings are also manufactured in three series. Generally the diameters and lengths of the rollers are equal Needle bearings operate without a cage and their diameter is very small in comparison with their length. They are used for piston pin bearings in large I C engines and for gear mounting in transmission units Bearings with tapered rollers are used for machine tool and automotive applications Such bearings permit the carrying of large unidirectional end thrust, which cannot be done with bearings having cylindrical rollers. The apex angle of the roller must be less than the angle of repose for lubricated steel on steel in order to prevent severe end thrust being set up by the radial load The usual angle of apex is 6° to 7°

Ball and straight roller bearings may be used singly if desired Due to the end thrust developed by the radial load acting on them, tapered roller bearings are used in pairs These bearings are so arranged that the cones point in opposite directions

Ball and roller bearings for thrust load only are used to some extent in machine tools in combination with radial sliding bearings Fig 9-13 shows a self aligning, spherical seated ball thrust bearing which is used for usual thrust applications the most common being the crane hook swivels

9-16. Selection of ball and roller bearings:

The selection of a ball or roller bearing for a given installation depends upon the following five factors.

(a) the load carrying capacity

(b) the speed of the shaft in r.p.m.

(c) the type of service✓

(d) the anticipated life of the bearing

(e) the proportion of thrust to radial load.

The capacity of the bearing decreases as the speed increases. If a ball bearing operates continuously, its life expectancy measured in hours, will obviously be shorter than if operated intermittently.

on Timken bearings is converted into thrust which is taken by the other member of the pair; as a result additional radial load must be added to the radial load due to external forces.

If a bearing A carries a radial load P, the magnitude of thrust on bearing B due to radial load $P = \dfrac{0{\cdot}34 \times P}{K_{TA}}$.

Calculated radial load on bearing $B = 0{\cdot}66 \times$ radial load on bearing $B + K_{TB}$ (thrust on bearing B due to radial load $P \pm$ external thrust).

Examples:

1. *A Timken bearing supporting one end of a wormshaft of a speed reducer turning at 1,800 r.p.m. is to carry a radial load of 80 kg and a thrust load of 200 kg. The desired life of the bearing is 12,000 hours. Determine the rated capacity of the bearing to be selected. Application factor may be taken as unity.*

$$\text{Speed factor} = \frac{6{\cdot}46}{(1800)^{0\cdot3}} = 0{\cdot}681.$$

$$\text{Life factor} = \frac{(12000)^{0\cdot3}}{11{\cdot}05} = 1{\cdot}51.$$

Since thrust load pre-dominates, steep angle bearing will be used for which K_T equals 0·75.

$$\begin{aligned}\text{Calculated load} &= 0{\cdot}66 \times 80 + 0{\cdot}75 \times 200\\ &= 203 \text{ kg.}\end{aligned}$$

$$\text{Rated load} = \frac{203 \times 1{\cdot}51 \times 1}{0{\cdot}681} = 450 \text{ kg.}$$

2. *Two bearings, A and B, constituting a pair, carry loads of 200 kg and 80 kg respectively, owing to external forces. Determine the calculated radial load for bearing B at the operating speed and desired life.*

$$\begin{aligned}\text{Thrust on } B \text{ due to radial load on } A &= \frac{0{\cdot}34 \times 200}{1{\cdot}5}\\ &= 45{\cdot}3 \text{ kg.}\end{aligned}$$

We assume $K_{TB} = 1{\cdot}5$.

Calculated radial load on bearing B at operating speed and desired life $= 0{\cdot}66 \times 80 + 1{\cdot}5 \times 45{\cdot}3$
$= 120{\cdot}5$ kg.

Exercises:

1. *The shaft of an electric motor which rotates at 1,440 r.p.m. is to be supported by a pair of Timken roller bearings. Each bearing is*

R = actual radial load

y = an empirical factor depending on the size and type of bearing

A = axial thrust load

$$S = \text{life factor} = \frac{(\text{desired hours of life})^{1/3}}{7{\cdot}94}$$

$$T = \text{speed factor} = \frac{7{\cdot}94}{(\text{r.p.m.})^{1/3}}.$$

Elevated temperatures, even of short duration, cause a reduction in load carrying capacity of the bearing.

In case of Timken bearing the scheme is to find the load for a speed of 500 r.p.m. and a life of 3,000 hours which will be equivalent to the actual load at the given speed and desired hours of life. *In addition to speed factor and life factor* we apply a third factor called the application factor to allow for the special conditions met with in the service to which the bearing under consideration is to be applied.

Required rating at 500 r.p m. will be equal to

$$\frac{\text{calculated load} \times \text{life factor} \times \text{application factor}}{\text{speed factor}} \quad \ldots\ldots \quad \text{(v)}$$

$$\text{Life factor for Timken bearing} = \frac{(\text{hours of life})^{0{\cdot}3}}{11{\cdot}05}$$

$$\text{Speed factor for Timken bearing} = \frac{6{\cdot}46}{(\text{speed})^{0{\cdot}3}}$$

$$\text{Calculated load} = 0{\cdot}66 \times \text{actual radial load} + K_T \times \text{thrust load} \quad \ldots\ldots \text{(vi)}$$

where $K_T = \dfrac{\text{radial rating}}{\textit{thrust rating}} = 1{\cdot}5$ for standard Timken bearing and 0·75 for steep angle bearings where thrust load exceeds radial load.

If the calculated load comes out to be less than the actual radial load, we take the actual radial load in using equation (v).

Since Timken bearings must be used in pairs, any difference in the load on the two bearings of the pair causes an unbalanced thrust to act on the lightly loaded Timken bearing of the pair. When we calculate the radial load capacity of the bearing, this unbalanced thrust should be taken into consideration. As the rollers are conical in form, a certain portion of the radial load

The maximum safe shear stress for the shaft and the crank pin is 550 kg/sq cm. The maximum safe bearing pressures are 30 kg/sq cm for the main bearings and 70 kg/sq cm for the connecting rod big end bearing.

Ans. (a) 10 cm diameter; (b) 9 cm diameter.

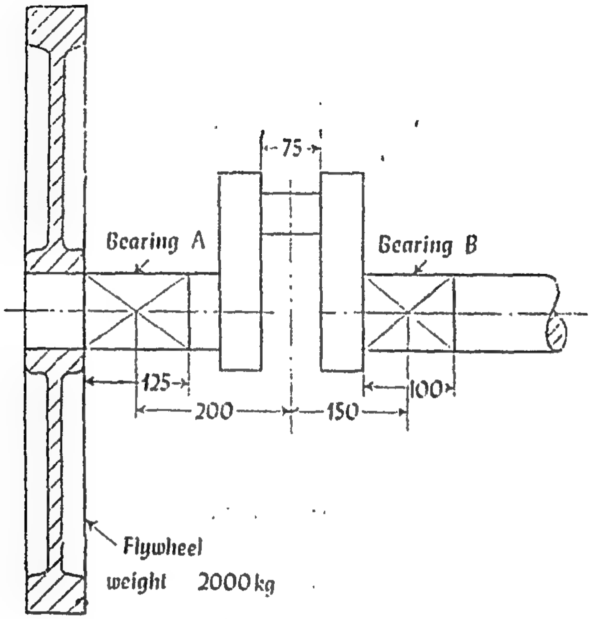


FIG. 9-14

3. A journal bearing for 75 mm diameter shaft is shown in fig. 9-10. The vertical and horizontal reactions at the bearing are $V = 1{,}300$ kg and $H = 1{,}850$ kg. The cap C is fixed by two studs A to the main bearing. The bearing is fixed to the vertical R.S.J. by four bolts B.

Design and prepare dimensioned sketches of the following: (a) cap C (b) stud A (c) bolt B.

Materials: Cap—cast iron; stud and bolts—mild steel.

Bearing pressure—15 kg/sq cm. Choose suitable stresses.

4. Fig. 9-15 shows a shaft mounted in two self-aligning bearings at B and C. Power is supplied to pulley A by a horizontal belt; this pulley is 90 cm in diameter and weighs 75 kg. Work is taken off at pulley D by a vertical belt; this pulley is 60 cm in diameter and weighs 50 kg.

For each belt, the tension ratio $\frac{T_1}{T_2}$ is 2·75. The input belt at A runs at 1,050 metre/min and its greater tension, which is on the underside, is 90 kg.

Calculate the force on each bearing and draw to scale the diagram of resultant bending moment for the shaft. Assume simple point support at each bearing.

to sustain a pure radial load of 400 kg and is to have a desired life of 12,000 hours. Determine the rated capacity of the bearing.

Ans. 830 *kg.*

2. *A shaft, mounted on two Timken roller bearings 35 cm apart, carries at its middle the gear, which causes a radial load of 1,000 kg and a thrust load of 320 kg on the shaft when running at 920 r.p.m. Determine the rated capacity of the bearing for a desired life of 10,000 hours. The application factor may be taken as 1·33.*

Ans. 1,800 kg.

3. *A herringbone gear used in a paper machine is mounted on a pairs of Timken bearings. Bearing A carries a radial load of 1,000 kg and an axial thrust of 400 kg. Bearing B carries a radial load of 360 kg. The speed of the shaft is 150 r.p.m. The desired life is 16,000 hours. Specify the rated capacities of bearings A and B. Timken application factor may be taken as 1·33.*

Ans. 2,100 kg; 880 kg.

EXAMPLES IX

1. A 100 cm diameter overhung pulley, weighing 50 kg transmits a torque of 5,000 kg cm. The belt pull is vertically downwards and the tension on the tight side is double the tension on the slack side. The distance between the centre of the pulley and the bearing is 150 mm Determine the diameter and length of the pedestal bearing assuming a maximum shear stress of 600 kg/sq cm in the journal and length of bearing double its diameter

If the two holding down bolts are 10 cm apart, calculate the diameter of the bolts and the thickness of the cap assuming permissible stress in the bolts 350 kg/sq cm and that in the cap 150 kg/sq cm Give a neat dimensioned sketch of the cap.

2. A single cylinder single acting horizontal steam engine with a 30 cm stroke is to run at 120 r p m. The crankshaft arrangement is shown in fig. 9-14. The flywheel weighs 2,000 kg and this weight may be assumed to be entirely supported by bearing *A*.

The greatest thrust developed in the connecting rod is 4,800 kg and this may be assumed to be maintained during the first half of the working stroke. The connecting rod is to be 50 cm long between centres

Determine suitable diameters for:

(a) the main bearings

(b) the crank pin.

The maximum safe shear stress for the shaft and the crank pin is 550 kg/sq cm. The maximum safe bearing pressures are 30 kg/sq cm for the main bearings and 70 kg/sq cm for the connecting rod big end bearing.

Ans. (a) 10 cm diameter; (b) 9 cm diameter.

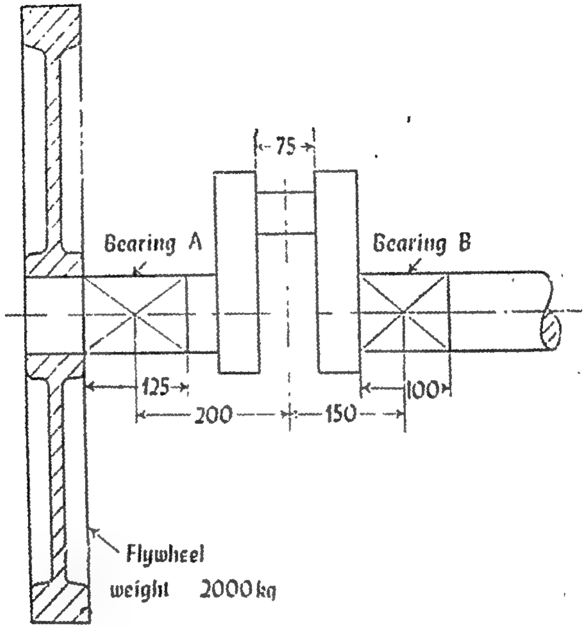


FIG. 9-14

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Design and prepare dimensioned sketches of the following: (a) cap C (b) stud A (c) bolt B.

Materials: Cap—cast iron; stud and bolts—mild steel.

Bearing pressure—15 kg/sq cm. Choose suitable stresses.

4. Fig. 9-15 shows a shaft mounted in two self-aligning bearings at B and C. Power is supplied to pulley A by a horizontal belt; this pulley is 90 cm in diameter and weighs 75 kg. Work is taken off at pulley D by a vertical belt; this pulley is 60 cm in diameter and weighs 50 kg.

For each belt, the tension ratio $\frac{T_1}{T_2}$ is 2·75. The input belt at A runs at 1,050 metre/min and its greater tension, which is on the underside, is 90 kg.

Calculate the force on each bearing and draw to scale the diagram of resultant bending moment for the shaft. Assume simple point support at each bearing.

Determine suitable diameter for the shaft assuming it to be of solid steel having a working shear stress of 280 kg/sq cm

Calculate a suitable width for the bearings assuming that the pressure must not exceed 17 kg/sq cm.

Ans 5 cm; 6 5 cm.

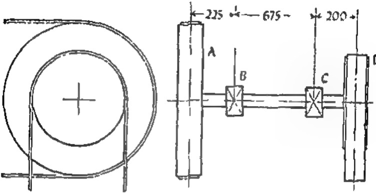


Fig. 9-15

5 Fig 9-16 shows some particulars of the big end of the connecting rod of a steam engine. The maximum load on the connecting rod is 30,000 kg Allowable bearing pressure on the crank pin is 70 kg/sq cm Length of the crank-pin is $1\frac{1}{4}$ times its diameter Maximum stress in the bolt is 400 kg/sq cm Permissible stresses in the strap are 200 kg/sq cm in shear and 400 kg/sq cm in tension Design and draw dimensioned neat sketches giving sectional elevation and plan In each of these views show the bolts complete with nuts, the lubricator and any other parts and details that are omitted in the sketch

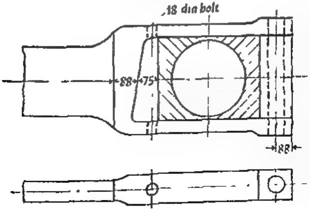


Fig. 9-16

6. The connecting rod cap of an engine sustains a maximum load of 13,000 kg and is made of cast steel having an ultimate strength in tension of 5,600 kg/sq cm. The diameter of the crank pin is 14 cm and its axial length is 15 cm.

Assuming a factor of safety of 5 for the cap, the safe working stress for the bolt of 700 kg/sq cm and a suitable thickness for the brasses, design and prepare a working drawing of the cap.

7. The torque transmitted by an engine crank shaft is 15,000 kg cm, when the speed is 240 r.p.m. The flywheel overhangs at a distance of 30 cm centre line of bearing to centre line of wheel and the maximum load on the bearing is 2,500 kg. Determine suitable dimensions for the bearing if the allowable bearing pressure is 25 kg/sq cm and maximum shear stress in the shaft is 350 kg/sq cm. Oil cooling is used and the maximum temperature of the bearing is not to exceed 40°C. Calculate the quantity of oil required in litres per hour if the inlet temperature of the oil is 20°C. Specific gravity of oil 0·8 and specific heat 0·4. Weight of flywheel is 1,000 kg. Coefficient of friction between shaft and bearing 0·01.

Ans. 8 cm diameter; 12·5 cm length.

8. A ship travelling at 30 km/hour requires 3,200 h.p. at the propeller. The diameter of the propeller shaft is 30 cm. Determine the number of collars required if the bearing pressure is not to exceed 3 kg/sq cm. The outside diameter of the thrust collar may be taken as 1·6 times the diameter of the propeller shaft. How will you determine the thickness of the collar? Calculate the amount of heat developed at the bearing.

Ans. 9 collars; shear consideration.

9. In a screw clamp, the clamping pad which may be of cast iron is provided with a conical bearing surface to take up the screw thrust of 5,000 kg. The pad is prevented from leaving the pivot by means of a set screw 8 mm diameter cleared through and counterbored in the face of the pad and tapped into the conical end of the spindle. Determine the outside diameter of the conical pivot if the bearing pressure is limited to 1,200 kg/sq cm. Also, determine the torque required to overcome friction at the conical pivot of the pad if the semi-angle of the pivot cone is 60°. $\mu = 0{\cdot}15$.

Ans. 2·8 cm; 1,160 kg cm.

10. Describe how the friction between the journal and bearing varies from the instant at which rotation of the journal begins until high speed is attained. You may assume that a plentiful supply of oil is available and the journal has been at rest for some time.

Also sketch the curve showing how the coefficient of friction varies with bearing characteristic number $\dfrac{ZN}{p}$.

An oil with an absolute viscosity Z of 50 centipoise is used for a bearing 5 cm diameter. This oil is satisfactory when the bearing pressure is 11 kg/sq cm, shaft speed 200 r.p.m. and bearing clearance 0·0187 cm. If the shaft speed is increased to 320 r.p.m., at what pressure should the bearing now operate?

When the bearing was re-conditioned, a clearance of 0·0225 cm had to be given. The pressure had to be 11 kg/sq cm, but the speed 320 r.p.m. What change must be made in the oil?

Determine suitable diameter for the shaft assuming it to be of solid steel having a working shear stress of 220 kg/sq cm.

Calculate a suitable width for the bearings assuming that the pressure must not exceed 17 kg/sq cm.

Ans. 5 cm; 6.5 cm.

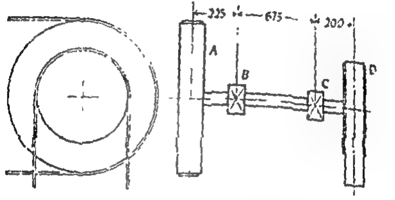


Fig. 9-15

5. Fig. 9-16 shows some particulars of the big end of the connecting rod of a steam engine. The maximum load on the connecting rod is 30,000 kg. Allowable bearing pressure on the crank pin is 70 kg/sq cm. Length of the crank-pin is $1\frac{1}{4}$ times its diameter. Maximum stress in the bolt is 400 kg/sq cm. Permissible stresses in the strap are 200 kg/sq cm in shear and 400 kg/sq cm in tension. Design and draw dimensioned neat sketches giving sectional elevation and plan. In each of these views show the bolts complete with nuts, the lubricator and any other parts and details that are omitted in the sketch.

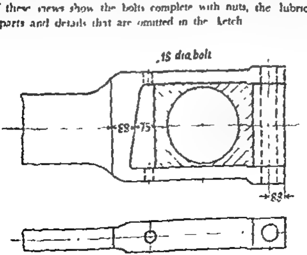


Fig. 9-16

CHAPTER X

STRUTS AND COLUMNS

10-1. Introduction:

Machine parts which are subjected to compressive forces are termed as *columns or struts*. In general, a column or a pillar is fixed at both ends and is vertical; struts may be inclined, one or both ends fixed rigidly, or one or both ends hinged or pin-jointed. The failure of such members may occur by pure compression, or by buckling or by combination of pure compression and buckling, depending upon a slenderness ratio of the column. *The slenderness ratio is defined as a ratio of the length of the column, to the least radius of gyration of the section about centroidal axis.* Piston rods, connecting rods, valve push rods, side links in forging machines, etc. are few examples of struts, found in machines.

10-2. Euler's Formula:

Euler's theory is the foundation of all strut theories. The critical load for a column or strut is given by

$$P_e = \frac{\pi^2 EI}{l^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

where P_e = critical load

E = modulus of elasticity

I = least second moment of area

l = length of the strut.

The above formula is valid only for very slender columns with rounded ends or pin-jointed ends, which are free to turn at the supports, and do not take into account the effect of direct compression.

$$\text{Safe working load on strut} = \frac{P_e}{\text{factor of safety}} \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

The value of factor of safety is usually 4 for a steady load or dead load and 8 to 10 when the machine part is alternately in tension and in compression as in piston rods or connecting rods.

The experiments have shown that Euler's formula gives reasonably accurate results for higher slenderness ratios. We notice that the crippling load is dependent on the dimensions of the strut, end connections and the modulus of elasticity.

11. The bore of the cylinder of a Diesel engine is 15 cm The maximum pressure in the cylinder = 42 kg/sq cm The piston is connected to the connecting rod by a floating gudgeon pin, which is hollow. Assuming a bearing pressure of 154 kg/sq cm and a bending stress of 840 kg/sq cm, design the gudgeon pin Assume $d_i = 0.75\, d_o$ and l/d ratio for bearing = 1.

(*Sardar Vallabhbhai Vidyapeeth, 1966*)

12. Describe briefly the procedure you will follow to design a thrust ball bearing Mention at least five applications of such a type of bearing.

(*Gujarat University, 1966*)

13. A rotary pump has a shaft of 5 cm diameter and is required to sustain a load of 400 kg If the maximum speed of the pump is 1,000 r.p.m., calculate the collar bearing dimensions and the horse power lost in friction Take $\mu = 0.04$ The bearing pressure is limited to 3.5 kg/sq cm

(Gujarat University, 1966)

14. The bearing cap is to withstand a load of 750 kg The journal is 6.75 cm in diameter and 11.25 cm long There are two fixing bolts one on each side of the shaft axis placed on 15 cm centres Suggest the suitable size of the cap bolts and thickness of the cap if there is 2.5 cm diameter hole in the cap

Permissible tensile stress intensity for the material of the bolt and cap is limited to 420 kg/sq cm

(Sardar Patel University, 1967)

15. The propeller shaft of a steamer at 100 r.p.m. is subjected to a torque of 250,000 kg cm and a thrust of 7,000 kg Suggest the suitable number of collars for the bearing Also suggest the probable thickness for each collar

Take outside diameter of the collar as 1.5 times the diameter of the shaft Permissible bearing pressure for the collar 4 kg/sq cm and f_s for collar and shaft 350 kg/sq cm (*Sardar Patel University, 1968*)

16. (a) What procedure will you follow while designing a journal bearing?

(b) Each bearing of an electrical motor sustains a radial load of 400 kg Assuming $\frac{l}{d}$ ratio of 1.1, determine the length of the bearing if the permissible bearing pressure is limited to 10 kg/sq cm

(c) A bearing cap is to withstand a load of 1,000 kg The journal is 10 cm ... bolts two on each side of the shaft ... thick- ... provid- ed for lubrication purposes

Permissible tensile stress intensity for the material of the bolt and cap is limited to 400 kg/sq cm (Sardar Patel University, 1969)

17. Explain the importance of bearing characteristic number in the design of bearings and show a sketch how the coefficient of friction varies with the bearing index in different states of lubrication

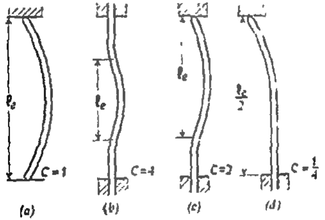
State the factors that govern the rate of feed for forced lubrication.

(Sardar Patel University, 1970)

Since there is little variation in the moduli of elasticity among different grades of steel, there is no advantage in using high strength alloy steel instead of structural steel. The strut can be made stronger by increasing the second moment of area and the least radius of gyration, which can very often be accomplished without any increase in cross sectional area by placing the material of the strut as far as possible from neutral axis. Thus, tubular sections are more economical as struts than solid sections.

10-3. End fixity coefficients:

Fig. 10-1 illustrates theoretical end connections which may be present in machine columns. The contact ends of round end columns are laterally guided so that they remain in vertical alignments, but the ends are not perfectly frictionless and free turning cannot be realised. Fig. 10-1(*b*) shows a fixed end column in



End connections for columns
Fig. 10-1

which the effective length l_e is equal to half the actual length of the column l. Such a column will resist four times the buckling load that can be carried by a round end column having the same length and same cross section. Fig. 10-1(*c*) shows a column in which a lower end is fixed and upper end is guided so that the vertical alignment of the column is maintained. It can be proved that the effective length of such a column is $\frac{l}{\sqrt{2}}$. Fig. 10-1(*d*) shows a column with lower end fixed and upper end free. Such a column is weakest of all the above-mentioned columns. The effective length of such a column is twice the actual length of the column

The load carrying capacity of such a column is one-fourth of that having guided ends.

In order to consider the effect of end connections, Euler's formula is written as

$P_e = \dfrac{n\,\pi^2 EI}{l^2}$ where n is known as end fixity coefficient whose theoretical values are given below:

End connections	Value of end fixity coefficient n or C
Both ends fixed	4
One end fixed, other end guided	2
Both ends guided or hinged	1
One end fixed, other end free	0·25

Sometimes the end fixity coefficient is denoted by the letter C.

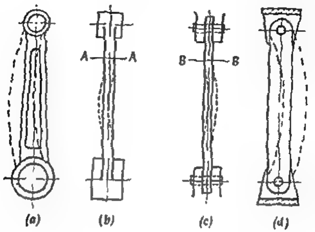
In practice the values of the above coefficients are modified. In arriving at the design values of end-fixity coefficient, it is necessary to consider the degree to which the actual support approaches the ideal one. Because of the flexibility of the supporting members it is doubtful whether a column in a machine has ideally fixed ends, even when welded in place. So the design value of end fixity coefficient for a fixed end column should at the most be limited to 3·5 instead of ideal value 4. Generally the value 3 is preferred.

10·4. Radius of gyration and plane of bending:

The end connections for machine columns such as connecting rods will vary with the plane of buckling. For example, the column may be pin-connected in one plane of buckling but may be considered to have fixed ends in plane perpendicular to this plane. In such cases *the plane of failure will be that plane for which the combination of end fixity and bending resistance is least, i.e. the plane for which the product nI is the least.*

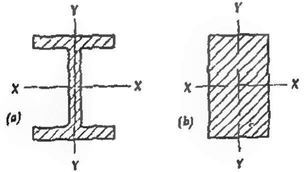
When a column is cylindrical or square there is little difficulty in selecting the critical plane because it depends only upon the end connections. However, in machines, the majority of columns are such that they have different radii of gyration as well as different degrees of end fixity in perpendicular planes, the commonest example being the connecting rod of an engine having I section or rectangular section. Thus, when we design

such a column, we should aim at equal buckling strength in both the planes.



Buckling of links in two perpendicular planes
Fig 10-2

Let us consider the buckling of a connecting rod. The gudgeon pin and crank pin give it freedom to bend in a plane perpendicular

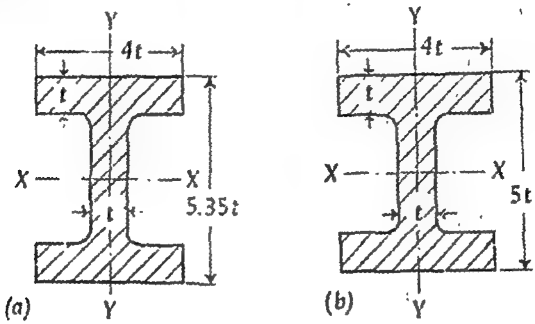


X-X = *Neutral axis, free ends column*
Y-Y = Neutral axis, fixed ends column
Fig 10-3

to their *axes as shown in fig.* 10-2(*a*), while at the same time tend to stiffen it in the plane of their axes as shown in fig 10-2(*b*). Thus, we see that the connecting *rod* behaves as a *fixed-ended* column for buckling in the plane of the axes of the pins while in the plane perpendicular to their axes, it behaves as a column hinged at its end. Fig. 10-2(*c*) and Fig. 10-2(*d*) show the behaviour

of the forging link, having a rectangular section, in two planes at right angles. If the rod and link were to be equally resistant to buckling in both the planes for the sections shown in fig. 10-3, the relation $I_{xx} = 4I_{yy}$ should be satisfied.

The above relation will be satisfied for a rectangular section if breadth of a section is one-half the depth. The proper proportions for the I section are derived as follows. Fig. 10-4(a) shows the various proportions of I section in terms of t, the thickness of a web as well as flanges.



Ideal I section for a column

FIG. 10-4

$$I_{xx} = \frac{1}{12}\left[4 \times 5{\cdot}35^3 - 3 \times 3{\cdot}35^3\right] t^4 = \frac{518}{12}\, t^4.$$

$$I_{yy} = \frac{1}{12}\left[2 \times 4^3 + 3{\cdot}35 \times 1^3\right] t^4 = \frac{131{\cdot}35}{12}\, t^4.$$

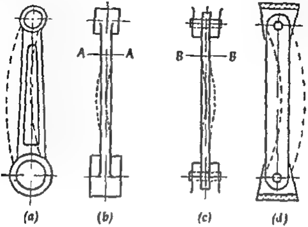
$$\therefore \quad \frac{I_{xx}}{I_{yy}} = \frac{518}{131{\cdot}35} = 3{\cdot}95.$$

Thus, the above proportions will be most satisfactory for the I section of a rod theoretically. In actual practice for fix end columns, the end fixity coefficient is taken between 3 and 3·5. I section with proportions shown in fig. 10-4(b) will be found satisfactory as it gives the value of the ratio $\frac{I_{xx}}{I_{yy}}$ as 3·2.

10-5. Rankine's formula:

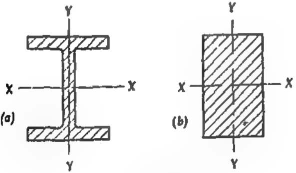
Euler's formula is only applicable for long struts and many struts encountered in machines are of such proportions that Euler's formula may not be applicable so that other formulas have to be used for comparatively short struts. Many empirical formulas

such a column, we should aim at equal buckling strength in both the planes.



Buckling of links in two perpendicular planes
FIG. 10-2

Let us consider the buckling of a connecting rod. The gudgeon pin and crank pin give it freedom to bend in a plane perpendicular



X-X = Neutral axis, free ends column
Y-Y = Neutral axis, fixed ends column
FIG. 10-3

to their axes as shown in fig. 10-2(a), while at the same time tend to stiffen it in the plane of their axes as shown in fig. 10-2(b). Thus, we see that the connecting rod behaves as a fixed-ended column for buckling in the plane of the axes of the pins while in the, plane perpendicular to their axes, it behaves as a column hinged at its end. Fig. 10-2(c) and Fig. 10-2(d) show the behaviour

have been suggested to represent the experimental results. One of the most important of these formulas is Rankine's formula. This formula was originally devised to fit experimental results but some attempt has been made to rationalise it. It i assumed that the actual buckling load P of a strut is connected with the maximum direct compressive load P_c of the material, and the buckling load Pe related to Euler's formula by the equation

$$\frac{1}{P} = \frac{1}{P_c} + \frac{1}{P_e}.$$

For a strut with rounded ends, $P_e = \dfrac{\pi^2 EI}{l^2}$

$$\text{so that } \frac{1}{P} = \frac{1}{P_c} + \frac{l^2}{\pi^2 EI}$$

$$= \frac{\pi^2 EI + P_c l^2}{\pi^2 EI P_c}$$

$$\text{or} \qquad P = \frac{\pi^2 EIP_c}{\pi^2 EI + P_c l^2}.$$

Direct compressive load $P_c = f_c A$ where f_c is the yield stress in compression.

$$P = \frac{Af_c}{1 + \dfrac{A f_c l^2}{\pi^2 EI}} = \frac{A f_c}{1 + \dfrac{f_c}{\pi^2 E}\left(\dfrac{l}{k}\right)^2} = \frac{A f_c}{1 + a\left(\dfrac{l}{k}\right)^2}$$

where k is the least radius of gyration and a, the constant depending upon the material and the end fixity coefficient.

$$\text{Safe working load on the column} = \frac{\text{actual bucking load}}{\text{factor of safety}}.$$

The values of f_c and a for different materials and different end fixity coefficients are given below

Material	f_c yield stress in compression kg/sq cm	Values of a for various end fixity coefficients 1	4	2	n
C.I.	5,600	$\frac{1}{1600}$	$\frac{1}{6400}$	$\frac{1}{3200}$	$\frac{1}{n \times 1600}$
M.S	3,300	$\frac{1}{7500}$	$\frac{1}{30000}$	$\frac{1}{15000}$	$\frac{1}{n \times 7500}$
Dry timber	500	$\frac{1}{750}$	$\frac{1}{3000}$	$\frac{1}{1500}$	$\frac{1}{n \times 750}$

When Rankine's formula is expressed in the form

$$P = \frac{f_c A}{1 + \alpha \left(\frac{l}{d}\right)^2}$$

where d is the diameter or least width of the section and α, another constant, the formula is known as Gordon's formula.

The following table is the approximate guide for the application of suitable formula for the design of struts and columns in various machines, when slenderness ratio is known.

Formula to be used	Slenderness ratio
Compression formula	Upto 40
Rankine's formula	40 — 120
Euler's formula	Greater than 120

10-6. Tetmajer's formula:

As a result of numerous experiments on columns of different materials, Tetmajer suggested the following general expression for the buckling load P on hinged columns.

$$P = Af_c\,(1 - a\lambda + b\lambda^2)\ \text{kg}$$

where A is the area of cross section in sq cm, f_c is stress intensity at yield point in compression kg/sq cm, λ is the slenderness ratio and a and b are constants.

The above relation is a parabolic one. The value of the constant b vanishes for all materials except for cast iron and hence, the relation between the buckling load and the slenderness ratio becomes linear one.

The following table gives the useful information for application of Tetmajer's formula:

Material	f_c kg/sq cm	E kg/sq cm	λ maximum value	Tetmajer's expression $f_c\,(1 - a\lambda + b\lambda^2)$ kg/sq cm
Cast iron	7,760	10×10^5	80	$7760 - 120\lambda + 0.53\lambda^2$
Mild steel	3,100	21×10^5	105	$3100 - 11.4\lambda$
Medium steel	3,350	22×10^5	89	$3350 - 6.2\lambda$
Nickel steel	4,700	21×10^5	86	$4700 - 23.0\lambda$
Timber	293	1×10^5	100	$293 - 1.94\lambda$
Wrought iron	3,030	20×10^5	112	$3030 - 12.9\lambda$

If the value of the slenderness ratio exceeds that given in above table, Euler's formula is to be used. Mostly Tetmajer's formula is used for checking the stresses. The use of Tetmajer's formula is more prevalent in European countries, where metric system is used. According to ten Bosch the factor of safety 3·5 to 5 is sufficient for Tetmajer's formula.

Examples:

1. The eccentric rod to drive the valve mechanism of a steam engine carries a maximum compressive load of 1,150 kg, the length of the rod being 150 cm. Assuming the eccentric rod to be a hinged at both ends, determine the diameter of the rod at the middle. If the section of the rod were to be rectangular, what would be the dimensions of the cross section if the depth of the section is 1·75 times the thickness?

Take the factor of safety to be 40. Modulus of elasticity is $2\cdot 1 \times 10^6$ kg/sq cm.

The eccentric rod carries a maximum compressive load of 1,150 kg. The factor of safety is 40. Therefore, the strut is to be designed for a buckling load of 1150 × 40 = 46,000 kg. From the data, Euler's formula is suggested.

$$\text{Buckling load} = \frac{\pi^2 EI}{l^2}$$

$$46000 = \frac{\pi^2 \times 2\cdot 1 \times 10^6 \times I}{150^2}$$

or $$I = 50 \text{ cm}^4.$$

If d cm be the diameter of the solid rod, then

$$\frac{\pi}{64} d^4 = 50$$

or $$d = \sqrt[4]{\frac{50 \times 64}{\pi}} = 5\cdot 65 \text{ cm, we adopt 6 cm.}$$

If h be the depth of the rectangular section, then

$h = 1\cdot 75\,w$, where w is the width of the section.

$$\therefore \quad I = \frac{1}{12} w h^3.$$

$$\therefore \quad 50 = \frac{1}{12} \times w \times (1\cdot 75 w)^3$$

or $w = 3\cdot 24$ cm; we adopt 4 cm.

Depth of the section will be $4 \times 1\cdot 75 = 7$ cm.

2. In a certain water works installation the water is pumped against a head of 165 metre. The bore of the reciprocating pump is 45 cm. The

unsupported length of the piston rod is 140 cm. Determine the diameter of the piston rod by using Rankine's formula, taking the factor of safety to be 10. Take $f_c = 3{,}300$ kg/sq cm and Rankine constant $a = \frac{1}{17500}$.

Water is pumped against a head of 165 metre. This head of water is equivalent to $\frac{165 \times 100}{1000} = 16{\cdot}5$ kg/sq cm.

Maximum compressive load on piston rod $= \frac{\pi}{4} \times 45^2 \times 16{\cdot}5$
$= 26{,}260$ kg.

As the factor of safety is 10, the buckling load on the column will be $26260 \times 10 = 262{,}600$ kg.

If d cm be the diameter of the solid rod, by using Rankine's formula, we get

$$262600 = \frac{3300 \times \frac{\pi d^2}{4}}{1 + \frac{1}{17500}\left[\frac{140 \times 4}{d}\right]^2}.$$

The above equation after simplification gives us an equation $d^4 - 101{\cdot}5d^2 - 1850 = 0$, which is a quadratic in d^2.

After solving, we get $d = 10{\cdot}9$ cm; we adopt $d = 12$ cm.

3. *The rectangular steel link of a toggle type screw jack (see fig. 11-10) of 1 tonne capacity is subjected to a maximum compressive load of 360 kg. Determine the cross sectional dimensions of the link, which is 45 cm long, from the following data:*

Width three times the thickness; factor of safety 5; $f_c = 3{,}300$ kg/sq cm.

Rankine constant $a = \frac{1}{7500}$.

The maximum compressive load on the link $= 360$ kg. As the factor of safety is 5, the link should be designed for a buckling load of $360 \times 5 = 1{,}800$ kg.

Let t be the thickness of the link, then width of the link will be $3t$. Area of the cross section of the link will be $3t^2$.

Assuming that the buckling of the link takes place in the plane of the link, the second moment of area of the link will be equal to

$$\tfrac{1}{12} \times t \times (3t)^3 = 2{\cdot}25t^4.$$

$$\therefore \qquad k^2 = \frac{2{\cdot}25t^4}{3t^2} = 0{\cdot}75t^2.$$

If the value of the slenderness ratio exceeds that given in above table, Euler's formula is to be used. Mostly Tetmajer's formula is used for checking the stresses. The use of Tetmajer's formula is more prevalent in European countries, where metric system is used. According to ten Bosch the factor of safety 3·5 to 5 is sufficient for Tetmajer's formula.

Examples:

1. The eccentric rod to drive the valve mechanism of a steam engine carries a maximum compressive load of 1,150 kg, the length of the rod being 150 cm. Assuming the eccentric rod to be a hinged at both ends, determine the diameter of the rod at the middle. If the section of the rod were to be rectangular, what would be the dimensions of the cross section if the depth of the section is 1·75 times the thickness?

Take the factor of safety to be 40. Modulus of elasticity is $2 \cdot 1 \times 10^6$ kg/sq cm.

The eccentric rod carries a maximum compressive load of 1,150 kg. The factor of safety is 40. Therefore, the strut is to be designed for a buckling load of $1150 \times 40 = 46{,}000$ kg. From the data, Euler's formula is suggested.

$$\text{Buckling load} = \frac{\pi^2 EI}{l^2}$$

$$46000 = \frac{\pi^2 \times 2 \cdot 1 \times 10^6 \times I}{150^2}$$

or $$I = 50 \text{ cm}^4$$

If d cm be the diameter of the solid rod, then

$$\frac{\pi}{64} d^4 = 50$$

or $$d = \sqrt[4]{\frac{50 \times 64}{\pi}} = 5 \cdot 65 \text{ cm, we adopt 6 cm.}$$

If h be the depth of the rectangular section, then

$h = 1 \cdot 75x$, where x is the width of the section.

$$\therefore \quad I = \frac{1}{12} x h^3$$

$$\therefore \quad 50 = \frac{1}{12} \times x \times (1 \cdot 75 x)^3$$

or $x = 3 \cdot 21$ cm, we adopt 4 cm.

Depth of the section will be $4 \times 1 \cdot 75 = 7$ cm.

2. In a system a water [illegible] the water is pumped against a head of 16·5 metre. The bore of the reciprocating pump is 45 cm. The

where $n = \dfrac{\text{length of connecting rod}}{\text{length of crank}}$

r = length of crank

w = weight of reciprocating parts per sq cm of piston area

ω = angular velocity of the crank in radian/sec.

w = 0·028 kg/sq cm of piston area.

$$n = \frac{0{\cdot}30}{0{\cdot}0625} = 4{\cdot}8 \qquad r = \frac{0{\cdot}125}{2} = 0{\cdot}0625 \text{ metre.}$$

$$\omega = \frac{1500 \times 2\pi}{60} = 157 \text{ radian/sec.}$$

∴ Inertia pressure intensity at the beginning of the stroke

$$= \frac{0{\cdot}028}{9{\cdot}81} \times 0{\cdot}0625 \times 157^2 \left[1 + \frac{1}{4{\cdot}8}\right] = 5{\cdot}3 \text{ kg/sq cm.}$$

At the beginning of the power stroke, resulting load on the piston will be obtained by subtracting inertia load from the explosion load.

Load on piston $= \dfrac{\pi}{4} \times 10^2(25 - 5{\cdot}3) = 1{,}545$ kg. As the factor of safety is 7, the connecting rod will be designed for a buckling load of $7 \times 1545 = 10{,}800$ kg.

Let t be the thickness of the flange and web, width of flange will be $4t$ and height of the section will be $5t$. Area of the section will be $= 11t^2$.

Second moment of area for buckling in plane of the rod $= 35t^4$.

$$\therefore \qquad k^2 = \frac{35t^4}{11t^2} = 3{\cdot}18t^2.$$

By Rankine's formula

$$10800 = \frac{3300 \times 11t^2}{1 + \dfrac{1}{7500} \times \dfrac{(30)^2}{3{\cdot}18t^2}}.$$

By solving, we get $t = 0{\cdot}58$ cm; we adopt 6 mm.

Width of the flange $= 4t = 4 \times 6 = 24$ mm.

Height of the section $= 5t = 5 \times 6 = 30$ mm.

5. *Determine the diameter of the connecting rod, of 150 cm length, subjected to an axial compressive load of 18,000 kg, taking it to be freely hinged at the ends. Take the factor of safety to be 7.* $E = 21{\cdot}5 \times 10^5$ *kg/sq cm.*

Buckling load $= \dfrac{f_c A}{1 + a\left(\dfrac{l}{k}\right)^2}$

$$\therefore \qquad 1800 = \frac{3300 \times 3t^2}{1 + \dfrac{1}{7500} \times \dfrac{2025}{0{\cdot}75t^2}}.$$

On solving the above equation, we get $t = 0{\cdot}61$ cm; we adopt 7 mm. Width of the link $= 3 \times 7 = 21$ mm.

When the buckling of the link takes place in the plane perpendicular to the plane of the link, the second moment of area will be $= \frac{1}{12} \times 2{\cdot}1 \times 0{\cdot}7^3 = 0{\cdot}06$ cm^4.

Area of the section $= 2{\cdot}1 \times 0{\cdot}7 = 1{\cdot}47$ sq cm.

$k^2 = \dfrac{I}{A} = \dfrac{0{\cdot}06}{1{\cdot}47} = 0{\cdot}0408$ cm^2.

For buckling in the plane at right angles to the plane of the link, the link behaves as a fixed ended strut, the Rankine constant will be $\dfrac{1}{30000}$.

Buckling load $= \dfrac{1{\cdot}47 \times 3300}{1 + \dfrac{1}{30000} \times \dfrac{45^2}{0{\cdot}0408}} = 1{,}830$ kg

As the calculated load is more than 1,800 kg, the link is safe for buckling in plane at right angles to the plane of the link.

Note: The economical section for equal strength in buckling in both the planes could be obtained by adopting proportions $w : t :: 2 : 1$

4. *Determine the dimensions of I section connecting rod for a petrol engine from the following data:*

Diameter of the piston = 10 cm
Weight of reciprocating parts 0·028 kg/sq cm of piston area
Length of the connecting rod = 30 cm
R.P.M. of engine = 1,500
Maximum explosion pressure = 25 kg/sq cm
Stroke length = 12·5 cm.

The proportions for the I section may be taken as 30 mm × 24 mm with 6 mm web and 6 mm flange. Take the factor of safety as 7.

The maximum value of inertia pressure will be at the beginning of the stroke and is equal to $\dfrac{w}{g} r\omega^2 \left[1 + \dfrac{1}{n}\right]$ kg/sq cm,

$$\text{Allowable stress} = \frac{3600}{2 \cdot 5} = 1{,}440 \text{ kg/sq cm.}$$

Let d cm be the diameter of the rod. According to Rankine's formula we get

$$7000 = \frac{1440 \times \frac{\pi}{4} d^2}{1 + \frac{1}{7500} \times \left(\frac{200}{d}\right)^2}.$$

From the above equation we get d as 3·12 cm; we adopt 3·2 cm.

As the slenderness ratio is $\frac{50 \times 4}{3 \cdot 2} = 62 \cdot 6$, the use of Rankine's formula is justified.

Let us consider the effect of variable loading. As the ultimate strength is 5600 kg/sq cm, the endurance limit for reversed bending is $0 \cdot 5 \times 5600 = 2{,}800$ kg/sq cm.

The equivalent normal stress due to variable loading with a maximum stress of 1,440 kg/sq cm (in fact it will be little less), a mean stress will be $\frac{1440}{2} = 720$ kg/sq cm and a variable stress of 720 kg/sq cm, will be

$$f_{en} = f_a + \frac{f_y f_m K}{f_e \times ABC}$$

$$= 720 + \frac{3600}{2800} \times \frac{720 \times 1 \cdot 9}{0 \cdot 7 \times 0 \cdot 85 \times 1}$$

where $A = 0 \cdot 7$ for axial loading

$B = 0 \cdot 85$ for size effect

$C = 1$ since actual stress concentration factor is used.

$\therefore$ Equivalent normal stress $= 3{,}490$ kg/sq cm

Thus the actual design factor according to Soderberg will be $\frac{3600}{3490} = 1 \cdot 03$ if the diameter were to be 3·2 cm. This design factor is inadequate hence we should increase the diameter.

Let us find out the diameter according to Soderberg criterion, keeping in mind the effect of buckling. As the design factor is 2·5, we design for a design factor of 4 to account for buckling.

Let d cm be the diameter of the column. As the load varies from zero to 7,000 kg, the mean stress as well as the variable stress

Let us use Euler's formula. Let d cm be the diameter of the connecting rod.

$$\therefore \quad 18000 \times 7 = \frac{\pi^2 \times \frac{\pi}{64} d^4 \times 21{\cdot}5 \times 10^5}{150^2}.$$

$$\therefore \quad d = 7{\cdot}3 \text{ cm}.$$

Slenderness ratio $\lambda = \dfrac{150 \times 4}{7{\cdot}3} = 82$. As the slenderness ratio is 82, Euler's formula cannot be used. Therefore, we check the diameter of the rod by Tetmajer's formula.

Yield stress $f_c = 3100 - 11{\cdot}4\lambda = 3100 - 11{\cdot}4 \times 82 = 2{,}165$ kg/sq cm. Factor of safety $= 7$.

$\therefore$ Working stress $= \dfrac{2165}{7} = 309$ kg/sq cm.

Stress induced $= \dfrac{18000}{\frac{\pi}{4} \times 7{\cdot}3^2} = 430$ kg/sq cm.

As induced stress is more than the working stress, the design is not safe. We increase the diameter of the connecting rod to 9 cm.

New slenderness ratio $= \dfrac{150 \times 4}{9} = 66{\cdot}7$.

New value of the permissible stress $= \frac{1}{7}\,[3100 - 11{\cdot}4 \times 66{\cdot}7]$
$= 334$ kg/sq cm.

New value of the stress induced $= \dfrac{18000}{\frac{\pi}{4} \times 9^2} = 282$ kg/sq cm.

As induced stress is less than the working stress, the design is safe. We adopt 9 cm as the diameter of the connecting rod.

Note. This illustrative example explains the use of Tetmajer's formula

6. The link which is 50 cm in length is to be designed for a design factor of 2·5 and is to support an axial compressive load that varies from zero to 7,000 kg. Determine the diameter of the link considering buckling only. Determine the diameter considering varying stresses and using the Soderberg line.

The link is made of AISI 1030 for which the ultimate strength is 5,600 kg/sq cm and yield stress is 3,600 kg/sq cm. Take end fixity coefficient as 1. Take stress concentration factor as 1·9.

We apply Rankine's formula.

$$\text{Allowable stress} = \frac{3600}{2 \cdot 5} = 1{,}440 \text{ kg/sq cm.}$$

Let d cm be the diameter of the rod. According to Rankine's formula we get

$$7000 = \frac{1440 \times \frac{\pi}{4} d^2}{1 + \frac{1}{7500} \times \left(\frac{200}{d}\right)^2}.$$

From the above equation we get d as 3·12 cm; we adopt 3·2 cm.

As the slenderness ratio is $\frac{50 \times 4}{3 \cdot 2} = 62 \cdot 6$, the use of Rankine's formula is justified.

Let us consider the effect of variable loading. As the ultimate strength is 5600 kg/sq cm, the endurance limit for reversed bending is $0 \cdot 5 \times 5600 = 2{,}800$ kg/sq cm.

The equivalent normal stress due to variable loading with a maximum stress of 1,440 kg/sq cm (in fact it will be little less), a mean stress will be $\frac{1440}{2} = 720$ kg/sq cm and a variable stress of 720 kg/sq cm, will be

$$f_{en} = f_a + \frac{f_y\, f_m\, K}{f_e \times ABC}$$

$$= 720 + \frac{3600}{2800} \times \frac{720 \times 1 \cdot 9}{0 \cdot 7 \times 0 \cdot 85 \times 1}$$

where $A = 0 \cdot 7$ for axial loading

$B = 0 \cdot 85$ for size effect

$C = 1$ since actual stress concentration factor is used.

$\therefore$ Equivalent normal stress $= 3{,}490$ kg/sq cm

Thus the actual design factor according to Soderberg will be $\frac{3600}{3490} = 1 \cdot 03$ if the diameter were to be 3·2 cm. This design factor is inadequate hence we should increase the diameter.

Let us find out the diameter according to Soderberg criterion, keeping in mind the effect of buckling. As the design factor is 2·5, we design for a design factor of 4 to account for buckling.

Let d cm be the diameter of the column. As the load varies from zero to 7,000 kg, the mean stress as well as the variable stress

will be $\frac{7000}{\frac{\pi}{4}d^2} = \frac{8900}{d^2}$ kg/sq cm.

According to Soderberg equation we get

$$\frac{1}{4} = \frac{8900}{d^2}\left[\frac{1}{3600} + \frac{1 \cdot 9}{2800 \times 0 \cdot 7 \times 0 \cdot 85 \times 1}\right]$$

$$900 = \frac{8900}{d^2}\left[1 + \frac{1 \cdot 9 \times 3600}{2800 \times 0 \cdot 595}\right]$$

or $$d = \sqrt{\frac{8900 \times 5 \cdot 12}{900}} = 7 \cdot 15 \text{ cm.}$$

We adopt 7·2 cm.

Now we see that the slenderness ratio is $\frac{50 \times 4}{7 \cdot 2} = 27 \cdot 8$ and we can reduce the diameter to 6·5 cm and we shall see that the design factor of 2 5 will be maintained

Exercises:

1. Determine the diameter of a compressive link of a valve mechanism which may be considered a column 50 cm long. The estimated axial load is 366 kg.

Assume the following

Factor of safety 2 5, end fixity coefficient 3 0 and modulus of elasticity 2·1 × 10⁶ kg /sq cm

Ans 1 cm diameter

2. Determine the diameter of the piston rod of the hydraulic cylinder of 10 cm bore when the maximum hydraulic pressure in the cylinder is limited to 140 kg/sq cm The length of the piston rod is 122 cm. The end fixity coefficient may be taken as 2 Factor of safety should be taken as 5. Ans. 4 8 cm diameter

3. A part of a nail heading machine is to be circular in cross section and is to carry a load of 2,730 kg. The length of the part is 20 cm. Assuming the permissible stress to be 1,470 kg/sq cm, determine the diameter of the column, taking it to be freely hinged at the ends, by using Rankine's formula. Ans 2 cm.

4. The piston rod of a steam engine may be considered a column having the end restraint coefficient as 3 5. Determine the diameter of the piston rod from the following data:

Diameter of the cylinder 45 cm
Maximum net pressure of steam on piston 5·6 kg/sq cm
Distance from piston to crosshead 150 cm
Factor of safety 4
Elastic limit stress 2,800 kg/sq cm.

Ans. 6 cm diameter.

5. *The dump cylinder piston rod of an excavator carries a negligible load when lifting and dumping but carries a maximum tensile load and compressive load of 27,000 kg during bull — and back dozing respectively. The unsupported length of piston rod is 75 cm. Determine the minimum diameter of the piston rod when the permissible stress intensity is limited to 1,050 kg/sq cm. The end-fixity coefficient may be taken as 2.*

Ans. 6 cm.

6. *Design a connecting rod dimensions at mid length for a petrol engine from the following data:*

Diameter of piston 9 cm

Length of connecting rod 30 cm

The maximum explosion pressure 20 kg/sq cm

The rod is of I section, of width 4t and depth equal to 5t where t is the thickness of a web and flanges.

Take $f_c = 3{,}300$ *kg/sq cm, factor of safety 5 and constant a in Rankine's formula* $= \dfrac{1}{7500}$.

Ans. $t = 5$ *mm;* $w = 20$ *mm;* $h = 25$ *mm.*

7. *The connecting rod of a Diesel engine has to withstand a maximum load of 96,200 kg. Using Tetmajer's relation suggest the suitable dimension for (a) circular section and (b) rectangular section having ratio of sides as 1·8. Factor of safety 12.* $E = 2{\cdot}2 \times 10^6$ *kg/sq cm. Assume that it is freely hinged at the ends.*

Ans. 22 cm; 27 × 15 cm.

8. *Determine the dimensions of the cross section of a rectangular connecting rod for a double acting steam engine 50 cm diameter and [illegible] stroke. The steam pressure is 17·5 kg/sq cm. Length of the conn. rod 150 cm. Assume the material of the connecting rod to be mild st[eel]. Take the factor of safety to be 12.* *Ans.* $t = 8{\cdot}5$ *cm;* $w = 17$ *cm.*

9. *The piston rod of a reciprocating pump is subjected to a maximum axial compressive load of 29,000 kg. The length of the piston rod*

will be $\dfrac{7000}{\frac{\pi}{4}d^2} = \dfrac{8900}{d^2}$ kg/sq cm.

According to Soderberg equation we get

$$\frac{1}{4} = \frac{8900}{d^2}\left[\frac{1}{3600} + \frac{1 \cdot 9}{2800 \times 0 \cdot 7 \times 0 \cdot 85 \times 1}\right]$$

$$900 = \frac{8900}{d^2}\left[1 + \frac{1 \cdot 9 \times 3600}{2800 \times 0 \cdot 595}\right]$$

or $$d = \sqrt{\frac{8900 \times 5 \cdot 12}{900}} = 7 \cdot 15 \text{ cm.}$$

We adopt 7·2 cm.

Now we see that the slenderness ratio is $\dfrac{50 \times 4}{7 \cdot 2} = 27 \cdot 8$ and we can reduce the diameter to 6·5 cm and we shall see that the design factor of 2·5 will be maintained.

Exercises:

1. Determine the diameter of a compressive link of a valve mechanism which may be considered a column 50 cm long. The estimated axial load is 366 kg.

Assume the following.

Factor of safety 2·5; end fixity coefficient 3·0 and modulus of elasticity $2 \cdot 1 \times 10^6$ kg/sq cm.

Ans. 1 cm diameter

2. Determine the diameter of the piston rod of the hydraulic cylinder of 10 cm bore when the maximum hydraulic pressure in the cylinder is limited to 140 kg/sq cm. The length of the piston rod is 122 cm. The end fixity coefficient may be taken as 2. Factor of safety should be taken as 5. *Ans. 4·8 cm diameter*

3. A part of a rail heading machine is to be circular in cross section and is to carry a load of 2,730 kg. The length of the part is 20 cm. Assuming the permissible stress to be 1,470 kg/sq cm, determine the diameter of the column, taking it to be freely hinged at the ends, by using Rankine's formula. *Ans. 2 cm.*

4. The piston rod of a steam engine may be considered a column having the end restraint coefficient as 3·5. Determine the diameter of the piston rod from the following data:

r = radius of gyration of the section
l = length of the column
E = modulus of elasticity.

The above formula is known as the *secant formula.* Problems exist in machine design in which there is deliberate eccentricity. This formula has the advantage that it applies to all column lengths. In design, however, it is difficult to employ because one must know all the dimensions of the column before the formula can be applied.

EXAMPLES X

1. A piston rod for an air cylinder is to be designed for an axial load of [illegible],700 kg. The rod when extended has a length of 50 cm. Although one end of the rod is fastened more or less rigidly to the piston and the other end of the rod is pinned to a member which is constrained in a guide, the value of end fixity coefficient may be taken as 1. Determine the size of the rod to be used for a factor of safety of 2·5 with a material having a yield point of 2,800 kg/sq cm.

Ans. 24 mm.

2. A steam engine has a cylinder diameter of 27·5 cm and the maximum net steam pressure of 8·7 kg/sq cm. The actual distance from the piston to the crosshead centre is 65 cm. Determine the diameter of the piston rod assuming that the yield point stress is 3,600 kg/sq cm. Rankine constant $= \frac{1}{30000}$ and factor of safety 1·5. State the assumptions made in design.

Ans. 33 mm.

3. The following data refer to an internal combustion engine:

Diameter of the piston	68 mm
Weight of reciprocating parts	1·6 kg
Length of connecting rod	30 cm
Stroke	125 mm
Maximum explosion pressure	35 kg/sq cm.

Suggest the suitable cross sectional dimensions for I section connecting rod at its mid length if the factor of safety is taken as 5.

Take f_c = 3,200 kg/sq cm. Ans. 6 mm × 24 mm × 30 mm.

4. The connecting rod of an internal combustion engine is 100 cm long and is subjected to an axial load of 2,000 kg. It can be considered as a strut with the ends free to turn on the crank pin and the gudgeon pin. However, it may be considered as a fixed strut in the direction of the axis of these pins.

Calculate the dimensions of the rectangular sectioned connecting rod, for equal strength against buckling in both the planes. Factor of safety may be taken as [illegible]

Ans. Thickness 2 cm; width 4 cm.

is 140 cm. Assuming the material of the piston rod to be mild steel, determine the diameter of the piston rod, taking it to be freely hinged at the ends. Take the factor of safety to be 10. Ans. 12 cm.

10. The eccentric rod of a horizontal steam engine is subjected to an axial compressive load of 1,500 kg. The length of the eccentric rod is 140 cm. Assuming the factor of safety to be 25, determine the cross section of the eccentric rod at the middle. Take $E = 2{\cdot}1 \times 10^6$ *kg/sq cm.*
Ans. 0·8 cm × 1·5 cm.

11. A water tank of 12 cu metre capacity is supported on four columns of I section. The weight of the tank when empty is 3,600 kg. The height of the column is 350 cm. Suggest the minimum value of the second moment of area of the section. Assume factor of safety to be 5. $E = 2{\cdot}1 \times 10^6$ *kg/sq cm.* Ans. 114 cm^4.

12. The maximum thrust in a connecting rod is 10,000 kg. The length of the rod is 90 cm between centres. Yield point stress is 3,200 kg/sq cm. Taking a factor of safety of four based on the yield stress and the Rankine constant for hinged ends as $\frac{1}{7500}$, *determine the diameter of the rod.*

Why is an I section usually preferred to a round section in the case of connecting rods? Ans. 5·5 cm.

13. Determine the diameter for the piston rod of the hydraulic cylinder 10 cm in diameter. The maximum pressure of fluid in the cylinder is 150 kg/sq cm. The length of the rod is 80 cm. The permissible stress is 400 kg/sq cm. The end fixity coefficient is 3. Ans. 6·5 cm

14. What value of the slenderness ratio would result in the same safe load from the Euler's equation as well as that from Rankine's equation for a design factor of 3?

15. A link of rectangular cross section is likely to buckle in two mutually perpendicular planes, the effective length for buckling in one plane being 1·4 times the length in the other plane. What should be the relation between the width and the thickness of the link for the same resistance to buckling in each plane? Ans. Width = 1·4 times the thickness.

16. The connecting rod of an engine is a strut with the ends free to turn on the crank pin and the gudgeon pin. In the direction of the axis of these pins, however, the strut may be considered to approach fixed-end condition, the end fixity coefficient being 3. Determine the ratio of the depth to the thickness of a rectangular connecting rod if the rod were to be equally strong in both the planes of buckling. Ans. 1·732.

(a) Determine the diameter of cylinder bore required, assuming that overall friction due to stuffing box and piston packing is equivalent to 10% of maximum force.

(b) Determine the thickness of a cylinder, assuming that it is seamless steel tubing. The allowable tensile stress is 200 kg/sq cm.

(c) Determine the diameter of the piston rod of mild steel having length of 75 cm. Its upper end is pin connected to the operating lever and the piston end of the rod is fixed ended. The factor of safety may be taken as 2·5. Ans. (1) 21 cm; (b) 0·8 cm; (c) 2·2 cm.

9. A locomotive coupling rod is of I cross section. The maximum thrust in the rod is 10 tonnes. The rod is 250 cm between centres. Neglecting friction at the pins, if the maximum intensity of stress in the rod equals 10 kg/sq mm, determine the thickness of the web and flanges, assuming depth as 10 cm.

Take $E = 2{\cdot}1 \times 10^6$ kg/sq cm.

10. A push rod 45 cm long is made of mild steel tube. One end is connected to the valve lever by a pin joint with the forked end on the lever. The other end carries a roller of 3 cm diameter and 15 mm wide. The maximum valve load is 450 kg. Assume a factor of safety 4.

Design and prepare a dimensioned drawing of the push rod with the end connections.

Permissible bearing pressure on pins 100 kg/sq cm

Permissible tensile stress in pins 850 kg/sq cm

Permissible compressive stress in tube 900 kg/sq cm

5. Determine the diameter for the piston rod of the hydraulic cylinder shown in fig. 3-7. The rod is to be a round steel forging. The cylinder is 8 cm in diameter and the maximum hydraulic pressure is 200 kg/sq cm.

The following assumptions should be made:

(i) The rod will not fail by any method other than column action. The length of the rod is 105 cm.

(ii) *The connecting rod is fixed in one plane and pin ended in the plane* at right angles to the plane of the figure.

(iii) The piston end of the rod is fixed in all planes

(iv) To account for eccentricities and load variations, design the rod for a maximum load of three times the actual hydraulic load.

The permissible stress may be taken as 1,500 kg/sq cm.

Ans [redacted] cm

6. A petrol engine has a bore of 6" (15 cm) and the stroke — bore ratio [redacted] 1:1. The estimated maximum pressure in the cylinder [redacted] 450 psi (31.5 kg/sq cm) and the engine runs at 1,100 r.p.m. The ratio of the connecting rod length to the crank radius is 5. Design the section of the connecting rod, assuming it is of mild steel and having the cross section as shown in fig. 10-6. Take a factor of safety 8 and $E = 30 \times 10^6$ psi (2.1×10^6 kg/sq cm)

(M. S. University of Baroda, 1959)

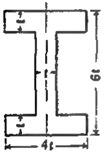


Fig. 10-6

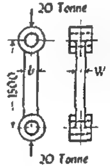


Fig. 10-7

7. *A steel strut, 1,500 mm long as shown in fig. 10-7 is rectangular in cross* section. The compressive load to be carried is 20 tonne. Determine the following:

(a) the relation between *b* and *W* required for equal strengths of the strut in failure by buckling in either plane,

(b) the required dimensions *b* and *W*, assuming a factor of safety 4, $E = 2{\cdot}1 \times 10^6$ kg/sq cm and elastic limit of steel as 2,500 kg/sq cm.

Ans [redacted] cm; 10 cm.

8. The force analysis of a 7 tonne air operated arbor press shows that the piston rod for the operating cylinder must exert a maximum force of 2,000 kg. The air pressure in the cylinder is 7 kg/sq cm.

(a) Determine the diameter of cylinder bore required, assuming that overall friction due to stuffing box and piston packing is equivalent to 10% of maximum force.

(b) Determine the thickness of a cylinder, assuming that it is seamless steel tubing. The allowable tensile stress is 200 kg/sq cm.

(c) Determine the diameter of the piston rod of mild steel having length of 75 cm. Its upper end is pin connected to the operating lever and the piston end of the rod is fixed ended. The factor of safety may be taken as 2·5. Ans. (1) 21 cm; (b) 0·8 cm; (c) 2·2 cm.

9. A locomotive coupling rod is of I cross section. The maximum thrust in the rod is 10 tonnes. The rod is 250 cm between centres. Neglecting friction at the pins, if the maximum intensity of stress in the rod equals 10 kg/sq mm, determine the thickness of the web and flanges, assuming depth as 10 cm.

Take $E = 2{\cdot}1 \times 10^6$ kg/sq cm.

10. A push rod 45 cm long is made of mild steel tube. One end is connected to the valve lever by a pin joint with the forked end on the lever. The other end carries a roller of 3 cm diameter and 15 mm wide. The maximum valve load is 450 kg. Assume a factor of safety 4.

Design and prepare a dimensioned drawing of the push rod with the end connections.

Permissible bearing pressure on pins 100 kg/sq cm

Permissible tensile stress in pins 850 kg/sq cm

Permissible compressive stress in tube 300 kg/sq cm.

Ans. 17 mm outside diameter; 13 mm inside diameter.

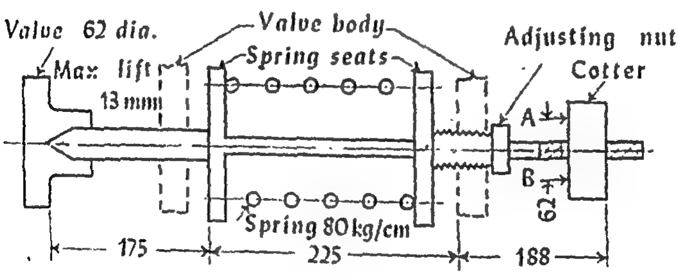


Fig. 10-8

11. Fig. 10-8 shows a valve spindle of a spring loaded safety valve. The diameter of the valve is 62 mm and the valve is adjusted to lift at 17 kg/sq cm. The maximum lift of the valve is 13 mm. A handle (not shown in the figure) applies the force through the cotter for the purposes of lifting the valve by hand. The spring scale of the spring is 80 kg/cm.

If the spindle is made of phosphor bronze, design and prepare a dimensioned drawing of the spindle.

13. Fig. 10-9 shows the valve gear arrangement of a Diesel engine. Pin at *F* for the follower and the pin *G* for the roller are supported only on one side. Pin at *E* for the tappet lever is supported on both the sides. The maximum force exerted by the spring is 68 lb (30 kg). Design and draw the following

(a) tappet lever and the push rod

or

(a) oscillating follower and the push rod

Assume the suitable materials and stresses. (Gujarat University, 1956)

14. Vibration is used as a means of transporting material, such as sand, in a conveyor system. The frame of the conveyor is vibrated by means of a connecting rod and crankshaft. The speed of the crank is 200 r.p.m. and crank length is 5 cm. The connecting rod is 60 cm long and the frame when completely loaded weighs 3,800 kg. The stress allowed in the connecting rod is 560 kg/sq cm. Determine the forces at inner — and outer dead centres and the diameter of the rod. Ans 9,000 kg, 8,170 kg; 5 5 cm.

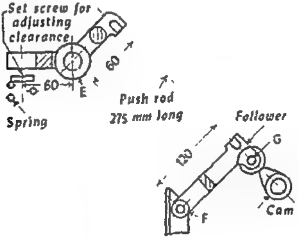


Fig. 10-9

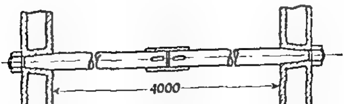


Fig. 10-10

15. A combined pump and piston rod for a direct-acting pumping engine is shown in fig 10-10. The rod is subjected to a load of 30 tonnes alternately tensile and compressive, and is in two parts connected by a sleeve and cotters. Design the rod and cottered joints for the following conditions

$f_t = 6$ kg/sq mm; $f_s = 4{\cdot}5$ kg/sq mm; $f_b = 10{\cdot}5$ kg/sq mm. The diameter of the rod must be determined by Rankine's formula for hinged joints, allowing a factor of safety 4. The unsupported length of the rod may be taken as 360 cm.

Ans. 15 cm.

16. A three-throw single-acting reciprocating pump is to deliver 25,000 litres per hour against a suction head of 3 metre and delivery head of 60 metre. The mean plunger velocity is to be about 30 metre per minute and the stroke is to be 1·25 times the bore. Assuming delivery volume to be 80 per cent of swept volume, calculate bore and stroke dimensions and r.p.m.

Assuming pressures are constant during the stroke, deduce the maximum force in each connecting rod, each rod being four times crank length.

Calculate a minimum cross-section for the connecting rod (assuming uniform) its proportions being taken as those of an I section $10t \times 6t$ with t web and t flanges, assuming a maximum compressive stress of 430 kg/sq cm. Calculate, also, a suitable diameter for the connecting rod cap bolts at the crank pin allowing 300 kg/sq cm as permissible tensile stress.

17. A mechanical clutch is remotely operated by a steel rod of rectangular section, which is equally strong in both the planes of buckling and is 120 cm long between the knuckle-jointed ends. The rod axis and the knuckle pin are horizontal and the rod is un-supported except at the ends. Find out the dimensions of the cross section of the rod for carrying a compressive load of 220 kg. The allowable stress for the material of the rod is 800 kg/sq cm.

18. A connecting rod 100 cm long may be considered a strut with the ends free to turn on the crank pin and the gudgeon pin. In the direction of the axes of these pins, however, it may be considered as having fixed ends. The connecting rod is subjected to an axial thrust of 1,500 kg. Suggest the suitable dimensions of rectangular sectioned connecting rod. Safe stress may be taken as 980 kg/sq cm. What will be the value of the tensile load that can be sustained by the connecting rod if the above given value of the safe stress is not to be exceeded?

(Sardar Vallabhbhai Vidyapeeth, 1965)

19. The following particulars relate to a two cylinder locomotive, in whose cylinders expansion is simple:

Cylinder bore— 45·72 cm

Stroke— 66·04 cm

Working pressure— 14·04 kg per sq cm, by gauge

Length of connecting rod between centres—1·651 m

Find (a) size of gudgeon pin and crank pin bearings, if length of pins is 1·25 times the diameter and if permissible bearing pressures in these are respectively 210·6 and 105·3 kg per sq cm;

(b) breadth and depth of rectangular cross-section connecting rod for equal strength in the plane of operation and in a plane transverse to it. Yield stress for material of rod is 4,212 kg per sq cm and assume

13. Fig. 10-9 shows the valve gear arrangement of a Diesel engine. Pin at F for the follower and the pin G for the roller are supported only on one side. Pin at E for the tappet lever is supported on both the sides. The maximum force exerted by the spring is 63 lb (30 kg). Design and draw the following:

(a) tappet lever and the push rod

or

(a) oscillating follower and the push rod

Assume the suitable materials and stresses (Gujarat University, 1956)

14. Vibration is used as a means of transporting material, such as sand, in a conveyor system. The frame of the conveyor is vibrated by means of a connecting rod and crankshaft. The speed of the crank is 200 r.p.m. and crank length is 5 cm. The connecting rod is 60 cm long and the frame when completely loaded weighs 3,700 kg. The stress allowed in the connecting rod is 560 kg/sq cm. Determine the forces at inner — and outer dead centres and the diameter of the rod. Ans. 9,000 kg, 8,170 kg; 5.5 cm.

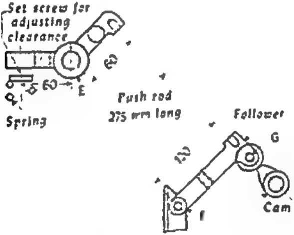


Fig. 10-9

Fig. 10-10

15. A combined pump and piston rod for a direct-acting pumping engine is shown in Fig. 10-10. The rod is subjected to a load of 30 tonnes alternately tensile and compressive, and is in two parts connected by a sleeve and cotters. Design the rod and cottered joints for the following conditions.

CHAPTER XI

POWER SCREWS

11-1. Introduction:

A power screw is a device used in machinery to change rotary motion into linear motion. This type of screw is much used in presses, clamps, air craft control surface actuators, jacks, valves, vices, testing machines, lathes, broaching machines and many other machine tools to move machine parts against resisting forces. The essential elements are (i) a screw and (ii) a nut threaded to engage the screw. A torque is applied to one of these elements, causing it to rotate and move either itself or the other element in an axial direction. In most power screws, the screw rotates in bearings while the nut has axial motion against the resisting axial forces. In some screws the nut is stationary while the screw rotates and moves axially against the resisting forces. In some cases, the nut rotates while the screw moves axially with no rotation.

The materials commonly used are steel for the screw and bronze or brass for the nut. To reduce the amount of bronze bimetallic nuts are used in the form of a steel or a cast iron shell lined with bronze by the centrifugal method.

11-2. Form of threads:

Fig. 11-1 illustrates three thread forms (square threads, Acme threads and Buttress threads) used for transmitting power.

The square thread will transmit power without any side thrust but is difficult to cut when the lead is long, on account of the difficulty in clearing tool from the groove. They cannot be used conveniently with split or half nuts on account of the difficulty of disengagement. The Acme thread, though not as efficient as the square thread, is easier to cut or mill, is stronger than square thread and permits the use of a split nut which can be used to take up wear. In trapezoidal thread the angle is 30° instead of 29°, which is the case for Acme threads.

The Buttress thread form combines the higher efficiency of the square thread and the ease of cutting and adaptability to a split nut of the Acme thread; furthermore it is stronger than other forms because of greater thickness at the base of the thread.

It is employed in jack screws where power is transmitted in one direction only. Sometimes 10° modified square threads are used.

factor of safety as 6. Neglect inertia forces and assume 'a' for hinged ends as $\frac{1}{7500}$.

Maximum thrust in connecting rod takes place when crank is 30° from inner dead centre.

(Gujarat University, 1965)

20. A piston rod of a steam engine may be considered to be a column. Determine the diameter of the rod from the following data:

Diameter of the cylinder 45 cm

Maximum net steam pressure 8 kg/sq cm

Distance from piston to crosshead 2 metre

End restraint coefficient 2.6

Permissible stress 1,000 kg/sq cm

(Sardar Patel University, 1970)

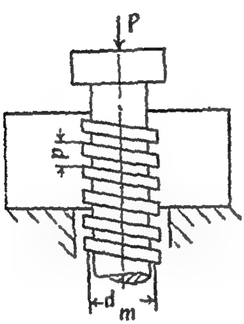
where ϕ is the friction angle which is equal to $\tan^{-1} \mu$, where μ is the coefficient of friction between the threads.

The helix angle α is given by

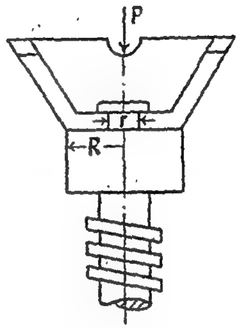
$$\alpha = \tan^{-1} \frac{\text{lead}}{\pi d_m} \quad \ldots\ldots \text{(ii)}$$

The efficiency of the square threads is given by the expression

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} \quad \ldots\ldots \text{(iii)}$$



Square threaded screw
FIG. 11-2



Collar friction
FIG. 11-3

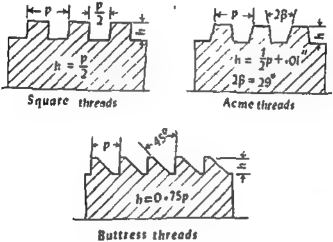
When efficiency of the screw is less than 50%, it is said to be self locking. The self locking screw requires a positive torque to lower the screw or to loosen the screw if it has been turned tight against the resistance. It can be proved from principles of mechanics that for self locking of a square thread the friction angle should be greater than the helix angle. In design of screw jacks and many other power screws application, the self locking property is intentionally introduced to prevent reverse motion when the effort is removed.

However the use of multithread screw is preferred in presses. Here the self locking property is not required; on the contrary having pressed the part or material appreciable turning moment is to be applied to the screw in order to reverse it. Hence if non self-locking screw is used the screw under the action of the reaction of the body pressed rotates itself in the direction opposite to that of the pressing force.

In the preceding equations, only the friction in the thread is taken into account. Some additional torque must be applied in order to overcome the thrust or collar bearing friction. Fig. 11-3

Multiple-threaded screws are employed when a comparatively large axial movement is required without much reduction of area at the root of the threads. Multiple threads will increase the efficiency and the travelling speed of the sliding element but the mechanical advantage may be reduced and the self locking property may be lost.

The pitch of a screw thread is the distance between adjacent crests, while the lead is the distance the nut will advance axially for one turn of the screw. *The pitch and lead are alike in single threaded screws; the lead is twice the pitch in double threaded and three times the pitch in triple threaded screws.*



Various forms of power threads

Fig. 11-1

Power screws with multiple threads are employed when it is desired to secure a large lead with fine threads or high efficiency.

11-3. Force analysis:

(a) *Square thread:*

Fig. 11-2 shows a simplified square threaded power screw with single thread having a mean diameter d_m, a pitch p, a helix angle α and loaded by the axial compressive force P. It can be proved from first principles that the torque T required to overcome the thread friction and raise the load is given by

$$T = \frac{P\,d_m}{2} \tan\,(\alpha + \phi) \qquad \text{...(i)}$$

where β is taken as one-half the thread angle, which is $14\frac{1}{2}°$ in case of Acme threads and 15° in case of trapezoidal threads. Strictly speaking β should be the pressure angle in a plane normal to the thread, rather than the pressure angle in the diametral plane, the relation being

$$\tan \beta = \tan 14{\cdot}5° \cos\alpha \text{ or } \tan 15° \cos\alpha \quad \ldots\ldots\ldots\ldots\text{(viii)}$$

Since α is small for power screws, $\cos \alpha$ may be taken to be unity and β is taken to be 14·5° for Acme threads and 15° for trapezoidal threads.

If there is some friction at the pivot, the total torque to turn the screw will be the sum of that from equation (viii) and that at the pivot.

The efficiency of the Acme thread or trapezoidal thread is given by,

$$\eta = \frac{\tan \alpha \ (1 - \mu \sec \beta \tan \alpha)}{\tan \alpha + \mu \sec \beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

11·4. Design of a screw:

The screw is subjected to a heavy compressive or tensile stress depending upon the method of mounting the screw and the manner of transmitting the desired power. It is also subjected to the torsional shear stresses induced by the external turning moment applied, though a part of this turning moment may be used in overcoming the friction of the bearings, depending upon the arrangement of the nut and screw. In addition the threads of both screw and nut are subjected to shear.

If the normal (direct and compressive) and shear stresses on the section of the screw were to be the functions of the same dimension, it would have been possible to write the equation in such a form that direct solution for the core diameter of the screw was possible. Unfortunately this is not the case for power screws. The section of the screw is a function of core diameter, mean diameter, helix angle, the coefficient of friction and the pivot bearing friction diameter. The helix angle cannot be calculated until the mean diameter and pitch or lead are known. For this reason the screw diameter must be selected by a trial and error solution.

The following procedure for the design of the screw is suggested:

shows a typical thrust collar having R and r as the outer and inner radii of the collar. If μ_1 be the coefficient of the collar friction then torque required *to overcome friction* at the collar surface is given by

$$T_1 = \frac{2}{3}\mu_1 P \frac{R^3 - r^3}{R^2 - r^2} \quad \text{.......................} \quad (iv)$$

The above equation is, sometimes, written as

$$T_1 = \frac{\mu_1 P d_c}{2} \quad \text{.......................} \quad (v)$$

where d_c is called the mean friction diameter of the collar.

The total torque is obtained by adding the results of eq. (i), and (iv) or (v). The efficiency will be given by (when we consider the collar friction)

$$\eta = \frac{\tan\alpha\,(1-\mu\tan\alpha)}{\tan\alpha + \mu + \mu_1 \frac{d_c}{d_m}(1-\mu\tan\alpha)} \quad \text{..............} \quad (vi)$$

The coefficient of friction for the threads of translation screw is found to be independent of the load and speed within the *ranges used in common practice. It shows little variation for* different combinations of commercial materials. It is found to depend on quality of material, workmanship in cutting threads and lubrication.

The coefficient of thread friction varies from 0·10 to 0·15 depending upon the workmanship and condition of lubrication. The coefficient of collar friction may be taken as the same as for thread friction. The coefficient of friction for the starting condi*tions may be taken as 33% more than the value* for running conditions. C. W. Ham and D. G. Ryan give the following values of coefficient of friction for the thrust collars of power screws:

Material	*Running friction*	*Starting friction*
Soft steel on cast iron	0 121	0 170
Hardened steel on cast iron	0 092	0 147
Soft steel on bronze	0·084	0 105
Hardened steel on bronze	0 063	0 081

(*b*) *Acme threads and Trapezoidal threads:*

Torque required to overcome friction and raise the load, when pivot friction is neglected, is given by

$$T = P\frac{d_m}{2}\left[\frac{\cos\beta\tan\alpha + \mu}{\cos\beta - \mu\tan\alpha}\right] \quad \text{.....................} \quad (vii)$$

where β is taken as one-half the thread angle, which is $14\frac{1}{2}°$ in case of Acme threads and 15° in case of trapezoidal threads. Strictly speaking β should be the pressure angle in a plane normal to the thread, rather than the pressure angle in the diametral plane, the relation being

$$\tan\beta = \tan 14{\cdot}5° \cos\alpha \text{ or } \tan 15° \cos\alpha \ldots\ldots\ldots\ldots\text{(viii)}$$

Since α is small for power screws, $\cos\alpha$ may be taken to be unity and β is taken to be 14·5° for Acme threads and 15° for trapezoidal threads.

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$$\eta = \frac{\tan\alpha\,(1 - \mu \sec\beta \tan\alpha)}{\tan\alpha + \mu \sec\beta} \ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

11·4. Design of a screw:

The screw is subjected to a heavy compressive or tensile stress depending upon the method of mounting the screw and the manner of transmitting the desired power. It is also subjected to the torsional shear stresses induced by the external turning moment applied, though a part of this turning moment may be used in overcoming the friction of the bearings, depending upon the arrangement of the nut and screw. In addition the threads of both screw and nut are subjected to shear.

If the normal (direct and compressive) and shear stresses on the section of the screw were to be the functions of the same dimension, it would have been possible to write the equation in such a form that direct solution for the core diameter of the screw was possible. Unfortunately this is not the case for power screws. The section of the screw is a function of core diameter, mean diameter, helix angle, the coefficient of friction and the pivot bearing friction diameter. The helix angle cannot be calculated until the mean diameter and pitch or lead are known. For this reason the screw diameter must be selected by a trial and error solution.

The following procedure for the design of the screw is suggested:

The axial load on the screw is generally known; by using a reasonable factor of safety the core diameter or the root diameter of the screw is obtained. If the screw is under compression and if the unsupported length of the screw between the load and the nut is more than *ten times* the calculated core diameter, the factor of safety must be increased to 20 per cent to allow for the column action of the screw and the core diameter should be re-calculated. Then by referring to table of power threads, an outside diameter can be selected for square or Acme or trapezoidal threads with the required root area and the pitch or lead. We decide whether to use single — or multiple threaded screw. The angle of helix is obtained by the formula

$$\alpha = \tan^{-1}\frac{\text{lead}}{\pi d_m} \quad \ldots \quad \text{(i)}$$

The mean diameter of the screw, d_m, is taken as the outside diameter less the depth of the thread for square threads. The suitable value of the coefficient of friction is assumed. The torque required to overcome friction at the thread is calculated by the equation

$T = P\dfrac{d_m}{2}\tan(\alpha + \phi)$. The shear stress, f_s, induced in core diameter, d_c, is calculated by the equation

$$f_s = \frac{16T}{\pi d_c^3} \quad \ldots \quad \text{(ii)}$$

or

$$d_c = 1{\cdot}72\sqrt[3]{\frac{T}{f_s}} \quad \ldots \quad \text{(iii)}$$

The trapezoidal threads are commonly adopted for power screws. The angle of thread is 30°. *Tr* 60 × 9 means the outside diameter or nominal diameter of the screw is 60 mm and pitch of the thread = 9 mm. Tr means trapezoidal threads

Core diameter = Nominal diameter − pitch − clearance.

Mean diameter = Nominal diameter − $\dfrac{\text{pitch}}{2}$.

Clearance is 0·5 mm for nominal diameter upto 110 mm and 1 mm for nominal diameter more than 110 mm.

The table of trapezoidal threads is given on page 442:

If the same section of the screw is subjected to both torsion and compression and if the calculated value of the torsional shear

where β is taken as one-half the thread angle, which is $14\frac{1}{2}°$ in case of Acme threads and 15° in case of trapezoidal threads. Strictly speaking β should be the pressure angle in a plane normal to the thread, rather than the pressure angle in the diametral plane, the relation being

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$$\eta = \frac{\tan \alpha \,(1 - \mu \sec \beta \tan \alpha)}{\tan \alpha + \mu \sec \beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ix)}$$

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The screw is subjected to a heavy compressive or tensile stress depending upon the method of mounting the screw and the manner of transmitting the desired power. It is also subjected to the torsional shear stresses induced by the external turning moment applied, though a part of this turning moment may be used in overcoming the friction of the bearings, depending upon the arrangement of the nut and screw. In addition the threads of both screw and nut are subjected to shear.

If the normal (direct and compressive) and shear stresses on the section of the screw were to be the functions of the same dimension, it would have been possible to write the equation in such a form that direct solution for the core diameter of the screw was possible. Unfortunately this is not the case for power screws. The section of the screw is a function of core diameter, mean diameter, helix angle, the coefficient of friction and the pivot bearing friction diameter. The helix angle cannot be calculated until the mean diameter and pitch or lead are known. For this reason the screw diameter must be selected by a trial and error solution.

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The axial load on the screw is generally known; by using a reasonable factor of safety the core diameter or the root diameter of the screw is obtained. If the screw is under compression and if the unsupported length of the screw between the load and the nut is more than *ten times* the calculated core diameter, the factor of safety must be increased to 20 per cent to allow for the column action of the screw and the core diameter should be re-calculated. Then by referring to table of power threads, an outside diameter can be selected for square or Acme or trapezoidal threads with the required root area and the pitch or lead. We decide whether to use single — or multiple threaded screw. The angle of helix is obtained by the formula

$$\alpha = \tan^{-1}\frac{\text{lead}}{\pi d_m} \quad \ldots \quad \text{(i)}$$

The mean diameter of the screw, d_m, is taken as the outside diameter less the depth of the thread for square threads. The suitable value of the coefficient of friction is assumed. The torque required to overcome friction at the thread is calculated by the equation

$T = P\frac{d_m}{2}\tan(\alpha + \phi)$. The shear stress, f_s, induced in core diameter, d_c, is calculated by the equation

$$f_s = \frac{16T}{\pi d_c^3} \quad \ldots \quad \text{(ii)}$$

or

$$d_c = 1{\cdot}72\sqrt[3]{\frac{T}{f_s}} \quad \ldots \quad \text{(iii)}$$

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Mean diameter = Nominal diameter − $\frac{\text{pitch}}{2}$.

Clearance is 0·5 mm for nominal diameter upto 110 mm and 1 mm for nominal diameter more than 110 mm

The table of trapezoidal threads is given on page 442.

If the same section of the screw is subjected to both torsion and compression and if the calculated value of the torsional shear

Trapezoidal threads

Nominal diameter d mm	Core diameter dc mm	Mean diameter dm mm	Core area ac sq cm	Pitch p mm
10	6·5	8·5	0·33	3
12	8·5	10·5	0·57	3
14	9·5	12	0·71	4
16	11·5	14	1·04	4
18	13·5	16	1·43	4
20	15·5	18	1·89	4
22	16·5	19·5	2·14	5
24	18·5	21·5	2·69	5
26	20·5	23·5	3·30	5
28	22·5	25·5	3·89	5
30	23·5	27	4·34	6
32	25·5	29	5·11	6
34	27·5	31	5·94	6
36	29·5	33	6·83	6
38	30·5	34·5	7·31	7
40	32·5	36·5	8·30	7
42	34·5	38·5	9·35	7
44	36·5	40·5	10·46	7
46	37·5	42·0	11·04	8
48	39·5	44	12·25	8
50	41·5	46	13·53	8
52	43·5	48	14·86	8
55	45·5	50·5	16·26	9
58	48·5	53·5	18·47	9
60	50·5	55·5	20·03	9
62	52·5	57·5	21·65	9
65	54·5	60	23·33	10
68	57·5	63	25·97	10
70	59·5	65	27·81	10
72	61·5	67	29·71	10
75	64·5	70	32·67	10
78	67·5	73	35·78	10
80	69·5	75	37·94	10
82	71·5	77	40·15	10
85	72·5	79	41·28	12
88	75·5	82	44·77	12
90	77·5	84	47·17	12
92	79·5	86	49·64	12
95	82·5	89	53·46	12
98	85·5	92	57·41	12
100	87·5	94	60·13	12
105	92·5	99	67·20	12
110	97·5	104	74·66	12
115	100	108	78·54	14
120	105	113	86·59	14
125	110	118	95·03	14
130	115	123	103·87	14
135	120	128	113·1	14
140	125	133	122·72	14
145	130	138	132·73	14
150	133	142	138·93	16
155	138	147	149·57	16
160	143	152	160·61	16
165	148	157	172·03	16
170	153	162	183·85	16
175	158	167	196·07	16

The axial load on the screw is generally known; by using a reasonable factor of safety the core diameter or the root diameter of the screw is obtained. If the screw is under compression and if the unsupported length of the screw between the load and the nut is more than *ten times* the calculated core diameter, the factor of safety must be increased to 20 per cent to allow for the column action of the screw and the core diameter should be re-calculated. Then by referring to table of power threads, an outside diameter can be selected for square or Acme or trapezoidal threads with the required root area and the pitch or lead. We decide whether to use single — or multiple threaded screw. The angle of helix is obtained by the formula

$$\alpha = \tan^{-1}\frac{\text{lead}}{\pi d_m} \quad \ldots \quad \text{(i)}$$

The mean diameter of the screw, d_m, is taken as the outside diameter less the depth of the thread for square threads. The suitable value of the coefficient of friction is assumed. The torque required to overcome friction at the thread is calculated by the equation

$T = P\frac{d_m}{2}\tan(\alpha + \phi)$. The shear stress, f_s, induced in core diameter, d_c, is calculated by the equation

$$f_s = \frac{16T}{\pi d_c^3} \quad \ldots \quad \text{(ii)}$$

or

$$d_c = 1{\cdot}72\sqrt[3]{\frac{T}{f_s}} \quad \ldots \quad \text{(iii)}$$

The trapezoidal threads are commonly adopted for power screws. The angle of thread is 30°. *Tr* 60×9 means the outside diameter or nominal diameter *of the screw is 60 mm and* pitch of the thread is 9 mm. Tr means trapezoidal threads.

Core diameter = Nominal diameter — pitch — clearance.

Mean diameter = *Nominal* diameter — $\frac{\text{pitch}}{2}$

Clearance is 0·5 mm for nominal diameter upto 110 mm and 1 mm for nominal diameter more than 110 mm.

The table of trapezoidal threads is given on page 442:

If the same section of the screw is subjected to both torsion and compression and if the calculated value of the torsional shear

The values of allowable bearing pressures for severe combination of materials and speeds are given on page 443.

The bearing pressure intensity is restricted due to wear consideration. It is customary to restrict the work lost in friction to 100 kg cm per second.

The ratio of $\frac{h}{d_c}$ is selected from 1·2 to 2·5 for unsplit nuts and from 2·5 to 3·5 for split nuts. Nuts on the screws of screw cutting lathe are split so that the nut can be separated from the driving screw when plain turning or boring is carried out.

As a rule the threads will be safe in shear but the check should be made in order to be sure. As the threads of the nut shear at the major diameter while the threads of the screw shear at the minor diameter, the shear area will be less for the threads on the screw than for the thread in the nut. Since the screw and nut are made of different materials, it becomes necessary to calculate separately the length of engagement required for the screw and nut.

$$l_{screw} = \frac{p\ P}{\pi d_c\, t_c f_{s\ screw}} \quad \text{(ii)}$$

$$l_{nut} = \frac{p\ P}{\pi\, d\, t f_{s\ nut}} \quad \text{(iii)}$$

where d_c = core diameter of the screw

t_c = thread thickness at core diameter

d = outer diameter of the screw

t = thread thickness at outer diameter.

The outer diameter of the nut is usually made twice the diameter of the screw. However it can be checked for direct axial load (tensile or compressive) combined with torsion. The nut can be treated as a hollow shaft. The permissible stress for phosphor bronze nut may be taken from 600 to 700 kg/sq cm in tension and compression and 250 to 350 kg/sq cm in shear.

Generally the nut, which is made of bronze is not integral with the frame. It may be fixed in the body by interference fit. If it is fitted in the body without interference fit, care must be taken to see that it does not rotate with the thread of the screw. There must be sufficient friction grip, between the nut and the

stress is greater than one-third of the tensile or compressive stress in the screw, it is advisable to re-calculate the diameter of the screw using 10 per cent greater factor of safety or we calculate the principal stress due to combined effect of direct and shear stresses, and the values of the principal stress or maximum shear stress should be within safe limits. Finally the screw should be checked as a column by taking suitable end connections. The end fixity coefficient of a column will depend on method of mounting the screw in a mechanism Screw supports with ratio $\frac{h}{d_c} = 1{\cdot}5$ to 2, should be classed as hinged columns.

11-5. Design of a nut:

The most important dimension in the design of a nut is the height of the nut, which depends upon the amount of bearing surface required between the thread surfaces. The permissible bearing stress depends on the speed of sliding and pressure. The projected bearing surface of one thread is nearly equal to the area of the annular surface, the larger diameter of which is the outside diameter of the screw and the smaller diameter will be the core diameter. The required height h of the nut is

$$h = \frac{4\,pP}{\pi(d^2 - d_c^2)\,f_b} \qquad \ldots (1)$$

where p = pitch of the thread

P = axial load

d = outer diameter of the screw

d_c = core diameter

f_b = allowable bearing pressure.

Service	Material		Permissible bearing pressure
	Screw	Nut	kg/sq cm
Hand press	Steel	Bronze	175 – 225
Jack screw	,,	C.I.	126 – 175
,,	,,	Bronze	112 – 175
Hoisting screw	,,	C.I.	42 – 70
,,	,,	Bronze	56 – 98
Lead screw	,,	,,	10 – 16

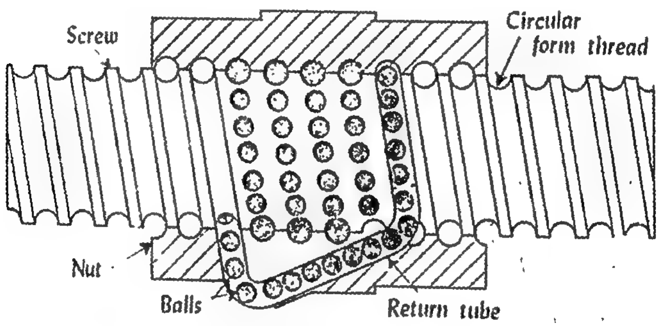
lates the balls to the leading end of the nut (Fig. 11-6). The function of ball screws is similar to that of trapezoidal screw and nut but with the following important advantages:

(i) High efficiency
(ii) Low starting friction
(iii) Accurate positioning.

Because of low coefficient of friction, both rotary to linear functions are *reversible;* that is a linear motion of either the screw or nut can cause other member to rotate.

Threads on most types are machined or rolled. When machined, they are rough cut, hardened and then ground to obtain highest

position accuracy,
efficiency,
load capacity and
life.



Ball Screw
Fig. 11-6

Rolled threads, on the other hand, are cheaper to produce and offer considerable cost savings if quantities are sufficiently high.

Load life characteristics of ball screws are similar to those of ball bearings. Because most of the loading is in thrust, ball screws operate similarly to a series of angular contact ball bearings. The sides of the ball grooves extend almost to the mid point of the ball.

The ball screw can be rated for static loading as well as dynamic loading. The dynamic load carrying capacity is usually of greater importance than the static load capacity, since fatigue is the primary cause of failure, the load-life rating of a ball screw is based on the number of times a ball rolls over a particular point in the race way and the load on that point as each ball passes over it. *Life varies inversely as the cube of the load.* Doubling the rated load cuts life to one eighth: reducing the load in half increases life eight times. This rated value is only about one-fifth the actual average life.

frame. Generally, such type of nuts may be provided with a flange, which acts as a collar. The friction torque of the collar is given by

$$T_1 = \tfrac{2}{3}\mu_1 P\left[\frac{R^3-r^3}{R^2-r^2}\right] \qquad \text{.......................(iv)}$$

where P = axial load on the screw

μ_1 = coefficient of the collar friction

R = outer radius of the collar

r = inner radius of the collar.

With usual notations, the thread friction torque of the screw is given by

$$T = P\frac{dm}{2}\tan\,(\alpha+\phi) \qquad \text{.............(v)}$$

Let T kg cm be the thread friction torque and N the operating speed in r.p.m. of the nut.

Energy lost in friction $= T \times 2\pi N$ kg cm/minute If n be the number of threads in a nut then work lost in friction per thread per minute will be $\frac{T\times 2\pi N}{n}$ kg cm/thread/minute. Bearing area per thread $= \frac{\pi}{4}[d^2 - d_c^2]$ where d and d_c are respectively nominal and core diameter of screw in cm

$$\therefore \text{ Energy lost in friction} = \frac{\dfrac{T\times 2\pi N}{60\times n}}{\dfrac{\pi}{4}[d^2-d_c^2]}$$

$$= \frac{TN}{7{\cdot}5n\,[d^2-d_c^2]} \text{ kg cm/sec/sq cm of bearing surface.}$$

The energy given by the above equation should not exceed 100 kg cm. Thus from wear considerations the operating speed of the nut can be determined if the height of the nut has been decided earlier. If the operating speed is known, the minimum height of the nut can be determined.

If the friction torque between the flange of the nut and the contact surface of the body is greater than thread friction torque, the nut will be prevented from turning with the screw. If the thread friction torque is more than the collar friction torque,

α = helix angle $= \tan^{-1}\dfrac{p}{\pi d_m} = \tan^{-1}\dfrac{0{\cdot}5}{\pi \times 2{\cdot}75} = 3{\cdot}3°.$

ϕ = friction angle $= \tan^{-1}\mu = \tan^{-1} 0{\cdot}14 = 8{\cdot}1°.$

Torque required to overcome friction at the nut will be equal to $P\dfrac{d_m}{2}\tan(\alpha+\phi) = 3000 \times \dfrac{2{\cdot}75}{2}\tan 11{\cdot}4° = 820$ kg cm.

Torque required to overcome friction at the pad = 350 kg cm.

Total torque to be applied at the handle = 820 + 350
= 1,170 kg cm.

$\therefore$ Effective length of the handle $= \dfrac{1170}{30} = 39$ cm.

If d cm be the diameter of the handle, then

$$\frac{\pi}{32} d^3 \times 1000 = 1170$$

or

$$d = \sqrt[3]{\frac{1170}{1000} \times \frac{32}{\pi}} = 2{\cdot}28 \text{ cm; we adopt } 2{\cdot}3 \text{ cm.}$$

If f_s be the shear stress induced in the screw, then

$$\frac{\pi}{16}(2{\cdot}5)^3 f_s = 1170$$

or

$$f_s = \frac{1170 \times 16}{\pi \times 2{\cdot}5^3} = 383 \text{ kg/sq cm.}$$

$$\text{Direct compressive stress} = \frac{3000}{\frac{\pi}{4} \times 2{\cdot}5^2} = 610 \text{ kg/sq cm.}$$

$$\text{Principal stress} = \frac{610 \pm \sqrt{610^2 + 4 \times 383^2}}{2}$$

$$= 794 \text{ kg/sq cm and } -184 \text{ kg/sq cm.}$$

The value of the induced stress is within safe limits.

Dimensions of the screw: Nominal diameter 3 cm, single start square thread 5 mm; nut of 5 cm height having 6 cm outside diameter. Lever 40 cm of effective length having 23 mm diameter.

2. *A vertical two start square threaded screw of 10 cm mean diameter and 2 cm pitch supports a vertical load of 1,800 kg. The nut for the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 60 teeth. The mechanical efficiency of the wheel and pinion is 90%. The axial thrust of the vertical screw is taken on a collar bearing 10 cm inside diameter and 25 cm outside diameter for which a uniform pressure condition may be assumed. If the coefficient of friction*

Examples:

1. The compressive load on the nut and screw clamp shown in fig. 11-7 is 3,000 kg. Calculate the diameter of the screw, height of the nut and the dimensions of the handle if a force of 30 kg is required to be applied at the end of a handle to operate the screw. Assume the following:

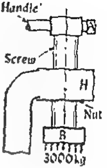


Fig. 11-7

Safe compressive stress for screw

= 1,200 kg/sq cm

Bearing pressure for screw and nut

= 175 kg/sq cm.

Coefficient of screw thread friction = 0 14

Frictional torque of pad B = 350 kg cm.

Bending stress in the handle = 1,000 kg/sq cm.

The screw is subjected to a direct compressive stress and torsional shear stress. In order to find the diameter of the screw, we must consider the principal stress. Let us find the diameter of the screw taking the lower value of the stress, say 850 kg/sq cm

If d be the diameter of the screw at the bottom of the thread,

$$\frac{\pi}{4} d^2 \times 850 = 3000$$

or $$d = \sqrt{\frac{3000}{850} \times \frac{4}{\pi}} = 2{\cdot}12 \text{ cm, we adopt } 2{\cdot}5 \text{ cm.}$$

We adopt single start square threads having 5 mm pitch.

$\therefore$ Mean diameter of the screw $= 25 + 2{\cdot}5 = 27{\cdot}5$ mm.

Outside diameter of the screw $= 25 + 5 = 30$ mm.

Bearing area of each thread $= \frac{\pi}{4}[3^2 - 2{\cdot}5^2] = 2{\cdot}16$ sq cm.

If n be the number of threads, then

$$n \times 2{\cdot}16 \times 175 = 3000$$

or $$n = \frac{3000}{2{\cdot}16 \times 175} =$$ [redacted] threads.

From stability point of view, we adopt 10 threads The height of the nut will be $10 \times 5 = 50$ mm. The outside diameter of the nut is generally twice the outside diameter of the screw. In the present case it will be $2 \times 3 = 6$ cm.

$$\text{Number of threads required in nut} = \frac{1800}{473} = 3{\cdot}8.$$

For stability purpose, we adopt 6 threads.

The height of the nut will be $6 \times 2 = 12$ cm.

3. *It is required to determine the pressure P that the screw of the bracket clamp shown in fig. 11-8 can exert so that the compressive stress in the screw shall not exceed 1,000 kg/sq cm. Assume square threads of 6 mm pitch. Determine the bearing pressure coming upon the threads, also the efficiency of the screw assuming the coefficient of friction 0·12. Determine the maximum stress produced in the section AB when the load upon the screw has the magnitude determined earlier. Determine the size of the two bolts used to fasten the bracket clamp to the platen, assuming that the permissible tensile stress is 1,400 kg/sq cm.*

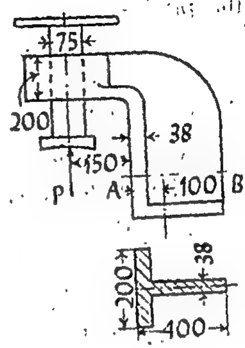


FIG. 11-8

The screw is subjected to direct compressive stress and torsional shear stress. In order to find the load, we should consider the principal stress. As torsional shear stress is not known, we calculate the load by considering the lower value of the stress, say 700 kg/sq cm.

The outside diameter of the screw is 75 mm. The pitch of the threads is 6 mm.

Inner diameter of the screw $= 75 - 6 = 69$ mm.

The maximum pressure P = area at the core of the screw × allowable stress intensity

$$= \frac{\pi}{4} \times 6{\cdot}9^2 \times 700 = 26{,}000 \text{ kg.}$$

Bearing pressure on threads:

$$\text{Number of threads in the nut} = \frac{200}{6} = 33.$$

$$\text{Bearing area per thread} = \frac{\pi}{4}(7{\cdot}5^2 - 6{\cdot}9^2) = 6{\cdot}79 \text{ sq cm.}$$

$$\therefore \text{ Bearing pressure} = \frac{26000}{33 \times 6{\cdot}79} = 1{,}160 \text{ kg/sq cm.}$$

for the vertical screw is 0·15 and for the collar is 0·2, determine the minimum diameter of the pinion shaft and the length of the nut. The permissible value of the shear stress in the shaft material is limited to 550 kg/sq cm. Allow the bearing pressure as 15 kg/sq cm for threads.

As the square thread is two start, the lead of the thread is twice the pitch.

α = helix angle $= \tan^{-1}\dfrac{2p}{\pi d_m} = \tan^{-1}\dfrac{2 \times 2}{\pi \times 10} = 7° - 15'$.

ϕ = friction angle $= \tan^{-1}\mu = \tan^{-1} 0{\cdot}15 = 8° - 32'$.

As the friction angle is greater than the helix angle, the arrangement is self locking.

Torque required to overcome friction at the thread surfaces

$= P\dfrac{d_m}{2} \tan(\alpha + \phi) = 1800 \times \dfrac{10}{2} \tan(15° - 47')$

$= 2{,}542$ kg cm.

Torque required to overcome friction at the collar

$= \frac{2}{3}\mu P\left[\dfrac{R^3 - r^3}{R^2 - r^2}\right] = \frac{2}{3} \times 0{\cdot}2 \times 1800\left[\dfrac{12{\cdot}5^3 - 5^3}{12{\cdot}5^2 - 5^2}\right]$

$= 3{,}340$ kg cm.

Total torque required at the gear wheel $= 2542 + 3340$
$= 5{,}882$ kg cm.

The mean radius of the pinion is less than that of the gear wheel and since torque is proportional to the radius, the torque on the pinion will be less than 5,882 kg cm. As the radius of the gear wheel is proportional to the number of teeth, the torque will be proportional to the number of teeth in the gear As the mechanical efficiency is 90%, the torque on the pinion shaft will be

$$5882 \times \frac{60}{80} \times \frac{100}{90} = 4{,}900 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 550 = 4900.$$

$\therefore\ d = \sqrt[3]{\dfrac{4900}{550} \times \dfrac{16}{\pi}}$ 3·56 cm; we adopt 3·6 cm.

Bearing area per thread $= \pi \times 10 \times 1 = 31{\cdot}4$ sq cm.

Bearing load that can be supported by each thread will be
$31{\cdot}4 \times 15 = 473$ kg.

The load of 26,000 kg is eccentric with respect to the section and the eccentricity is $15 + 14{\cdot}8 = 29{\cdot}8$ cm.

Moment of inertia of section about neutral axis

$$= \int_{-25\cdot2}^{11\cdot5} 3{\cdot}8x^2\, dx + \int_{11\cdot5}^{14\cdot8} 20x^2\, dx = 34{,}200 \text{ cm}^4.$$

Tensile modulus of section $= \dfrac{34200}{14{\cdot}8} = 2{,}310$ cm^3.

Compressive modulus of section $= \dfrac{34200}{25{\cdot}2} = 1{,}310$ cm^3.

Maximum tensile stress due to bending $= \dfrac{26000 \times 29{\cdot}8}{2310}$

$= 325$ kg/sq cm.

Maximum compressive stress due to bending $= \dfrac{26000 \times 29{\cdot}8}{1310}$

$= 596$ kg/sq cm.

Direct tensile stress on the section $= \dfrac{26000}{213} = 120$ kg/sq cm.

Maximum tensile stress in the top of the flange $= 335 + 120$

$= 455$ kg/sq cm.

Maximum compressive stress in the bottom of the leg

$= 596 - 120 = 476$ kg/sq cm.

The distance of the fibre of the zero stress can be obtained. If y be the distance of the fibre of zero stress from the c.g., we have

$$120 = \frac{26000 \times 29{\cdot}8 \times y}{34200}$$

or $\quad y = 5{\cdot}3$ cm.

Thus, the fibre of zero stress is at a distance of $14{\cdot}8 + 5{\cdot}3 = 20{\cdot}1$ cm from the top of the flange of the section.

4. *Design a screw jack for lifting a load of 5,500 kg through a height of 25 cm. The screw is to be made of steel for which Rankine constants are 3,300 kg/sq cm and* $\dfrac{1}{7500}$ *for column pin jointed at both ends. Use a factor of safety 4. The nut is made of gun metal for which the allowable shear stress is 250 kg/sq cm and the allowable bearing stress in screw threads 100 kg/sq cm. The nut is fixed in the cast iron housing and the screw is turned in the nut by means of a tommy bar inserted in suitable holes in the head of the screws. Some form of swivelling device is to be provided on the top to prevent the load from turning while being raised. The coefficient of friction of the threads and collar may be taken as 0·14.*

Efficiency:

φ = friction angle $= \tan^{-1}\mu = \tan^{-1} 0{\cdot}12 = 6° - 54'$.

α = helix angle $= \tan^{-1}\dfrac{p}{\pi d_m} = \tan^{-1}\dfrac{6}{\pi \times 72} = 1° - 36'$.

As the friction angle is greater than the helix angle, the arrangement is self locking. The efficiency of the screw is given by

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\tan 1° - 36'}{\tan 8° - 30'} = \frac{0{\cdot}0278}{0{\cdot}1495} = 0{\cdot}186, \text{ i.e. } 18{\cdot}6\%.$$

Torque required to overcome friction at the thread surfaces

$$= P\frac{d_m}{2}\tan(\alpha+\phi) = \frac{26000 \times 7{\cdot}2}{2} \times 0{\cdot}1495 = 13{,}600 \text{ kg cm.}$$

If f_s be the shear stress induced in the screw, then

$$\frac{\pi}{16} \times 6{\cdot}9^3 \times f_s = 13600$$

or

$$f_s = \frac{13600 \times 16}{\pi \times 6{\cdot}9^3} = 209 \text{ kg/sq cm.}$$

The principal stress in the screw will be

$$\frac{700 + \sqrt{700^2 + 4 \times 209^2}}{2} = 757 \text{ kg/sq cm.}$$ This value is less than the permissible value of 1,000 kg/sq cm.

Size of the bolts:

Due to load of 26,000 kg, the bolts are subjected to tensile loading. The tilting of the bracket is resisted by the extensional load produced in the bolts. If F be the load produced in each bolt, then

$$2 \times F \times 30 = 26000 \times 35$$

or

$$F = \frac{26000 \times 35}{2 \times 30} = 15{,}150 \text{ kg}$$

The permissible tensile stress intensity is 1,400 kg/sq cm.

$\therefore$ Minimum area required at the bottom of the thread $= \dfrac{15150}{1400}$

$= 10{\cdot}8$ sq cm.

From the table of metric threads, we adopt M 45 diameter bolts.

Stress in section AB:

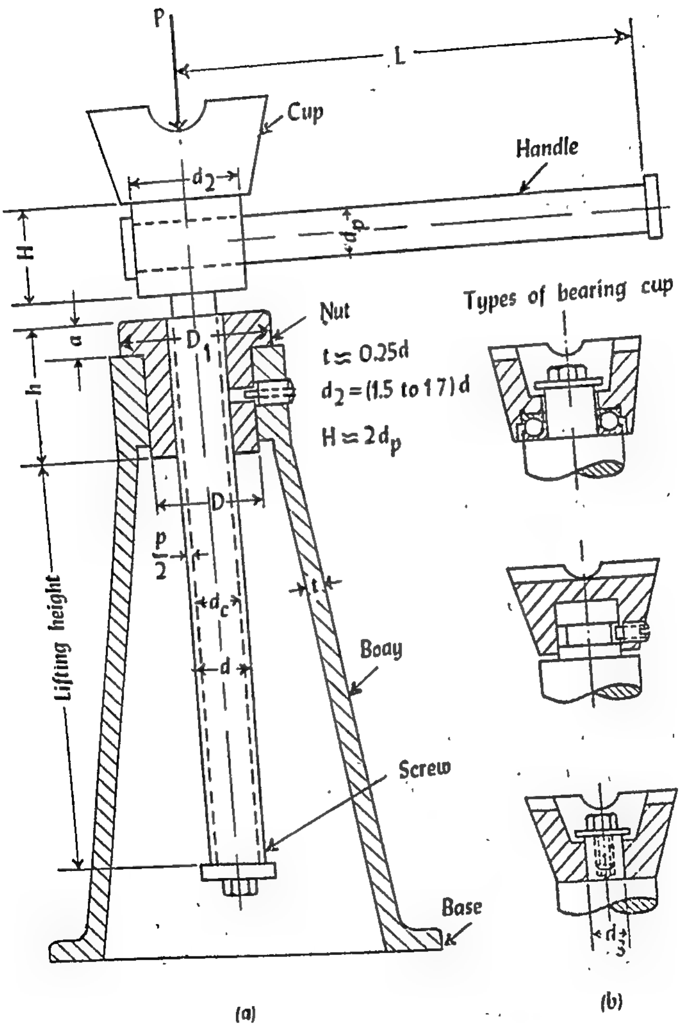
Area of section $AB = 20 \times 3{\cdot}8 + 3{\cdot}8 \times 36{\cdot}2 = 213$ sq cm.

If x be the c.g. of the section from the top of the flange, then $x \times 213 = 20 \times 3{\cdot}8 \times 1{\cdot}9 + 3{\cdot}8 \times 36{\cdot}2 \times 21{\cdot}9$.

$\therefore \quad x = 14{\cdot}8$ cm.

$$\therefore \quad T = \frac{5500 \times 4{\cdot}2}{2} \tan (11^\circ - 34')$$

$$= 2{,}420 \text{ kg cm.}$$



Bottle type screw jack

FIG. 11-9

The screw jack of bottle type, as shown in fig 11-9, consists of a screw at the top of which load to be lifted is placed. The load is prevented from rotation by providing a swivel. The nut is press-fixed in the main body which is usually made of cast iron. The effort is applied by means of a tommy bar inserted in suitable holes in the head of the screw. The screw is under direct compression due to load and torsional shear stresses will be set up when it is being raised. When the load is raised through the maximum lift it is subjected to buckling also. The load may not be applied centrally due to carelessness of the worker as a result the bending stresses are induced in the screw.

The friction between the screw head and cup over the circular supporting surface can be reduced either by diminishing the radius of cup supporting surface or by replacing sliding friction by rolling friction; for this purpose a thrust ball bearing is introduced. The end of the screw is provided with a washer that prevents *it from being screwed out of the nut completely through negligence*

The screw will be made of mild steel for which the ultimate strength is 4,500 kg/sq cm. As the screw is likely to get very rough handling we should take higher factor of safety.

First of all, we determine the diameter of the screw from consideration of the direct compressive load taking the lower value of the stress and then we check for the principal stress.

We take $f_c = 550$ kg/sq cm.

If d cm be the core diameter of the screw, then

$$\frac{\pi}{4} d^2 \times 550 = 5500$$

or
$$d = \sqrt{\frac{5500}{550} \times \frac{4}{\pi}} = 3{\cdot}68 \text{ cm, we adopt } 3{\cdot}8 \text{ cm.}$$

We assume square threads having pitch of 8 mm.

The outside diameter of the screw $= 38 + 8 = 46$ mm.

The mean diameter of the screw $= \dfrac{38 + 46}{2} = 42$ mm.

$$\alpha = \text{helix angle} = \tan^{-1} \frac{p}{\pi d_m} = \tan^{-1} \frac{8}{3{\cdot}14 \times 42} = 3° - 36'.$$

$$\varphi = \text{friction angle} = \tan^{-1}\mu = \tan^{-1} 0{\cdot}14 = 7°\text{-}58'.$$

As the friction angle is greater than the helix angle, the arrangement is self locking. The frictional torque, T, required to overcome friction at the thread surfaces $= P \dfrac{d_m}{2} \tan (\alpha + \varphi)$.

or $$d = \sqrt[3]{\frac{4460}{1250} \times \frac{32}{\pi}} = 3\cdot25 \text{ cm; we adopt } 3\cdot5 \text{ cm.}$$

Let us keep the height of the screw head $= 2 \times 3\cdot5 = 7$ cm.

Let us check the screw head for compressive stress as it has been weakened by two holes of 3·5 cm diameter holes.

The area of the cross section of head through holes equals

$$\frac{\pi}{4} \times 8^2 - 8 \times 3\cdot5 - 3\cdot5\,(8 - 3\cdot5) = 8\cdot5 \text{ sq cm.}$$

Compressive stress intensity $= \dfrac{5500}{8\cdot5} = 646$ kg/sq cm, which is within safe limits.

Let us check the screw for buckling.

The lift of the jack = 25 cm. The height of the head = 7 cm.

The length of the screw, which is unsupported will be 25 + 7 = 32 cm. When the maximum lift is attained the screw can be looked upon as a strut fixed at the lower end and free at the top. The end fixity coefficient for this column will be 0·25. Therefore,

Rankine constant will be $\dfrac{1}{7500 \times 0\cdot25} = \dfrac{1}{1875}$.

$$\text{Rankine load for the screw} = \frac{f_c.A}{1 + \frac{1}{a}\left(\frac{l}{k}\right)^2} = \frac{3300 \times \frac{\pi}{4} \times 3\cdot8^2}{1 + \frac{1}{1875}\left(\frac{32 \times 4}{3\cdot8}\right)^2}$$
$$= 23{,}200 \text{ kg.}$$

The factor of safety is 4.

$\therefore$ Permissible buckling load $= \dfrac{23200}{4} = 5{,}800$ kg.

As the value of the buckling load is greater than the value of the specified load, the screw is safe against buckling.

Design of a nut:

Bearing area per thread $= \dfrac{\pi}{4}\,[4\cdot6^2 - 3\cdot8^2] = 5\cdot24$ sq cm.

Minimum number of threads in the nut $= \dfrac{5500}{100 \times 5\cdot24} = 10\cdot5.$

In order to provide stability, we adopt 12 threads giving a height of the nut as 9·6 cm. The outer diameter of the nut is taken as twice the outer diameter of the screw. We take $2 \times 4\cdot6 = 9\cdot2$ cm as the outer diameter of the nut.

Let us check the screw for the principal stress.

Direct compressive stress $= \dfrac{5500}{\dfrac{\pi}{4} \times 3{\cdot}8^2} = 484$ kg/sq cm.

Maximum torsional shear stress $= \dfrac{2420}{\dfrac{\pi}{16} \times 3{\cdot}8^3} = 225$ kg/sq cm.

Maximum principal stress $= \dfrac{484 + \sqrt{484^2 + 4 \times 225^2}}{2}$

$= 652$ kg/sq cm.

Factor of safety $= \dfrac{4500}{652} = 6{\cdot}9$, which is adequate.

Before, we can check the screw for buckling, we decide upon the dimensions of the head of the screw, which will be made integral with the screw spindle. It is general practice to make the diameter of the head of the screw *1 75 times the outside diameter of* the screw. The diameter of the head of the screw will be equal to $1{\cdot}75 \times 4{\cdot}6 = 8$ cm.

Two holes, at right angles to each other, will be bored for inserting the handle to rotate the screw One hole would have been sufficient for inserting the handle but it is convenient to change the position of the handle after a quarter revolution

The seat for the cup is made 8 cm in diameter. This cup is provided at the top of the head to prevent the load from rotating. The cup is fitted to the head by means of 1·5 cm pin which remains a loose fit in the cup. The thickness of the cup will be 1 cm, height 5 cm and the outer diameter of the cup at the top will be 14 cm. Frictional torque, T, at the cup will be $\frac{2}{3}\,\mu P\left(\dfrac{R^3 - r^3}{R^2 - r^2}\right)$.

$T = \frac{2}{3} \times 0\ 14 \times 5500\left(\dfrac{4^3 - 0\ 75^3}{4^2 - 0\ 75^2}\right) = 2{,}040$ kg cm.

The portion of the screw below the handle is subjected to a torque of 2,420 kg cm, while the portion above the handle is subjected to a torque of 2,040 kg cm. Total torque to be applied by the handle $= 2420 + 2040 = 4{,}460$ kg cm. Let us assume the permissible stress in the handle to be 1,250 kg/sq cm.

If d cm be the diameter of the handle, then

$$\frac{\pi}{32}\,d^3 \times 1250 = 4460$$

If x be the distance, from the centre of the base, at which the resultant meets the base, we have

$$x \times 5500 = 80 \times 68{\cdot}5$$

or
$$x = \frac{80 \times 68{\cdot}5}{5500} = 1 \text{ cm.}$$

Thus, we see that the body is stable against toppling.

5. *A machine slide weighing 300 kg is elevated by a two start Acme thread (29° thread angle) 4 cm diameter, 0·6 cm pitch, at the rate of 0·6 metre/min. If the coefficient of friction be 0·12, calculate the horse power of the motor to drive the slide. The end of the screw is carried on a thrust collar 3 cm inside and 5·5 cm outside diameter.*

The screw is two start with 0·6 cm pitch. During one rotation of the screw, slide advances by $0{\cdot}6 \times 2 = 1{\cdot}2$ cm. In one minute the slide is elevated by $0{\cdot}6 \times 100 = 60$ cm.

The speed of rotation of the screw $= \dfrac{60}{1{\cdot}2} = 50$ r.p.m.

Mean diameter of the thread $= 4 - 0{\cdot}3 = 3{\cdot}7$ cm.

$$\alpha = \text{helix angle} = \tan^{-1}\frac{\text{lead}}{\pi d_m} = \tan^{-1}\frac{1{\cdot}2}{\pi \times 3{\cdot}7} = 5^\circ - 55'.$$

Since thread angle is 29°, the modified coefficient of friction will be $0{\cdot}12 \sec 14\tfrac{1}{2}^\circ = 0{\cdot}124$.

ϕ = friction angle $= \tan^{-1} 0{\cdot}124 = 7^\circ\text{-}4'$.

Torque required to overcome friction at the thread surfaces

$$= P\frac{d_m}{2}\tan(\alpha + \phi) = 300 \times \frac{3{\cdot}7}{2}\tan(12^\circ\text{-}59')$$

$= 128$ kg cm.

Torque required to overcome friction at the collar surface

$$= \tfrac{2}{3}\,\mu P\left[\frac{R^3 - r^3}{R^2 - r^2}\right] = \tfrac{2}{3} \times 0{\cdot}12 \times 300\left[\frac{2{\cdot}75^3 - 1{\cdot}5^3}{2{\cdot}75^2 - 1{\cdot}5^2}\right]$$

$= 79$ kg cm.

Total torque required to overcome friction at threads and collar $= 128 + 79 = 207$ kg cm.

$$\text{H.P.} = \frac{TN}{71620} = \frac{207 \times 50}{71620} = 0{\cdot}144.$$

6. *Following data apply to the machinists' clamp:*

Outside diameter of the screw 1·4 cm; root diameter 0·95 cm; pitch (single thread) = 0·4 cm; collar friction radius = 0·6 cm; collar friction coefficient = 0·15; screw friction coefficient = 0·15, thread angle 30°.

The collar of the nut, which is made of gunmetal, rests on the casting of the main body and it may be crushed under the load being lifted. The outer diameter of the collar is taken as 12·5 cm. Crushing area between the collar of the nut and the body will be $\frac{\pi}{4}$ $[12{\cdot}5^2 - 9{\cdot}2^2] = 56{\cdot}2$ sq cm.

Crushing stress intensity $\frac{5500}{56{\cdot}2} = 98$ kg/sq cm, which is reasonably low.

Thickness of the collar will be adopted 15 mm.

Area that resists shearing of the collar $= \pi \times 9{\cdot}2 \times 1{\cdot}5$
$= 43{\cdot}5$ sq cm.

Shear stress induced in the collar $= \frac{5500}{43{\cdot}5} = 126{\cdot}5$ kg/sq cm which is less than 250 kg/sq cm which is the permissible stress.

Design of the body:

Length of the threaded portion of the screw = lift − height of the nut = 25 − 9·6 = 34·6 cm.

∴ The height of the body will be taken as 40 cm.

The thickness of the casting will be 15 mm.

The diameter of the body at the top will be 15 cm.

The inside diameter of the body at the base will be 18 cm.

The bottom flange diameter will be taken as 30 cm.

The thickness of the flange will be 25 mm.

Lever:

The diameter of the lever is calculated as 3·5 cm.

Let us assume that two persons apply load at the end of a lever.

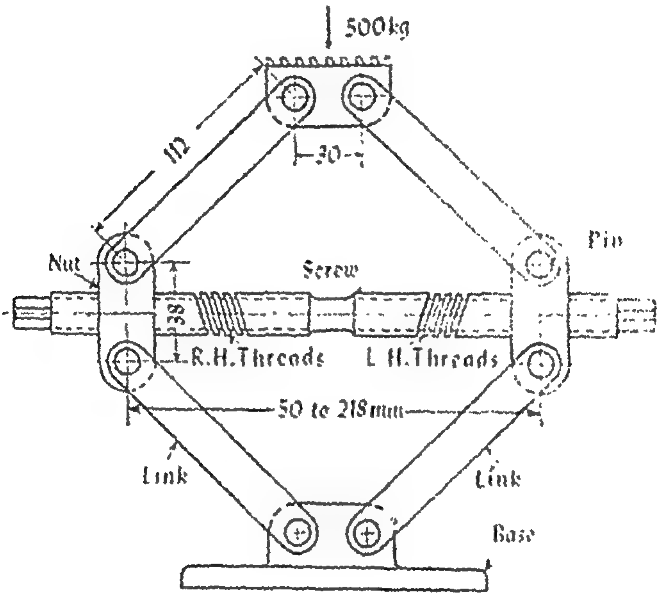
The operation is intermittent so 40 kg can be applied by a person.

The effective length of the handle will be $\frac{4460}{2 \times 40} = 55{\cdot}5$ cm.

Allowing some length for gripping, we adopt 70 cm as the length of the handle.

When the load is at the topmost position and when the effort is being applied at the end of the handle, the body is likely to topple. The effort will be acting at a distance of 40 + 25 + 3·5 = 68·5 cm from the base of the body.

From the dimensions given in the figure, we can assume that the distance between the nuts varies from 5 to 21·8 cm. When the jack is in top position the distance between the centre lines of the nuts will be 5 cm. The height of the line joining two pins in the load platform from the line joining two pins in the base when the jack is in top position will be $3{\cdot}8 + 2\sqrt{11{\cdot}2^2 - 1^2} = 26$ cm. The similar distance, when the jack is in bottom position will be $3{\cdot}8 + 2\sqrt{11{\cdot}2^2 - 9{\cdot}4^2} = 16$ cm.



Toggle screw jack

Fig. 11-10

The lift of the jack will be $26 - 16 = 10$ cm.

The maximum load in the screw will occur when the jack is in bottom position. The load in the screw will be tensile one. When the jack is in bottom position, the link will be inclined at an angle $\cos^{-1}\dfrac{9{\cdot}4}{11{\cdot}2} = 33° - 24'$ to the horizontal. The component of force in the link along the screw will be $\dfrac{500}{2 \times \tan 33° - 24'}$ $= 379$ kg. As this pull acts at two pin joints at the nut, the

Assume that the machinist can comfortably exert a maximum force of 12 kg on the handle whose radius is 13 cm. Calculate the maximum clamping force that can be developed between the jaws of the clamp and the efficiency of the clamp.

The equivalent coefficient of friction $\mu' = \dfrac{\mu}{\cos \dfrac{30°}{2}} = 1{\cdot}04\,\mu.$

$$= 1{\cdot}04 \times 0{\cdot}15 = 0{\cdot}156.$$

φ = friction angle $= \tan^{-1} 0{\cdot}156 = 8°{\cdot}42'.$

Mean diameter of the screw $= \dfrac{1{\cdot}4 + 0{\cdot}95}{2} = 1{\cdot}175$ cm.

α = helix angle $= \tan^{-1} \dfrac{0{\cdot}4}{\pi \times 1{\cdot}175} = 6°{\cdot}10'.$

Let P be the maximum value of the clamping force.

Torque required to overcome friction at the thread

$$= P \cdot \frac{d_m}{2} \tan (\alpha + \varphi)$$

$$= P \times \frac{1{\cdot}175}{2} \tan(14°{\cdot}52') = 0{\cdot}1559P \text{ kg cm.}$$

Torque required to ovecome friction at the collar

$$= 0{\cdot}6 \times P \times 0{\cdot}15 = 0{\cdot}09P \text{ kg cm}$$

Total torque required to overcome friction $= 0{\cdot}2459P.$

Torque supplied $= 13 \times 12 = 156$ kg cm

$\therefore$ $$156 = 0{\cdot}2459P$$

or $$P = \frac{156}{0{\cdot}2459} = 634 \text{ kg.}$$

$$\eta = \frac{\text{useful work done}}{\text{energy supplied}} = \frac{634 \times 0{\cdot}4}{156 \times 2\pi}$$

$$= 0{\cdot}259, \text{ i.e. } 25{\cdot}9\%.$$

7. Fig. 11-10 shows a toggle jack to lift a load of 500 kg. The jack is operated by a lever 38 cm long. The eight links are symmetrical and 12 cm long. Determine (a) the lift of the jack, (b) the dimensions of the square threaded screw if the permissible tensile stress intensity is limited to 900 kg/sq cm; assume pitch of the threads as 6 mm, (c) the maximum torque to be applied to the screw shaft and (d) the length of the nut if the bearing pressure is limited to 200 kg/sq cm. Assume the coefficient of thread friction to be 0·15.

How will you design the pins and symmetrical links?

If n be number of threads in the nut, then

$$n \times 1{\cdot}8 \times 200 = 758$$

or $$n = \frac{758}{1{\cdot}8 \times 200} = 2{\cdot}1.$$

To have good stability and also to prevent rocking of the screw in the nut, we adopt four threads in the nut.

The pins are designed on 374 kg load and they are in double shear. Links are designed as struts which are hinged at both ends for buckling in the vertical plane and they are fixed at both ends for buckling in the plane at right angles to vertical plane.

(Please refer example 3 on page 420.)

8. *A 26 mm trapezoidal screw is 40 cm long between the nut and collar. The axial load is 2,400 kg and the torque in the screw between the nut and collar is 1,000 kg cm. The load is applied repetitively from zero to maximum. Assume the end fixity coefficient as 1. Take the actual stress concentration factor for threads in axial loading as 2·8 and for the threads in torsion as 2. Determine the factor of safety if a steel with a yield point 3,500 kg/sq cm is used and the endurance limit for the material for reversed bending is 2,200 kg/sq cm.*

As the nominal diameter of a trapezoidal screw is 26 mm, from the table we find the core diameter as 20·5 mm. The slenderness ratio is $\frac{4 \times 40}{2{\cdot}05} = 78$. Hence Rankine's formula is applicable.

For end fixity coefficient 1, Rankine's constant is $\frac{1}{7500}$. Hence the column effect is $1 + \frac{1}{7500} \times 78^2 = 1{\cdot}815$.

The equivalent compressive stress due to an axial load of 2,400 kg is $\frac{2400 \times 1{\cdot}815}{\frac{\pi}{4} \times 2{\cdot}05^2} = 1{,}220$ kg/sq cm.

As the load varies from zero to maximum, the equivalent normal stress due to the variable loading, with a maximum stress f_{en}, of 1,220 kg/sq cm, a mean stress of $\frac{1220}{2} = 610$ kg/sq cm and a variable stress of 610 kg/sq cm is (with usual notations)

$$f_{en} = f_a + \frac{f_y \, K \, f_m}{f_e \, ABC}$$

$$= 610 + \frac{3500 \times 2{\cdot}8 \times 610}{2200 \times 0{\cdot}7 \times 0{\cdot}85 \times 1} = 5{,}170 \text{ kg/sq cm.}$$

total tensile pull in the screw will be $2 \times 379 = 758$ kg. The screw is also subjected to torsional shear stress.

If d_c be the core diameter of the screw, then

$$\frac{\pi}{4} d_c^2 \times 900 = 758$$

or
$$d_c = \sqrt{\frac{758}{900} \times \frac{4}{\pi}} = 1{\cdot}1 \text{ cm.}$$

In order to account for the torsional shear stress, we adopt the core diameter as 16 mm. As the pitch of the thread is 6 mm, the outer diameter of the screw will $16 + 6 = 22$ mm and the mean diameter will be $16 + 3 = 19$ mm.

$$\alpha = \text{helix angle} = \tan^{-1} \frac{p}{\pi d_m} = \tan^{-1} \frac{6}{\pi \times 1{\cdot}9} = 5° - 42'.$$

$$\phi = \text{friction angle} = \tan^{-1} \mu = \tan^{-1} 0{\cdot}15 = 8° - 32'.$$

Torque required to overcome friction at both the surfaces will be $2 \times P \times \frac{d_m}{2} \tan (\alpha + \phi) = 2 \times 758 \times \frac{1{\cdot}9}{2} \tan (14° - 14')$

$$= 365 \text{ kg cm.}$$

Note: Toggle screw jacks are operated from one end only. However, provision is made to operate the jack from either end as shown in fig 11-10. If the screw is operated from both the sides, the portion of the screw between the nuts is subjected to tensile load only while the each end of the screw outside the nut is subjected to torque due to friction at the thread. If the jack is to be operated from one end, then the screw portion within the nuts is subjected to a friction torque at nut on the operating side in addition to tensile load.

Let us check the screw for principal stress.

$$\text{Maximum value of direct tensile stress} = \frac{758}{\frac{\pi}{4} \times 1{\cdot}6^2}$$

$$= 377 \text{ kg/sq cm.}$$

$$\text{Maximum torsional shear stress} = \frac{365}{\frac{\pi}{16} \times 1{\cdot}6^3} = 466 \text{ kg/sq cm.}$$

The maximum value of the torsional shear stress on the portion of screw between the nuts $= \frac{466}{2} = 233$ kg/sq cm.

$$\text{Principal stress} = \frac{377 \pm \sqrt{377^2 + 4 \times 233^2}}{2} = 488 \text{ kg/sq cm}$$

tensile and 111 kg/sq cm compressive. Thus the design is safe.

$$\text{Bearing area per thread} = \frac{\pi}{4} (2{\cdot}2^2 - 1{\cdot}6^2) = 1{\cdot}8 \text{ sq cm.}$$

(a) Determine the diameter of the root of the screw for an allowable stress in compression of 425 kg/sq cm.

(b) Determine the size of the lever to raise the load, the permissible stress in the material being 1,300 kg/sq cm.

(c) Determine the force required at the end of 150 cm long lever to raise the load.

(d) Determine the twisting moment exerted at the root of the thread.

(e) Determine the efficiency of the screw and collar.

Ans. (a) 6·5 cm; (b) 5 cm; (c) 91·5 kg;
(d) 9,500 kg; (e) 18·1%.

3. *A 20 cm gate valve is subjected to a maximum unbalanced pressure of 10 kg/sq cm. The stem of the valve is made of steel having square threads of 6 mm pitch. The coefficient of friction between wheel and its seat is 0·2, between nut and stem 0·15 and between wheel and its seat 0·12. The friction radius of the wheel is 4 cm.*

Determine the diameter of the valve stem assuming the permissible stress intensity of 550 kg/sq cm.

Calculate the pull that must be applied at the rim of 40 cm hand wheel to open the valve. *Ans.* 25 mm; 12 kg.

4. *A screw for the transmission of power is to propel the engaging nut at 20 metre/minute against a resistance of 2,800 kg. The quadruple square threaded screw is to have 1 thread per 2·5 cm. The root diameter is 4 cm. The collar lying between the screw and the source of power has an inside diameter of 6·5 cm and an outside diameter of 12·5 cm. Assuming the coefficient of friction to be 0·1, determine:*

(i) *the horse power required to drive the screw,*

(ii) *the efficiency of the screw and collar,*

(iii) *the resultant tensile or compressive stress and shearing stress in the root of the screw.*

Ans. (i) 19·4 h.p.; (ii) 64·5%;
(iii) 571 kg/sq cm; 347 kg/sq cm; 459 kg/sq cm.

5. *Design a square thread screw for a screw jack, which is to raise and support a load of 4,000 kg. The maximum lift is to be 45 cm. The elastic strength for the screw material may be taken as 2,400 kg/sq cm in tension and compression and 1,570 kg/sq cm in shear. The material of the nut is to be phosphor bronze for which the elastic strength may be taken*

where $A = 0{\cdot}7$ for axial loading

$B = 0{\cdot}85$ to take into account for the size

and $C = 1$ since actual stress concentration factor is used.

$$\text{Shearing stress due to torsion} = \frac{1000}{\frac{\pi}{16} \times 2{\cdot}05^3} = 600 \text{ kg/sq cm.}$$

The equivalent torsional shear stress, f_{es}, due to the variable loading with a maximum of 600 kg/sq cm, a mean stress of 300 kg/sq cm and a variable stress of 300 kg/sq cm is

$$f_{es} = 300 + \frac{3500 \times 0{\cdot}6 \times [\text{illegible}] \times 300}{2200 \times 0{\cdot}6 \times 0{\cdot}85 \times 1}$$

$$= 1{,}430 \text{ kg/sq cm.}$$

(The factor 0·6 is used to reduce the data to torsional shear loading.)

The equivalent shear stress due to the variable loading, according to maximum shear stress theory is

$$\sqrt{\left(\frac{5170}{2}\right)^2 + 1430^2} = 2{,}960 \text{ kg/sq cm.}$$

If N be the design factor or factor of safety, then

$$\frac{0{\cdot}5 \times 3500}{N} = 2960$$

or
$$N = \frac{0{\cdot}5 \times 3500}{2960}$$
$$= 0{\cdot}595.$$

Thus the proposed design is unsatisfactory. A suitable design factor will be from 1·5 to 2.

Exercises :

1. *A square thread screw 5 cm diameter is used to exert a force of 9,000 kg in a shaft straightening press. The maximum unsupported length of the screw is 45 cm. The pitch of the thread is 10 mm. (a) What is the equivalent compressive stress in the screw considering column effect only? (b) What torque is necessary to turn the screw against the load for $\mu = 0{\cdot}15$? (c) What is the efficiency of the screw?*

Ans. (a) 1,510 kg/sq cm, (b) 4,700 kg cm; (c) 30·8%

2. *A square, single threaded screw jack has 12 mm pitch. It is to lift 13 tonnes. The friction radius of the collar is 2·5 cm. The coefficient of friction between the threads of the screw and base is 0·15 and that between the screw and collar is 0·13.*

of the screw. Assume single start trapezoidal threads. Neglect buckling effect. Safe bearing pressure, for low velocities of rubbing, 175 kg/sq cm; safe fc for material of the screw 800 kg/sq cm.

11. *The moving head of a 25 tonne hydraulic testing machine is supported by two trapezoidal threaded screws. The screws are under the action of tensile load. Suggest the suitable size of the screw if the permissible stress is limited to 900 kg/sq cm. Take the coefficient of friction as 0·1.*

Determine the height of the nut if the bearing pressure is limited to 150 kg/sq cm.

Ans. 55 mm; 13 threads.

12. *A screw spindle with trapezoidal threads is used for reciprocating motion of a forked connecting link subjected to a maximum load of 2,500 kg. Suggest the suitable size of the spindle if the principal stress in the screw is not to exceed 500 kg/sq cm. The coefficient of friction is 0·15. Determine the height of the nut if the bearing pressure is limited to 100 kg/sq cm. What will be the horse power lost in friction if the spindle rotates at 100 r.p.m?*

Ans. 40 cm; 50 mm; 0·9.

13. *The lead screw of a lathe has trapezoidal threads. To drive the tool carriage the screw has to exert an axial force of 1,500 kg. The thrust is carried by the collar. The length of the lead screw is 150 cm. The coefficient of friction at collar and nut are 0·1 and 0·15 respectively.*

Assume the factor of safety to be 6.

Suggest the suitable size of the screw and the minimum number of threads in the nut if 40 kg/sq cm is the permissible bearing stress intensity.

The permissible stress intensity is limited to 500 kg/sq cm.

Ans. Tr 55 × 9; 7 cm.

14. *The trapezoidal screw of a small hand punch press is to exert a maximum load of 2,500 kg. Suggest the suitable size of the screw, the size of the body where the screw is located, the dimensions of the tommy bar operating the screw and the collar dimensions.*

The coefficient of friction for threads is 0·15 and for collar 0·1. The unsupported length of the screw is 25 cm. Bearing pressure for the nut and pad is 100 kg/sq cm. The permissible stress in the screw is limited to 1,000 kg/sq cm.

Ans. Tr 36 × 6; height of nut 54 mm; outside diameter of nut 72 mm; outside diameter of collar 70 mm.

as 1,250, 1,150 and 1,050 kg/sq cm in tension, compression and shear respectively. The bearing pressure between the threads of the screw and nut may be taken as 175 kg/sq cm. The coefficient of friction of the threads and collar may be taken as 0·14.

6. *The screw of a toggle press is driven by a gear G and turns at 80 r.p m. The cross head C moves along the screw in opposite directions against the axial force of 2 tonnes when the press is operated. Determine the horse power required to drive the gear, assuming the coefficient of friction in the threads to be 0 11 and neglect the friction in the bearings. The screw is 6 cm diameter with single start square threads of 1 cm pitch. Also, determine the maximum tensile stress and shear stress induced in the screw.* *Ans.* 2 07 *h.p.; 15 kg/sq cm; 64 kg/sq cm.*

7. *The two screws that support the moving head of 33,000 kg hydraulic testing machine have Tr 60 × 9 threads. Each one exerts half the load in tension. The nuts on these screws are used only for adjustments and do not turn under load (the force is supplied by oil pressure).*

(a) *Determine the tensile stresses in these screws under the maximum load of 33,000 kg*

(b) *Determine the number of threads required in the nuts if the permissible bearing pressure is 150 kg/sq cm.*

8. *The lead screw of a lathe has 5 cm Acme thread having 1 cm pitch. To drive the lood carriage this screw must exert an axial force of 400 kg The thrust is carried on a collar having a mean friction radius of 2 cm. The lead screw revolves at 40 r p.m.*

Determine the h.p. required to drive this screw and the efficiency of the screw and collar, assuming a coefficient of friction of 0·15 for the threads and 0·12 for the thrust collar.

Ans. 0·184; 19%.

9. *An axial load of 80 tonnes acts on a square threaded screw of inside diameter 9 cm. The depth of thread is 1·5 cm. A bearing pressure between nut and screw of 150 kg/sq cm is allowed. Obtain the length of nut necessary If the coefficient of friction between screw and nut is 0 12, calculate the stress in the screw due to torsion and end load.*

Ans. 39 *cm; 630 kg/sq cm (in shear); 1,250 kg/sq cm.*

10. *The screw and the nut of a screw clamp for securing work to, a machine table are shown in fig 11-6. Total compressive load on the screw is 6 tonnes. Calculate the height 'H' of the nut and the diameter*

The maximum force of 27 kg can be exerted by the operator.

The following values of permissible stresses are to be adopted:

Handle 1,050 kg/sq cm

Main body 1,050 kg/sq cm

Screw 1,300 kg/sq cm

Bearing pressure for the nut 175 kg/sq cm.

Ans. (i) 20 mm (ii) 26 mm outside diameter, (iii) =25 mm (iv) 70 mm.

2. A screw clamp with a gap of 13 cm and a maximum distance between jaws of 20 cm is required. It may be assumed that the maximum effort likely to be exerted at the end of the handle of effective length 30 cm is 30 kg.

Design the handle, screw, clamping pad at the end of the screw and the section of the frame at the middle of the gap assuming it to be of I section of proportions as $10t \times 6t \times t$ where t is the thickness of the web as well as flanges. Also, determine the height of the nut for the screw clamp.

Choose your own materials and suitable values for the stresses.

Show by means of a neat sketch the clamp in section with all dimensions marked on it.

3. The screw of a shaft straightener similar to one shown in fig. 11-7 is subjected to a maximum load of 3,000 kg. The screw has 75 mm outside diameter and the pitch is 10 mm.

(i) Determine the force required at the rim of 40 cm diameter hand wheel assuming that the coefficient of thread and collar friction is 0·13 and that the mean diameter of the collar is 7 cm. Square threads are cut on the screw.

(ii) If the height of the nut is 15 cm, determine the bearing pressure on the threads and shear stress in the threads.

(iii) Determine the maximum compressive stress in the screw.

(iv) Determine the efficiency of the straightener.

Ans. (i) 81kg (ii) 18·2 kg/sq cm; 19·6 kg-sq cm (iii) 102 kg/sq cm (iv) 29·6%

4. A screw press is to exert a force of 5,000 kg with applied torque of 5,000 kg cm. The maximum movement of the screw from the nut is 45 cm. Design the screw and nut and prepare a neat dimensioned drawing of the screw press.

The following materials are suggested:

Screw: steel

Nut : bronze

Choose your own values for the stresses.

5. The screw of a small hand punch press is shown in fig. 11-11. If the maximum force required for punching is 5,500 lb (2,750 kg), determine the size D, d, d_1, t and l if the punch is operated by one man.

The following data apply:

Collar diameter................$1\frac{1}{2}''$ (4 cm)

Coefficient of thread friction......0·14

Coefficient of collar friction........0·1.

15. The compressive load on the nut and screw clamp shown in fig. 11-7 is 2,500 kg. Calculate the diameter of the screw, height of the nut and the dimensions of the handle if a force of 30 kg is required to be applied at the end of a handle to operate the screw.

Safe compressive stress for screw	*= 1,200 kg/sq cm*
Bearing pressu for screw and nut	*= 175 kg/sq cm*
Coefficient of screw thread friction	*= 0·14*
Friction torque of pad B	*= 350 kg cm*
Bending stress in the handle	*= 1,000 kg/sq cm.*

Ans. Single start square thread of 5 mm pitch on outside diameter of 2·5 cm; height of the nut 5 cm; 22 mm diameter; effective length of handle 32 cm.

16. Design a screw and a nut for a 2 tonne screw press. Screw is of mild steel and the material of the nut is bronze. Use your own stresses for the materials used. *(University of Bombay, 1964)*

17. A 30 mm trapezoidal screw [illegible] 100 cm long between the nut and collar. The axial load is 1,200 kg and the torque in the screw between the nut and collar is 1,500 kg cm. Assuming that it is a hinged ended column, and neglecting the threads and stress concentration, determine the factor of safety if a steel with a yield point in tension of 3,500 kg/sq cm is used. Also determine the actual factor of safety if the load is applied repetitively from zero to maximum if the actual stress concentration factor for the threads in axial loading is 2·9 and the actual stress concentration factor for the threads in torsion is 2. The endurance limit for the material for reversed bending is 2,400 kg/sq cm

EXAMPLES XI

1. The following data refer to a C type of clamp Throat size 20 cm, gap 30 cm, lead of double start square thread [illegible] mm, length of the hand lever 30 cm; size of T section for clamp body $6t \times 4t$ where t [illegible] the thickness of a web and flanges.

Determine the following dimensions and clearly show the arrangement of attaching the pad to the screw:

(i) diameter of the handle
(ii) size of the screw
(iii) cross section of the main body
(iv) height of the nut.

8. Design a bottle jack (screw jack of bottle type) for a maximum load of $1\frac{1}{2}$ tons (1,500 kg) and a lift of 12" (30 cm). The head of the screw is provided with a plate and a thrust bearing to enable the screw to rotate independently of the plate while supporting the load. Materials used: cast iron for the body of the bottle, gun metal for the nut and mild steel for the screw.

Give a neat dimensioned sketch of the joint you design.

(Gujarat University, 1955)

9. The spindle screw of a screw jack moves in a nut. The screw has square threads, single start and pitch $\frac{1}{3}$ inch (8·5 mm). The effort is applied at the end of a single lever of effective length 16 inches (40 cm) and lateral movement of head is prevented. The load does not rotate but is carried on a swivel head, the bearing surface of which has a mean radius of $1\frac{1}{2}$ times that of the thread; μ for the thread is 0·12 and μ for swivel head and spindle 0·1. The load to be lifted is 1,325 lb (600 kg). Design (i) the spindle screw, (ii) nut, (iii) the lever and (iv) the swivel head.

Prepare a neat dimensioned drawing in two views. Assume suitable materials and stresses. (Poona University, 1956)

10. Design a toggle jack as shown in fig. 11-10 to lift a load of 200 lb (90 kg). Calculate also the force to be applied at the end of 12 in. (30 cm) lever in order to operate it. The eight links are each 4 in. (10 cm) long and the links, pins and spindle are all made of mild steel for which the safe stresses may be assumed as 10,000 psi (700 kg/sq cm) in tension, 8,000 psi (560 kg/sq cm) in shear and 15,000 psi (1,050 kg/sq cm) in crushing.

Draw fully dimensioned assembly drawing giving at least two views.

(Gujarat University, 1957)

11. The screw press similar to one shown in fig. 11-8 has a cast iron *T* section frame with a steel screw having a square thread screwing into a bronze bush. The following is a list of design requirements and permissible stresses:

Maximum load exerted by screw—3 tonnes

Maximum torque applied to screw through the hand wheel 1,200 kg cm

Gap = 20 cm.

Cast iron frame:

Maximum tensile stress — 150 kg/sq cm

Maximum compressive stress — 700 kg/sq cm

Screw:

Maximum compressive stress — 700 kg/sq cm

Maximum shear stress — 450 kg/sq cm

Bush:

Maximum shear stress — 200 kg/sq cm.

The bearing pressure between screw and bushing must not exceed 100 kg/sq cm. The screw must not unwind.

Coefficient of friction between screw and bush: $\mu = 0{\cdot}12$.

Calculate suitable dimensions for the screw and bush, find the efficiency of the screw and design a suitable section *AB* for the frame.

Bearing pressure for the threads 2,500 psi (175 kg/sq cm)
Elastic limits for the material of the screw and handle:
34,000 psi (2,300 kg/sq cm) in tension and compression
22,000 psi (1,540 kg/sq cm) in shear.
Give sketches of working drawings of the screw and handle

(M. S. University of Baroda, 1954)

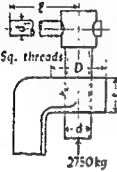


FIG. 11-11

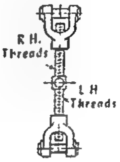


FIG. 11-12

6 The tie bar coupling, the outlines of which are shown in fig 11-12 has to be designed for a maximum static load of 30 tonnes. The adjustment of the length of the tie-bar is made by rotating the screw

Making suitable assumptions, calculate the dimensions of the coupling Maximum adjustment for length required is 10 cm

Safe stress in steel in tension	1,050 kg sq cm
Safe stress in steel in shear	– 700 kg sq cm
Safe stress in steel in crushing	– 1,050 kg/sq cm

Coefficient of friction between steel and steel is 0 2. Make a neat dimensioned sketch of the coupling you design

7 A simple hand operated screw jack is to be capable of raising a maximum load of 2 tons (2,000 kg) through a vertical distance of 6 in (15 cm) With the screw in its lowest position, the over all height of the jack is to be 1 ft (30 cm).

Design a suitable jack, assuming the steel used in its construction to have an ultimate tensile strength of 35 ton/sq in (55 kg/sq mm) and yield stress of 24 ton/sq in (37 8 kg/sq mm).

The greatest manual effort required to operate the jack must not be more than 20 lb (9 kg)

Assume a coefficient of friction of 0 10. (The mechanical efficiency of a screw and nut $= \dfrac{\tan\alpha}{\tan(\alpha+\phi)}$ where α is the helix angle and ϕ the angle of friction.)

Give a fully dimensioned sketch of the jack. (A.M.I.C.E., 1954)

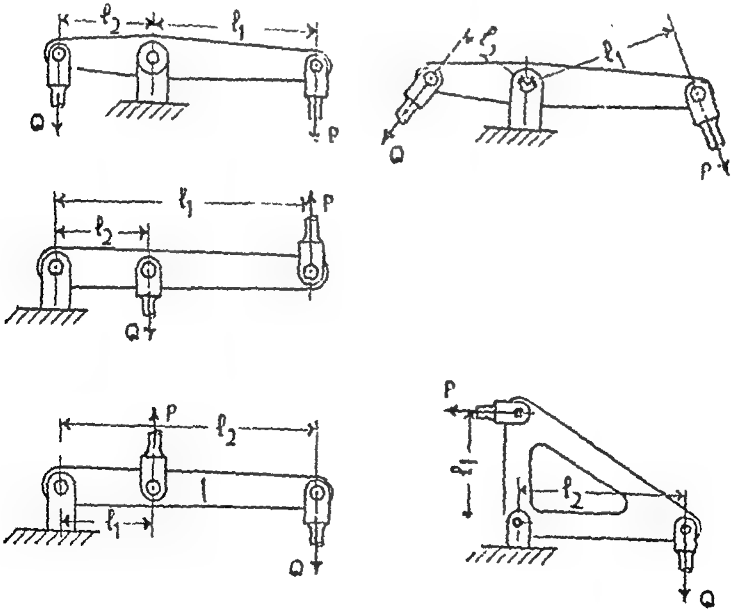
CHAPTER XII

LEVERS

12-1. Introduction:

A rigid rod which is capable of turning about a fixed point is called a lever. The lever may be straight or curved and the forces exerted on or by the lever may be parallel or may be inclined to one another.

The levers may be classified as one arm lever, two arm lever, cranked lever or angular lever, cross lever and compound lever. Fig. 12-1 shows various kinds of levers.



Various forms of levers
FIG. 12-1

The perpendicular distances of the forces from the fulcrum are called the *arms of the lever.* The ratio of the arm of the effort to the arm of the resistance is called the *leverage.*

12. Fig 11-13 shows the spindle of 20 cm steam stop valve used for a maximum pressure of 14 kg/sq cm

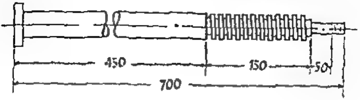


FIG. 11-13

Design: (i) The diameter of spindle, material M S., factor of safety 5, using Rankine formula

(ii) The pitch of the screw The outside diameter of the screw to be equal to or slightly less than the spindle diameter. The coefficient of friction between screw and nut = 0 08 The mean torque on hand wheel = 1,500 kg cm.

(iii) The number of threads in the gun metal nut Bearing pressure 150 kg/sq cm

(iv) Size of arms in a 4 arm 40 cm diameter hand wheel

13 A power screw on a machine has a single start square thread which engages with a non-rotating bronze nut The axial force on the nut is 2,000 lb (900 kg) and the core diameter is 1 in (2 5 cm) Find·

(a) suitable thread dimensions, (b) the length of the nut, (c) the efficiency of the screw if the coefficient of friction between screw and nut is 0 12 and (d) the h p. required to move the nut with a velocity of 10 ft/min (3 metre/min)

The allowable stresses are
Maximum shear stress in screw 6,000 psi (420 kg/sq cm)
Maximum shear stress in bronze 4,000 psi (280 kg/sq cm)
Maximum bearing pressure 200 psi (14 kg/sq cm).

(Manchester University, 1959)

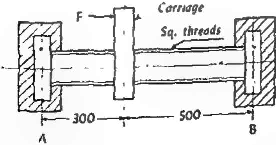


FIG. 11-14

14. Fig. 11-14 shows a schematic diagram of lead screw on a metal working lathe. The screw is mounted in two thrust bearings A and B. The force F=250

The area of cross section of the rod $B = \frac{\pi}{4} \times 1{\cdot}3^2$

$= 1{\cdot}327$ sq cm.

Tensile stress in tie rod will be $\frac{600}{1{\cdot}327} = 452$ kg/sq cm.

The pins, A, B and C are of 13 mm diameter. The pins A and B are in double shear while that at C is in single shear.

Shear stress in pin at $C = \frac{100}{1{\cdot}327} = 75{\cdot}2$ kg/sq cm.

Shear stress in pin at $B = \frac{600}{2 \times 1{\cdot}327} = 225{\cdot}6$ kg/sq cm.

Shear stress in fulcrum pin $A = \frac{500}{2 \times 1{\cdot}327} = 188{\cdot}5$ kg/sq cm

Bearing area of pin at B = diameter of pin $\times$ thickness of lev

$= 1{\cdot}3 \times 1{\cdot}8 = 2{\cdot}34$ sq cm.

Bearing stress $= \frac{600}{2{\cdot}34} = 256$ kg/sq cm.

2. *Fig. 12-3 shows a handle for turning the spindle of a large va. An effort of 35 kg is applied at each end of the handle. The hand attached to the valve spindle by a round tapered key. Calculate the di sions of the handle and the mean diameter of the tapered key. Allow tensile stress for handle and key = 1,100 kg/sq cm. Shear stress fo 550 kg/sq cm.*

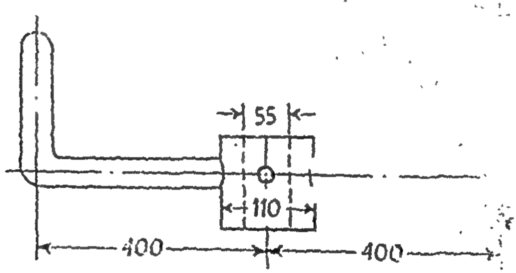


FIG. 12-3

The torque on the shaft $= 35 \times 2 \times 40 = 2{,}8$

The tapered key is in double shear. If d_1 be th meter of the tapered key then

$$2 \times \frac{\pi}{4} d_1^2 \times 550 \times \frac{5{\cdot}5}{2} = 2800$$

If a great leverage is required for any purpose, that is, if a large resistance is to be overcome by a small effort, the arm of the effort must be *very much* greater than the arm *of the resistance*. To obtain a great leverage without the use of *excessively long* levers, compound levers are used. The *compound* levers may be made up of straight pieces, which may be *attached with* one another by means of pin-joints. Instead of having a *number* of jointed levers, bell cranked levers may be used.

The leverage of compound levers is the product of leverages of various levers. Levers are very much used in engineering. In many applications in engineering levers are used to change the direction of forces i e. rocker arms in I. C. engines and bell crank levers in railway signals.

12-2. General procedure for Design of Levers:

The principle of a lever is practically the principle of moments. A lever is acted upon by the reaction of the fulcrum and by two other single forces or two sets of forces, one causing a turning moment in one and the other a turning moment in the opposite direction. Generally, one set of forces acting on one arm of the lever is specified. The forces acting on the other arm of the lever are obtained by taking moment about the fulcrum.

The lever arms may be equal; in that case the effort applied and the resistance overcome are numerically equal if we neglect the effect of friction at the journal bearing. When the arms are unequal, the effort applied and the resistance overcome are in the inverse proportion of the length of the respective arms.

The reaction at the fulcrum will be the resultant of P and Q. When P and Q are parallel and the same direction, then the reaction at the fulcrum, will be the sum of P and Q. When P and Q are parallel and in opposite directions, the reaction at the fulcrum will be the difference of two forces P and Q. When the lines of action of P and Q are inclined to one another, the reaction at the fulcrum is determined by parallelogram of forces and is equal to the resultant of P and Q. The line of action of reaction passes through the intersection of the lines of action of P and Q and also through the fulcrum. If the forces P and Q act at right angles to two arms of the lever, which include an

The area of cross section of the rod $B = \frac{\pi}{4} \times 1{\cdot}3^2$
$= 1{\cdot}327$ sq cm.

Tensile stress in tie rod will be $\frac{600}{1{\cdot}327} = 452$ kg/sq cm.

The pins, A, B and C are of 13 mm diameter. The pins A and B are in double shear while that at C is in single shear.

Shear stress in pin at $C = \frac{100}{1{\cdot}327} = 75{\cdot}2$ kg/sq cm.

Shear stress in pin at $B = \frac{600}{2 \times 1{\cdot}327} = 225{\cdot}6$ kg/sq cm.

Shear stress in fulcrum pin $A = \frac{500}{2 \times 1{\cdot}327} = 188{\cdot}5$ kg/sq cm.

Bearing area of pin at B = diameter of pin $\times$ thickness of lever
$= 1{\cdot}3 \times 1{\cdot}8 = 2{\cdot}34$ sq cm.

Bearing stress $= \frac{600}{2{\cdot}34} = 256$ kg/sq cm.

2. *Fig. 12-3 shows a handle for turning the spindle of a large valve. An effort of 35 kg is applied at each end of the handle. The handle is attached to the valve spindle by a round tapered key. Calculate the dimensions of the handle and the mean diameter of the tapered key. Allowable tensile stress for handle and key = 1,100 kg/sq cm. Shear stress for key 550 kg/sq cm.*

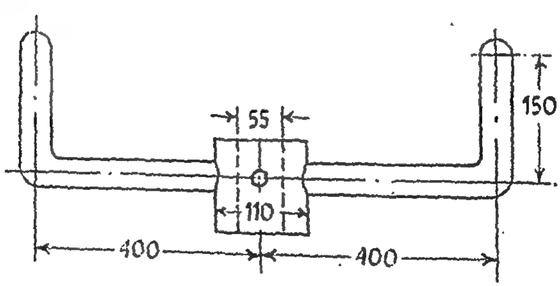


FIG. 12-3

The torque on the shaft $= 35 \times 2 \times 40 = 2{,}800$ kg cm.

The tapered key is in double shear. If d_1 be the mean diameter of the tapered key then

$$2 \times \frac{\pi}{4}\, d_1^2 \times 550 \times \frac{5{\cdot}5}{2} = 2800$$

or $$d_1 = \sqrt{\frac{2800 \times 2 \times 4}{2 \times \pi \times 550 \times 5 \cdot 5}} = 1 \cdot 1 \text{ cm.}$$

Assuming that the bending moment arm of the handle extends upto the axis of the spindle, the maximum bending moment on the arm will be $40 \times 35 = 1{,}400$ kg cm.

The arm is also subjected to a twisting moment also, the magnitude of which depends on the point of application of the load of 35 kg. The value of twisting moment will not exceed $35 \times 10 = 350$ kg cm. We neglect the effect of twisting at present, and design the lever on bending alone.

If d cm be the diameter of the handle, then

$$\frac{\pi}{32} d^3 \times 1100 = 1400$$

or $$d = \sqrt[3]{\frac{1400}{1100} \times \frac{32}{\pi}} = 2 \cdot 46 \text{ cm};$$ we adopt the diameter of the handle as 2·7 cm. The diameter has been increased to allow for the effect of torsional shear

Exercises

1. A casting is being raised by a 90 cm crow-bar which is supported at 5 cm from the end where it takes the weight. If the casting weighs 1 tonne and half this is taken by the crow-bar, what force must be applied at the end of the bar in order to raise the casting? What will be the minimum diameter of the crow-bar if the permissible stress in the material is limited to 850 kg/sq cm? *Ans. 27·8 kg; 3·2 cm.*

2. A 25 mm hand reamer is being operated by hand pressure at each end of a lever 60 cm long. The hole is 25 mm long and each of the six teeth is taking a cut of 0·005 cm. If the cutting pressure at the teeth is 7,000 kg/sq cm of cut, estimate the diameter of the lever if the permissible stress is limited to 1,050 kg/sq cm *Ans. 15 mm.*

3. A symmetrical body weighing 600 kg is lifted from one end by an arrangement shown in fig. 12-4. If a force P is applied to raise the body, design the lever.

Given

allowable bearing pressure for pins 70 kg/sq cm

allowable tensile stress for pins and lever 700 kg/sq cm.

or $$d_1 = \sqrt{\frac{2800 \times 2 \times 4}{2 \times \pi \times 550 \times 5 \cdot 5}} = 1 \cdot 1 \text{ cm.}$$

Assuming that the bending moment arm of the handle extends upto the axis of the spindle, the maximum bending moment on the arm will be $40 \times 35 = 1{,}400$ kg cm.

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Exercises

1. A casting is being raised by a 90 cm crow-bar which is supported at 5 cm from the end where it takes the weight. If the casting weighs 1 tonne and half this is taken by the crow-bar, what force must be applied at the end of the bar in order to raise the casting? What will be the minimum diameter of the crow-bar if the permissible stress in the material is limited to 850 kg/sq cm? *Ans.* 27·8 kg; 3·2 cm.

2. A 25 mm hand reamer is being operated by hand pressure at each end of a lever 60 cm long. The hole is 25 mm long and each of the six teeth is taking a cut of 0 005 cm. If the cutting pressure at the teeth is 7,000 kg/sq cm of cut, estimate the diameter of the lever if the permissible stress is limited to 1,050 kg/sq cm *Ans.* 15 mm.

3. A symmetrical body weighing 600 kg is lifted from one end by an arrangement shown in fig 12-4. If a force P is applied to raise the body, design the lever.

Given

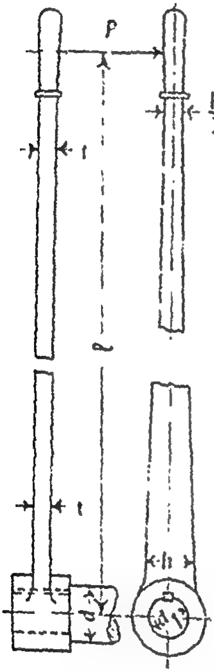
allowable bearing pressure for pins 70 kg/sq cm

allowable tensile stress for pins and lever 700 kg/sq cm.

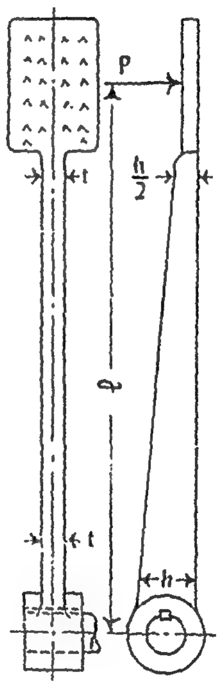
in greatest and 25 mm in smallest diameter and 125 mm long. The force exerted by a man may be normally taken as 30 kg.

Let d_1 be the diameter of the shaft in the eye of the lever. This part of the shaft is subjected to twisting moment Pl only. If the permissible stress for the shaft material is known, the diameter d_1 of the shaft is obtained by the usual torsion formula

$$Pl = \frac{\pi}{16} d_1^3 f_s \quad \text{.....................................(ii)}$$



Hand lever
FIG. 12-8



Foot lever
FIG. 12-9

If the shaft overhangs the bearing, then it is subjected to a twisting moment Pl and a bending moment Px where x is overhung from centre of lever to centre of nearest bearing of the shaft. The equivalent twisting moment, according to Guest formula, will be $P\sqrt{l^2+x^2}$. If d be the diameter of the shaft in the bearing, then

$$\frac{\pi}{16} d^3 f_s = P\sqrt{l^2+x^2} \quad \text{.....................................(iii)}$$

6. A hand lever with knurled handle (fig. 12-7) is used to overcome the resistance of 100 lb (50 kg) as shown. Design and prepare a dimensioned drawing of the lever if it is made of mild steel.

(Poona University, 1960)

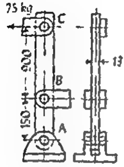


FIG. 12-6

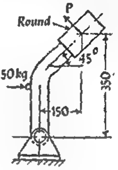


FIG. 12-7

12-3. Hand lever:

Fig. 12-8 shows a hand lever commonly adopted in various machineries.

Let P = maximum force exerted at the handle
l = effective length of the lever
t = thickness of the lever at the boss
h = height of the lever at the boss.

We assume that the moment arm of the lever extends upto the centre. This assumption gives stronger section of the lever near the boss

Maximum bending moment on the lever $= Pl$.

Modulus of section $= \frac{1}{6}\, t\, h^2$.

By equating the applied bending moment to resisting moment of the lever, we get

$P\,l = \frac{1}{6}\, t\, h^2 f$ .... (1)

where f is the permissible stress for the lever material.

From the above equation, the dimensions of the lever at the boss can be calculated. Generally $t = \frac{1}{2}h$ and the thickness of the lever is kept constant, while its depth varies. The least depth near the grip of the handle should be half its greatest depth, which is near the boss. The part grasped by hand may be 32 mm

The handle of the lever is generally of circular section and is subjected to bending only. The pressure on the handle may be taken to act at $\frac{2}{3}$ of its length. The maximum bending moment may be taken as $\frac{2}{3}$ *Pl* where *P* is the maximum applied force at the effective length $\frac{2}{3}l$.

If *d* be the diameter of the handle of the lever, then

$$\frac{2}{3} P\, l = \frac{\pi}{32} d^3 f \quad \text{.......................... (i)}$$

where *f* is the permissible bending stress for the lever material. The arm portion of the lever is generally made of rectangular section having constant thickness and more or less constant depth. The arm is subjected to constant twisting moment of the magnitude $\frac{2}{3}$ *Pl* and the varying bending moment which is maximum near the boss and it may be taken as *Pr*, if we assume that the moment arm extends upto the axis of the shaft.

At present time, there is insufficient information on the subject of the combined bending and twisting of rectangular shafts to enable an equivalent bending or twisting moment to be calculated with sufficient accuracy. Therefore, indirect procedure will be adopted

It is worth noticing that for the usual proportions of the cranked lever, the twisting moment never exceeds the maximum bending moment. If these were combined in the case of a circular shaft, the equivalent bending moment would be never greater than 21% of the simple bending moment. So we design the arm for 25% more bending moment, and determine the dimensions for the thickness and width of the arm and then check for the stresses induced. The maximum value of the principal stress or the maximum shear stress should not exceed the permissible limit.

The torsional shear stress caused by $\frac{2}{3}$ *Pl* has a maximum value at the middle of the long side and somewhat smaller maximum at the middle of the short side of the rectangular section. *The maximum torsional shear stress will be at the end of a smallest radius vector of the section.* The bending stress has a maximum value of uniform amount along the whole short side. Evidently this maximum bending stress combines itself with the shear stress at the middle of the short side to form the principal stress or maximum shear stress, which must be computed to ascertain whether the arm of the cranked lever is safe.

If Rankine's formula were to be the design criterion, then

$$\frac{\pi}{16}\, d^3 f = P\,(l + \sqrt{l^2 + x^2}) \quad \text{(iv)}$$

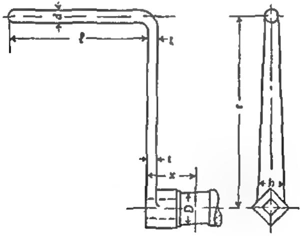
The eye of the lever may have a thickness 0·3d and a length of the eye may vary from d to 1·5d.

12-4. Foot lever:

Fig. 12-9 shows a foot lever. The foot-plate is about 20 cm by 12 cm by 16 mm thick. In designing this lever the maximum force P is taken to be 80 kg. The design procedure for calculating the various dimensions of the lever and shaft diameter is exactly similar to one outlined for the design of a hand lever.

12-5. Cranked lever:

Fig. 12-10 shows a cranked lever or a winch handle. This lever may be operated either by one person or by two persons.



Cranked lever
FIG. 12-10

When it is operated by one person, the maximum force may be taken as 40 kg and the length of the handle may be from 25 to 30 cm. When the cranked lever is worked by two persons, the maximum force will be doubled and the length of the handle may be 50 cm. The radius r is usually from 40 to 45 cm and the height of the shaft from the ground may be from 90 cm to 100 cm.

$$\frac{\pi}{16} d^3 \times 200 = 1500$$

or $$d = \sqrt[3]{\frac{1500}{200} \times \frac{16}{\pi}} = 3{\cdot}4 \text{ cm; we adopt } 3{\cdot}5 \text{ cm.}$$

$$\text{Twist of shaft per metre length} = \frac{2f_s l}{Gd}$$

$$= \frac{2 \times 200 \times 100}{84 \times 10^4 \times 3{\cdot}5}$$

$$= 0{\cdot}0136 \text{ radian.}$$

Distance moved by the point of application of the load will be equal to $0{\cdot}0136 \times 3 = 0{\cdot}0408$ metre i.e. 4·08 cm.

Let us assume that the section of the lever is rectangular having the thickness t equal to $\frac{3}{8}h$.

Maximum bending moment $= 15 \times 100 = 1{,}500$ kg cm.

$$1500 = \tfrac{1}{6} \times \tfrac{3}{8} h \times h^2 \times 850$$

or $$h = \sqrt[3]{\frac{1500 \times 8 \times 6}{850 \times 3}} = 3{\cdot}1 \text{ cm; we adopt } 3{\cdot}5 \text{ cm and}$$

$$t = 1{\cdot}5 \text{ cm.}$$

The diameter of the eye of the lever may be taken as 7 cm. Length of the eye may be taken as 5·5 cm.

2. *A foot lever is 60 cm from the centre of the shaft to the point where the load acts. The load is 90 kg. Calculate the diameter of the shaft, the dimensions of the sunk rectangular key and the depth of the lever assuming it to be of rectangular section and the width of the lever being $\frac{3}{8}$ of the depth.*

Let us assume the following stresses:

f_s for shaft and key 560 kg/sq cm; f_t for lever 1,050 kg/sq cm.

Torque on the shaft $= 90 \times 60 = 5{,}400$ kg cm.

If d_1 cm be the diameter of the shaft on which the lever is keyed, then

$$\frac{\pi}{16} d_1^3 \times 560 = 5400$$

or $$d_1 = \sqrt[3]{\frac{5400 \times 16}{560 \times \pi}} = 3{\cdot}7 \text{ cm; we adopt 4 cm.}$$

According to the usual proportions for key, we adopt the following dimensions:

Length of the key = 7 cm; thickness of the key 1 cm and width of the key 1·3 cm.

The following formulas shall be useful in checking the stresses in the section of the arm near the boss.

For a square section of side a,

$$T = \tfrac{2}{9}\, a^3 f_s \quad \text{......(ii)}$$

For a rectangular section of thickness t and depth h,

$$T = \tfrac{2}{9}\, ht^2 f_s \quad \text{......(iii)}$$

For an elliptical section of major axis h and minor axis b,

$$T = \frac{\pi}{16}\, b^2 h f_s \quad \text{......(iv)}$$

The journal of the shaft is subjected to a twisting moment Pr and a bending moment $P\,(\tfrac{3}{8}l + x)$. The equivalent twisting moment will be $P\sqrt{r^2 + (\tfrac{3}{8}l + x)^2}$. When the value of permissible shear stress is known, the diameter of the journal can be obtained.

The cranked lever is subjected to very rough usage and must be made strong enough to stand it.

In practice the following proportions are adopted:

$D = 3{\cdot}2$ cm to $3{\cdot}8$ cm for one person

$D = 3{\cdot}8$ cm to $4{\cdot}5$ cm for two persons

$d = 2{\cdot}5$ cm for one person

$d = 3{\cdot}8$ cm for two persons

Thickness of eye $= \tfrac{3}{4}D$ to $\tfrac{1}{2}D$

Length of eye $= 5$ cm to $6{\cdot}5$ cm

Section of arm at eye 2 cm $\times$ 5 cm.

The procedure outlined for the design of the arm of the cranked lever may be adopted for the design of overhung cranks in engines.

Examples:

1 A hand lever is mounted on a shaft The maximum force of 15 kg is applied by the worker at the moment arm of 1 metre. Determine the diameter of the solid shaft if the permissible shear stress is limited to 200 kg/sq cm. What is the twist of the shaft per metre length? What is the distance moved by the point of application of the load due to elasticity of the shaft? The modulus of rigidity is $8{\cdot}4 \times 10^5$ kg/sq cm. The length of the shaft is 3 metre. Determine the section of the lever near the boss if the permissible stress is limited to 850 kg/sq cm.

Torque on the shaft $= 15 \times 100 = 1{,}500$ kg cm.

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16}\, d^3 \times 200 = 1500$$

or $$d = \sqrt[3]{\frac{1500}{200} \times \frac{16}{\pi}} = 3{\cdot}4 \text{ cm; we adopt } 3{\cdot}5 \text{ cm.}$$

$$\text{Twist of shaft per metre length} = \frac{2f_s l}{Gd}$$

$$= \frac{2 \times 200 \times 100}{84 \times 10^4 \times 3{\cdot}5}$$

$$= 0{\cdot}0136 \text{ radian.}$$

Distance moved by the point of application of the load will be equal to $0{\cdot}0136 \times 3 = 0{\cdot}0408$ metre i.e. 4·08 cm.

Let us assume that the section of the lever is rectangular having the thickness t equal to $\frac{3}{8}h$.

Maximum bending moment $= 15 \times 100 = 1{,}500$ kg cm.

$$1500 = \tfrac{1}{6} \times \tfrac{3}{8}\, h \times h^2 \times 850$$

or $$h = \sqrt[3]{\frac{1500 \times 8 \times 6}{850 \times 3}} = 3{\cdot}1 \text{ cm; we adopt } 3{\cdot}5 \text{ cm and}$$

$$t = 1{\cdot}5 \text{ cm.}$$

The diameter of the eye of the lever may be taken as 7 cm. Length of the eye may be taken as 5·5 cm.

2. *A foot lever is 60 cm from the centre of the shaft to the point where the load acts. The load is 90 kg. Calculate the diameter of the shaft, the dimensions of the sunk rectangular key and the depth of the lever assuming it to be of rectangular section and the width of the lever being $\frac{3}{8}$ of the depth.*

Let us assume the following stresses:

f_s for shaft and key 560 kg/sq cm; f_t for lever 1,050 kg/sq cm.

Torque on the shaft $= 90 \times 60 = 5{,}400$ kg cm.

If d_1 cm be the diameter of the shaft on which the lever is keyed, then

$$\frac{\pi}{16}\, d_1^3 \times 560 = 5400$$

or $$d_1 = \sqrt[3]{\frac{5400 \times 16}{560 \times \pi}} = 3{\cdot}7 \text{ cm; we adopt 4 cm.}$$

According to the usual proportions for key, we adopt the following dimensions:

Length of the key = 7 cm; thickness of the key 1 cm and width of the key 1·3 cm.

The outside diameter of the boss will be 8 cm.

If t be the thickness of the lever and h the depth, then modulus of section of the lever $= \frac{1}{6}th^2 = \frac{1}{6} \times \frac{3}{8}h^3 = \frac{h^3}{16}$.

$\therefore \quad \frac{h^3}{16} \times 1050 = 5400$

or $$h = \sqrt[3]{\frac{5400 \times 16}{1050}} = 4\text{·}35 \text{ cm; we adopt } 4\text{·}5 \text{ cm.}$$

Thickness of the lever $= 2$ cm.

3. *The cranked lever as shown in fig. 12-10 is operated by a single person exerting a maximum pressure of 40 kg at a distance of one-third length of the handle from its free end. The length of the handle is 30 cm and the length of the crank arm is 40 cm. Determine the necessary size of the journals if the overhung is 10 cm. Also, determine the diameter of the handle. The permissible value of the shear stress for the shaft material is limited to 550 kg/sq cm and the permissible tensile stress for the lever is 850 kg/sq cm.*

The journal of the shaft is subjected to a twisting moment of $40 \times 40 = 1{,}600$ kg cm and a bending moment of $40\ [\frac{2}{3} \times 30 + 10] = 1{,}200$ kg cm. The equivalent twisting moment will be equal to $\sqrt{1600^2 + 1200^2} = 2{,}000$ kg cm

If d cm be the diameter of the journal, then

$\frac{\pi}{16}\, d^3 \times 550 = 2000$

or $$d_1 = \sqrt[3]{\frac{2000}{550} \times \frac{16}{\pi}} = 3\text{·}9 \text{ cm, we adopt 4 cm.}$$

Maximum bending moment on the handle $= 40 \times \frac{2}{3} \times 30$
$= 800$ kg cm.

If d_1 cm be the diameter of the handle, then

$\frac{\pi}{32}\, d_1^3 \times 850 = 800$

or $$d_1 = \sqrt[3]{\frac{800 \times 32}{850 \times \pi}} = 2\text{·}12 \text{ cm, we adopt } 2\text{·}2 \text{ cm.}$$

Exercises:

1. *Make a sketch design of a hand lever for a brake making the length of the handle 90 cm, effective overhang from the nearest bearing being 15 cm. Assuming that the greatest pull on the ends is 36 kg and*

that the maximum stress in shear, tension and compression is 630 kg/sq cm determine the diameter of the spindle.

2. *Make a sketch design of a foot brake lever, length of the lever being 90 cm. Assume that the greatest load that can come on the foot plate is 80 kg effective overhang from the nearest bearing being 15 cm. Determine the diameter of the shaft if the value of the maximum shear stress is limited to 500 kg/sq cm.*

3. *The distance between the centres of two bearings, which support a pedal lever brake shaft, is 37 cm. The effective length of the pedal is 23 cm, and that of the lever, which is in the same plane but at the opposite side of the shaft, is 10 cm. The latter is 20 cm from one bearing and the former is 15 cm from the other bearing. Determine the diameter of the shaft if the permissible shear stress is not to exceed 560 kg/sq cm. Determine the dimensions of the pedal lever at the boss if the breadth is four times the thickness. The value of the permissible stress in the material of the lever is not to exceed 630 kg/sq cm.*

4. *Fig. 12-11 shows a cranking lever for a Diesel engine. P is a pin, on which a force of 50 kg is applied at a point F, 162 mm away from the centre line of the crank web C. Crank radius is 275 mm. S is a boss, 30 mm long having a square hole to connect the lever with the square end of the crank shaft. Design and prepare the drawing of the cranking lever.*

Material used — mild steel.

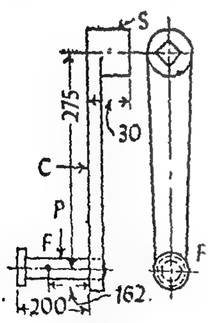


Fig. 12-11

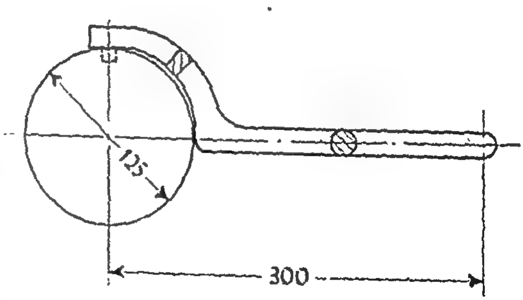


Fig. 12-12

5. *A mild steel hook wrench (fig. 12-12) has a force of 50 lb (22·5 kg) applied at the handle. Design and prepare a dimensioned drawing.*

(*Poona University, 1960*)

The outside diameter of the boss will be 8 cm.

If t be the thickness of the lever and h the depth, then modulus of section of the lever $= \frac{1}{6}th^2 = \frac{1}{6} \times \frac{3}{8}h^3 = \frac{h^3}{16}$.

$$\therefore \quad \frac{h^3}{16} \times 1030 = 5400$$

or $$h = \sqrt[3]{\frac{5400 \times 16}{1050}} = 4{\cdot}35 \text{ cm; we adopt } 4{\cdot}5 \text{ cm.}$$

Thickness of the lever $= 2$ cm.

3. The cranked lever as shown in fig. 12-10 is operated by a single person exerting a maximum pressure of 40 kg at a distance of one-third length of the handle from its free end. The length of the handle is 30 cm and the length of the crank arm is 40 cm. Determine the necessary size of the journals if the overhung is 10 cm. Also, determine the diameter of the handle. The permissible value of the shear stress for the shaft material is limited to 550 kg/sq cm and the permissible tensile stress for the lever is 850 kg/sq cm.

The journal of the shaft is subjected to a twisting moment of $40 \times 40 = 1{,}600$ kg cm and a bending moment of $40\ [\frac{2}{3} \times 30 + 10] = 1{,}200$ kg cm. The equivalent twisting moment will be equal to $\sqrt{1600^2 + 1200^2} = 2{,}000$ kg cm

If d cm be the diameter of the journal, then

$$\frac{\pi}{16}\, d^3 \times 550 = 2000$$

or $$d_1 = \sqrt[3]{\frac{2000}{550} \times \frac{16}{\pi}} = 3{\cdot}9 \text{ cm, we adopt } 4 \text{ cm}$$

Maximum bending moment on the handle $= 40 \times \frac{2}{3} \times 30$
$= 800$ kg cm

If d_1 cm be the diameter of the handle, then

$$\frac{\pi}{32}\, d_1^3 \times 850 = 800$$

or $$d_1 = \sqrt[3]{\frac{800 \times 32}{850 \times \pi}} = 2{\cdot}12 \text{ cm; we adopt } 2{\cdot}2 \text{ cm.}$$

Exercises:

1. Make a sketch design of a hand lever for a brake making the length of the handle 90 cm, effective overhang from the nearest bearing being 15 cm. Assuming that the greatest pull on the ends is 36 kg and

The maximum steam load, at which the valve blows off is $\frac{\pi}{4} \times 7{\cdot}5^2 \times 13 = 550$ kg.

As the leverage is $\frac{80}{8} = 10$, the magnitude of the dead load to be kept at the end of the lever will be $= \frac{550}{10} = 55$ kg.

As the steam load and dead load are parallel and opposite to each other, the reaction of the fulcrum on the lever must be vertically downwards and is equal to $550 - 55 = 495$ kg.

The pin at the valve spindle is first of all designed from bearing considerations. Let us assume that the width of the lever is equal to the diameter of the pin. If d_p be the diameter of the pin, then

$$d_p \times d_p \times 245 = 550$$

or

$$d_p = \sqrt{\frac{550}{245}} = 1{\cdot}5 \text{ cm.}$$

The pin is in double shear. If f_s be the shear stress induced in the pin, then $2 \times \frac{\pi}{4} \times 1{\cdot}5^2 \times f_s = 550$.

$\therefore \quad f_s = \frac{550 \times 4}{2 \times \pi \times 1{\cdot}5^2} = 156$ kg/sq cm which is very much less than the permissible value.

As the load at the fulcrum pin is not very much different from that at the spindle pin, we take both the pins of the same diameter to facilitate the interchangeability of parts.

We provide 2 mm thick gun metal bushes at both the pin holes to take up the wear. The outside diameter of the boss is kept twice the diameter of the hole. We adopt 3 cm.

The maximum bending moment$=55\ (80-8)=4{,}000$ kg cm.

Let h be the depth of the lever. The thickness of the lever is 1·5 cm.

$$\therefore \quad 4000 = \tfrac{1}{6} \times 1{\cdot}5 \times h^2 \times 700$$

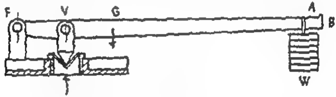
or

$$h = \sqrt{\frac{4000 \times 6}{1{\cdot}5 \times 700}} = 4{\cdot}8 \text{ cm.}$$

As the lever is weakened at that section by the pin hole, we take the depth as 6 cm to account for the presence of the hole. The maximum bending stress induced at that section will be

12-6. Lever of a safety valve:

A safety valve is attached to steam boilers and is opened by the pressure of the steam when this pressure reaches a certain point upto which it is safe to subject the boiler; when this safe pressure is exceeded the valve is opened, allowing the steam to escape with a consequent reduction of pressure. The valves are weighted either indirectly by levers or directly by weights and springs. It is the first method with which we are concerned. Fig. 12-13 shows a lever safety valve, pivoted at the end F. The valve is attached to lever at point V close to the F. The centre of gravity of the lever is at a point G and the valve is held on its seat against the upward steam pressure by the weight W hung from a point on the lever. The weight W and its distance from the fulcrum are so adjusted that when the steam pressure acting upwards on the valve reaches a certain value, it overcomes the downward force exerted on the valve by the weight W kept at the end of the lever. As a result, the valve opens and steam escapes, until its pressure falls to the working value and then the valve closes again.



Lever safety valve
FIG. 12-13

This lever is one arm lever and its design procedure is explained by an illustrative example.

Example.

1. A lever safety valve is 7.5 cm in diameter. It is required to blow at 13 kg/sq cm gauge. Design the mild steel lever of rectangular cross section for the following permissible stresses.

700 kg/sq cm in tension, 525 kg/sq cm in shear and 245 kg/sq cm in bearing.

The pin is made of the same material as that of the lever. Distance from the fulcrum to the weight of the lever is 80 cm. Distance between the fulcrum and pin connecting the valve spindle links to the lever is 8 cm.

Sketch and dimension a suitable rocker lever.

Ans. 30 mm × 15 mm.

3. *The sketch of a Diesel engine valve rocker arm is shown in fig. 12-16 in which C is the pin, B is the tappet end and A the valve end. The total load at A is 120 kg and that at B is 180 kg. The safe bearing pressure at the pin may be taken 45 kg/sq cm. The rocker arm is to be of I section with an allowable stress of 450 kg/sq cm. The distance "a" is 7 cm and the distance "b" is 6 cm. A 15 mm tapped hole at B to receive the tappet is to be provided. Design the rocker arm and the pin at the fulcrum C and give a neat sketch of the rocker arm designed.*

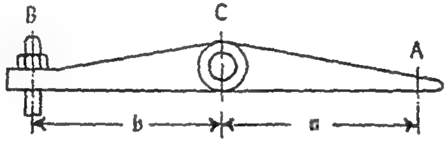


FIG. 12-16

12-8. Angular levers:

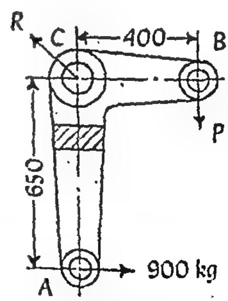
In engineering, we come across levers whose arms are inclined at a certain angle. When the angle between two arms is a right angle, the lever is known as a bell crank lever. They are used in governors of Hartnell type, the drive for the air pump of a condenser, radial valve gears, etc. The angular levers known as rocker arms are used for operating valve gears in internal combustion engines. Generally, the angle between two arms is 135°. The section of the arms of bell crank lever or rocker arm may be rectangular, elliptical or *I* form. The principles outlined in section 12-2 may be applied for the design of bell crank levers and rocker arms. We consider the design of such arms by illustrative examples.

(A) Bell crank lever:

Examples:

1. *A bell crank lever to drive the condenser air pump is shown in fig. 12-17. A force of 900 kg acts at A as shown in figure.*

Determine (*a*) *the force at B and C,*
(*b*) *the diameter of pins at A, B and C,*
(*c*) *the section of the lever near the fulcrum.*



Bell crank lever
FIG. 12-17

equal to $\dfrac{4000 \times 12 \times 3}{(6^2 - 1{\cdot}9^2) \times 1{\cdot}5} = 462$ kg/sq cm, which is within safe limits.

Exercises:

1. *Fig. 12-14 shows the lever of a lever loaded safety valve. The maximum load when the valve blows off freely is 575 kg. Design the lever for the following permissible stresses: 350 kg/sq cm in tension; 210 kg/sq cm in shear, 500 kg/sq cm in crushing. The allowable bearing pressure for the pins may be taken as 200 kg/sq cm. Give a dimensioned sketch of the lever*

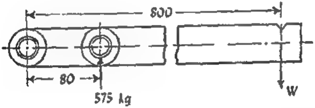


Fig. 12-14

2. *A lever loaded safety valve has a diameter of 8 cm and the blow off pressure of the boiler is 14 kg/sq cm. The weight on the lever is not to exceed 60 kg*

Design and prepare dimensioned sketch of the lever designed by you. Also give sketch of the fulcrum.

Materials — Lever and fulcrum — Mild steel, Weight — Cast iron

Choose suitable stresses. (*Sardar Patel University, 1968*)

12-7. Rocker arm for Diesel engine (Straight arm):

The rocker arms for Diesel engine of the over head valve type are treated as two arm lever because the fulcrum is in the middle. They are made generally of cast steel or malleable iron but good cast iron may be used if the permissible stress is confined to 140 kg/sq cm. The commonly adopted section for the lever is *I* section. Generally, the lever is made approximately of uniform strength by tapering towards the ends both the width and depth while the thickness of the web and flanges are kept constant

If the end of the rocker arm is to be operated by the cam directly, it is forked to receive the roller. The tappet end consists of a

The length of the pin = 5 cm.

We provide for 3 mm thick bush and the radial thickness of the eye will be taken as half the diameter of the pin. The outer diameter of the eye will be

$3{\cdot}8 + 2 \times 0{\cdot}3 + 2 \times \frac{1}{2} \times 3{\cdot}8 = 8{\cdot}2$ cm; we adopt 8·5 cm.

The diameter of the pin at A will be $= 3{\cdot}8\sqrt{\dfrac{900}{1470}} = 3$ cm.

The length of the pin will be 3·8 cm. We provide 3 mm thick bush for pin A. The outside diameter of the eye will be 6·5 cm. As the forces at B and C do not differ much, the same sized pin can be adopted at B and C to reduce the spares.

The section of the arm is obtained by considering the bending moment. We assume that the arm of the bending moment on the lever extends upto the centre of the fulcrum. This assumption results in stronger section. The section of the arm is rectangular having thickness as $\frac{3}{8}$ of the depth.

Modulus of section will be $\frac{1}{6}t\ h^2 = \frac{1}{6} \times \frac{3}{8}h^3 = \dfrac{h^3}{16}$.

Maximum bending moment = 900 × 65 = 58,500 kg cm.

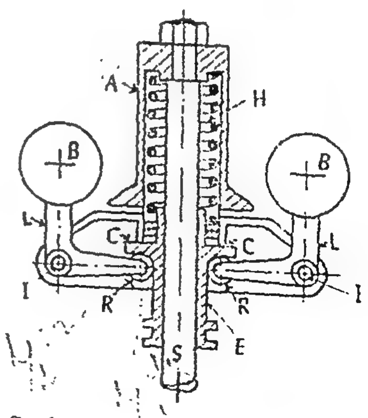
$$\therefore \quad 58500 = \frac{h^3}{16} \times 850$$

or $$h = \sqrt[3]{\frac{58500 \times 16}{850}} = 10{\cdot}2 \text{ cm; we adopt 11 cm.}$$

The thickness of the lever will be $\dfrac{3}{8} \times 10{\cdot}2 = 3{\cdot}82$ cm.

We adopt 4 cm as the thickness of the lever.

2. *In a Hartnell governor the length of the ball arm is 19 cm, that of the sleeve arm is 14 cm and the weight of each ball is 2·5 kg. The distance of the pivot of each bell crank lever from the axis of rotation is 17 cm and the speed when the ball arm is vertical is 300 r.p.m. The speed is to increase 3% for a lift of 1·5 cm of the sleeve. Determine the stiffness of the spring. Design the bell crank lever. Assume your own values for the permissible stresses.*



Spring loaded Hartnell governor

FIG. 12-18

The lever consists of a steel forging, turning on a pin at the fulcrum. The following data apply both to the pins and lever:

safe stress in tension	*850 kg/sq cm*
safe stress in shear	*625 kg/sq cm*
bearing pressure for the pin	*85 kg/sq cm.*

The force at B can be obtained by taking moment about the fulcrum C. If P be the force at B, then

$$P \times 40 = 900 \times 65$$

or $$P = \frac{900 \times 65}{40} = 1{,}470 \text{ kg.}$$

The reaction at the fulcrum will be $R = \sqrt{1470^2 + 900^2}$
$= 1{,}724$ kg.

The fulcrum pin is in double shear and it should also be designed from bearing considerations. Let us design the pin from bearing considerations and then check it for shear. Let $\frac{l}{d}$ ratio for the pin be 1·25. If d_p be the diameter of the pin, then length of the pin will be $1{\cdot}25\, d_p$. The projected area will be $1{\cdot}25 d_p^2$.

$\therefore \quad 1{\cdot}25 d_p^2 \times 85 = 1724$

or $$d_p = \sqrt{\frac{1724}{1{\cdot}25 \times 85}} = 4{\cdot}05 \text{ cm; we adopt } 4{\cdot}5 \text{ cm.}$$

The length of the pin will be $= 5$ cm.

The shear stress induced in the pin will be $\dfrac{1724}{2 \times \frac{\pi}{4} \times 4{\cdot}5^2}$

$= 54{\cdot}5$ kg/sq cm which is well below 625 kg/sq cm.

A brass bush 3 mm thick will be inserted in the boss of the lever so that the renewal will be simple, when wear occurs. As the bush is 3 mm thick the internal diameter of the hole in the lever will be $4{\cdot}5 + 2 \times 0{\cdot}3 = 5{\cdot}1$ cm. The outside diameter of the boss will be twice the diameter of the hole. In our case it will be $2 \times 5{\cdot}1 = 10{\cdot}2$ cm; we can adopt 10 cm.

By assuming $\frac{l}{d}$ *ratio of* 1·25 *for the pin* B, we get the diameter of the pin B from the equation

$$1{\cdot}25 d^2 \times 85 = 1470$$

or $$d = \sqrt{\frac{1470}{1{\cdot}25 \times 85}} = 3{\cdot}7 \text{ cm, we adopt } 3{\cdot}8 \text{ cm.}$$

The pin is in double shear. The shear stress induced in pin will be $\dfrac{86}{2 \times \frac{\pi}{4} \times 1^2} = 54{\cdot}9$ kg/sq cm which is well within safe limits.

The length of the pin in the fulcrum will be 2 cm.

The boss diameter will be twice the diameter of the pin i.e. 2 cm.

Maximum bending moment $= 69 \times 14 = 966$ kg cm.

The material of the lever is assumed to be cast steel for which the permissible stress is 850 kg/sq cm.

Assuming a rectangular section, having thickness equal to $\frac{3}{8}$ of the depth and if h be the depth of the section and t the thickness, the modulus of section will be $\frac{1}{6} \times \frac{3}{8} h^3 = \dfrac{h^3}{16}$.

$$\therefore \quad 966 = \frac{h^3}{16} \times 850$$

or $$h = \sqrt[3]{\frac{966}{850} \times 16} = 2{\cdot}7 \text{ cm.}$$

The thickness of the lever will be 1·2 cm.

The material of the ball is cast iron and it is screwed at the end of the lever. The density of cast iron is 7·8 gm/cu cm. If r be the radius of the ball, then

$$2{\cdot}5 \times 1000 = \tfrac{4}{3} \times \pi r^3 \times 7{\cdot}8$$

or $$r = \sqrt[3]{\frac{2{\cdot}5 \times 1000 \times 3}{4 \times \pi \times 7{\cdot}8}} = 4{\cdot}25 \text{ cm.}$$

The end of the lever will be screwed upto the centre of the ball.

Maximum centrifugal force on the ball $= 50{\cdot}8$ kg.

Maximum bending moment on the screwed end of the lever will be equal to $50{\cdot}8 \times 4{\cdot}25 = 216$ kg cm.

If d_c be the diameter at the bottom of the thread, we get

$$\frac{\pi}{32} d_c^3 \times 850 = 216$$

or $$d_c = \sqrt[3]{\frac{216 \times 32}{\pi \times 850}} = 1{\cdot}38 \text{ cm.}$$

From table of metric thread, we adopt M 18.

Let P be the force exerted by the spring and F be the centrifugal force on one ball. Then, taking moment about the fulcrum of the bell crank lever, we get

$$F \times 19 = \frac{P}{2} \times 14.$$

The centrifugal force $= \frac{W}{g} \times \frac{r}{100} \times \omega^2 = \frac{W}{g} \times \frac{r}{100} \times \left[\frac{2\pi N}{60}\right]^2$

where W is the weight of each ball in kg, r the radius of ball path in cm and N the speed in r.p.m. On substitution of values, we get

$$\frac{2.5}{9.81} \times \frac{r}{100} \times 19\left[\frac{2\pi N}{60}\right]^2 = \frac{P}{2} \times 14$$

or $\quad P = 0.00007586 N^2 r$ kg.

At 300 r.p.m., $r = 17$ cm.

$\therefore \quad P_1 = 0.00007586 \times 300^2 \times 17 = 116$ kg.

For 3% increase in speed, $N = 309$ r.p.m

and $r = 17 + 1.5 \times \frac{19}{14} = 19.04$ cm.

$\therefore$ At 309 r.p.m., $P_2 = 0.00007586 \times 309^2 \times 19.04 = 138$ kg.

$\therefore$ Spring stiffness $= \frac{138 - 116}{1.5} = 14.7$ kg/cm.

The maximum load on the roller arm of bell crank lever $= \frac{138}{2} = 69$ kg. By taking moment about the fulcrum of the bell crank lever, we get the maximum value of the centrifugal force on the ball arm.

Maximum centrifugal force $= \frac{69 \times 14}{19} = 50.8$ kg.

The reaction at the fulcrum $= \sqrt{69^2 - 50.8^2} = 86$ kg.

We have neglected the gravity effects of the arms and balls.

The bell crank lever will be pivoted at the fulcrum pin and the pin will be supported in the eye which is integral with the casing for the spring. The fulcrum pin will be designed from bearing considerations and checked for shear. Assuming $\frac{l}{d}$ ratio of 2 and the bearing pressure of 70 kg/sq cm, we get

$$2d \times d \times 70 = 86$$

or $\quad d = \sqrt{\frac{86}{70 \times 2}} = 0.8$ cm; we adopt 1 cm.

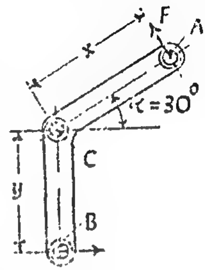
an I section between the boss and the ring, such that the depth of the section is about 2·25 times the flange width.

Make the necessary calculations and draw a dimensioned sketch of the lever, giving sectional views where required.

(Bombay University, 1967)

4. Fig. 12-19 shows a bell crank lever used on job shop printing press. The material of the lever is mild steel. A force F of 100 kg is applied at A in the direction shown. The lever arms 'X' and 'Y' are 20 cm and 15 cm respectively. Design the lever and pins at A, B and C.

(Sardar Vallabhbhai Vidyapeeth, 1960)



Angular lever
FIG. 12-19

(B) Rocker arm for exhaust valve:

The rocker arm is generally of *I* section and it is subjected to bending due to applied load. In order to calculate the bending moment, it is assumed that the arm of the lever extends from the point of application of load to the centre of the pivot of the rocker. This assumption results in a slightly stronger section near the boss. One end of the rocker arm is forked to receive a roller, which is carried on a pin which is free to revolve in a eye in order to have uniform wear. The ratio of length to diameter of the roller and fulcrum pin is taken as 1·25 and the permissible bearing pressure is taken from 35 to 60 kg/sq cm. 3 mm thick brass bush is pressed into the boss of the fulcrum and the eyes of the forked ends to provide for wear. The outside diameter of the boss is usually taken as twice the diameter of the fulcrum pin. The radial thickness of each eye of the forked end is taken as half the diameter of the pin. The outside diameter of the roller is taken at least slightly larger than the outer diameter of the eye. No phosphor bronze bush is provided for the roller as after sufficient service it is to be discarded due to wear at the profile. As some little clearance is provided between the roller and inside of the fork, it is desirable that pin should be checked for bending as explained earlier. Some cut away portion should be provided to clear the nose of the cam. The amount of cut away portion to be provided is obtained by trial and error method.

The maximum load on the roller will be half the maximum spring load. The pin will be fixed in the forked end of the bell crank lever and the roller will be free to move on the pin. We design the pin from bearing considerations. We assume $\frac{l}{d}$ ratio as 1·5 and bearing pressure as 70 kg/sq cm.

$\therefore \quad 1{\cdot}5d \times d \times 70 = 69$

or $$d = \sqrt{\frac{69}{70 \times 1{\cdot}5}} = 0{\cdot}83 \text{ cm};$$ we adopt the diameter of the roller pin as 1 cm and the length as 1·5 cm. The thickness of the eye of the fork will be 8 mm.

The outer diameter of the eye will be 2·6 cm.

The outer diameter of the roller will be 2·8 cm (slightly larger than the outer diameter of the eye).

Exercises:

1. A spring loaded governor of the Hartnell type has arms of equal length. The weights rotate in a circle of 30 cm diameter when the sleeve is in mid position and the weight arms are vertical. The equilibrium speed for this position is 400 r p m. The maximum sleeve movement is to be 4 cm and the maximum variation of speed allowed is 4% from the mean speed. The weight of the ball is 3·2 kg. Assuming your own values of the stresses, design the spring and bell crank lever for the governor

2. A right angled bell crank lever has arms 105 cm and 75 cm centres and the longer arm carries a load of 5,500 kg The shaft transmits a torque of 157,000 kg cm through one bearing neck, the bearing being close to the lever.

Adopt any suitable method of making the lever and comment on your design. Choose your materials and working stresses. Draw two views of the lever and add the leading dimensions.

3. Design a bell crank lever having both arms of equal length The stress in the material of the lever is not to exceed 150 kg/sq cm The free ends of the arms terminate in a circular ring, of 75 mm external diameter and 25 mm hole diameter to receive pins. The common end of the arms is formed into a boss of suitable diameter to receive a pin of 40 mm diameter.

A load of 180 kg is applied at one of the free ends with an effective arm length of 400 mm between the ring and boss centres. The arms have

For the tappet end of the rocker arm, we adopt 18 mm stud whose core area is 1·92 sq cm and compressive stress intensity will not exceed $\frac{200}{1 \cdot 92} = 105$ kg/sq cm. It is provided with a lock nut. The outer diameter of the circular end will be $2 \times 1 \cdot 8 = 3 \cdot 6$ cm and the depth of the end will also be 3·6 cm.

Let us consider the rectangular section, with the proportions $t = \frac{3}{8}d$. The modulus of section will be $\frac{1}{6} \times \frac{3}{8}d \times d^2 = \frac{d^3}{16}$ cm³.

$$\therefore \quad \frac{d^3}{16} \times 840 = 3000$$

or $$d = \sqrt[3]{\frac{3000 \times 16}{840}} = 3 \cdot 84 \text{ cm; we adopt 4 cm.}$$

Thickness of the lever $= 1 \cdot 5$ cm.

In order to determine the section of the arm, we adopt the following proportions for the *I* section:

Thickness of the web and flange $= t$.

Breadth of the section $= 2 \cdot 5t$.

Height of the section $= 6t$.

The modulus of section will be $= \frac{\frac{1}{12}[2 \cdot 5t\,(6t)^3 - 1 \cdot 5t\,(4t)^3]}{3t}$

$$= 12 \cdot 33t^3.$$

Maximum bending moment $= 200 \times 15 = 3{,}000$ kg cm

$$\therefore \quad 3000 = 12 \cdot 33t^3 \times 840$$

or $$t = \sqrt[3]{\frac{3000}{12 \cdot 33 \times 840}} = 0 \cdot 66 \text{ cm; we adopt 8 mm.}$$

Breadth of the section $= 2$ cm.

Height of the section $= 5$ cm.

The load on the roller is 200 kg. We adopt here a bearing pressure of 50 kg/sq cm and ratio of length to diameter 1·25.

$$\therefore \quad 50 \times 1 \cdot 25d \times d = 200$$

or $$d = \sqrt{\frac{200}{50 \times 1 \cdot 25}} = 1 \cdot 7 \text{ cm; we adopt 2 cm.}$$

The length of pin $= 2 \times 1 \cdot 25 = 2 \cdot 5$ cm.

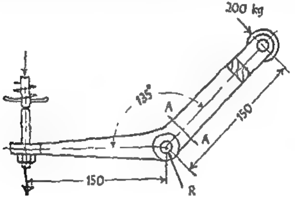
The thickness of each eye $= 1 \cdot 3$ cm

The overall diameter of the eye will be $2 + 2 \times 0 \cdot 2 + 2 = 4 \cdot 4$ cm.

The other end of the rocker arm is made circular to receive the tappet, which is a stud provided with a lock nut. The outside diameter of the circular arm is twice the diameter of the stud and the depth of the section is also equal to twice the diameter of the stud.

Example:

1. Fig. 12-20 shows the rocker arm for an exhaust valve of a gas engine. Total maximum force on the roller is 200 kg. Design (i) the pin R and (ii) the cross section of the arm at section AA. Allowable bearing pressure at the pin may be assumed to be 50 kg/sq cm. The section of the arm may be of I or rectangular form. The rocker arm is made of forged steel for which permissible stress is 840 kg/sq cm.



Exhaust valve gear rocker arm
Fig. 12-20

As the arms of the lever are equal, the loads at two ends of the rocker arm are equal. The reaction Q at the fulcrum is obtained by the formula

$$Q = \sqrt{200^2 + 200^2 - 2 \times 200 \times 200 \cos 135^\circ} = 360 \text{ kg}.$$

Assuming a ratio of length to diameter for the pin as 1·25, we get

$$50 \times 1{\cdot}25 d \times d = 360$$

$$\therefore \quad d = \sqrt{\frac{360}{50 \times 1{\cdot}25}} = 2{\cdot}4 \text{ cm, we adopt } 2{\cdot}5 \text{ cm}$$

The length of the pin $= \dfrac{360}{50 \times 2{\cdot}5} = 2{\cdot}88$ cm, we adopt 3 cm

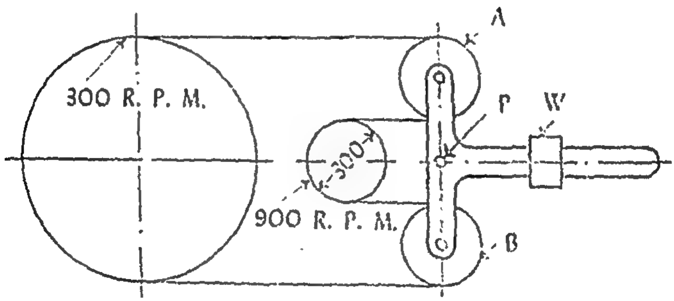
The external diameter of the boss will be 5 cm and the internal diameter will be 2·9 cm (bush 2 mm thick).

the lever will be $2(T_1 - T_2)\,x$, where x is the distance between the fulcrum of the lever and the centre of the intermediate pulley. The lever is kept in equilibrium position by adjusting the sliding counter-weight W. If y be the distance of the sliding weight from the fulcrum, then

$$Wy = 2\,(T_1 - T_2)\,x$$

$$\therefore \quad y = \frac{2\,(T_1 - T_2)\,x}{W} \qquad \text{(i)}$$

The design data include the power to be transmitted, speed ratio and the size of either driver or driven pulley. As the speed ratio is known, the size of the other pulley can be calculated. The angle of lap may be taken as 180° and the coefficient of friction as 0·25 to 0·3. With these data, tensions in the tight and slack sides can be calculated. When forces acting on pins, i.e. $2T_1$ and $2T_2$ are known, the position of the sliding weight can be determined. By drawing the polygon of forces the reaction at the fulcrum can be obtained.



Belt transmission dynamometer
FIG. 12-21

Freely revolving intermediate pulleys A and B are equal in diameter. The diameter of intermediate pulley will be equal to half the difference of the driver and driven pulleys. When the diameter of the intermediate pulley is known, the distance 'x' can be calculated. When the materials of the pins and lever are known, by assuming the suitable values of the permissible stresses, the sizes of fixed pins, fulcrum pin and lever can be obtained by the methods explained in this chapter. The sizes of the pins are determined from bearing considerations and then checked for strength.

Exercises:

1. The exhaust valve rocking lever of a Diesel engine similar to one shown in fig. 12-20 has arms 17·5 cm long. The exhaust valve is 10 cm diameter and the gas pressure when the valve begins to open, is 3·5 kg/sq cm. The initial load may be taken as 0 6 kg/sq cm of valve area, and the valve inertia and friction losses as 25 kg. Design a suitable lever in malleable cast iron, including the details at the tappet end and at the roller end. Choose your own values of the working stresses.

2. Design a rocker arm for operating an exhaust valve of a gas engine. The maximum force on the roller is limited to 220 kg. The effective length of each of the two arms is 15 cm and the angle included is 135°. The load and effort are applied at right angles to the lever arms. The opening of the valve is adjusted by 1 cm stud and a check nut. The rocker arm is made of forged steel, the permissible stresses for which are 700 kg/sq cm in tension and 560 kg/sq cm in shear. The bearing pressure between the pins and bores at the fulcrum and fork end is 55 kg/sq cm. The pin may be taken to have the same stresses as those of the lever. Width of roller may be taken as 25 mm.

Calculate at least the following dimensions and stresses and make a neat sketch or drawing.

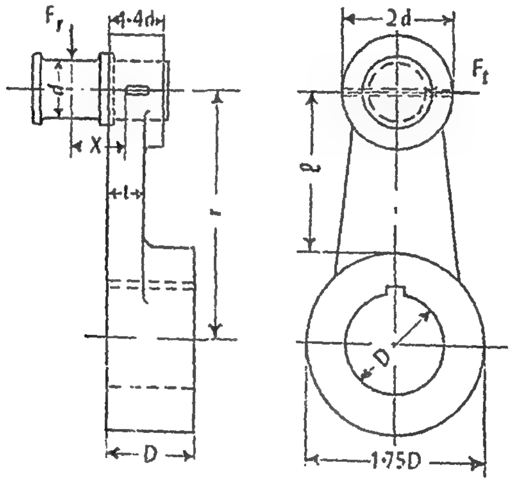
(i) Diameter and length of fulcrum pin (ii) Shear stress induced in the fulcrum pin (iii) Diameter and length of bores in the fork (iv) Shear stress induced in the roller pin in the fork (v) Section of lever near fulcrum.

3. A push rod 375 mm (14 76") long has forked end on one side and spherical one on the other side. It is connected to the tappet lever. Design the push rod and the tappet lever if the maximum resistance at the valve is 90 kg (198 lb). Choose suitable materials and permissible stresses. *(Bombay University, 1960)*

(C) *Lever design for Transmission Dynamometers:*

The transmission dynamometers are used where power must be measured under service conditions, as in a machine tool performing the function. It may be the belt dynamometer as shown in fig. 12-21 or epicyclic train dynamometer as shown in fig. 16-9. In belt transmission dynamometer the driver and driven pulleys revolve about fixed axes but the intermediate pulleys revolve on pins fixed to the lever which in turn pivots about the fulcrum on the fixed frame. If T_1 and T_2 be the tensions in the tight and slack sides of the belt respectively, the disturbing torque on

along the connecting rod resolves in two components: tangential component and radial component. In any arbitrary position of the crank, due to the tangential force, the crank arm will be subjected to transverse shear, bending and twisting, while due to radial component it will be subjected to direct stress and bending. It will be laborious to consider all these straining actions in several positions of the crank. Therefore, we shall consider the strength of the crank arm in two positions, when the crank is in dead centre position and when the crank and connecting rod are at right angles.



Overhung crank
FIG. 12-22

When the crank is at dead centre position, there is no tangential component of the force in the connecting rod and any section of the crank is subjected to direct and flexural stresses. If the section were to be rectangular of thickness t and width b, the direct stress will be $\frac{F_c}{bt}$. If x be the distance of the centre of gravity of the section from the line of action of the load, the bending moment on the section will be $F_c x$. The maximum bending stress on the section will be

$$\frac{F_c x}{\frac{1}{6}bt^2}=\frac{6F_c x}{bt^2}.$$

Bending moment at the shaft journal $= 11000 \times 35$
$= 384{,}000$ kg cm.

Twisting moment at the shaft journal $= 11000 \times 45$
$= 495{,}000$ kg cm.

The equivalent moment by Rankine formula,

$$Te = 384000 + \sqrt{384000^2 + 495000^2} = 1{,}011{,}000 \text{ kg cm.}$$

If D cm be the diameter of the shaft journal, then

$$\frac{\pi}{16} D^3 \times 630 = 1011000$$

or $$D = \sqrt[3]{\frac{1011000 \times 16}{\pi \times 630}} = 20{\cdot}2 \text{ cm; we adopt 21 cm}$$

diameter shaft.

Exercises:

1. *Design an overhung crank-pin journal to carry a maximum force along the connecting rod of 70 tonnes. Allow a bearing pressure of 60 kg/sq cm of projected area and a maximum bending stress of 550 kg/sq cm. Show by sketches how the pin may be fixed in the arms.*

Ans. $d = 26$ *cm;* $l = 47$ *cm.*

2. *The tangential force on the crank pin of an overhung crank is 25,000 kg. Find the diameter and length of the crank pin, (a) if the pin is to be designed from strength point of view, the permissible stress being 630 kg/sq cm, (b) if the bearing pressure is not to exceed 56 kg/sq cm. Assume* $\frac{l}{d}$ *ratio to be 1·25.*

Ans. (a) *16·5 cm; 21 cm.* (b) *19 cm; 24 cm.*

The resultant maximum stress will be $F_r\left[\frac{1}{bt}+\frac{6x}{bt^2}\right]$.

The resultant maximum stress should not exceed the permissible limit. So far as this straining action is concerned, the arm must have a uniform section

When the crank and connecting rod are at right angles, there is no radial component of the force in the connecting rod. There acts on the section a bending moment $F_t l$, a twisting moment $F_t x$ and a transverse shear stress of maximum value $\frac{3F_t}{2bt}$. The torsional shear stress due to twisting moment $F_t x$ has a maximum at the middle of the long side and a some what smaller maximum at the middle of the short side The bending stress due to $F_t l$ has a maximum value of uniform magnitude along the whole short side. This maximum bending stress combines itself with the shear stress at the middle of the short side to form maximum shear stress. The maximum transverse shear stress has a maximum at the middle of the long side and thus shear stress is added directly to the maximum torsional shear occuring at the same point

When the crank is in any position, the section of the crank arm is subjected to direct stress together with bending and torsional stresses.

The permissible value of the stress varies from 350 kg/sq cm for cast steel to 700 kg/sq cm for steels

12-10. Design of a crank pin (overhung crank):

The dimensions of crank pins are based on the strength required to resist all the forces to which they will be subjected and on the intensity of bearing pressure permissible between pin and journal. The pressure must be low enough to maintain the film of lubricant otherwise excessive heating and wear will result The allowable bearing pressure varies from 20 kg/sq cm for small high speed engines to 70 kg/sq cm for large low speed engines.

We consider the design of crank pin, which is equally strong in bending and bearing

Let P = force acting on the crank pin

l = length of the crank pin

d = diameter of the crank pin

f = permissible tensile stress intensity in the pin material

p = safe bearing pressure on the projected area.

Bending moment at the shaft journal $= 11000 \times 35$
$= 384{,}000$ kg cm.

Twisting moment at the shaft journal $= 11000 \times 45$
$= 495{,}000$ kg cm.

The equivalent moment by Rankine formula,

$$Te = 384000 + \sqrt{384000^2 + 495000^2} = 1{,}011{,}000 \text{ kg cm.}$$

If D cm be the diameter of the shaft journal, then

$$\frac{\pi}{16} D^3 \times 630 = 1011000$$

or $$D = \sqrt[3]{\frac{1011000 \times 16}{\pi \times 630}} = 20{\cdot}2 \text{ cm};$$ we adopt 21 cm diameter shaft.

Exercises:

1. Design an overhung crank-pin journal to carry a maximum force along the connecting rod of 70 tonnes. Allow a bearing pressure of 60 kg/sq cm of projected area and a maximum bending stress of 550 kg/sq cm. Show by sketches how the pin may be fixed in the arms.

Ans. $d = 26$ *cm;* $l = 47$ *cm.*

2. The tangential force on the crank pin of an overhung crank is 25,000 kg. Find the diameter and length of the crank pin, (a) if the pin is to be designed from strength point of view, the permissible stress being 630 kg/sq cm, (b) if the bearing pressure is not to exceed 56 kg/sq cm. Assume $\frac{l}{d}$ *ratio to be 1·25.*

Ans. (a) 16·5 cm; 21 cm. (b) 19 cm; 24 cm.

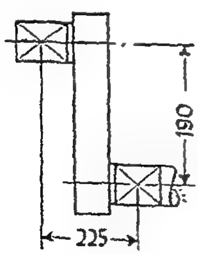


FIG. 12-23

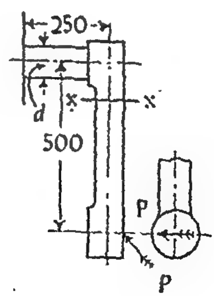


FIG. 12-24

3. Design an overhung crank with pin and shaft of a steam engine shown in fig. 12-23. The load on the crank pin is 3,000 kg, when the

crank and connecting rod are at right angles. Allowable bearing pressure on the pin may be taken upto 70 kg/sq cm and for main bearing 30 kg/sq cm. Assume suitable stresses for the design. Draw the crank with pin and shaft which you have designed.

4. In fig 12-24, is shown an overhung crank which has to support horizontal load P of ½ tonne as shown. Calculate the diameter of the shaft 'd' required and obtain the dimensions of the web at the cross section XX. You may allow a shear stress of 450 kg/sq cm in the shaft and a tensile stress of 700 kg/sq cm in the web

5. The crank pin of an overhung crank is acted upon by a force of 3,000 kg at right angles to the crank. The length of the crank is 20 cm. Distance of the centre line of the crank pin to the adjacent bearing is 25 cm. Assuming the shaft and pin to be made of carbon steel, having an ultimate strength of 4,500 kg/sq cm, calculate (a) the dimensions of the pin and (b) the diameter of the crank shaft

Assume your own value of the factor of safety. Safe bearing pressures for the crank pin and shaft journal are 70 and 15 kg/sq cm respectively

12-11. Miscellaneous Examples:

Let us consider some illustrative examples, involving the principles of design of lever, considered in this chapter

Examples:

1. Fig. 12-25 shows a cross lever to operate the twin cylinder double acting pump. The force of 600 kg acts upwards, while that of 400 kg acts downwards

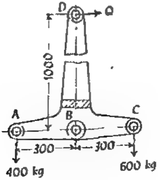


FIG. 12-25

Suggest the suitable cross sectional dimensions of the cross lever and pin sizes for A, B, C and D. The permissible bending stress in the material of the lever is 600 kg/sq cm. The bearing pressure is limited to 150 kg/sq cm.

Let Q be the effort required.

Taking moment about the fulcrum of the lever, we get

$Q \times 100 = (600 + 400)\ 30$

or $\qquad Q = 300$ kg.

The worst condition arises when one side of the pump does not work. At that time the effort required will be given by the equation $Q \times 100 = 600 \times 30$

or $\qquad Q = 180$ kg.

Reaction at the fulcrum $= \sqrt{180^2 + 600^2} = 624$ kg when only one side of the pump operates.

When both sides operate, the reaction at the fulcrum will be $\sqrt{300^2 + 200^2} = 360$ kg.

Thus the fulcrum pin is to be designed for a maximum load of 624 kg. The pins A, B, C and D are to be designed for the loads as shown below:

Pin	Load (kg)
A	624
B	400
C	600
D	300

The dimensions of the pins A and C will be the same and the dimensions of the pin B and D will be the same.

Bearing pressure for pins A, B, C and D is limited to 150 kg/sq cm. Maximum load for the design of pins A and C is to be taken as 624 kg.

Bearing area required $= \dfrac{624}{150} = 4{\cdot}16$ sq cm.

We assume the $\dfrac{l}{d}$ ratio as 1·25 $\quad \therefore \quad 1{\cdot}25d^2 = 4{\cdot}16$

or $\quad d = \sqrt{\dfrac{4{\cdot}16}{1{\cdot}25}} = 1{\cdot}84$ cm; we adopt the diameter of pins A and C as 2 cm. Length of the bearing $= \dfrac{4{\cdot}16}{2} = 2{\cdot}08$ cm; we adopt 2·2 cm.

crank and connecting rod are at right angles. Allowable bearing pressure on the pin may be taken upto 70 kg/sq cm and for main bearing 30 kg/sq cm. Assume suitable stresses for the design. Draw the crank with pin and shaft which you have designed.

4. In fig. 12-24, is shown an overhung crank which has to support horizontal load P of ½ tonne as shown. Calculate the diameter of the shaft 'd' required and obtain the dimensions of the web at the cross section XX. You may allow a shear stress of 450 kg/sq cm in the shaft and a tensile stress of 700 kg/sq cm in the web.

5. The crank pin of an overhung crank is acted upon by a force of 3,000 kg at right angles to the crank. The length of the crank is 20 cm. Distance of the centre line of the crank pin to the adjacent bearing is 25 cm. Assuming the shaft and pin to be made of carbon steel, having an ultimate strength of 4 500 kg/sq cm, calculate (a) the dimensions of the pin and (b) the diameter of the crank shaft.

Assume your own value of the factor of safety. Safe bearing pressures for the crank pin and shaft journal are 70 and 15 kg/sq cm respectively.

12-11. Miscellaneous Examples:

Let us consider some illustrative examples, involving the principles of design of lever, considered in this chapter.

Examples:

1. Fig. 12-25 shows a cross lever to operate the twin cylinder double acting pump. The force of 600 kg acts upwards, while that of 400 kg acts downwards.

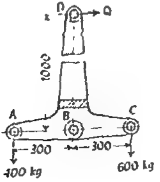


FIG. 12-25

Suggest the suitable cross sectional dimensions of the cross lever and pin sizes for A, B, C and D. The permissible bending stress in the material of the lever is 600 kg/sq cm. The bearing pressure is limited to 150 kg/sq cm.

Let Q be the effort required.

Taking moment about the fulcrum of the lever, we get

$Q \times 100 = (600 + 400)\ 30$

or $\qquad Q = 300$ kg.

The worst condition arises when one side of the pump does not work. At that time the effort required will be given by the equation $Q \times 100 = 600 \times 30$

or $\qquad Q = 180$ kg.

Reaction at the fulcrum $= \sqrt{180^2 + 600^2} = 624$ kg when only one side of the pump operates.

When both sides operate, the reaction at the fulcrum will be $\sqrt{300^2 + 200^2} = 360$ kg.

Thus the fulcrum pin is to be designed for a maximum load of 624 kg. The pins A, B, C and D are to be designed for the loads as shown below:

Pin	Load (kg)
A	624
B	400
C	600
D	300

The dimensions of the pins A and C will be the same and the dimensions of the pin B and D will be the same.

Bearing pressure for pins A, B, C and D is limited to 150 kg/sq cm. Maximum load for the design of pins A and C is to be taken as 624 kg.

Bearing area required $= \dfrac{624}{150} = 4{\cdot}16$ sq cm.

We assume the $\dfrac{l}{d}$ ratio as $1{\cdot}25 \quad \therefore \quad 1{\cdot}25 d^2 = 4{\cdot}16$

or $\quad d = \sqrt{\dfrac{4{\cdot}16}{1{\cdot}25}} = 1{\cdot}84$ cm; we adopt the diameter of pins A and C as 2 cm. Length of the bearing $= \dfrac{4{\cdot}16}{2} = 2{\cdot}08$ cm; we adopt $2{\cdot}2$ cm.

Thus, the dimensions of the pins A and C may be made 20 mm diameter and 22 mm bearing length. We adopt 3 mm thick bush. The diameter of the hole in the lever will be $20 + 6 = 26$ mm. The outside diameter of the boss will be 52 mm.

It should be remembered that the direction of the load will be reversed consequently the loads will be changed hence the pins at B and C must be identical. Thus, pins at A, B and C will be identical and so the holes in the lever will be identical.

It should be noted that the lever is subjected to fatigue stresses. Let us fix the thickness of the two horizontal arms.

Assume the ratio of depth to thickness for horizontal arm as 3:1.

Modulus of section $= \frac{1}{6} \times t \times (3t)^2 = 1{\cdot}5t^3$, if t is the thickness of the lever.

Maximum B.M. $= 600 \times 30 = 18{,}000$ kg cm.

Modulus of section $= \dfrac{18000}{600} = 30$ cm³.

$$\therefore \qquad 1{\cdot}5t^3 = 30$$

or
$$t = \sqrt[3]{\frac{30}{1{\cdot}5}} = 2{\cdot}71 \text{ cm; we adopt 3 cm.}$$

Depth of the section $= 3 \times 3 = 9$ cm

Section of the vertical arms

B.M $= 300 \times 100 = 30{,}000$ kg cm

Modulus of section $= \dfrac{30000}{600} = 50$ cm³

The thickness of the lever is kept the same i.e. 3 cm. If d be the depth of the section, then

$$\tfrac{1}{6} \times 3 \times d^2 = 50$$

or
$$d = \sqrt{\frac{50 \times 6}{3}} = 10 \text{ cm}$$

We adopt 3 cm as the thickness and depth as 10 cm.

2 Fig 12-26 shows a bench shearing machine used to shear mild steel bars of cross section 5 mm × 3 mm. The ultimate shear strength of the bar material is 4,000 kg/sq cm. Suggest the suitable cross sectional dimensions of the lever

Bearing pressure of the pins A and B and fulcrum pin is not to exceed 200 kg/sq cm. The tensile stress in the lever and the link AB is not to exceed 800 kg/sq cm.

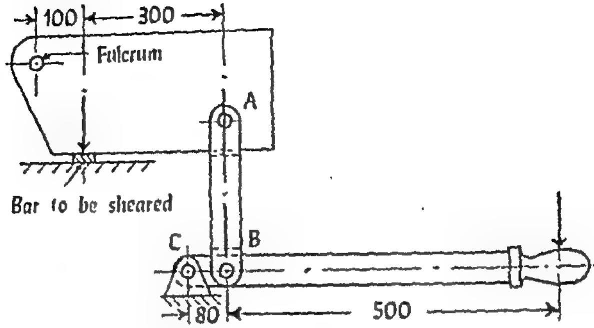


Fig. 12-26

Area to be sheared $= 0\cdot5 \times 0\cdot3 = 0\cdot15$ sq cm.

Maximum shear force $= 0\cdot15 \times 4000 = 600$ kg.

Load in the link $= \dfrac{600}{4} = 150$ kg, as the leverage for the blade is 4.

Pins B and A are in double shear.

Assuming $\dfrac{l}{d}$ ratio of 1, the diameter of the pin may be taken as 1 cm.

The dimensions of the pins at A, B and C will be the same.

The area of the pin $= \dfrac{\pi}{4} \times 1^2 = 0\cdot785$ sq cm.

Shearing stress in the pin $= \dfrac{150}{2 \times 0\cdot785} = 95\cdot5$ kg/sq cm.

Effort required to operate the lever $= \dfrac{150 \times 8}{58} = 20\cdot8$ kg; we adopt 21 kg.

Outside diameter of the boss = 3 cm.
Maximum B.M. $= 21 \times 50 = 1{,}050$ kg cm.
Permissible stress = 800 kg/sq cm.
If t cm be the thickness of the lever, then

modulus of section $= \dfrac{\frac{1}{12}[t \times 3^3 - t \times 1^3]}{1\cdot5} = \frac{26}{18}t$ cm^3.

$$\therefore \quad \frac{26t}{18} \times 800 = 1050$$

Thus, the dimensions of the pins A and C may be made 20 mm diameter and 22 mm bearing length. We adopt 3 mm thick bush. The diameter of the hole in the lever will be 20 + 6 = 26 mm. The outside diameter of the boss will be 52 mm

It should be remembered that the direction of the load will be reversed consequently the loads will be changed hence the pins at B and C must be identical. Thus, pins at A, B and C will be identical and so the holes in the lever will be identical.

It should be noted that the lever is subjected to fatigue stresses. Let us fix the thickness of the two horizontal arms.

Assume the ratio of depth to thickness for horizontal arm as 3:1.

Modulus of section $= \frac{1}{6} \times t \times (3t)^2 = 1{\cdot}5t^3$, if t is the thickness of the lever.

Maximum B.M. $= 600 \times 30 = 18{,}000$ kg cm.

Modulus of section $= \dfrac{18000}{600} = 30\ \text{cm}^3$.

$\therefore \quad 1{\cdot}5t^3 = 30$

or $\quad t = \sqrt[3]{\dfrac{30}{1{\cdot}5}} = 2{\cdot}71$ cm; we adopt 3 cm.

Depth of the section $= 3 \times 3 = 9$ cm.

Section of the vertical arms

B.M. $= 300 \times 100 = 30{,}000$ kg cm

Modulus of section $= \dfrac{30000}{600} = 50\ \text{cm}^3$

The thickness of the lever is kept the same i.e. 3 cm. If d be the depth of the section, then

$\frac{1}{6} \times 3 \times d^2 = 50$

or $\quad d = \sqrt{\dfrac{50 \times 6}{3}} = 10$ cm

We adopt 3 cm as the thickness and depth as 10 cm

2. Fig. 12-26 shows a bench shearing machine used to shear mild steel bars of cross section 5 mm × 3 mm. The ultimate shear strength of the bar material is 4,000 kg/sq cm. Suggest the suitable cross sectional dimensions of the lever.

Bearing pressure of the pins A and B and fulcrum pin is not to exceed 200 kg/sq cm. The tensile stress in the lever and the link AB is not to exceed 800 kg/sq cm.

Shear area of the key $= \frac{30 \times 4}{100} = 1{\cdot}2$ sq cm.

Tangential force on the key $= \frac{185}{0{\cdot}75} = 247$ kg.

Shear stress induced $= \frac{247}{1{\cdot}2} = 206$ kg/sq cm, which will be within limits.

Crushing stress $= \frac{247}{3 \times 0{\cdot}15} = 550$ kg/sq cm, which is safe.

Let us consider another method of mounting the hand wheel on the shaft by transverse taper key of dimension, d, for which permissible stress in key may be taken as 600 kg/sq cm.

$$\therefore \quad \frac{\pi}{4} d^2 \times 600 \times 1{\cdot}5 = 185$$

$$\text{or} \quad 100d^2 = \frac{185 \times 4}{9\pi} = 26{\cdot}2$$

$d = 0{\cdot}51$ cm, we adopt $0{\cdot}52$ cm key. Such a dimensioned key will weaken the shaft.

The third alternative arrangement of mounting the hand wheel on the shaft will be by means of a set screw.

$$\text{The diameter of the set screw} = \sqrt[2{\cdot}3]{\frac{\text{Tangential force}}{132}}$$

$$= \sqrt[2{\cdot}3]{\frac{247}{132}} = 1{\cdot}36 \text{ cm.}$$

This size is too large and hence we adopt two set screws of 1 cm diameter each.

We have taken 3 arm handwheel.

B.M. on each arm $= \frac{185}{3} = 62$ kg cm.

Assume 200 kg/sq cm as the flexural stress for the material of the flywheel.

Modulus of section required $= \frac{62}{200} = 0{\cdot}31$ cm^3.

Elliptical section with major axis twice the minor axis is adopted.

$$\therefore \quad \frac{\pi}{64} a^3 = 0{\cdot}31$$

$$\therefore \quad a = \sqrt[3]{\frac{0{\cdot}31 \times 64}{\pi}} = 1{\cdot}85 \text{ cm.}$$

or　　$t = \frac{1030 \times 18}{26 \times 800} = 0{\cdot}91$ cm; we adopt 1 cm as thickness, and 3 cm as the depth of the lever.

The diameter of the circular cross sectional link *AB* may be taken as 1 cm.

Tensile stress in the link $= \frac{150}{0{\cdot}785} = 191$ kg/sq cm.

The part grasped by hand may be 32 mm in greatest and 25 mm in the smallest diameter and 125 mm long The maximum force exerted by a man may be taken as 35 kg.

3. A hand wheel is to be designed for a torsion testing machine required to test to destruction standard test pieces upto 12 mm diameter for materials developing maximum shear stress of 5,000 kg/sq cm. The hand wheel is mounted on a shaft which is coupled to the test piece through a reduction gear of 25·1 and having an efficiency of 40%. The operator is capable of applying with ease a force of 12 kg to the hand wheel.

Design the hand wheel, shaft and key assuming appropriate safe stresses. Sketch two views of the hand wheel

Maximum torque on a test piece $= \frac{\pi}{16} \times 1{\cdot}2^3 \times 5000$

$= 1{,}760$ kg cm.

Ideal torque on the hand wheel $= \frac{1760}{25} = 74$ kg cm as the gear ratio is 25:1

Efficiency of transmission is 40%. Hence the actual torque to be applied by the hand wheel $= \frac{74}{0{\cdot}4} = 185$ kg-cm.

Minimum diameter of the hand wheel $= \frac{185}{12} = 15{\cdot}4$ cm

We adopt 20 cm diameter hand wheel, with three arms.

If d be the diameter of the solid shaft on which the hand wheel is mounted, then $\frac{\pi}{16} \times d^3 \times 450 = 185$, (where permissible shear stress for the shaft material is taken as 450 kg/sq cm).

or　　$d = \sqrt[3]{\frac{185 \times 5{\cdot}1}{450}} = 1{\cdot}26$ cm; we adopt 15 mm diameter shaft. The hub of the hand wheel will be 30 mm in diameter. The dimensions of the key will be 30 mm × 4 mm × 3 mm.

Outside diameter of the hub $= 2 \times 4{\cdot}5 = 9$ cm.

Let us consider the cross section of the lever. We assume that the thickness is 3/8th of the depth. Hence the modulus of section will be $\frac{1}{6} \times \frac{3}{8} d \times d^2 = \frac{d^3}{16}$ cm^3.

Let us consider the bending moment on the horizontal arm. B.M. $= 625\ (60 - 4{\cdot}5) = 3{,}400$ kg cm. B.M. on vertical arm of the bell crank lever where it joins hub $= 2500\ (15 - 4{\cdot}5) = 2{,}630$ cm.

$$\therefore \quad \frac{d^3}{16} \times 800 = 3400 \text{ or } \quad d = \sqrt[3]{\frac{3400 \times 16}{800}} = 4{\cdot}05 \text{ cm.}$$

We adopt $d = 4{\cdot}5$ cm and thickness will be $1{\cdot}5$ cm.

Thus providing $\frac{1}{6} \times 4{\cdot}5^2 \times 1{\cdot}5 = 5{\cdot}1$ cm^3 as the modulus of section and the resisting moment will be $800 \times 5{\cdot}1 = 4{,}050$ kg cm. Hence the design is safe. *The thickness of the lever at the boss is increased by providing two welded washers of 2 cm thick on each side* so as to provide the necessary bearing length, for the pin, of 5 cm.

Design of a bracket bolt:

1. Due to horizontal load, the bracket bolts are subjected to direct tensile load, which is the same for each bolt.
2. Due to vertical load, the bracket bolts are subjected to tensile loading due to tilting of the bracket about the lower edge and direct shear load, which is the same for each bolt.
   When permissible stress intensity is known and resultant tension load is calculated, the size of the bolt can be fixed.

Design of the bracket:

Due to horizontal load the bracket is subjected to direct tensile stress, while the bracket is stubjected to flexural stresses, due to vertical load. When the permissible stress is known, the size can be fixed.

EXAMPLES XII

1. A band brake for a crane is required to hold a load of 450 kg. This load is taken on a rope wound round the crane barrel which is of 40 cm diameter. The brake drum, which is fixed to the barrel shaft is of 60 cm diameter. The band embraces three quarter of the circumference of the drum and the coefficient of friction between band and drum is 0·3. The brake is to be applied by a hand

We adopt 2 cm × 1 cm as dimensions of the cross section of the arm at the hub and those at the rim as 1·4 cm × 0·7 cm.

4. Fig. 12-27 shows a safety device. The door opens when the load on it reaches 2,500 kg. Design the bell crank lever if the mechanical advantage of the lever is 4 and the length of the shorter arm is 15 cm.

Bearing pressure for the pin may be taken as 125 kg/sq cm and the bending stress for the lever may be taken as 800 kg/sq cm. Also suggest the procedure for designing the bolts required to fasten the fulcrum bracket, and the bracket.

As the load on the door is 2,500 kg, the effort at the end of the horizontal arm will be $\frac{2500}{4} = 625$ kg as the mechanical advantage is 4.

Resultant load on the fulcrum pin $= \sqrt{2500^2 + 625^2}$
$= 2{,}580$ kg.

Bearing pressure is limited to 125 kg/sq cm. Hence minimum projected bearing area $= \frac{2580}{125} = 22\cdot5$ sq cm. Let us assume $\frac{l}{d}$ ratio to be 1·3.

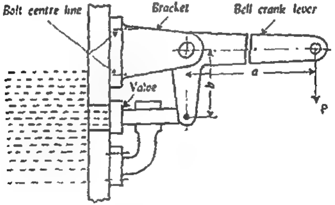


FIG 12-27

$\therefore \quad 1\cdot3d^2 = 22\cdot5$

or $\quad d = \sqrt{\frac{22\cdot5}{1\cdot3}} = 4\cdot15$ cm; we adopt 4·5 cm

Length $= 5\cdot5$ cm thus providing bearing area of 24·8 sq cm and the bearing pressure $= \frac{2580}{24\cdot8} = 104$ kg/sq cm.

lever above the drum and the operating force (vertically downwards) must not exceed 25 kg.

Design and sketch a suitable mild steel operating lever, having one end of the band attached to its fulcrum, assuming a working stress in bending of 1,000 kg/sq cm.

Devise and sketch a suitable catch for locking the brake in the "ON" position.

2. *A beam scale, has to be designed to weigh upto 500 kg The scale pan* measures 900 mm × 900 mm Making suitable assumptions, calculate the leading dimensions of the beam and the knife edges. The beam is made of cast steel and the knife-edge inserts of case hardened carbon steel

Safe stress for steel in tension is 1,100 kg/sq cm

Safe stress for steel in shear is 700 kg/sq cm

Make a neat sketch of the beam showing the knife edges and the central hanger *in position and put in all dimensions.*

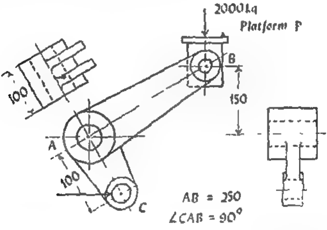


Fig 12-28

3. Fig. 12·28 shows a bell crank lever for a garrage jack to lift a load of 2,000 kg. The load platform *P* remains horizontal in all positions of the jack. The lift is 15 cm If the lever is made of carbon steel and is horizontal in the lower most position of the jack, design and prepare a dimensioned drawing of the bell crank lever.

4. A machine part of the shape shown in fig. 12·29 has a load of 600 kg acting at end *A*. The allowable stress in bending is 400 kg/sq cm Determine the dimensions at section *XX* assuming bending only, if $d = 4t$.

Using the calculated dimensions, *obtain the stress at XX*, taking into consideration the torsional effect of loading

5. Fig. 12·30 shows the arrangement of a brake lever rod and indicates the maximum loading. The ends *A* and *B* are freely supported in bearings.

Compute the load on each bearing giving the direction in each case. Determine a suitable diameter for the rod if it is of steel with a working shear stress of 550 kg/sq cm. The equivalent torque formula $Te = \sqrt{M^2 + T^2}$ may be used.

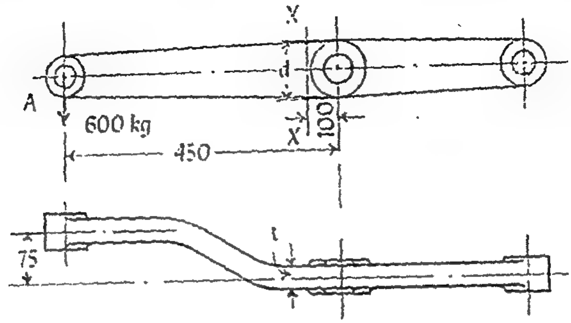


FIG. 12-29

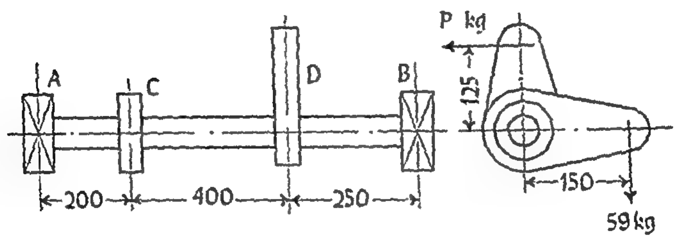


FIG. 12-30

Decide suitable dimensions for the two lever arms C and D and give a working sketch of the rod; the bending stress must not exceed 850 kg/sq cm.

6. In a single throw crank-shaft the crank pin dia is 125 mm and the shaft dia 100 mm. The bearings, which are equally spaced on each side of the crank, are at 90 cm centres. The centre distance of the crank webs is 20 cm. The crank length is 30 cm and the webs are 20 cm by 8 cm. If a load of 4,000 kg acts on the crank, calculate the stresses in the crank pin and in the left web when (a) the crank is in dead centre position and (b) the crank and connecting rod are at right angles to each other. You may assume that the torque is transmitted from the left hand end of the crank.

7. A single purchase winch crab is used to lift a load of 600 lb (270 kg). The drum on which $\frac{1}{2}$ in. (13 mm) diameter rope is wound has a diameter $7\frac{1}{2}''$ (19 cm). The gear ratio is 10:1, the diameter of the pinion shaft being $1\frac{1}{4}''$ (30 mm). The pinion shaft is squared at the end to receive a handle. Design a suitable handle if the maximum force at the end of the handle is limited to

3o lb (14 kg) and the maximum stress in it is limited to 12,000 psi (840 kg/sq cm) Assume the efficiency of the winch to be 65%.

(M. S University of Baroda, 1960)

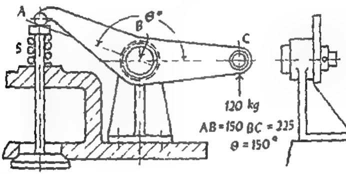
8 A steam engine has a stroke of 12" (30 cm) and works with an overhung crank The maximum tangential force on the crank is about 7,500 lb (3,400 kg) and the distance between the centres of the crank pin and the main bearings is 11" (28 cm).

Design the crank and draw two views to a suitable scale The following values of the allowable stresses and bearing pressures are available:

f_s = 4,400 psi (300 kg/sq cm), f_t = 6,000 psi (420 kg/sq cm)

Bearing pressure = 1,000 psi (70 kg/sq cm) for crank pin and 200 psi (14 kg/sq cm) for the main bearing (Bombay University, 1960)

9. The rocker arm for operating an inlet valve of an I.C. engine is shown in fig. 12-31 The rocker arm is pivoted to the pin *B* which in turn is mounted on the bracket A spring is provided to overcome the forces at the end *A*



Rocker arm

FIG. 12-31

Design.

(a) the rocker arm assuming it to be of rectangular cross section,

(b) the pin 'B'

Material and safe stresses.

The rocker arm to be of forged steel having safe stresses:

f_t = 12,000 psi (840 kg/sq cm); f_s = 9,000 psi (630 kg/sq cm), f_b = 12,000 psi (840 kg/sq cm).

The pin 'B' is to be of steel having the same stresses as above.

Allowable bearing pressure may be taken from 750 psi to 1,000 psi (42 to 70 kg/sq cm) of projected area.

Make a neat sketch of the arrangement and enter principal dimensions.

(Bombay University, 1960)

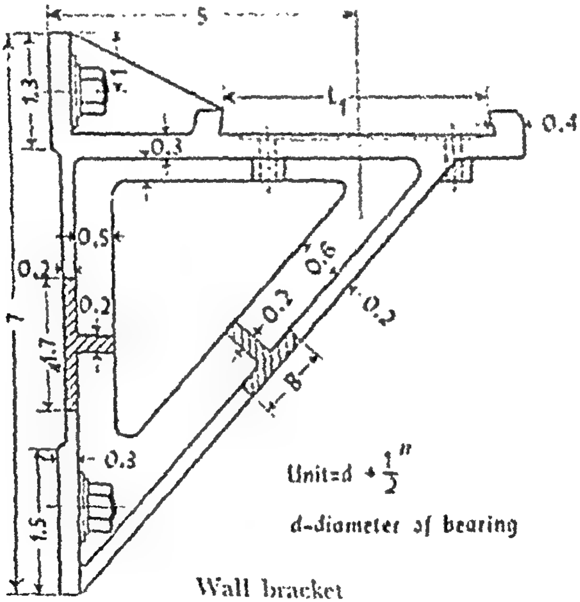
10. Fig. 12-32 is a diagram of a hand operated mechanism in which the plunger at *D* is required to exert a horizontal pressure. The actuating lever *ABC* is pivoted at *O* and is operated by the manual force *P* applied at *A*. The

CHAPTER XIII

BRACKETS

13-1. Bracket:

The bearings for workshop and mill shafting generally require fixing to the walls or overhead beams for which purpose a number of standard forms of cast iron brackets are used. The brackets are generally of two types: (i) wall bracket (fig. 13-1) and (ii) pillar bracket (fig. 13-2). The pedestal bearings may either be cast with the bracket as shown in fig. 13-3 or bolted to it in the form of a separate plummer block.



Wall bracket

Fig. 13-1

The pillar bracket shown in fig. 13-2 has a minimum amount of overhung. It may be used to support a horizontal shaft from a pillar where there is no wall in the way of wheels or pulleys on the shaft. The pillar bracket may be used as wall bracket if the

load is transmitted to the pedal axle at two points 8.75 cm away from each other. The material used for the axle, crank, and pedal axle has the following safe stresses:

$f_t = 980$ kg per sq cm.
$f_s = 560$ kg per sq cm.
$f_b = 63$ kg per sq cm

Design the crank and the pedal axle and show clearly the fixing arrangements for main axle and pedal axle with crank and pedal respectively.

(Sardar Vallabhbhai Vidyapeeth, 1966)

13. ABC is a bell crank lever used in a centrifugal governor. The length of the arm BA is 20 cm and that of arm BC is 15 cm. At B there is a hole to fit a pin for supporting it on a pivot. At A a force of 150 kg perpendicular to BA is applied to it. Find the force required at C perpendicular to BC and the reaction at B. Propose a suitable cross section for the arm BA. Assume a square section and bending stress 140 kg per sq cm.

(Sardar Vallabhbhai Vidyapeeth, 1966)

14. Design an overhung crank pin journal to carry a maximum force along the connecting rod of 50 tonnes, allowing a bearing pressure of 56 kg/sq cm of projected area and a maximum bending stress of 560 kg/sq cm. The crank pin must have equal strength in bearing and bending.

Show by sketches, how the pin may be fixed in the arms.

(Sardar Patel University, 1967)

15. The operating mechanism of a manually operated 25 tonne screw press comprises a hand lever, single spur gear reduction and power screw elements.

Indicate by a sketch the general arrangement of the power mechanism.

(Sardar Patel University, 1968)

16. A lever safety valve is loaded with a dead weight of 20 kgf acting at 90 cm from the valve axis. The valve is subjected to a pressure of 10 kgf/cm² and its area is 20 cm². Assuming the lever to be of rectangular section, depth equal to three times thickness, design the lever, two pins and their forks. Draw only lever, pin forks and their connection with the body and valve disc.

$f_t = 600$ kgf/cm², $f_s = 400$ kgf/cm² $f_c = 800$ kgf/cm² and $f_b = 90$ kgf/cm²

(Bombay University, 1969)

17. Design a bell crank lever to raise a vertical load of 500 kg acting through a pin at the forked ends, by a horizontal rod connected to the lever through a pin at the other end. The lever consists of a steel forging, turning on the pin at the fulcrum. Mechanical advantage of the lever is 4, the load arm being 20 cm long.

The following data apply both to pins and lever:

$f_t = 800$ kg/sq cm and $f_s = 700$ kg/sq cm

Bearing pressure for pins 100 kg/sq cm

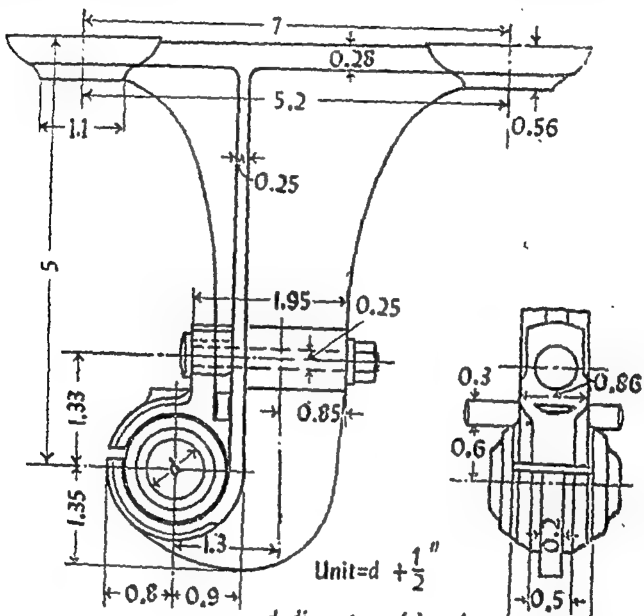
Sketch the lever designed by you.

(Sardar Patel University, 1970)

shaft does not carry wheels or pulleys which may interfere with the wall.

13-2. Hangers:

When the shaft is to be supported from a ceiling or from the overhead beam, the pedestal is modified in form and is known as hanger (fig. 13-4). The hanger shown in fig. 13-4 is known as *J* hanger and the bearing is supported on one side only.

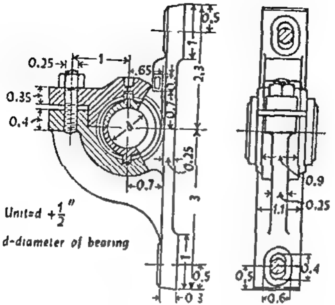


J hanger

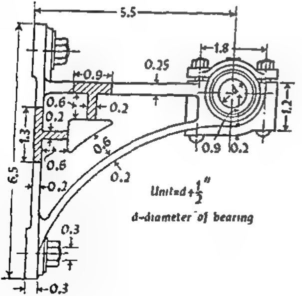
Fig. 13-4

13-3. Wall box:

It is used when a transmission shaft passes from one workshop to another through a dividing wall. When wall boxes are built during its erection, they may have broad outside flanges on each face of the wall to prevent movement. All wall boxes giving a clear opening through a wall should be provided with internal flanges to which wrought iron plates can be bolted to form a fire-proof barrier.



Pillar bracket
FIG 13-2



Wall bracket with integral bearing
FIG. 13-3

the lower edge. The magnitude of the tensile load in each bolt will be proportional to its position from the tilting edge. We assume that the entire tilting action of the bracket is resisted by two upper bolts, the assumption being on the safer side.

Direct shear load on each bolt $= \dfrac{3000}{4} = 750$ kg.

If T be the tensile load in each of the top bolt, then

$2T(400 + 50) = 3000 \times 500$

or $$T = \frac{3000 \times 500}{2 \times 450} = 1{,}670 \text{ kg.}$$

$$\text{Principal load} = \frac{1670 + \sqrt{1670^2 + 4 \times 750^2}}{2} = 2{,}105 \text{ kg.}$$

Minimum cross sectional area at the bottom of the thread will be equal to $\dfrac{2105}{700} = 3$ sq cm.

From metric table, we adopt $M24$ bolt having 3·53 sq cm area at the bottom of the thread.

We assume that the moment arm of the bracket extends upto the wall. This assumption gives stronger section for the bracket.

Maximum bending moment at the section $AA = 3000 \times 50$
$= 150{,}000$ kg cm.

Let us assume the depth of the section as 30 cm. If t be the thickness of the bracket, then

$150000 = \frac{1}{6} \times t \times 30^2 \times 700$

or $$t = \frac{150000 \times 6}{700 \times 30 \times 30} = 1 \cdot 5 \text{ cm.}$$

The thickness calculated from the strength point of view will be modified from rigidity considerations.

The maximum shear stress due to transverse shear of 3,000 kg will be $\dfrac{1 \cdot 5 \times 3000}{1 \cdot 5 \times 30} = 100$ kg/sq. cm, which is well within limits.

2. *Fig. 13-6 gives some dimensions of a solid forged bracket to carry a vertical load of 1,500 kg applied through the centre of the hole. The square flange is secured to the flat side of a vertical stanchion by four M24 bolts. Calculate suitable diameter D and d for the arms of the bracket.*

Estimate the greatest shearing force on a fixing and also find the tensile load on each top bolt. Mention the assumptions made in the estimations and state whether the bolts are of adequate size. For all parts, assume working stresses of 1,050 kg/sq cm in tension and 600 kg/sq cm in shear.

13-4. Design considerations:

The proportions for the end of the bracket depend upon the size of the bearing fixed to it. The overhung of the bracket depends upon the length of the base of the bearing and the width of the bracket will depend upon the width of the bearing to be mounted. When the bearing is integral with the bracket as shown in fig 13-3, the proportions of the steps, cap and cap bolt are the same as for an ordinary pedestal.

The projecting arm of the bracket is subjected to bending and shearing forces. *T* section of the arm offers the most economical form for strength and stiffness, which is absolutely necessary for the bracket.

The fixing bolts for the wall and pillar brackets are subjected to direct shear stresses and tensile stresses due to tilting action of the bracket about the edge of the flange as a hinge. On the bolts of the hanger, there are tensile stresses only; hence other things being equal, the bolts for the hanger should be smaller than for a wall bracket. The design of such bolts have been considered in more detail in article 5-17.

The usual proportions for brackets and hangers have been given on respective figures, the unit being $d + 13$ mm where d is the diameter of the shaft It is doubtful if the makers of the brackets are able to calculate their necessary section It seems that the manufactures are guided by previous experience and the brackets are abnormally strong

Examples:

1. *Fig 13-5 shows a steel bracket fixed to a wall by four bolts as shown The bracket carries a pedestal bearing A load of 3,000 kg acts vertically Design the bolts and the cross section of the bracket at AA. The permissible tensile and shear stress intensities for the bolt and bracket materials are 700 and 550 kg/sq cm respectively*

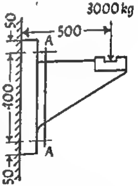


Fig. 13-5

The bolts are subjected to direct shear load, which will be equally shared by all the four bolts. In addition, the bolts are subjected to extensional load due to tilting of the bracket about

or $d = \sqrt[3]{\frac{31400}{1000} \times \frac{32}{\pi}} = 6{\cdot}8$ cm; we adopt 7 cm.

The bracket bolts are subjected to extensional load which are due to tilting action of the bracket.

If F be the force in a bolt situated at a unit distance from the tilting edge, then

$$2F\,(3{\cdot}75^2 + 23{\cdot}75^2) = 1500 \times 30$$

$$F = 39 \text{ kg.}$$

$\therefore$ Tensile load on each top bolt $= 39 \times 23{\cdot}75 = 920$ kg.

The bolts are subjected to direct shear load of the magnitude $\frac{1500}{4} = 375$ kg. As the shear load does not pass through the centre of gravity of group of bolts, bolts are subjected to secondary shear load. As all bolts are situated at equal distances from the centre of gravity of group of bolts, the magnitude of secondary shear force is the same on each bolt. If Q be the magnitude of the secondary shear force, then

$$Q \times 4 \times 10 \times \sqrt{2} = 25 \times 1500$$

$$\therefore \qquad Q = \frac{1500 \times 25}{4 \times 10 \times \sqrt{2}} = 663 \text{ kg.}$$

On a bolt having the greatest shearing force, the primary and secondary shear forces will be inclined at an angle of 45°.

$\therefore$ Maximum shear force on a fixing bolt

$$= \sqrt{663^2 + 375^2 + 2 \times 663 \times 375 \cos 45°}$$

$$= 965 \text{ kg.}$$

3. *A horizontal pull of 500 kg is exerted by the belting on one of the C.I. wall bracket which carry a factory line shaft. At a point 5 cm from the wall, the bracket has a T-section of dimensions shown in fig. 13·7. Calculate the maximum stresses in the flange and web of the bracket due to the pull of the belting.*

The section is subjected to the direct tensile stress and the bending stresses, which are tensile and compressive, due to the eccentricity of the load. Maximum tensile stresses due to bending occur in the flange and compressive stresses occur in the web.

The area of the section:

Area of the flange $= 6{\cdot}2 \times 1{\cdot}2 = 7{\cdot}44$ sq cm.
Area of the web $= 0{\cdot}9 \times 7{\cdot}6 = 6{\cdot}84$ sq cm.
Total area $= 7{\cdot}44 + 6{\cdot}84 = 14{\cdot}28$ sq cm.

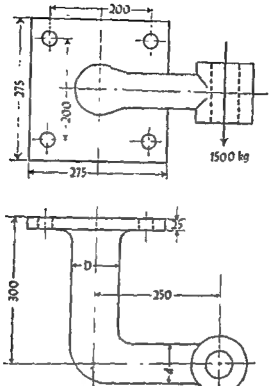


FIG. 13-6

The section of the arm of the bracket having "D" as the diameter is subjected to a bending moment of $1500 \times 27{\cdot}5 =$ 41,500 kg cm and twisting moment of $1500 \times 25 = 37{,}500$ kg cm. The equivalent torque is $\sqrt{41500^2 + 37500^2} = 55{,}700$ kg cm. If D be the diameter of the arm, then $\frac{\pi}{16} D^3 \times 600 = 55700$

or $\quad D = \sqrt[3]{\dfrac{55700 \times 16}{600 \times \pi}} = 7{\cdot}8$ cm; we adopt 8 cm

Maximum bending moment on the arm of the bracket having "d" as diameter is $1500(25 - 4) = 31{,}400$ kg cm. By equating the applied bending moment to the resisting moment of the bracket arm, we get

$$\frac{\pi}{32} d^3 \times 1000 = 31400$$

Direct tensile stress in the bracket due to pull of belting

$$= \frac{500}{14 \cdot 28} = 35 \text{ kg/sq cm.}$$

$\therefore$ Maximum tensile stress in the flange $= 83 + 35$
$= 118$ kg/sq cm.

Maximum compressive stress in the web $= 190 - 35$
$= 155$ kg/sq cm.

Determination of the fibre of zero stress:

If x be the distance of the fibre of zero stress from c.g. then

$$35 = \frac{190 \times x}{6 \cdot 1}$$

or $$x = \frac{35 \times 6 \cdot 1}{190} = 1 \cdot 12 \text{ cm.}$$

$\therefore$ The fibre of zero stress is at a distance of $2 \cdot 7 + 1 \cdot 12 = 3 \cdot 92$ cm from the top of the flange.

Exercises:

1. The critical section AB of a wall bracket carrying a plummer block is shown in fig. 13-8. State whether the section is safe or not if the permissible tensile stress in the material is limited to 150 kg/sq cm.

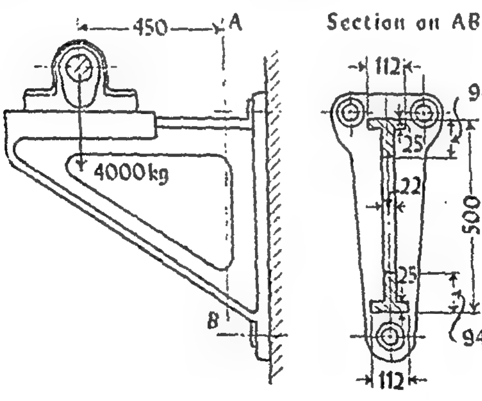


FIG. 13-8

2. The bearing of a shaft is supported on a cast iron beam, which may be considered to be a beam simply supported at the ends as shown in fig. 13-9. Suggest the suitable I section for the beam if the permissible tensile stress in the beam is limited to 140 kg/sq cm. The maximum load on the bearing is 2,200 kg. The width and depth of the I section may be

Determination of the centre of gravity:

If x be the distance of c.g. from the top of the flange, then $x \times 14{\cdot}28 = 7{\cdot}41 \times 0{\cdot}6 + 6{\cdot}81 \times 5$

$\therefore \quad x = 2{\cdot}7$ cm.

The neutral axis passes through the centre of gravity of the section.

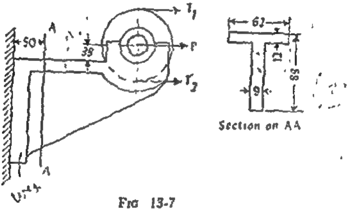


FIG 13-7

Determination of moment of inertia of the section:

$$Ig = \int_{-6\cdot1}^{1\cdot5} 0{\cdot}9x^2dx + \int_{1\cdot5}^{2\cdot7} 6{\cdot}2x^2dx$$

$$= 70 + 34{\cdot}4 = 104{\cdot}4 \text{ cm}^4.$$

Modulus of section for max. tensile stress $= \dfrac{104{\cdot}4}{2{\cdot}7}$ $38{\cdot}7$ cm³

Modulus of section for max. compressive stress

$$= \frac{104{\cdot}4}{6{\cdot}1} = 17{\cdot}15 \text{ cm}^3.$$

Bending moment exerted on the section $= P \times e$, where P is the load and e is eccentricity of the load

$$e = 3{\cdot}8 + 2{\cdot}7 = 6{\cdot}5 \text{ cm}$$

B.M. on the section $= 500 \times 6{\cdot}5 = 3{,}250$ kg cm.

Maximum flexural tensile stress in the flange $= \dfrac{3250}{38{\cdot}7}$

$= 83$ kg/sq cm

Maximum flexural compressive stress in the web $= \dfrac{3250}{17{\cdot}15}$

$= 190$ kg/sq cm.

4. *For the crane runway bracket shown in fig. 13-11, determine the maximum tensile and compressive stresses produced in the section XX when the magnitude of load P be 1,100 kg. Also, determine the stress produced in two M23 bolts used in fastening the bracket to the roof truss.*

Ans. 86·5 kg/sq cm; — 140 kg/sq cm; 516 kg/sq cm.

5. *A bracket of mild steel for a guide pulley is shown in fig. 13-12. Design the bracket and prepare a dimensioned drawing if the tension T is 400 lb (180 kg) and the pin diameter of the guide pulley is 1·25" (30 mm).*

(Poona University, 1960)

EXAMPLES XIII

1. Design and prepare dimensioned drawing of the bracket (fig. 13-13) for a crane runway, including the bolts for fastening it. The load (max.) lifted by the crane is 2,000 kg. Material—cast steel.

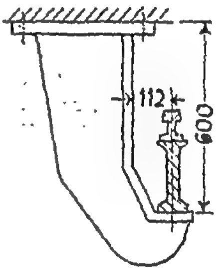


Fig. 13-13

2. Fig. 13-14 shows a C.I. bracket to carry a shaft and a belt pulley. The bracket is fixed to the main body by four standard bolts. The tensions on the slack and tight sides of the belt are 250 and 500 kg respectively.

Calculate (i) diameter D_1 of the shaft, (ii) diameter D_2 and thickness of the eye, (iii) diameter d of the bolt and (iv) cross section of the bracket at AA.

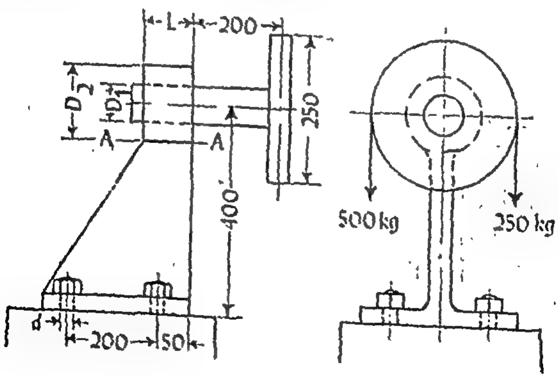


Fig. 13-14

Safe f_t for bolts 560 kg/sq cm; safe f_s for shafts 450 kg/sq cm; safe f_t for C.I. 280 kg/sq cm and safe bearing pressure for shaft 10 kg/sq cm.

taken as 5 and 8 times the thickness of the web respectively. The thickness of the flanges and web are the same.

Ans. Thicknees of web and flanges 33 mm.

FIG 13-9

3 The wall bracket made of cast iron carries a plummer block and the critical section for the bracket is shown in fig. 13-10, If the permissible tensile stress intensity for cast iron is limited to 140 kg/sq cm, determine the magnitude of the load P. State also the maximum value of the compressive stress. *Ans. 1,600 kg; 630 kg/sq cm.*

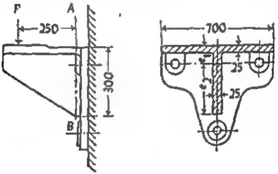


FIG 13-10

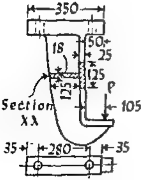


FIG. 13-11

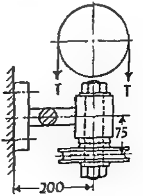


FIG 13-12

3. Fig 13-15 shows a bracket for a guide pulley. *A* is cast iron flange, *B* is a tube 3 mm thick, *C* is a cast iron fork and pin is of mild steel. Design and prepare a dimensioned drawing of the bracket.

Safe stresses: Cast iron 200 kg/sq cm
M S. tube 530 kg/sq cm
Pin 700 kg/sq cm

Permissible bearing pressure 150 kg/sq cm

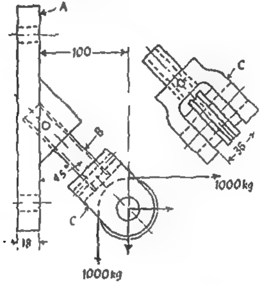


Fig 13-15

4 Determine the sizes of the bolts to be used in fastening the bracket shown in fig 13-16 and the dimensions of the section *XX* Assume permissible stress in bolts 550 kg/sq cm and in brackets 200 kg/sq cm

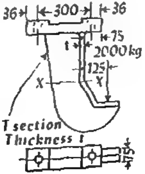


Fig. 13-16

Give a dimensioned sketch of the bracket.

Note: In order to give initial tension in the belt the actual length of the belt will be slightly less than the calculated length. The decrease in length depends on the initial tension to be given to the belt and the material of the belt. The reduction in length for the leather belt may vary from 0·25 to 1% of the calculated length.

(C) Ratio of driving tensions and power transmitted:

Let

T_1 = tension in the tight side of the belt

T_2 = tension in the slack side of the belt

μ = the coefficient of friction between belt and pulley

θ = the angle of lap on small pulley measured in radians.

Then

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad \text{(vi)}$$

$(T_1 - T_2)$ is known as the effective tension in the belt.

Horse power transmitted will be $\frac{(T_1 - T_2)\,V}{4500}$ where V is the speed of belt in metre per minute and T_1 and T_2 are measured in kg.

$$\text{Width of the belt} = \frac{T_1}{F} \qquad \text{(vii)}$$

where F = safe tension per unit width.

At high speeds, the centrifugal tension becomes of consequence and should be included when determining the width of the belt. The centrifugal tension in the belt is given by

$$T_c = \frac{wv^2}{g} \qquad \text{(viii)}$$

where w = weight per unit length of the belt

v = linear speed of the belt per second.

When we consider the effect of centrifugal tension, we have

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta} \qquad \text{(ix)}$$

The horse power transmitted, when the effect of centrifugal tension is taken into account, is given by

$$\text{h.p.} = \frac{(T_1 - T_c)\left(1 - \frac{1}{e^{\mu\theta}}\right) v}{75} \qquad \text{(x)}$$

The velocity, at which maximum horse power is transmitted, is given by

CHAPTER **XIV**

BELTS AND PULLEYS

14-1. Introduction:

The power transmission from one shaft to another shaft is accomplished either by flexible connectors or by direct contact. Where the distance between the shaft axes is large, flexible connectors or belts are usually employed. Owing to the slip and creep of such connectors, the angular velocity ratio of the driving and driven members is not constant.

Belt drives fall into two classifications: *flat belt drive* in which the width of the connector is appreciably larger than the thickness and the connector operates on pulleys and *V or rope drive* in which shaped belting material or rope operates in grooved wheels or sheaves.

When the driver and the driven members rotate in the same direction, the drive is said to be the open belt drive while in the crossed belt drive the directions of rotation of the driver and driven members are opposite. Both these drives are used for power transmission between parallel shafts. For satisfactory flat belt operation, the law of belting should be observed which states that *the centre of the belt approaching the pulley must be in a plane perpendicular to the axis of rotation*

14-2. Material for Belts:

The most common flat belt material is oak tanned leather, although chrome leather and other materials are used extensively Chrome leather belts are suitable for exposure to steam, water or oil Double or treble ply belts are made by cementing various thicknesses of hides

Fabric belts are made from canvas or cotton duck folded to three or more layers or plies and stitched together In order to render the belt water-proof it is treated with linseed oil. They are used for intermittent service under hot ... attention is given to their main-

... ber belts are made from layers

... impregnated with compounded

rubber and ther ...

outside weather ...

leather belting ...

$$v = \sqrt{\frac{gT}{3w}} \quad \ldots\ldots (xi)$$

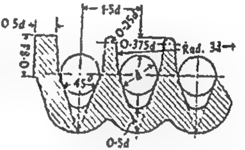
where T = maximum permissible tension in a given belt.

Maximum horse power is transmitted when one-third of the maximum tension is utilized as centrifugal tension or conversely when the tight side driving tension is equal to twice the centrifugal tension.

(D) Rope drive with grooved pulleys:

Ropes of cotton, manilla or hemp fitting into circumferential grooves on the pulleys are often used for power transmission. The drive may be single or multiple rope. The main advantage in using the multiple rope system is the continuity of power even though one rope may break. The tightening of the ropes is not required frequently.

V groove
Fig. 14-2



Proportions for a rope groove
Fig. 14-3

For horizontal transmission, the slack side is always on the top so that any sag in the rope due to its own weight tends to wedge the rope in the groove as a result, the angle of lap is increased. For vertical transmission the pressure of the rope in the grooves of the lower pulley tends to be reduced and, in consequence, initial tensioning of the rope is necessary.

The pulley groove angle has an angle of 40° to 50° between the faces so that the rope bears on the two side faces as shown in fig. 14-2. Fig. 14-3 shows the usual proportions for the grooved pulley in terms of rope diameter.

The rope tension ratio, when the centrifugal tension effect is considered, for the grooved pulley is given by

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta \operatorname{cosec}\frac{\alpha}{2}} \quad \ldots\ldots (xii)$$

where α = groove angle.

The horse power transmitted by n number of ropes is

$$\text{h p} = \frac{n(T_1 - T_2)V}{4500} \quad \ldots\ldots (xiii)$$

(iv) Silent operation
(v) Simplicity of care and servicing
(vi) Possibility of transmitting power between the shafts arranged in a variety of ways in space.

Disadvantages of flat belt drives:

(i) Inconstancy of the velocity ratio
(ii) Comparatively large size
(iii) Stretching of the belt calling for resewing when the centre distance is constant, or using a tensioning device.

14-4. Design procedure for flat belts:

The following is the sequence of design calculations for a flat belt:

(i) Select type of belt to suit service conditions.
(ii) Determine the diameter of the smaller pulley.
(iii) Determine the diameter of the other pulley.
(iv) Determine the belt thickness. The relation $\frac{D_{min}}{t}$ determines the bending stresses in the belt when the belt runs around the smaller pulley. As these stresses reduce the service life of the belt as well as the effective tension, the minimum value of $\frac{D_{min}}{t}$ is suggested. The following table gives the value of $\frac{D_{min}}{t}$ and allowable values of stress in tne belt material due to effective tensions.

Type of belt	$\frac{D_{min}}{t}$ min	D_{min} 25	30 Allowable	40 effec tive	60 stress	100 kg/sq cm
Leather	25	17	19	21	24	26
Rubber	30	—	20	21	22	22
Woven Cotton	25	15	16	17	18	19

The value of the belt *thickness* is then rounded off to the standard values.

(v) Determine the belt speed which should lie between the limits 10 and 20 metre/sec. If the speed is lower the diameter of the pulleys must be increased.

Method of belt joining	Percentage efficiency
Cemented by belt maker	100
Cemented	98
Wire laced by machine	90
Wire laced by hand	82
Raw hide laced	[illegible]
Metal belt hooks	35

(F) Multiple ply leather belts:

Theoretical considerations indicate that if the thickness of the belt is doubled by making the belt two ply, the horse power capacity of the belt should be doubled. However, test results indicate that a two ply belt will have its capacity increased by 70% of the single ply belt having the same thickness and that a three ply belt will have its capacity increased by 125% instead of 200%.

(G) Stresses in a belt:

In addition to the stresses caused by the working tension, the belt is subjected to the stress due to bending on the pulley.

$$\text{Bending stresses} = E \times \frac{t}{D} \qquad \text{(xvi)}$$

where t = thickness of the belt

D = diameter of the pulley on which the belt wraps

E = modulus of elasticity of the belt material.

[illegible]

In addition to bending stresses, there are centrifugal stresses in the belt material which cause a reduction in the angle of contact and the horse power rating of the belt drive.

The highest stresses arise at the point where the belt runs into the smaller pulley and are equal to

$$f = \frac{T_1}{bt} + \frac{Et}{D} + \frac{wv^2}{g} \qquad \text{(xvii)}$$

where b = width of the belt

In practice however belt drives are calculated not for strength but for the horse power transmitted.

(H) Advantages and Disadvantages of flat belt drive:

Advantages:

(i) Simplicity and low cost

(ii) Smoothness of operation, ability to absorb shocks due to elasticity of the belt and protect the driven mechanisms against breakage in case of sudden overloads owing to belt slipping

(iii) Possibility to transmit power over considerable distances between the axes of the driving and driven shafts

The V belt is of trapezoidal cross section. The cross section of the pulley grooves is of the same shape. The cross sections and pitch lengths of the V belts are standardized. The included angle of the profile is 40°. The diameters of the pulleys for V-belt drives are also standardized. The calculations of a V belt drive are confined to the selection of a belt of standard profile and length and afterwards number of belts are fixed for the transmission of given power. The number of belts should not exceed 8 to 12; otherwise the next larger belt section should be used. The belt speed should range from 5 to 30 metre/second.

The following table gives the useful data for *V* belt drive (All :– dimensions *in the table are in mm.)*

b		5	6	8	10	13	17	20	25	32	40	50
a		3	4	5	6	8	11	12·5	16	20	25	32
Minimum diameter of pulley		22	32	45	63	90	125	180	250	355	500	710
Length of belt	from	150	212	296	420	585	832	1100	1650	2303	3230	4600
	to	860	1262	1916	2820	4275	6332	9540	14050	18063	18080	18100

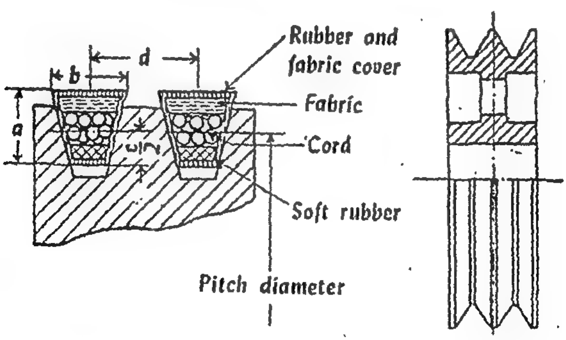


FIG. 14-4
Details of a V-groove

14-6. Design of V-flat drives:

On some drives with a speed ratio of 3 to 1 or more, and where the centre distance is no greater than the difference in

(vi) Determine the centre distance which should be greater than 1·5 to 2 $(D_1 + D_2)$.

(vii) Determine the length of the belt.

(viii) Determine the angle of contact on the smaller pulley which should not be under 2·5 radian. If this condition is not complied with the centre distance must be increased.

(ix) Determine the value of allowable effective stress depending on the ratio $\frac{D_{min}}{t}$. Then this value is reduced suitably corresponding to the service conditions of the drive.

(x) Determine the required cross sectional area of the belt; hence determine the width of the belt and round it off to the standard value.

(xi) Finally determine the load on the drive shafts. Operation of a belt drive is characterized by losses of energy.

The basic type of losses are due to the following factors

(i) Slipping of the belt on pulleys

(ii) Internal friction between the particles of the belt if stresses vary

(iii) Friction of the belt and pulleys against air

(iv) Friction in the pulley bearings.

14·5. V belt drive:

Modern V belts are made of fabrics and cords moulded in rubber and covered with fabric as shown in fig 14·4 V belts are used for short centre industrial drives for medium and heavy power demands. They can be run at low or high speeds on sheaves with velocity ratios upto 10:1 and at comparatively short centre distances. If one belt should break, the remaining belts in the drive will carry the load until it is convenient to shut down for repairs. This drive on account of wedging effect of the belt in the sheave groove, causes less pull on the shaft than flat belts of the same general characteristics.

The speed ratio for V belt drive can be obtained by using the pitch diameters of the sheaves. The equations (xii) and (xiii) of art. 14·3 apply to V belt drive. The efficiency of the V belt drive is somewhat lower and its elastic creep is somewhat higher than in the flat belt drive.

In some respects the V belt drive is inferior to the flat belt drive. At present the employment of V belt drive is second only to gear drives.

Would changing the small pulley to a multiple V-pulley (groove angle 34° and coefficient of friction 0·25) using the same compressor pulley, and eliminating the idler pulley, provide a more effective drive with greater horse power capacity? Assume that the pitch diameter of V belt pulley and the pitch diameter of the large pulley remain the same as for flat belt arrangement. Assume, also, that the total of the maximum force in each belt is the same as for the flat belt.

When the belt is on the point of slipping the tension ratio is given by $\frac{T_1}{T_2} = e^{\mu\theta}$. As the coefficient of friction is not the same, we have to determine at which pulley the belt will slip.

For larger pulley, the angle of lap is 270° and the coefficient of friction is 0·25. Therefore the tension ratio for the slip to occur at larger pulley is $\frac{T_1}{T_2} = e^{0\cdot25 \times \frac{270}{180} \times \pi} = 3\cdot26.$

For smaller pulley, the angle of lap is 220° and the coefficient of friction is 0·30. Therefore the tension ratio for the slip to occur at smaller pulley is $\frac{T_1}{T_2} = e^{0\cdot30 \times \frac{220}{180} \times \pi} = 3\cdot16.$

Thus, we see that the horse power capacity of the drive is governed by smaller pulley.

Maximum tension T_1 = area of the belt × permissible stress intensity $= 1 \times 25 \times 20 = 500$ kg.

Minimum tension $T_2 = \frac{500}{3\cdot16} = 158$ kg.

Belt velocity $= \frac{900 \times \pi \times 0\cdot3}{60} = 14\cdot2$ metre/sec.

Horse power capacity $= \frac{(500 - 158)\ 14\cdot2}{75} = 65.$

For open belt arrangement with no idler, the angle of lap on smaller pulley is $= 180° - 2 \sin^{-1} \frac{150 - 30}{2 \times 150} = 132\cdot8°.$

Angle of lap on larger pulley $= 180° + 2 \sin^{-1} \frac{150 - 30}{2 \times 150}$
$= 227\cdot2°.$

Tension ratio for smaller pulley $= e^{\mu\theta / \sin \frac{\alpha}{2}}$

pulley diameters it is possible to replace the large grooved pulley by a flat faced pulley; the friction grip provided by the base of V belts on the large flat pulley will not then be less than the wedge grip in the smaller, grooved pulley.

The following are the advantages and disadvantages of the V-flat drive:

Advantages

(i) Reduction in initial cost of drive, particularly if the large flat pulley is already in existence

(ii) Ability to run on the same short centre distances as drives using two grooved pulleys

(iii) Less side wear on V belts as the flat pulley permits a certain amount of self alignment.

Disadvantages·

(i) The efficiency can not be equal to that of a V-V drive because there is no wedge grip on the flat pulley.

(ii) The belts must be operated at greater tension so reducing both V belt life and efficiency and increasing bearing loads

(iii) More maintenance attention required as V-flat drives are much more prone to slip.

V-flat drives are recommended for the following applications·

(i) Where a large flat pulley already exists, the cost of removal, grooving and re-erection or the cost of a grooved replacement pulley may appreciably outweigh the higher efficiency and longer belt life to be obtained by using two grooved pulleys.

(ii) Where machine makers already have stock pulley castings or special flywheel patterns available, V-flat principles are to be applied to cut down initial cost and to maintain the flywheel effect.

(iii) Air compressor drives

The following points should be remembered while designing V-flat drives:

(i) The face of the flat pulley must not be crowned, it should be as smooth as possible.

(ii) Centre distance should not be greater than the difference in pulley diameter, but care should be taken to see that it exceeds the sum of their radii

Examples:

1. A compressor is driven by a 900 r p m. motor by means of 10 mm by 25 cm flat belt. The motor pulley is 30 cm in diameter and the compressor pulley is 150 cm The shaft centre distance is 150 cm and an idler is used to make the angle of wrap on the smaller pulley 220° and on the larger pulley 270°. The coefficient of friction between the belt and large pulley is 0 25, and between the belt and small pulley is 0 3. The allowable stress in the belt is limited to 20 kg/sq cm. Determine the horse power capacity of the drive, neglecting the effect of centrifugal tension.

$$\text{Belt speed} = \frac{\pi \times 40}{100} \times 700 = 880 \text{ metre/minute.}$$

We have

$$\frac{(T_1 - T_2) \times 880}{4500} = 30 \quad \therefore T_1 - T_2 = \frac{30 \times 4500}{880} = 154 \text{ kg.}$$

$$\frac{T_1}{T_2} = e^{0.75} = 2{\cdot}12. \qquad \therefore 2{\cdot}12 T_2 - T_2 = 154.$$

$$\text{or} \qquad T_2 = \frac{154}{1{\cdot}12} = 137 \text{ kg.}$$

$$T_1 = 137 + 154 = 291 \text{ kg.}$$

If b cm be the width of the belt, then $b \times 0{\cdot}5 \times 23 = 291$

$$\text{or} \quad b = \frac{291}{0{\cdot}5 \times 23} = 26 \text{ cm.}$$

3. A rope drive is to transmit 350 h.p. from a pulley 120 cm diameter running at a speed of 300 r.p.m. The angle of lap may be taken as π radian. The groove half angle is 22½°. The ropes to be used are 5 cm in diameter. The weight of rope is 1·3 kg per metre length and each rope has a safe maximum pull of 220 kg. The coefficient of friction between rope and pulley is 0·3.

Determine the number of ropes required. Also, suggest the suitable size for the pulley shaft if it is made of steel with a shear stress of 400 kg/cm².

Let us consider the power transmitted by one rope.

$$\text{Velocity of the rope} = \frac{\pi \times 1{\cdot}2 \times 300}{60} = 18{\cdot}84 \text{ metre/sec.}$$

$$\text{Centrifugal tension} = \frac{wv^2}{g} = \frac{1{\cdot}3 \times 18{\cdot}84^2}{9{\cdot}81} = 47 \text{ kg.}$$

$$\text{Tension ratio} = e^{0{\cdot}3 \times \pi \times \operatorname{cosec} 22{\cdot}5^\circ} = 11{\cdot}2.$$

$$\text{H.P. transmitted per rope} = \frac{(220 - 47)}{75}\left[1 - \frac{1}{11{\cdot}2}\right] \times 18{\cdot}84$$
$$= 39{\cdot}7.$$

$$\text{Minimum number of ropes required} = \frac{350}{39{\cdot}7} = 8{\cdot}83.$$

We adopt 10 ropes.

The width of grooved pulley for such a drive will be

$1{\cdot}5 \times 5 \times 9 + 2 \times 2 \times 5 = 87{\cdot}5$ cm. (See fig. 14-3.)

We adopt 90 cm.

Let us take 100 cm as the overhung of the grooved pulley.

$$\text{Tension ratio for smaller pulley} = e^{\frac{0{\cdot}25}{\sin 22{\cdot}5^\circ} \times \frac{132{\cdot}8}{180} \times \pi}$$
$$= 7{\cdot}27.$$

$$\text{Tension ratio for larger pulley} = e^{\mu\theta} = e^{0{\cdot}25 \times \frac{227{\cdot}2 \times \pi}{180}}$$
$$= 2{\cdot}69.$$

Thus, although the capacity of the smaller pulley is increased, the larger pulley is now the design criterion.

As the limiting tension ratio with V-flat arrangement is less than that for flat belt arrangement, the horse power transmitting capacity of V-flat drive is less.

T_1 remains the same and the peripheral velocity of the belt is the same.

$$T_2 = \frac{500}{2{\cdot}69} = 186 \text{ kg}.$$

$$\textit{Horse power capacity of V-flat drive} = \frac{(500 - 186)\ 14{\cdot}2}{75}$$
$$= 59{\cdot}5.$$

2. *An electric motor drives an exhaust fan. A flat leather belt is to be used. The following data are known:*

	Motor pulley	*Fan pulley*
Diameter	*40 cm*	*160 cm*
Angle of wrap	*2·5 radian*	*3·78 radian*
Coefficient of friction	*0·3*	*0·25*
Speed	*700 r.p.m.*	
Power transmitted	*30 h.p.*	

The belt is 5 mm thick and the permissible stress is 23 kg/sq/cm. Calculate the width of the belt.

Since the angles of wrap and the coefficient of friction are different for different pulleys, we should calculate the value of $e^{\mu\theta}$ for each pulley and the pulley which governs the design is one with smaller value of $e^{\mu\theta}$. $e^{\mu\theta}$ for motor pulley will be $e^{0{\cdot}3\times2{\cdot}5} = e^{0{\cdot}75}$ and for the fan pulley will be $e^{0{\cdot}25\times3{\cdot}78} = e^{0{\cdot}945}$. Therefore, the smaller pulley governs the design, i.e. the smaller pulley is *transmitting* its maximum power with the belt on the point of slip while the larger pulley is not developing its maximum capacity.

5. *Determine the percentage increase in horse power capacity made possible in changing over from a flat belt to a V belt drive. The diameter of the flat pulley is the same as the pitch diameter of the grooved pulley. The pulley rotates at the same speed as the grooved pulley. The coefficient of friction for the flat belt and the V belt is the same, 0·35. The V belt pulley groove angle is 60°. The belts are of the same material and have the same cross sectional area. In each case the angle of wrap is 160°.*

Ans. 38·1%.

6. *State the advantages of V belts and mention a typical material for such a belt.*

A V belt drive is to be arranged between two shafts whose centres are 85 cm. The driving pulley is of 30 cm effective diameter and is to be supplied with 95 h.p. at 1,200 r.p.m. The follower pulley is to run at 450 r.p.m.

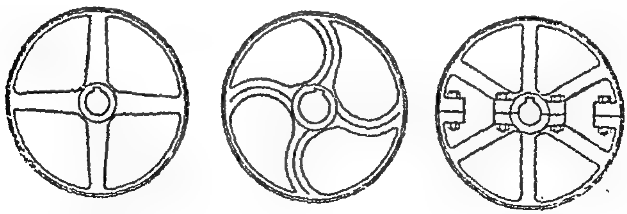
Determine the number of belts required, being given the following particulars:

Area of belt section	—	*4 sq cm*
Weight of belting	—	*1 gm/cu cm*
Safe working tensile stress	—	*21 kg/sq cm*
Coefficient of friction	—	*0·27.*
Groove angle of the pulley	—	*40°.*

Estimate the initial tension required in each belt.

14-7. Pulleys—Materials and types:

They are used to transmit power from one shaft to another shaft by means of belts or ropes. Because there is a certain amount



Straight arms Curved arms Split pulley

Various kinds of pulleys

Fig. 14-5

of slippage between both driver and driven pulley and the belt, they are not used when an exact velocity ratio is desired.

$$T_2 = \frac{(T_1 - T_c)}{11{\cdot}2} + T_c = \frac{(220 - 47)}{11{\cdot}2} + 47 = 62{\cdot}5 \text{ kg}.$$

Maximum bending moment on the shaft due to rope pull will be equal to 10 (220 + 62·5 + 2 × 47) × 50 = 188,250 kg cm.

$$\text{Twisting moment} = \frac{71620 \times 350}{300} = 83{,}500 \text{ kg cm}.$$

According to Guest's formula, equivalent twisting moment $T_e = \sqrt{(188250)^2 + (83500)^2} = 206{,}000$ kg cm.

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 400 = 206000$$

or
$$d = \sqrt[3]{\frac{206000}{400} \times \frac{16}{\pi}} = 13{\cdot}8 \text{ cm; we adopt 14 cm.}$$

Exercises:

1. A squirrel cage line-starting motor drives a compressor. The cast iron motor pulley is 32 cm in diameter and the motor speed is 910 r.p m. The belt has a cemented joints and is to run on 125 cm flywheel of a compressor. The horse power transmitted is 80 with a centre distance of 2·75 metre. Determine the width of a medium double leather belt. Assume the suitable values for the stress and the coefficient of friction

2 A 50 h p 940 r p.m. compensator started motor drives a pump at 320 r p.m The pump normally requires 45 h p but it is subjected to a peak loads of 180% of the full load. The centre distance is to be approximately 135 cm Determine the width of 10 mm thick belt if the permissible tensile stress intensity in the belt material is not to exceed 30 kg/sq cm The coefficient of friction may be taken as 0 3

3. An exhaust fan in a wood shop is driven by a belt from a compensator started squirrel cage motor running at 960 r p.m. A medium double leather belt 20 cm wide is used The safe tension in the belt material is 14 kg/cm width Determine the length and power transmission capacity of the drive if the centre distance be 150 cm The motor pulley is 40 cm in diameter and the diameter of the driven pulley is 150 cm.

4. A medium double leather belt transmits power to drive a mine fan which requires 20 h p at 950 r.p.m. The motor pulley 25 cm in diameter and the pulley turning with the fan is 90 cm in diameter The centre distance is 8 metre. Design a suitable belt for the drive. Assume your own values for the stresses.

5. Determine the percentage increase in horse power possible in changing over from a flat belt to a V belt drive. of the flat pulley is the same as the pitch diameter of the : The pulley rotates at the same speed as the grooved pulley. of friction for the flat belt and the V belt is the same, 0·35. pulley groove angle is 60°. The belts are of the same mate the same cross sectional area. In each case the angle of w.

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A V belt drive is to be arranged between two shafts whos. 85 cm. The driving pulley is of 30 cm effective diameter a supplied with 95 h.p. at 1,200 r.p.m. The follower pulley i. 450 r.p.m.

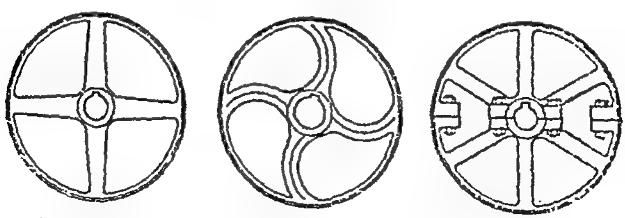
Determine the number of belts required, being given the particulars:

Area of belt section	—	*4 sq cm*
Weight of belting	—	*1 gm/cu cm*
Safe working tensile stress	—	*21 kg/sq cm*
Coefficient of friction	—	*0·27.*
Groove angle of the pulley	—	*40°.*

Estimate the initial tension required in each belt.

14-7. Pulleys—Materials and types:

They are used to transmit power from one shaft to anoth shaft by means of belts or ropes. Because there is a certain amoun



Various kinds of pulleys
FIG. 14-5

of slippage between both driver and driven pulley and the belt, they are not used when an exact velocity ratio is desired.

Pulleys are made of cast iron, pressed steel, welded steel, wood and paper in standard sizes as regards diameter, width of face and bore for the shaft. The principal parts of the pulley are the hub, which is adjacent to the shaft, the arms extending radially from the hub and the rim at the circumference. Instead of arms some pulleys have a solid web between the hub and rim. Some pulleys are made in two semi-circular halves and bolted or riveted together (fig. 14-5). When the belt speed exceeds 1,200 metre per minute the pulleys should be carefully balanced

The specifications for cast iron and mild steel flat pulleys have been recommended by IS: 1691-1960

14-8. Cast Iron Pulleys:

They are either solid or split The split pulleys are used for very long shafts where it would be troublesome to apply a new solid pulley when several other pulleys and bearings are already in place. Solid pulleys are connected to the shaft by means of plain sunk key. Sometimes headless set screws are also provided to prevent endwise motion Split pulleys are frequently so made that when two halves are bolted together, they grip the shaft firmly

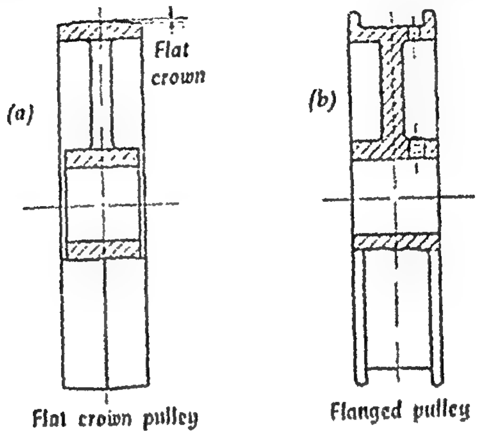
As the material of the pulley is comparatively weak in tension, the maximum safe rim speed is limited due to centrifugal stresses The maximum rim speed should be considered as about 1,500 metre/minute.

Varying thicknesses in any casting are responsible for an unequal cooling which tends to set up within the casting an internal stress of unknown magnitude. This must be carefully watched in the casting of the pulley and the internal stresses can be reduced or minimised by permitting the casting to cool slowly in the foundry sand or by proper annealing after its removal from the sand

Manufacturers classify pulleys as light duty and heavy duty. Light duty pulleys are constructed for use with thin belts while heavy duty pulleys are to be used with heavy belts

The crown is provided on the rim of the pulley [fig. 14-6(*a*)] to keep the belt in the centre of the width of the pulley. The taper which provides the height of the crown should not exceed 1 in 96 of the face width. Some pulleys are flanged [fig 14-6(*b*)] at the edges to hold the belt on the pulley but flanged pulleys'

rims chafe and wear the edges of the belt. The rim is also slightly tapered on the inside and the hub on the outside so as to facilitate the removal of the pattern from the mould.



Pulleys
FIG. 14-6

14·9. Design of a Cast Iron Pulley:

The parts to be considered in the design of cast iron pulleys are hub, arms and rim.

The diameter and length of the hub of a pulley are proportional to pulley diameter, face, bore and conditions of service. A formula that gives good average values in most cases is as follows:

Length of the hub $= \frac{\pi}{2} \times$ diameter of shaft. The minimum length of the hub is $\frac{2}{3}$ the width of the face of the pulley; it may be more for loose pulleys, but in no case exceeds the width of the face of the pulley. The outer diameter of the hub is equal to $1 \cdot 5 \times$ diameter of the shaft $+ 2 \cdot 5$ cm.

If the diameter of the hub is found to be more than twice the diameter of the shaft, which will be the case for small diameter shafts, in that case the length of the hub should be twice the diameter of the shaft.

If the diameter of the pulley is less than 20 cm, the rim is usually connected to the hub by means of a solid web of thickness equal to that of the rim measured at the middle of the pulley face.

For the pulley whose diameter is greater than twenty cm we employ arms. The number of arms may be taken as 4 for diameters from 20 to 60 cm, 6 for diameters from 60 cm to 150 cm and 8 for larger sizes. It is customary to use a double row of arms when the width of the face exceeds the diameter.

For larger size pulleys, the arms are generally curved as they are then less liable to fracture from internal stresses set up by the unequal rates of cooling of the rim and boss. With care, by avoiding abrupt changes of section and keeping the thickness as uniform as possible, a straight armed pulley can be cast free from internal stresses and this form is preferable being lighter and stronger than the curved form.

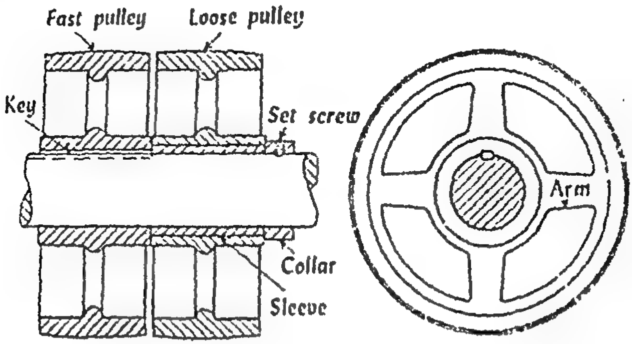
Fig. 14-7 shows four cross sections used for pulley arms. The one most commonly used is the elliptical form in which minor axis is 0 5 times the major axis. The arms taper from hub to rim.

Various sections for the arms
FIG. 14-7

The designer considers each arm as a cantilever beam fixed at the hub and supporting a concentrated load at the rim. At any time during rotation, about half the total number of arms are connected to a portion of the rim which is not in contact with the belt. Experiments by Prof. Benjamin indicate that the power is transmitted from rim to hub or vice versa, through only half the total number of arms at any given time. When we consider the bending moment on each arm the above fact must be taken into account. The length of the cantilever is taken as the radius of the pulley and this gives the stronger arm. Under the assumption that the arms act as cantilever beams, there is no bending moment and hence no bending stresses at the rim end, although the shearing stresses are present throughout the length of the arm For economy the arms are tapered from hub to rim, the usual taper is $\frac{1}{48}$ to $\frac{1}{32}$. These values give ample area on the outer end to resist shearing forces.

The width of the rim (width of pulley face) should be about 25% more than the width of the belt

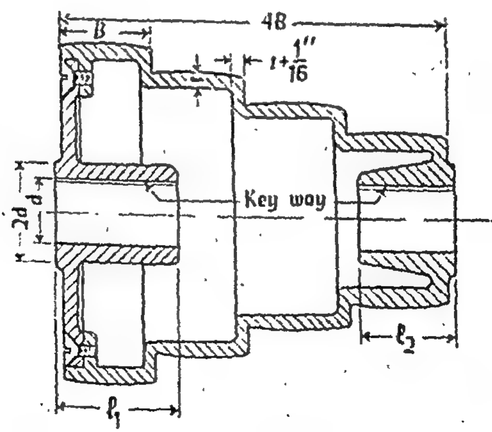
loose pulley, no power is transmitted. To enable the belt to mount easily the larger pulley rim is bevelled at the edge.



Fast and loose pulleys
FIG. 14-8

The loose pulley is generally mounted on a cast iron or gun metal sleeve with a collar at one end to prevent axial motion, the sleeve being secured to the shaft by means of a set screw. In order to relieve the tension in the belt during idle period, the loose pulley is made slightly smaller than the fast pulley. In order to reduce wear and friction, the extra long hub is provided. The hub should be symmetrical with the pulley face to prevent wobbling due to wear. Provision is made for the proper lubrication.

14-13. Speed Cone:



Speed cone
FIG. 14-9

For the pulley whose diameter is greater than twenty cm we employ arms. The number of arms may be taken as 4 for diameters from 20 to 60 cm, 6 for diameters from 60 cm to 150 cm and 8 for larger sizes. It is customary to use a double row of arms when the width of the face exceeds the diameter.

For larger size pulleys, the arms are generally curved as they are then less liable to fracture from internal stresses set up by the unequal rates of cooling of the rim and boss. With care, by avoiding abrupt changes of section and keeping the thickness as uniform as possible, a straight armed pulley can be cast free from internal stresses and this form is preferable being lighter and stronger than the curved form.

Fig. 14-7 shows four cross sections used for pulley arms The one most commonly used is the elliptical form in which minor axis is 0·5 times the major axis. The arms taper from hub to rim

Various sections for the arms
FIG. 14-7

The designer considers each arm as a cantilever beam fixed at the hub and supporting a concentrated load at the rim. At any time during rotation, about half the total number of arms are connected to a portion of the rim which is not in contact with the belt. Experiments by Prof Benjamin indicate that the power is transmitted from rim to hub or vice versa, through only half the total number of arms at any given time. When we consider the bending moment on each arm the above fact must be taken into account. The length of the cantilever is taken as the radius of the pulley and this gives the stronger arm. Under the assumption that the arms act as cantilever beams, there is no bending moment and hence no bending stresses at the rim end, although the shearing stresses are present throughout the length of the arm. For economy the arms are tapered from hub to rim, the usual taper is $\frac{1}{48}$ to $\frac{1}{32}$.

These values give ample area on the outer end to resist shearing forces.

The width of the rim (width of pulley face) should be about 25% more than the width of the belt

The velocity ratio between two shafts connected by belt is fixed by the diameters of the driver and driven pulleys. In workshop machines, we require variable velocity ratio. This can be effected by a system of pulleys as shown in fig 14-9. Such an arrangement is known as speed cone or stepped pulleys. Speed cones on two shafts are so arranged that the smallest step of one pulley is opposite to the largest step of the other. It is evident from the figure that the same belt must serve as the connector for each of the steps, and of course the belt tension should be the same whatever step the belt is driving from. As the belt is shifted, it engages pairs of steps of different diameters and in this manner may secure as many different speeds for the machine as there are pairs of steps. The diameters of the pairs of steps which work together must be so selected that they will give the desired speed ratio and permit the use of the same length of belt. This condition is satisfied exactly for the cross belts and very nearly for the open belts if the sum of the diameters of corresponding pulleys is constant. The "Burmester" method for a graphical solution is widely used to determine the diameter of the pulleys for open belts.

The graphical construction for the diameter of stepped pulleys is stated as follows

From *A* on a horizontal line *AB* (fig 14-10), draw a line *AC*, inclined at an angle of 45° with *AB*. Lay off *AS* on *AC* equal to the distance between centres of the shafts From *S* draw *ST* perpendicular to *AC*. Let *SK* equal to $\frac{1}{4}$ *AS* and with radius equal to *AK* draw an arc of a circle *XY*. From a convenient point *D* on *AC*, draw a vertical line *FDE*, and make *DE* equal to the given radius of a step on the cone, and *EF* equal to the given radius of the corresponding step on the other cone Draw *FG* and *EH* parallel to *AC*. From the point *G* on the arc, drop a vertical line cutting *EH* on *H* Through *H* draw a horizontal line *ML* touching *AC* at *M* Then if horizontal distances are measured from *M*, as *Ma*, *MH*, *MP*, to equal the radii of the pulleys on one cone, the corresponding vertical distances *ab*, *HG* and *PN* will be the radii of the corresponding steps on the other cone

If the radii of the two steps of any pair are to bear a certain ratio, as *ab*: *Ma*, from *M* draw a line at an angle with *ML* whose tangent equals that ratio, and from the point where it cuts the arc, as *b*, drop a vertical *ba* *Ma* and *ba* will be the radii required

The designer of a pair of cone pulley is concerned chiefly with the diameters. Rim and hub are proportioned in the same manner as for plain cast iron pulleys.

The given data usually include the r.p.m. of the driver, the several speeds of the driven pulley, the power to be transmitted

Radius of the pulley = 27·5 cm.

$\therefore$ Net pull in the belt $= T_1 - T_2 = \dfrac{5970}{27 \cdot 5} = 217$ kg.

For belt tension ratio, we have

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

where T_1 and T_2 are the tensions in tight and slack sides respectively, μ the coefficient of friction between belt and pulley and θ the angle of lap of the belt in radians.

$\therefore$ $\dfrac{T_1}{T_2} = e^{0 \cdot 35} = 2 \cdot 56.$

$\therefore$ $T_1 = 2 \cdot 56\, T_2.$

We have $T_1 - T_2 = 217$ kg. From above two equations we get $T_1 = 356$ kg and $T_2 = 139$ kg.

As the permissible tension is 25 kg per cm width, minimum width of the belt $= \dfrac{356}{25} = 14 \cdot 2$ cm; we adopt 15 cm.

For the width of the pulley face, we adopt 17 cm.

The hub length of the pulley as calculated is 6·5 cm, but this is too small so we adopt 12 cm.

Thickness of pulley rim $= \dfrac{d}{300} + 2$ mm $= \dfrac{550}{300} + 2$

$= 3 \cdot 83$ mm; we adopt 5 mm.

We assume that only half the number of arms are effective as the belt wraps itself around the pulley rim through 180°. Assuming that arms act as cantilevers and that the arms extend upto the centre of the pulley, bending moment o... arm ... by $\dfrac{5970}{4/2} =$ 2,985 kg cm. ... permissib... ...al is 150 kg/sq cm.

$\therefore$ Modulus ... n nec... of the arm = ... 3 cm

The sect... for

minor axis $\dfrac{a}{2}$

$\therefore$ $\dfrac{\pi a^3}{64} = 19 \cdot$...

25 h.p. at 300 r.p.m. Determine (a) the diameter of the shaft if the permissible value of the shear stress is limited to 500 kg/sq cm; (b) the dimensions of the arms at the hub assuming the allowable working stress in the arms to be 150 kg/sq cm; (c) the width of the pulley assuming that the angle of lap is 180°, that the coefficient of friction between the pulley surface and belt is 0·3 and that the permissible tension per cm width of the belt is 25 kg; (d) the length of the hub; (e) the diameter of the hub and (f) the dimensions of the key

Note: When the mean diameter of the pulley is not given, it is decided from the centrifugal stress considerations. The centrifugal stress induced in the rim is given by the formula

$f_t = \frac{\rho v^2}{g}$ where

ρ = density of the rim material
v = velocity of rim in metre/sec
g = acceleration due to gravity

In our present problem the diameter of the pulley is given.

$$\text{Torque on the pulley shaft} = \frac{71620 \times \text{h.p}}{\text{speed in r p m.}} \text{ kg cm}$$

$$= \frac{71620 \times 25}{300} = 5{,}970 \text{ kg cm.}$$

If d be the diameter of solid shaft, then

$$\frac{\pi}{16} d^3 \times 500 = 5970$$

or
$$d = \sqrt[3]{\frac{5970 \times 16}{500 \times \pi}} = 3\cdot96 \text{ cm, we adopt 4 cm}$$

The diameter of the hub for relatively small shafts is obtained by the formula:

Diameter of the hub = twice the diameter of the shaft.

∴ Diameter of the hub = 2 × 4 = 8 cm.

Length of the hub is obtained from the formula,

$$\text{Length of the hub} = \frac{\pi}{2} \times \text{diameter of shaft.}$$

$$\therefore \text{ Length of the hub} = \frac{\pi}{2} \times 4 = 6\cdot28 \text{ cm, we adopt 6·5 cm.}$$

Generally the length of the hub is normally kept $\frac{2}{3}B$ to B where B is the width of the pulley rim.

Let us determine the width of the rim.

Torque on the shaft = 5,970 kg cm.

Radius of the pulley $= 27{\cdot}5$ cm.

$\therefore$ Net pull in the belt $= T_1 - T_2 = \dfrac{5970}{27{\cdot}5} = 217$ kg.

For belt tension ratio, we have

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

where T_1 and T_2 are the tensions in tight and slack sides respectively, μ the coefficient of friction between belt and pulley and θ the angle of lap of the belt in radians.

$$\therefore \quad \frac{T_1}{T_2} = e^{0{\cdot}35} = 2{\cdot}56.$$

$$\therefore \quad T_1 = 2{\cdot}56\, T_2.$$

We have $T_1 - T_2 = 217$ kg. From above two equations we get $T_1 = 356$ kg and $T_2 = 139$ kg.

As the permissible tension is 25 kg per cm width, minimum width of the belt $= \dfrac{356}{25} = 14{\cdot}2$ cm; we adopt 15 cm.

For the width of the pulley face, we adopt 17 cm.

The hub length of the pulley as calculated is 6·5 cm, but this is too small so we adopt 12 cm.

Thickness of pulley rim $= \dfrac{d}{300} + 2 \text{ mm} = \dfrac{550}{300} + 2$

$= 3{\cdot}83$ mm; we adopt 5 mm.

We assume that only half the number of arms are effective as the belt wraps itself around the pulley rim through 180°. Assuming that arms act as cantilevers and that the arms extend upto the centre of the pulley, bending moment on each arm is given by $\dfrac{5970}{4/2} =$ 2,985 kg cm. The permissible stress in the arm material is 150 kg/sq cm.

$\therefore$ Modulus of section necessary for elliptical section of the arm $= \dfrac{2985}{150} = 19{\cdot}8 \text{ cm}^3$.

The section modulus for an ellipse whose major axis is a and minor axis $\dfrac{a}{2}$ is $\dfrac{\pi a^3}{64}$.

$$\therefore \quad \frac{\pi a^3}{64} = 19{\cdot}8.$$

or $a = \sqrt[3]{\frac{19.8 \times 64}{\pi}} = 7.35$ cm, we adopt 7.5 cm.

Minor axis of the ellipse = 3.8 cm.

Dimensions of the key:

Rectangular sunk key of dimensions 15 mm × 10 mm will be suitable.

Exercises:

1. A shaft 75 cm between bearings supports a 63 cm diameter pulley placed at 30 cm to the right of the left bearing. Determine the pulley bore, face of pulley and pulley arms of elliptical section if the pulley is of cast iron and transmits 20 h.p at 250 r.p.m

2. A cast iron pulley is to transmit 12 h p at 200 r p.m. Find the suitable diameter of the pulley, the face width of pulley, rim thickness, hub, bore, keyway and arms, which are of elliptical section and six in number. Assume that single belt is used for the transmission

3. A 5 cm diameter line shaft of a factory is run by an electric motor of 10 h p running at 500 r p m. The line shaft speed is 150 r.p.m. Design the pulley suitable to be mounted on the line shaft, the pulley on the motor being 25 cm diameter. Assume six elliptical arms of the line shaft pulley

4. Design a belt pulley to transmit 15 h p. at 180 r p.m. The velocity of the belt is not to exceed 500 metre/minute and the maximum tension is not to exceed 15 kg/cm width. The tension on the slack side is one half that on the tight side. Calculate all the dimensions of the pulley. Sketch neatly the sectional elevation and end view showing all leading dimensions.

5. A set of 3 step cone pulleys is required for a leather belt drive connecting two parallel shafts 4 metre apart and running in same directions. The driving shaft has a speed of 150 r p m and the driven shaft is required to run at 140, 200 and 260 r.p m. Maximum h p. to be transmitted is 5 and the load in the belt should not exceed 14 kg/cm width. Design the stepped pulleys and also specify suitable dimensions for the belt. Diameter of the smallest driven pulley = 20 cm

6. A line shaft is driven by a vertical belt from a motor placed directly below the shaft. The belt runs over a pulley 120 cm in diameter and weighing 120 kg. The distance from the centre line of the pulley which is overhung, to the centre line of the supporting bearing is, 40 cm. The

shaft transmits 24 h.p. when rotating at 200 r.p.m. Determine the suitable size of the belt and the diameter of the shaft if the permissible shear stress is limited to 450 kg/sq cm. Also, design a key for securing pulley to the shaft. Design also the overhung pulley suitable for the drive. Sketch arrangement.

7. *A cast iron pulley transmits 10 h.p. at 400 r.p.m. The diameter of the shaft on which the pulley is keyed is 30 mm. Calculate:*

(i) *diameter of the pulley if the hoop stress in the rim is not to exceed 45 kg/sq cm if the density of cast iron is 7·26 gm/cu cm,*
(ii) *dimensions of the four elliptical arms, allowing a bending stress of 150 kg/sq cm and major axis being twice the minor axis and*
(iii) *dimensions of the key allowing a shear stress of 650 kg/sq cm.*

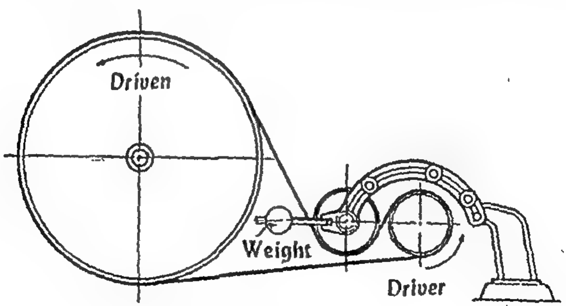
The width of the rim is 15 cm.

Give a neat dimensioned sketch of the pulley.

Ans. (i) 1 *metre diameter,* (ii) 4 *cm* × 2 *cm* (iii) 10 *mm* × 8 *mm* × 50 *mm.*

14-14. Short centre drive — Gravity Idlers:

High speed ratios and short centre distances decresae the arc of contact on the small pulley. In order to transmit the given horse power the initial tension should be high which may affect the life of the belt. The angle of contact on the pulley is increased by gravity idlers as shown in fig. 14-11. The idler is so located that



Gravity idler
FIG. 14-11

the angle of lap on smaller pulley is near about 240° and the clearance between the idler and the pulley should not be more than 4 to 5 cm. It is always located near the smaller pulley and on the slack side. It should never be crowned.

or $a = \sqrt[3]{\dfrac{19 \cdot 8 \times 64}{\pi}} = 7 \cdot 35$ cm; we adopt 7·5 cm.

Minor axis of the ellipse = 3·8 cm.

Dimensions of the key:

Rectangular sunk key of dimensions 15 mm × 10 mm will be suitable.

Exercises:

1. A shaft 75 cm between bearings supports a 63 cm diameter pulley placed at 30 cm to the right of the left bearing. Determine the pulley bore, face of pulley and pulley arms of elliptical section if the pulley is of cast iron and transmits 20 h p. at 250 r p.m

2. A cast iron pulley is to transmit 12 h p at 200 r.p m. Find the suitable diameter of the pulley, the face width of pulley, rim thickness, hub, bore, keyway and arms, which are of elliptical section and six in number. Assume that single belt is used for the transmission.

3. A 5 cm diameter line shaft of a factory is run by an electric motor of 10 h p running at 500 r p m. The line shaft speed is 150 r p m Design the pulley suitable to be mounted on the line shaft the pulley on the motor being 25 cm diameter Assume six elliptical arms of the line shaft pulley.

4. Design a belt pulley to transmit 15 h p. at 180 r p.m The velocity of the belt is not to exceed 500 metre/minute and the maximum tension is not to exceed 15 kg/cm width The tension on the slack side is one half that on the tight side Calculate all the dimensions of the pulley. Sketch neatly the sectional elevation and end view showing all leading dimensions.

5. A set of 3 step cone pulleys is required for a leather belt drive connecting two parallel shafts 4 metre apart and running in same directions. The driving shaft has a speed of 150 r p m and the driven shaft is required to run at 140, 200 and 260 r p m. Maximum h p. to be transmitted is 5 and the load in the belt should not exceed 14 kg/cm width Design the stepped pulleys and also specify suitable dimensions for the belt Diameter of the smallest driven pulley = 20 cm.

6. A line shaft is driven by a vertical belt from a motor placed directly below the shaft. The belt runs over a pulley 120 cm in diameter and weighing 120 kg. The distance from the centre line of the pulley which is overhung, to the centre line of the supporting bearing is, 40 cm The

The important feature of this drive is that the pressure between the belt and pulley is determined by the weight of the motor and its leverage and the sum of the belt tension increases or decreases as the load transmitted increases or decreases. Hence, at all loads except maximum load, the belt tension is much less than that required with an ordinary open belt drive as a result the life of the belt is increased.

Wherever possible the pulling side of the belt should pass between motor pulley and hinge point to reduce the effective moment of belt pull about the hinge point. In this case the motor can be mounted very near to the hinge.

When maximum power is exceeded, the belt will slip on the pulley, which will prevent overloading of the motor but may damage the belt.

A latter development by the American Pulley Co. depends mainly on reactive torque to adjust belt tension to suit the requirements of transmission. The reactive torque is equal and opposite to the applied torque. In this device, the pivot is placed close to the motor shaft (fig. 14-13) so that gravity causes only a small initial belt tension. In order to locate the pivot near the centre of the motor, the motor is mounted on a cradle so that the motor will swing about the point shown in figure. In this drive the belt tensions are built up as required by the load. The overloading may damage the belt or bearing.

This drive as well as Rockwood drive is limited to motors, where the power can enter through flexible leads. Such drives are great savers of space and belting, because they make very short drives possible with flat belts and plain pulleys.

Example:

1. *Fig. 14-12 shows a Rockwood drive. When the belt is on the point of slipping the maximum belt tension is 150 kg and the belt tension ratio* $e^{\mu\theta} = 3$. *The motor weight is 70 kg and the distance x and y are 8 cm and 25 cm respectively. Calculate the required moment arm c. Investigate the bearing loads at 50% of the maximum horse power and at no load.*

The belt tension ratio is 3 when the belt is on the point of slipping.

$$T_1 = 150 \text{ kg and therefore } T_2 = \frac{150}{3} = 50 \text{ kg}.$$

In order to determine the required motor moment arm c,

The advantages of gravity idlers are as follows·

(i) The belt may be relieved of initial tension when the drive is idle.

(ii) The initial tension of the belt may be regulated and may be maintained at the proper magnitude.

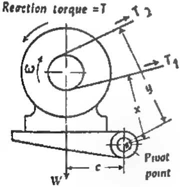
14-15. Special tension adjusting belt drive:

The Rockwood drive, fig 14-12, uses the weight of the motor to maintain belt tension practically independent of stretch. The motor is bolted to an intermediate base which is pivoted to the sub-base

The equilibrium equation for the moment is

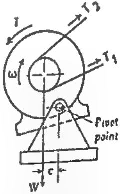
$T_1 \times x + T_2 \times y = W \times c +$ reaction torque.

From the figure, we see that the effect of the reaction torque is to rotate the motor anticlockwise about the pivot point and thus to increase the belt tension In this type of drive the lever arm of the weight is very large so that most of the belt tension is due to product of the weight of the motor and the moment arm c. By shifting the motor along the pivoted support, the lever arm can be adjusted so that belt tensions can be adjusted according to power requirements



Rockwood drive

Fig. 14-12



Drive as suggested by American Pulley Co.

Fig 14-13

When the motor has been properly balanced to maintain the required belt pull at the heaviest peak load, no further adjustment is necessary.

EXAMPLES XIV

1. Design a shafting of 20 metre length to transmit 15 h.p. at 180 r.p.m. required for various machine tools in the workshop.

Speed of the motor — 600 r.p.m.
Pulley for the motor — 30 cm × 20 cm.

The maximum bending moment on the shaft at the maximum torque is equal to 12,000 kg cm. Maximum load on bearing is 680 kg.

Safe working stress for shaft steel = 450 kg/sq cm.

Safe working stress for bolt steel = 250 kg/sq cm.

Design, also, the coupling and the main bearing required for the shaft and give approximate sizes of the pulley on the shaft and belt required to transmit the power. Prepare dimensioned drawings of (a) coupling (b) main bearing.

2. A 20 h.p. engine running at 375 r.p.m. is to drive a centrifugal pump requiring 15 h.p. and to run at 1,440 r.p.m. through a countershaft. Considering the engine pulley and pump pulley diameters as 90 cm and 20 cm respectively, design the countershaft and the two pulleys.

Assume the three plummer blocks as producing the conditions of built in ends and take:

Safe combined stress for torsion and bending = 400 kg/sq cm.
Maximum belt speed = 1,200 metre/minute.
Total belt pull = 3 times the net driving force.
Safe tensile stress in C.I. pulleys = 80 kg/sq cm.
Draw a dimensioned sketch of the shaft and pulleys.

3. Design a 120 cm diameter cast iron belt pulley transmitting 5 h.p. at 90 r.p.m. The tension in the belt is not to exceed 15 kg per cm width of belt. The pulley has six elliptical arms.

Tension on the tight side is double the tension on the slack side and the centrifugal tension in the belt is to be neglected.

Sketch two fully dimensioned views of the assembly.

4. A pulley of 90 cm diameter is to drive another pulley in the same direction with a speed reduction of 2·5 to 1 by means of a flat belt. The pulleys are spaced at 450 cm centres. The driving pulley is to deliver 24 h.p. at 400 r.p.m. Determine necessary width of belting if it is 6 mm thick of density 1 gm/cu cm and has a working tensile stress of 22 kg/sq cm. Assume a coefficient of friction 0·28.

Estimate the necessary initial tension for the belt, stating the assumptions made.

5. A cast iron pulley transmits 20 h.p. at 150 r.p.m. the diameter of the pulley being 70 cm. The pulley has four straight arms of elliptical cross section, the major axis being 2·5 times the minor axis.

Suggest the suitable cross sectional dimensions for the elliptical section of the arm (a) at the boss and (b) at the rim if the permissible stress in the arm is limited to 150 kg/sq cm.

(Sardar Patel University, 1969)

we take moment about the hinge point when maximum power is being transmitted.

$\therefore \quad 70 \times c = 150 \times 8 + 50 \times 25$

$$\therefore \quad c = \frac{150 \times 8 + 50 \times 25}{70} = 35 \text{ cm.}$$

When operating at one half the maximum load the tension ratio will not be three because the belt is not on the point of slipping. At half load, the net belt pull $T_1' - T_2'$ will be half of the net belt pull at full load.

$\therefore \quad T_1' - T_2' = \frac{1}{2}(T_1 - T_2) = \frac{1}{2}(150 - 50) = 50$ kg. . . . . . .(i)

The second equation will be the moment equation

$\therefore \quad T_1' \times 8 + T_2' \times 25 = 70 \times 35$ . . . .(ii)

From the above equations, we get $T_1' = 112{\cdot}3$ kg and $T_2' = 62{\cdot}3$ kg

When operating under no load, the net belt pull will be zero and the belt tensions will be practically equal.

$\therefore \quad T_1''(8 + 25) = 70 \times 35$ . . . .(iii)

$$\therefore \quad T_1'' = \frac{70 \times 35}{33} = 74 \text{ kg}$$

The total belt pull on the motor bearing $(T_1 + T_2)$ will be 200 kg at maximum load, 174·6 kg at half load and 148 kg at no load. With an open belt drive with fixed centres the sum of belt tensions will remain practically constant at all loads. Hence the bearing load will always be 200 kg, a value only reached with the Rockwood drive when the belt is operating at its maximum load.

Exercise:

1. A 10 h.p. 960 r.p.m. motor is arranged for a Rockwood drive as shown in fig. 14-12. The pulley diameter is 20 cm and the motor weight is 150 kg. The distance y and c are 40 cm and 50 cm respectively. The starting torque on the motor is 200% of the rating. Calculate the values of tensions T_1 and T_2 required for starting and the distance x from the hinge point to the line of action of the tension T_1. Also, determine the tensions in the belt at the rated load and no load.

CHAPTER XV

FLYWHEELS

15-1. Introduction:

A flywheel is a machine part which may be looked upon as a rotating energy reservoir which is attached to the shafts of particular types of machines. There are two classes of machines which require use of flywheels. In one type of class, where the operation is intermittent the flywheel absorbs energy from a power source during the greater portion of the operating cycle and delivers a large amount of stored energy as useful work in a very short portion of the cycle. This class includes punching machines, riveting machines, shearing machines, presses, crushers, etc. By application of flywheels to such machines the smaller power unit is required as a result sudden drain of power from the power lines is reduced. In other type of class, including steam engines, internal combustion engines, reciprocating compressors and pumps, the flywheel smoothens out the speed fluctuations caused by non-uniform flow of power from the piston during each energy cycle.

When energy is supplied to a machine at a variable rate and taken from the engine at a constant rate, the output shaft varies in speed. The use of flywheel would allow the engine to operate with the minimum of speed variation, which is an important factor in the design of all flywheels.

Flywheels may be formed of solid one piece section or they may be of sectional construction. Flywheels upto two metres in diameter are cast solid. The hub may be split up to avoid the cooling stresses. Flywheels ranging from two to five metre in diameters are cast in sections. The number of sections generally equals the number of arms in the wheel. The pieces composing the flywheel are joined at the hub either by through bolts or secured by using links shrunk into place.

15-2. Determination of a Weight of a Flywheel for given coefficient of Fluctuation of Speed:

The kinetic energy E of the flywheel of moment of inertia I rotating at ω radian per second is given by the equation

6. A plummer block is used to support a C I. pulley. Design the pulley, shaft and key with the following data:

H.P. transmitted = 20

R.P.M. = 200

Belt is vertical with angle of lap = 180°

$\mu = 0.3$

Weight of the belt material = 1 gm/cm³

Diameter of the pulley = 100 cm

Belt thickness = 10 mm

Allowable belt tension = 25 kg/sq cm

For C.I.:—f_t = 100 kg/sq cm, and
f_s = 80 kg/sq cm

For M S.: f_t = 700 kg/cm²,
f_s = 400 kg/sq cm and
f_{cr} = 1,000 kg/sq cm

Overhung of the pulley = 30 cm

Shock factor K_t = 1 0 and fatigue factor K_m = 1 5 for the shaft

Number of arms = 6

(Gujarat University, 1969)

7. Design a belt drive for a pump driven by a 10 h p Oil Engine running at 600 r p m. The pump r p m are 200. The engine pulley is 30 cm dia. belt thickness 0 8 mm and permissible stress in belt is limited to 30 kg/sq cm. The distance between the centres of engine pulley and pump pulley is 1 5 metres. Neglect slip and assume $\mu = 0.4$ and permissible tensile stress in C I arms not to exceed 140 kg/sq cm and the shear stress in M S = 300 kg/sq cm

Sketch the pump pulley

(Bombay University, 1969)

$E = \frac{1}{2}I\omega^2$ .... (i)

Differentiating the above equation, we get

$\delta E = I\omega\delta\omega$ .... (ii)

Equation (ii) is the important equation for the design of flywheels.

The following meanings are assigned to various symbols in equation (ii):

δE = maximum fluctuation of energy of the flywheel

I = moment of inertia of the flywheel about the axis of rotation

ω = mean speed of rotation of the flywheel

$\delta\omega$ = maximum permissible fluctuation in speed of flywheel

Equation (ii) can be modified to the form

$\delta E = I\omega^2 \frac{\delta\omega}{\omega}$ .... (iii)

$\frac{\delta\omega}{\omega}$ is known as the coefficient of fluctuation of speed. The value of the coefficient of fluctuation of speed depends upon the kind of the work the machine is required to do. It is greatest in machines such as punches where it is limited only by the danger of having the driving belt slip off the pulley. It is least for the engines driving alternating current generators running in parallel.

The reciprocal of the coefficient of fluctuation of speed is known as the coefficient of steadiness for the flywheel.

The moment of inertia of the flywheel, I, may be expressed by $\frac{W}{g}k^2$, where k is the radius of gyration. It is usual practice to consider the weight of the flywheel to be concentrated at the mean radius of the rim and finally to make adjustment for the flywheel effect of the hub and arm of the flywheel. The actual weight of the rim of the flywheel may be taken as approximately 10% less than that calculated by equation (ii) to allow for the flywheel effect of the arm and the hub.

If the flywheel is a disc type, the moment of inertia may be taken as $\frac{1}{2}Wr^2$ where r is the radius of the disc.

The maximum permissible values of coefficient of fluctuation of speed for different applications that have been found to give satisfactory operations vary considerably and have been given on page 576.

VALUES OF COEFFICIENT OF FLUCTUATION OF SPEED

Engines operating	$\frac{\delta\omega}{\omega}$
Crushing machinery	0·2
Hammers	0·2
Punches and shears	0·1 to 0·15
Pumping machinery	0·03 to 0·05
Machine shop drive	0·025 to 0·028
Engines with belt transmission	0·03
Gear wheel transmission	0·02
Textile machinery	0·025
Direct current generators	0·0065
Alternating current generators	0·0003 to 0·003

The general method of designing a flywheel is to determine the fluctuation of energy which varies for different classes of service. It may be found either graphically or analytically. The graphical method is more convenient when enough data are available to draw the energy diagram.

The mean diameter of the flywheel may be assumed or it may be fixed by the consideration of centrifugal stresses in the rim.

The centrifugal stress in the flywheel rim is given by the expression

$$f = \frac{\rho v^2}{g} \qquad \text{(iv)}$$

where ρ = density of the material

v = mean velocity

g = acceleration due to gravity.

When density is measured in kg/cu metre, the velocity in metre per second and the acceleration in metre/second², the stress will be in kg/sq metre. The density of cast iron is 7·26 gm/cu cm and that of cast steel 7·8 gm/cu cm.

With the help of equation (iv) when permissible stresses are known, by knowing the speed of the flywheel, the maximum limit for the diameter of the flywheel can be specified. It can also be fixed within certain limits by the general design of the machine. The use of a larger diameter flywheel will permit a smaller weight in the rim, with smaller cross sectional dimensions.

The practical design formula for use in metric unit will be $f = 0{\cdot}0737\, v^2$ kg/sq cm where v is the rim velocity in metre per second.

For cast iron, the flywheel rim speed is usually limited to 1,200 to 1,800 metres per minute. For steel flywheels, higher speeds can be adopted depending upon the construction of the flywheels.

The speed of the flywheel-shaft is known ordinarily and *the coefficient of speed fluctuation is known when the type of service is known.* When all these values are known, the weight of the rim and the cross sectional area required to obtain this weight can be calculated, when the decisions for material and diameter have been taken.

The ratio of the width to height of the rim section varies from 0·6 to 2. If a belt is run on a flywheel, the face width must be at least 2 to 3 cm wider than the belt.

The general procedure will be illustrated in more detail by considering numerical examples.

Note: It should not be taken for granted that a flywheel is always necessary. If some means can be found to reduce the energy fluctuation, the need for the flywheel is also reduced. In many machineries the flywheel effect is supplied by other rotating elements of the installation. We give some of the examples of the flywheel effect.

(i) The crankshaft and the equivalent mass of the connecting rod at the crank pin.

(ii) The rotating parts of the clutch mechanism in automotive installation.

(iii) Propellers and propeller reduction gears in air craft engines.

15·3. Flywheel for punches and shear:

In punches and shear the drive is generally produced by an electric motor so that the steady torque line is the input and it is the torque resistance which varies. For metal working machinery the energy required for a cycle may vary over wide limits and depends on the type of operation and particular set of tools that are used. *If we were to obtain the indicator diagram for punching a hole in a steel plate, it will be observed that the energy required to punch the hole of a specific diameter in a plate of given thickness increases as the radial clearance between punch and die is decreased.* Other factors such as sharpness of tools, and speed of operation will cause further variations in the

indicator diagrams. When more definite informations are lacking, the maximum force on the punch is calculated.

Maximum force on the punch = Area to be sheared × ultimate shear strength. ....(i)

Energy required to punch a hole is the average force multiplied by the thickness of the plate. ....(ii)

Average force = $\frac{1}{2}$ × maximum force on the punch.....(iii)

The maximum force is used in the design of all the parts of the machine. By mounting the flywheel in metal working machinery the horse power of the driving unit is reduced but the maximum stresses in the various components of the machines more or less remain unaffected. In order to keep the diameter and weight within reasonable limits, it is desirable to mount the flywheel on a high speed auxiliary shaft. [See note at the end of illustrative example 3.]

15-4. Engine flywheels:

There is an essential similarity between engines of a given type and consequently it has been found possible to generalise on the results from calculations of many engines and to establish values for the excess energy to be absorbed by the flywheel. This excess energy is found to be a certain proportion of the energy produced by an engine per revolution. The proportion of the energy to be absorbed depends on

(i) number of cylinders

(ii) regularity of the firing strokes

(iii) fuel used.

The following table gives some values of this proportion K.

(A) *Steam engines:*

% of cut-off	Single cylinder	Twin cylinders crank at 90°	Three cylinders crank at 120°
0·10	0·35	0·088	0·040
0·20	0·33	0·082	0·037
0·40	0·31	0·078	0·034
0·60	0·29	0·072	0·032
0·80	0·28	0·070	0·031
1·00	0·27	0·068	0·030

The practical design formula for use in metric unit will be $f = 0.0737\, v^2$ kg/sq cm where v is the rim velocity in metre per second.

For cast iron, the flywheel rim speed is usually limited to 1,200 to 1,800 metres per minute. For steel flywheels, higher speeds can be adopted depending upon the construction of the flywheels.

The speed of the flywheel-shaft is known ordinarily and *the coefficient of speed fluctuation is known when the type of service is known.* When all these values are known, the weight of the rim and the cross sectional area required to obtain this weight can be calculated, when the decisions for material and diameter have been taken.

The ratio of the width to height of the rim section varies from 0.6 to 2. If a belt is run on a flywheel, the face width must be at least 2 to 3 cm wider than the belt

The general procedure will be illustrated in more detail by considering numerical examples.

Note: It should not be taken for granted that a flywheel is always necessary If some means can be found to reduce the energy fluctuation, the need for the flywheel is also reduced In many machineries the flywheel effect is supplied by other rotating elements of the installation. We give some of the examples of the flywheel effect

(i) The crankshaft and the equivalent mass of the connecting rod at the crank pin

(ii) The rotating parts of the clutch mechanism in automotive installation

(iii) Propellers and propeller reduction gears in air craft engines

15-3. Flywheel for punches and shear:

In punches and shear the drive is generally produced by an electric motor so that the steady torque line is the input and it is the torque resistance which varies. For metal working machinery the energy required for a cycle may vary over wide limits and depends on the type of operation and particular set of tools that are used. *If we were to obtain the indicator diagram for punching a hole in a steel* [illegible]

between punch and die is decreased. Other factors such as sharpness of tools, and speed of operation will cause further variations in the

past a pair of successive poles, a shift of three phase degrees on a 16 pole generator will mean that the flywheel must be large enough to be capable of maintaining a steady speed within $\pm \frac{3}{8}$ of a degree.

Examples:

1. A machine, driven directly by an electric motor, has a cyclic load which is approximately represented by the torque angle graph in fig. 15-1. The speed of the machine must not vary during the cycle by more than $\pm$ 2 per cent of the mean speed of 500 r.p.m.

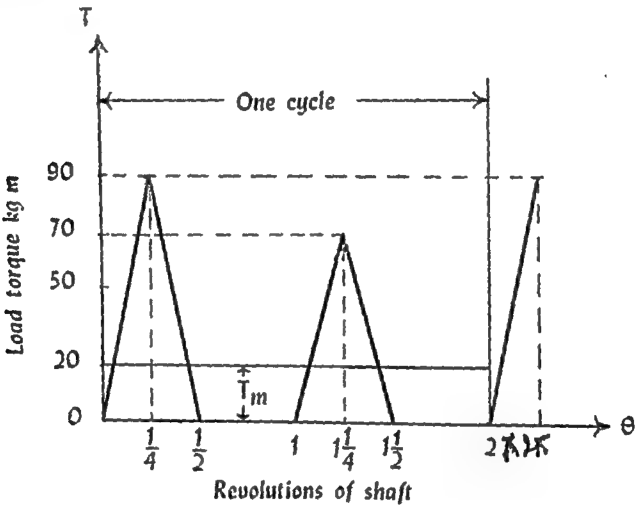


Fig. 15-1

Assuming the motor torque to be constant, calculate the necessary moment of inertia of the flywheel. If space considerations limit the diameter of the flywheel to 60 cm, determine suitable dimensions for the rectangular section rim if the breadth is twice the thickness. Neglect the inertia of the hub and spokes and assume the density of cast iron to be 7·25 gm/cu cm.

Determine the necessary horse power of the motor, allowing for a 25 per cent overload.

Select a suitable diameter for the steel output shaft assuming a working shear stress of 450 kg/sq cm.

The maximum torque on the output shaft is 90 kg metre.

(B) *Petrol engines:*

Engine types: Cylinders	Cycle	Single or double acting	Crankshaft	Firing frequency Angles of Crank turn	Average value of K
Single	4 stroke	single	—	720° regular	2·4
Single	2 stroke	single	—	360° regular	1·0
2-Twin	4 stroke	single	0°		
2-opposed	4 stroke	single	180°		
Single	4 stroke	double	—	180°-540° alternate	1·60
2-twin	4 stroke	single	180°		
Single	4 stroke	double	—	180° regular	0·2
2-twin or tandem	2 stroke or 4 stroke	double	180°		
4 in line	4 stroke	single	180°		
3 in line	4 stroke	single	120°	240° regular	0·7
4 in line	2 stroke	single	90°	90° regular	0·09
4 twin or tandem	4 stroke	double	90°		
8 in line	4 stroke	single	90°		
6 in line	4 stroke	single	120°	120° regular	0·12
V-12	4 stroke	single	120°	60° regular	0·02
V-16	4 stroke	single	90°	45° regular	0·01

(C) *Diesel Engines:*

The values of K given in table for petrol engines are to be increased by 25% for Diesel engines.

Thus if E is the energy developed by the engine per revolution, then fluctuation of energy $\delta E = K \times E$. When fluctuation of energy is known, the size of the flywheel can be calculated.

15·5. Flywheel for Electric generators:

When an engine drives an alternator, it is not the variation of speed within each cycle that is of prime concern but rather the phase shift that will be produced in a generated voltage as compared with a regular sinusoidal wave and this is of particular interest when alternators are to run in parallel. Since the flywheel is alternately absorbing and delivering energy the speed varies in each revolution as a result the flywheel mounted on the shaft is displaced ahead of and behind its mean position. Proper voltage regulation requires that the maximum angular displacement shall be limited to $2\frac{1}{2}$ or 3 electrical degrees. Since a complete electrical cycle transpires as the generator rotor turns

As the rim is rectangular in section, its dimension will be cm × 6 cm, providing 72 sq cm cross sectional area.

2. *A multi-cylinder engine is to run at a constant load at a speed 500 r.p.m. On drawing the crank effort diagrams to scales of cm = 250 kg metre and 1 cm = 30°, the areas above and below the ean torque line were measured and found to be sq cm in order:*

+ 1·60, − 1·72, + 1·68, − 1·91, + 1·97, − 1·62.

If the speed is to be kept within the limits 495 and 505 r.p.m., alculate the necessary moment of inertia of the flywheel.

Determine suitable dimensions for a cast iron flywheel with a rim vhose breadth is twice its radial thickness. Assume that 95% of the noment of inertia is to be provided by the rim. The density of cast iron is 7·25 gm/cu cm and its working stress in tension is 60 kg/sq cm.

The approximate shape of the crank effort diagram for a cycle is shown in fig. 15-2.

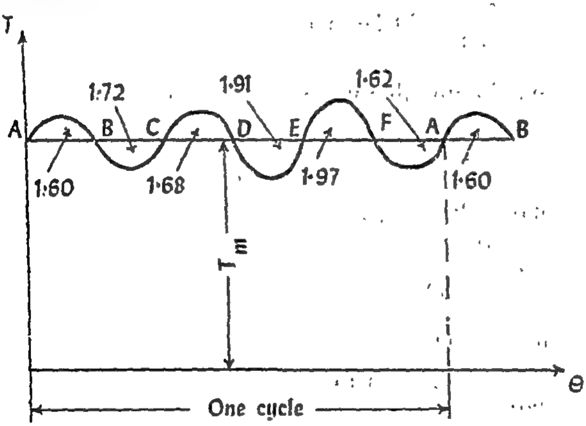


FIG. 15-2

Let E be the energy of the flywheel, in sq cm of crank effort diagram, at the beginning of the cycle.

The energies at salient points B, C, D, E and F of turning moment diagram will be $E + 1{\cdot}6$, $E - 0{\cdot}12$, $E + 1{\cdot}56$, $E - 0{\cdot}35$ and $E + 1{\cdot}62$ sq cm respectively. The maximum and minimum energies of the flywheel will be at F and E respectively. The maximum fluctuation of energy of the flywheel during a cycle will

If d cm be the diameter of the solid shaft then

$$\frac{\pi}{16}d^3 \times 450 = 90 \times 100$$

or $$d = \sqrt[3]{\frac{9000}{450} \times \frac{16}{\pi}} = 4{\cdot}67 \text{ cm; we adopt 5 cm.}$$

The energy required per cycle is given by the area under the torque angle graph which represents $90 \times \frac{\pi}{2} + 70 \times \frac{\pi}{2} = 80\pi$ kg metre of energy.

Number of cycles per minute $= \frac{500}{2} = 250$.

$\therefore$ Minimum h.p. of the driving motor $= \frac{250 \times 80\pi}{4500} = 14$.

As we have to allow for 25% overload, the rated h p. of the motor will be $14 \times 1{\cdot}25 = 17{\cdot}5$.

If T_m be the mean torque during a load cycle, then

$$T_m = \frac{80\pi}{2 \times 2\pi} = 20 \text{ kg metre.}$$

$\therefore$ δE = maximum fluctuation of energy $= 90 \times \frac{\pi}{2}\left[\frac{90-20}{90}\right]^2$

$= 86$ kg metre

(Areas of similar triangles are proportional to square of their altitudes)

$$\omega = \frac{500 \times 2\pi}{60} = 52{\cdot}5 \text{ radian/sec}$$

$$\delta\omega = \frac{4}{100} \times 52{\cdot}5 = 2{\cdot}12 \text{ radian/sec}$$

On substitution of values in equation $\delta E = I\omega\delta\omega$, we get

$$86 = \frac{Wk^2}{9{\cdot}81} \times 52{\cdot}5 \times 2{\cdot}12$$

or $$Wk^2 = \frac{86 \times 9{\cdot}81}{52{\cdot}5 \times 2{\cdot}12} = 7{\cdot}6 \text{ kg metre}^2$$

Assuming that the radius of gyration is same as the mean radius of the ~~flywheel, the~~ weight W of the flywheel rim will be $\frac{7{\cdot}6}{0{\cdot}3^2} = 84{\cdot}2$ kg. If a sq cm be the area of the cross section of the rim, then

$$a \times 2\pi \times 30 \times 7{\cdot}25 = 84{\cdot}2 \times 1000$$

or $$a = \frac{84{\cdot}2 \times 1000}{60 \times \pi \times 7{\cdot}25} = 61{\cdot}7 \text{ sq cm.}$$

If a be the cross sectional area of rim in sq. cm, then

$$\frac{\pi \times a \times 90 \times 7{\cdot}25}{1000} = 222$$

or
$$a = \frac{222 \times 1000}{\pi \times 90 \times 7{\cdot}25} = 107 \text{ sq cm.}$$

The rim section is a rectangle having breadth as twice the radial thickness. We adopt 16 cm wide and 8 cm thick section which provides 128 sq cm cross sectional area.

3. A punching machine is required to make 28 working strokes per minute and is to be capable of punching holes of 23 mm diameter in 18 mm thick plates of steel having an ultimate shear strength of 3,000 kg/sq cm. The punching of a hole is to occupy one-tenth of a revolution of the crank shaft of the machine.

Estimate the horse power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rectangular section rim of the flywheel, which is to revolve at 9 times the speed of the crank shaft.

The wheel is to be made of cast iron having a working tensile stress of 60 kg/sq cm and weighing 7·25 gm/cu cm. The diameter of the wheel must not exceed 140 cm owing to space considerations. It may be assumed that the hub and spokes provide 5 per cent of the rotational inertia of the wheel. Check the centrifugal stress.

Maximum punching force $= \pi \times 1{\cdot}8 \times 2{\cdot}3 \times 3000 = 39{,}200$ kg.

Energy required per stroke $= \frac{1}{2} \times 1{\cdot}8 \times 39200 = 35{,}200$ kg cm.

(We assume that the resisting shear force decreases uniformly from its maximum value to zero value as the hole is being punched.)

Number of cycles per minute = 28.

$$\therefore \text{ H.P. of the driving motor} = \frac{352 \times 28}{4500 \times 0{\cdot}95} = 2{\cdot}3.$$

The mean speed of the flywheel $= 28 \times 9 = 252$ r.p.m.

We adopt the mean diameter of the flywheel as 120 cm. The radius of gyration may be taken as 0·6 metre.

As the punching operation occupies one-tenth of a revolution of the crank shaft of the machine, during nine-tenth of the revolution of the crank shaft, the motor power is used to accelerate the speed of the flywheel; while during one-tenth revolution, when the actual punching operation is being performed, the energy stored up in the flywheel during the earlier part of the stroke is given out at the expense of its angular speed.

be equivalent to 1·97 sq cm of crank effort diagram. The energy scale of the diagram is

$$1 \text{ sq cm} = \frac{250 \times \pi}{6} = 131 \text{ kg metre.}$$

The maximum fluctuation of energy will be

$131 \times 1{\cdot}97 = 258$ kg metre.

Mean speed of the flywheel $= \frac{500 \times 2\pi}{60} = 52{\cdot}5$ radian/sec.

The maximum fluctuation of the speed of the flywheel

$$= \frac{(505 - 495)\, 2\pi}{60} = 1{\cdot}05 \text{ radian/sec.}$$

We have

$$\delta E = I\omega\, \delta\omega.$$

On substitution of values, we get

$$258 = I \times 52{\cdot}5 \times 1{\cdot}05$$

or $$I = \frac{258}{52{\cdot}5 \times 1{\cdot}05} = 4{\cdot}7 \text{ kg metre sec}^2.$$

The maximum diameter of the rim can be obtained from the centrifugal stress considerations. The centrifugal stress is given by the equation

$$f = 0{\cdot}0737\, v^2 \text{ kg/sq cm.}$$

On substitution of values, we have $60 = 0{\cdot}0737\, v^2$.

$$\therefore \quad v = \sqrt{\frac{60}{0{\cdot}0737}} = 28{\cdot}5 \text{ metre/sec}$$

If D cm be the outside diameter of the flywheel rim in metre,

then $$\frac{\pi D \times 505}{60} = 28{\cdot}5$$

or $$D = \frac{28{\cdot}5 \times 60}{\pi \times 505} = 1{\cdot}07 \text{ metre}$$

$\therefore$ Maximum radius of the flywheel $= \frac{1{\cdot}07}{2} = 0{\cdot}535$ metre.

$= 53{\cdot}5$ cm.

We adopt 45 cm as the mean radius of the flywheel. If W be the weight of the flywheel rim in kg, then

$$\frac{W}{9{\cdot}81} \times \left(\frac{45}{100}\right)^2 = 4{\cdot}7 \times 0{\cdot}95$$

or $$W = 4{\cdot}7 \times 0{\cdot}95 \times 9{\cdot}81 \times \frac{(100)^2}{(45)^2} = 222 \text{ kg.}$$

If a be the cross sectional area of rim in sq. cm, then

$$\frac{\pi \times a \times 90 \times 7{\cdot}25}{1000} = 222$$

or
$$a = \frac{222 \times 1000}{\pi \times 90 \times 7{\cdot}25} = 107 \text{ sq cm.}$$

The rim section is a rectangle having breadth as twice the radial thickness. We adopt 16 cm wide and 8 cm thick section which provides 128 sq cm cross sectional area.

3. A punching machine is required to make 28 working strokes per minute and is to be capable of punching holes of 23 mm diameter in 18 mm thick plates of steel having an ultimate shear strength of 3,000 kg/sq cm. The punching of a hole is to occupy one-tenth of a revolution of the crank shaft of the machine.

Estimate the horse power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rectangular section rim of the flywheel, which is to revolve at 9 times the speed of the crank shaft.

The wheel is to be made of cast iron having a working tensile stress of 60 kg/sq cm and weighing 7·25 gm/cu cm. The diameter of the wheel must not exceed 140 cm owing to space considerations. It may be assumed that the hub and spokes provide 5 per cent of the rotational inertia of the wheel. Check the centrifugal stress.

Maximum punching force $= \pi \times 1{\cdot}8 \times 2{\cdot}3 \times 3000 = 39{,}200$ kg.

Energy required per stroke $= \frac{1}{2} \times 1{\cdot}8 \times 39200 = 35{,}200$ kg cm.

(We assume that the resisting shear force decreases uniformly from its maximum value to zero value as the hole is being punched.)

Number of cycles per minute = 28.

$$\therefore \text{ H.P. of the driving motor} = \frac{352 \times 28}{4500 \times 0{\cdot}95} = 2{\cdot}3.$$

The mean speed of the flywheel $= 28 \times 9 = 252$ r.p.m.

We adopt the mean diameter of the flywheel as 120 cm. The radius of gyration may be taken as 0·6 metre.

As the punching operation occupies one-tenth of a revolution of the crank shaft of the machine, during nine-tenth of the revolution of the crank shaft, the motor power is used to accelerate the speed of the flywheel; while during one-tenth revolution, when the actual punching operation is being performed, the energy stored up in the flywheel during the earlier part of the stroke is given out at the expense of its angular speed.

The maximum fluctuation of the energy of the flywheel will be $\frac{9}{10} \times 352 = 317$ kg metre.

We assume that the permissible coefficient of fluctuation of speed is 0·1. As the flywheel rotates at 9 times the speed of the crank shaft, the mean speed of the flywheel shaft will be $28 \times 9 = 252$ r.p.m

$$\text{Mean speed of the flywheel} = \frac{252 \times 2\pi}{60} = 26{\cdot}4 \text{ radian/sec.}$$

Maximum fluctuation of speed of the flywheel will be $0{\cdot}1 \times 26{\cdot}4 = 2{\cdot}64$ radian/sec.

If W be the weight of the flywheel rim, then

$$\frac{W}{9{\cdot}81} \times 0{\cdot}6^2 \times 26{\cdot}4 \times 2{\cdot}64 = 317 \times 0{\cdot}95$$

$$\text{or } W = \frac{317 \times 0{\cdot}95 \times 9{\cdot}81}{0{\cdot}6^2 \times 26{\cdot}4 \times 2{\cdot}64} = 120 \text{ kg.}$$

If a sq cm be the area of the cross section of the rim, then $a \times \pi \times 120 \times 7{\cdot}25 = 1000 \times 120$

$$\text{or } a = \frac{1000 \times 120}{\pi \times 120 \times 7{\cdot}25} = 44 \text{ sq cm}$$

We adopt rim section as 9 cm × 5 cm, thus providing necessary area of 45 sq cm

The outside diameter of the flywheel rim will be 125 cm.

$$\text{The linear velocity of the rim will be } 1{\cdot}25 \times \pi \times \frac{252}{60} \times 1{\cdot}05$$

$$= 17{\cdot}3 \text{ metre/sec.}$$

$$\begin{aligned}\text{The centrifugal stress } &= 0{\cdot}0737v^2 \text{ kg/sq cm}\\ &= 0{\cdot}0737 \times 17{\cdot}3^2\\ &= 22 \text{ kg/sq cm}\end{aligned}$$

Note: In the problem discussed we notice that the flywheel is mounted on a high speed shaft. We can see that the higher the average speed of the flywheel for a given coefficient of fluctuation of speed, the smaller the flywheel need be. Therefore, it is desirable that the flywheel be attached to the shaft in the mechanism having the highest speed in order to reduce the size of the flywheel. However, there are limitations in the speed of the flywheel. Because of high stresses developed by centrifugal forces, these stresses must be considered in the selection of the flywheel speed. Also the gears connecting the flywheel and crank would have sudden loads imposed on them necessitating expensive gears. In most punch presses the flywheel is attached to the crank in order to eliminate the gears between the flywheel and crank.

Exercises:

1. *The flywheel of a punch must be capable of supplying 260 kg metre of energy in order that the machine may punch a hole. The flywheel is 125 cm in mean diameter and rotates at 150 r.p.m. when running at a normal speed. Determine the cross sectional area required for the rim of the cast iron flywheel if the coefficient of fluctuation of speed be limited to 0·15.* *Ans.* 80 sq cm.

2. *The turning moment diagram for a three cylinder engine is drawn to the following scales: crank displacement, 1 cm = 40°; turning moment, 1 cm = 700 kg metre. During one revolution of the crank the areas above and below the mean turning moment line taken in order are 0·60, 0·69, 0·64, 0·75, 0·78 and 0·58 sq cm. If the speed is to be kept within 1% of the mean speed which is 90 r.p.m. and the mean diameter of the flywheel is 210 cm, determine the minimum cross sectional area of the cast iron flywheel.* *Ans.* 420 sq cm.

3. *A single cylinder double acting pump is driven through gearing at 40 r.p.m. by an electric motor which gives a uniform torque. The resisting torque for each half revolution of the pump shaft may be assumed to follow a sine curve with a maximum value at 90° and 270° of 430 kg metre. Determine the diameter of the pump shaft if the permissible value of the shear stress is limited to 420 kg/sq cm. Determine the h.p. of the motor to drive the pump if the efficiency of transmission be 85%. Find what weight of flywheel will be required on the pump shaft to keep the speed within $1\frac{1}{2}$% of the mean speed if the mean radius of the flywheel is 120 cm. The flywheel effect of the motor armature and gear wheels is equivalent to 450 kg at a radius of 90 cm on the pump shaft. Suggest the suitable rim section for the cast iron flywheel.* *Ans.* 2,000 kg; 370 sq cm.

4. *A flywheel is required for a punching machine capable of punching 19 mm diameter holes through steel plates 16 mm thick. The machine shaft has a belt driven shaft at the rear which carries a flywheel and a pinion that meshes with a gear on the main shaft at the top of the machine which runs at $\frac{1}{10}$ th speed of the driven shaft. The ultimate shear strength of the material is 3,600 kg/sq cm. The hole is punched during $\frac{1}{10}$ revolution of the gear shaft. The preliminary lay out shows that the flywheel should have a mean diameter of about 80 cm. Determine the weight of the flywheel.*

5. *Design a cast iron flywheel for a four stroke cycle engine to develop 100 b.h.p. at 200 r.p.m. Assume that the work done in the power stroke is 1·25 times the average work during the whole cycle. The fluctuation of speed is to be limited to 4% and the diameter of the flywheel may*

The maximum fluctuation of the energy of the flywheel will be $\frac{9}{10} \times 352 = 317$ kg metre.

We assume that the permissible coefficient of fluctuation of speed is 0·1. As the flywheel rotates at 9 times the speed of the crank shaft, the mean speed of the flywheel shaft will be $28 \times 9 = 252$ r.p.m.

Mean speed of the flywheel $= \dfrac{252 \times 2\pi}{60} = 26{\cdot}4$ radian/sec.

Maximum fluctuation of speed of the flywheel will be $0{\cdot}1 \times 26{\cdot}4 = 2{\cdot}64$ radian/sec.

If W be the weight of the flywheel rim, then

$$\frac{W}{9{\cdot}81} \times 0{\cdot}6^2 \times 26{\cdot}4 \times 2{\cdot}64 = 317 \times 0{\cdot}95$$

$$\text{or } W = \frac{317 \times 0{\cdot}95 \times 9{\cdot}81}{0{\cdot}6^2 \times 26{\cdot}4 \times 2{\cdot}64} = 120 \text{ kg.}$$

If a sq cm be the area of the cross section of the rim, then

$$a \times \pi \times 120 \times 7{\cdot}25 = 1000 \times 120$$

$$\text{or } a = \frac{1000 \times 120}{\pi \times 120 \times 7{\cdot}25} = 44 \text{ sq cm.}$$

We adopt rim section as 9 cm × 5 cm, thus providing necessary area of 45 sq cm.

The outside diameter of the flywheel rim will be 125 cm.

The linear velocity of the rim will be $1{\cdot}25 \times \pi \times \dfrac{252}{60} \times 1{\cdot}05$

$$= 17{\cdot}3 \text{ metre/sec.}$$

$$\begin{aligned} \text{The centrifugal stress } &= 0{\cdot}0737 v^2 \text{ kg/sq cm} \\ &= 0{\cdot}0737 \times 17{\cdot}3^2 \\ &= 22 \text{ kg/sq cm.} \end{aligned}$$

Note: In the problem discussed we notice that the flywheel is mounted on a high speed shaft. We can see that the higher the average speed of the flywheel for a given coefficient of fluctuation of speed, the smaller the flywheel need be. Therefore, it is desirable that the flywheel be attached to the shaft in the mechanism having the highest speed in order to reduce the size of the flywheel. However, there are limitations in the speed of the flywheel. Because of high stresses developed by centrifugal forces, these stresses must be considered in the selection of the flywheel speed. Also the gears connecting the flywheel and crank would have sudden loads imposed on them necessitating expensive gears. In most punch presses the flywheel is attached to the crank in order to eliminate the gears between the flywheel and crank.

It can be seen from equation (ii) that the bending stress can be reduced by increasing the number of arms and it can be entirely eliminated by providing a web construction.

Although the equation (iii) is often used to calculate the stresses in the flywheel rim, care should be exercised in using it because the shrinkage stresses at the junction of rim and arms are very severe if the wheel is not carefully designed.

Wherever possible the cross section of the rim should be designed to withstand bending action which requires the radial depth of the rim larger than the width.

In order to reduce the shrinkage stresses the flywheel may be cast in sections and bolted together or the sections may be connected by shrink links or shrink rings. The bursting force to be resisted by the joint is $\frac{\rho v^2}{g} A$, where A is the area of the rim. When the permissible stress for the connecting member is known, the minimum cross sectional area for the bolt or shrink link or shrink ring can be calculated. The bolts at the hub are usually made of the same size as those at the rim.

15-7. Design of a hub:

The flywheel is mounted on a shaft which may transmit only the torque going to or coming from the flywheel. Manytimes the flywheel shaft transmits a torque greater than the flywheel torque. *The hub is designed as a hollow shaft transmitting the torque acting on the flywheel.* Usually the hub diameter is taken twice the shaft diameter while the length equals 2 to 2·5 times the shaft diameter.

15-8. Arms of a flywheel:

The arms of the flywheel are usually of elliptical cross section whose major axis is usually twice the minor axis and the major axis of the ellipse is in the plane of rotation to give the arms greater resistance to bending stresses and reduce the air resistance which may be considerable at a high velocity.

The stresses in the arms may be severe, due to the inertia of a heavy rim when sudden load changes occur. In addition the arms are subjected to complete reversal of stresses. Therefore the

not exceed 4 metres. Assume that the spokes and the hub provide 5% of the rotational inertia of the wheel and the allowable tensile stress in cast iron as 200 kg/sq cm. Cast iron weighs 7·25 gm/cu cm. Allowable shear stress in mild steel 400 kg/sq cm and tensile stress 800 kg/sq cm.

(Bombay University, 1968)

15-6. Stresses in a rim of flywheel:

The analysis of stresses in flywheel is rather complicated because of the effect of the centrifugal force in the rim and arms, stresses due to change of speed and load and unknown shrinkage stresses caused by unequal cooling rates of the metal in the hub, arms and rim.

The centrifugal stresses can be obtained, with considerable accuracy. If the arms have no restraining effects on the rim, the centrifugal stresses in the rim are given by

$$f_c = \frac{\rho v^2}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

where ρ = density of the material,

v = mean velocity of the rim,

g = acceleration due to gravity.

When the arms of the flywheel are rigid, the portion of the rim between the arms may be considered as a fixed beam with a uniform loading, equal in intensity to the centrifugal force. If we consider the portion of the rim as a straight beam fixed at both ends, the maximum bending stress will be

$$f_b = \frac{\pi^2 D \rho v^2}{g b n^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

where D = mean diameter

b = rim thickness

n = number of arms.

It should be remembered that the derivation of the bending stress by the beam theory is inaccurate.

Since the arms are neither completely rigid nor completely flexible the actual stress value shall lie between two extreme cases

According to Prof. Lanza (Transactions of A.S.M.E. Vol. 20 page 951), *the stress in the ordinary type of flywheels may be assumed to be equal to three-fourths of the hoop stress plus one-fourth of the bending stress.* Therefore, the resulting stress may be taken as

$$f = \tfrac{3}{4} f_c + \tfrac{1}{4} f_b = \tfrac{1}{4}\frac{\rho v^2}{g}\left[3 + \frac{\pi^2 D}{b n^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

No. of arms	Length of major axis at hub
6	$1{\cdot}3d$
8	$1{\cdot}2d$

If a flywheel rim is very heavy, the arms may be considered as a beam fixed at both ends. This assumption will give us equal bending moments at hub and rim, which has half the value of a simple cantilever. The dimension should be the same at both the places.

The usual procedure is to design the arm at the hub as a simple cantilever and it is tapered towards the rim. The usual taper of 1 in 50 is commonly used on the width and about half this amount on the thickness. The width at the rim, however, should never be less than 0·8 times the width at the hub.

Examples:

1. Design a cast iron flywheel to store 60,000 kg metre of energy at 180 r.p.m. The radius of gyration is 120 cm. Calculate

(i) *weight and thickness of the rim if width is 35 cm (Assume that 90 per cent of the energy is stored in the rim.),*

(ii) *diameter of the shaft if 360 horse power is transmitted,*

(iii) *diameter of six arms, cross section to be elliptical, minor axis 0·5 times the major axis; safe stress 150 kg/sq cm,*

(iv) *dimensions of the sunk rectangular key and*

(v) *dimensions of the boss of the flywheel.*

The angular velocity of the flywheel $= \dfrac{180 \times 2\pi}{60}$

$= 18{\cdot}8$ radian/sec.

If W be the weight of the flywheel rim, then

$$60000 \times 0{\cdot}9 = \tfrac{1}{2} \times \frac{W}{9{\cdot}81} \times 1{\cdot}2^2 \times (18{\cdot}8)^2$$

or
$$W = \frac{60000 \times 9{\cdot}81 \times 2 \times 0{\cdot}9}{1{\cdot}2 \times 1{\cdot}2 \times 18{\cdot}8 \times 18{\cdot}8} = 2{,}080 \text{ kg.}$$

If a sq cm be the area of cross section of the rim, then

$$a \times \pi \times 2 \times 120 \times 7{\cdot}25 = 2080 \times 1000$$

factor of safety should be at least 8. When a flywheel is used on metal cutting machineries subjected to severe shock, the factor of safety should be as high as 15.

The arms are designed to carry the maximum torque acting on the flywheel. The determination of maximum torque on the flywheel requires careful consideration. Each arm is assumed to act as a cantilever, which is fixed at the hub and carries its proportionate share of the load concentrated at the rim. This assumption regarding cantilever is permissible when the rim is flexible. This assumption may be taken when the rim is light

If T be the torque acting on the rim of mean diameter D, the maximum bending moment M on the arm at the hub of outside diameter d will be

$$M = \frac{T}{n} \times \frac{(D-d)}{D} \quad \ldots (i)$$

where n = number of arms.

If we assume that the arm of a cantilver extends upto the centre of the shaft, then

$$M = \frac{T}{n} \quad \ldots (ii)$$

If Z be the section modulus of the arm cross section at the hub, the maximum stress will be

$$f = \frac{M}{Z} \quad \ldots (iii)$$

While fixing the permissible stress for the flywheel arm it should be remembered that, the direct tensile stress due to radial expansion of the rim is present. The value of this stress is equal to the stress in the rim when it is free to expand due to centrifugal stress. The permissible tensile stress for cast iron should not exceed 140 kg/sq cm and if there be severe shock as in pumps the stress should not exceed 70 kg/sq cm

For an elliptical section, the modulus of section near the hub is $Z = \frac{\pi}{32} ab^2$ where a is the minor axis and b the major axis. The arms usually taper towards the rim. The cross sectional area at the rim should not be less than two thirds the area at the hub.

The usual number of arms is six but these may be eight, ter twelve depending upon the diameter and rim width of the flywh

The following proportions of the arms in terms of the s diameter are usually adopted for cast iron flywheel:

The minor axis will be 7·5 cm.

The dimensions of the elliptical cross section at the rim are 10 cm $\times$ 5 cm.

Let us verify whether we can arrange six arms at the hub or not. The circumference of the hub $= \pi \times 30 = 94{\cdot}2$ cm. As there are six arms. the maximum value of the major axis of the elliptical cross section should be limited to $\frac{94{\cdot}2}{6} = 15{\cdot}7$ cm. As this value is more than 15 cm, the arms can be accomodated.

2. *Design a cast iron flywheel for a four stroke engine to develop 150 h.p. (brake) at 150 r.p.m. The work done during the power stroke is 1·3 times the average work done during the whole cycle. The mean diameter of the flywheel may be taken as 3 metre. The total fluctuation of speed is to be limited to 5 per cent of the mean.*

Work done during a cycle $= \dfrac{150 \times 4500}{75} = 9{,}000$ kg metre.

Work done during the power stroke $= 1{\cdot}3 \times 9000$
$= 11{,}700$ kg metre.

Average resistance overcome per stroke $= \dfrac{9000}{4}$
$= 2{,}250$ kg metre.

Excess energy to be stored in the flywheel during the power stroke $= 11700 - 2250 = 9{,}450$ kg metre.

Mean speed of the flywheel $= \dfrac{150 \times 2\pi}{60} = 15{\cdot}7$ rad/sec.

Maximum speed fluctuation $= 0{\cdot}05 \times 15{\cdot}7 = 0{\cdot}785$ rad/sec.

If W kg be the weight of the flywheel rim, then

$$9450 = \frac{W}{9{\cdot}81} \times 1{\cdot}5^2 \times 15{\cdot}7 \times 0{\cdot}785$$

or $$W = \frac{9450 \times 9{\cdot}81}{1{\cdot}5^2 \times 15{\cdot}7 \times 0{\cdot}785} = 3{,}360 \text{ kg.}$$

Assuming a rectangular cross section of the rim, width being twice the thickness, we get

$$3360 \times 1000 = \pi \times 300 \times 2t \times t \times 7{\cdot}25$$

or $$t = \sqrt{\frac{3360 \times 1000}{\pi \times 300 \times 2 \times 7{\cdot}25}} = 15{\cdot}7 \text{ cm; we adopt 16 cm.}$$

Width of rim $= 2 \times 16 = 32$ cm.

Assuming that the turning moment diagram for a power stroke is a triangle, we get

or $$a = \frac{2000 \times 1000}{\pi \times 2 \times 120 \times 7{\cdot}25} = 380 \text{ sq cm.}$$

Thickness of the rim $= \frac{380}{35} = 10{\cdot}85$ cm; we adopt 11 cm.

Torque on the shaft $= \frac{71620 \times 360}{180} = 143{,}240$ kg cm.

Assuming permissible shear stress to be 400 kg/sq cm, the *diameter of the shaft can be obtained. If d be the diameter of the* solid shaft, then $\frac{\pi}{16} d^3 \times 400 = 143210$

or $$d = \sqrt[3]{\frac{143210 \times 16}{400 \times \pi}} = 12{\cdot}2 \text{ cm}$$

We adopt 14 cm as the diameter of the solid shaft

The dimensions of the rectangular sunk key should be checked for the stresses. The dimensions of the key may be taken as 30 cm $\times$ 3 cm $\times$ 2·5 cm.

The outside diameter of the boss may be taken as 30 cm and the length of the boss may be taken as 25 cm

The arms of the flywheel are subjected to bending moment due to inertia torque; its magnitude depends upon the rate at which the energy is either given out by the flywheel or stored by *the flywheel. The maximum value of the torque on which the* arms are to be designed is equal to the product of the moment of inertia of the flywheel and the maximum angular acceleration or retardation. Here we do not have the knowledge regarding the maximum angular acceleration. Hence we design on the mean torque transmitted

The flywheel has six arms. We design the arm as a cantilever whose moment arm extends upto the centre of the shaft. The maximum bending moment on each arm will be $\frac{143240}{6} = 24{,}000$ kg cm.

Modulus of section required $= \frac{24000}{150} = 160$ cm³.

If b cm be the major axis of the elliptical section at the hub, then $\frac{\pi}{32} \times 0{\cdot}5\, b^3 = 160$

or $$b = \sqrt[3]{\frac{160 \times 32}{\pi \times 0{\cdot}5}} = 14{\cdot}7 \text{ cm; we adopt 15 cm.}$$

The minor axis will be 7·5 cm.

The dimensions of the elliptical cross section at the rim are 10 cm × 5 cm.

Let us verify whether we can arrange six arms at the hub or not. The circumference of the hub $= \pi \times 30 = 94{\cdot}2$ cm. As there are six arms. the maximum value of the major axis of the elliptical cross section should be limited to $\frac{94{\cdot}2}{6} = 15{\cdot}7$ cm. As this value is more than 15 cm, the arms can be accomodated.

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If W kg be the weight of the flywheel rim, then

$$9450 = \frac{W}{9{\cdot}81} \times 1{\cdot}5^2 \times 15{\cdot}7 \times 0{\cdot}785$$

or $$W = \frac{9450 \times 9{\cdot}81}{1{\cdot}5^2 \times 15{\cdot}7 \times 0{\cdot}785} = 3{,}360 \text{ kg.}$$

Assuming a rectangular cross section of the rim, width being twice the thickness, we get

$$3360 \times 1000 = \pi \times 300 \times 2t \times t \times 7{\cdot}25$$

or $$t = \sqrt{\frac{3360 \times 1000}{\pi \times 300 \times 2 \times 7{\cdot}25}} = 15{\cdot}7 \text{ cm; we adopt 16 cm.}$$

Width of rim $= 2 \times 16 = 32$ cm.

Assuming that the turning moment diagram for a power stroke is a triangle, we get

$$T_{max} \times \frac{\pi}{2} = 11700$$

or $$T_{max} = \frac{11700 \times 2}{\pi} = 7{,}450 \text{ kg metre.}$$

Let us assume that the permissible shear stress in the shaft material is limited to 400 kg/sq cm. If d cm be the diameter of the solid shaft, then $\frac{\pi}{16} d^3 \times 400 = 7450 \times 100$

or $$d = \sqrt[3]{\frac{7450 \times 100 \times 16}{400 \times \pi}}$$

$= 21{\cdot}2$ cm; we adopt 22 cm

The outside diameter of the boss may be taken as 44 cm and the length may be taken as 35 cm.

$$\text{Resisting torque on the shaft} = \frac{150 \times 4500}{2\pi \times 150}$$
$$= 720 \text{ kg metre.}$$

Torque acting on flywheel $= 7450 - 720 = 6{,}730$ kg metre

We take six arms of elliptical section for the flywheel. The minor axis of the ellipse may be taken as 0·5 times the major axis.

$$\text{Maximum bending moment on each arm} = \frac{6730 \times 100}{6}\left[\frac{300-44}{300}\right]$$
$$= 96{,}000 \text{ kg cm.}$$

We assume that the permissible stress for the arm is 200 kg/sq cm.

If b cm be the major axis of the ellipse, we have

$$96000 = 200 \times \frac{\pi}{32} \times 0{\cdot}5b \times b^2$$

or $$b = \sqrt[3]{\frac{96000 \times 32}{200 \times \pi \times 0{\cdot}5}} = 22 \text{ cm.}$$

The minor axis of the ellipse will be 11 cm

The arm section at the rim will have 18 cm as the major ax and 9 cm as the minor axis.

It can be verified that the arms can be accomodated.

3. *An electric motor is employed to drive a rolling mill, the pou being supplied by 12, 4 cm diameter cotton ropes. The normal speeds motor and mill shaft are 360 r.p.m and 90 r.p.m. respectively. preliminary design of the flywheel for the drive suggests 780 sq cm as rim section. The centre of gravity of the rim section is 2·4 metre from flywheel axis. The flywheel, which is made of cast iron, is cast in*

Assume pulley diameter = 30 × rope diameter.

Ans. *12 ropes; area of cross section of the grooved rim = 690 sq cm; 4-M 48 bolts will suffice.*

6. Assuming the maximum stress in a flywheel rim can be approximated as suggested by Prof. Lanza, by adding $\frac{3}{4}$ *of the stress computed by considering the rim as a free rotating ring and* $\frac{1}{4}$ *of the stress computed by considering the rim as a straight beam of length equal to the arc between arms, fixed at both ends, and loaded uniformly with inertia forces, derive the equation for maximum stress. Take the density of cast iron as 7·26 gm/cu cm.*

Use the above equation to determine the maximum tensile stress in the thin rim of a cast iron flywheel running at 500 r.p.m. The rim has a mean radius of 150 cm, thickness 20 cm and width 30 cm. The flywheel has six arms. Comment on the result. How can you reduce the maximum stress in the rim without changing the speed of rotation?

EXAMPLES XV

1. A plate shearing machine requires 102 horse power for a period of 5 seconds during shearing operation. The motor to drive the machine has a maximum power rating of 95 horse power.

Two equal flywheels are to be provided to limit the speed fluctuation of the main shaft during shearing operations to 15 r.p.m. The maximum speed is to be 110 r.p.m.

Determine suitable dimensions for the rectangular section rim of each flywheel, assuming 5% of the rotational inertia to be provided by the hub and spokes. The wheels are to be of cast iron having density 7·25 gm/cu cm. The centrifugal stress must not exceed 55 kg/sq cm.

For space reasons the diameter of the wheels must not be more than 180 cm.

Give a working sketch, approximately to scale of a suitable wheel, estimating the remaining dimensions by judgment.

Ans. 26 cm × 13 cm by 150 cm mean diameter would suit.

2. A machine is to be driven at 450 r.p.m. by an electric motor which exerts a constant rorque. The load torque diagram on the machine varies cyclically as shown in fig. 15-3. Estimate the necessary horse power of the motor. Decide suitable dimensions for the rim of a flywheel to be mounted on the machine shaft to limit the cyclic speed fluctuation to ± 1·5% of the mean speed. The other rotating parts of the machine and motor have a combined moment of inertia of 0·6 kg metre^2. The flywheel rim is to have an outside diameter of 75 cm and is to be of rectangular section. The flywheel is made of cast iron for which the density is 7·25 gm/cu cm.

design ■ suitable rim, boss and arms. The rim speed should not exceed 1,250 metre/minute. Suggest the suitable size of the shaft.

3. Design the main dimensions of a flywheel for a punch press operating under the following conditions:

The press requires 15 h.p.; the complete punching cycle consists of 8 revolution, of the flywheel of which only three take place during the working portion of the cycle. There are 24 complete cycles per minute. The desirable mean diameter of the flywheel is 150 cm The coefficient of steadiness is 5

The flywheel effect due to weight of the boss and arms is 8 per cent of the total flywheel effect. The flywheel is to have 6 arms of elliptical cross section with the major axis twice the minor axis. The stress in the arm is not to exceed 100 kg/sq cm.

4 Design a flywheel for a punch press which must be brought to rest by one punching operation if the power has been shut off. The maximum work required consists in punching 30 mm hole in 20 mm thick mild steel plate for which ultimate shear strength is 3,500 kg/sq cm. The punch capacity is 24 holes per minute, and the speed ratio of the driving shaft to the eccentric shaft operating the punch is 9 . 1 In order to clear the floor, the wheel diameter cannot be larger than 105 cm. The wheel is keyed to the driving shaft, and the mechanical efficiency of the press and drive is 72%. Determine (a) the weight of the flywheel rim if the density of cast iron is 7·25 kg/dm^3 neglecting the energy stored in the arms and hub, (b) the maximum stress in the rim, and (c) the coefficient of steadiness of the driving shaft if the duration of the working stroke is one-half that of the idle stroke.

5. An electric motor is employed to drive a rolling mill, the power being transmitted by cotton ropes. The motor is rated at 260 h p , which is the normal power required, but is capable of taking a 40% overload for a short time. The mill is liable to require excess power upto 420 h p. for a maximum duration of 10 seconds. The mill shaft rope pulley must supply the excess power with a drop of speed not exceeding 10% of the mean speed. The normal speeds of motor and mill shaft are 400 r p.m and 90 r.p.m. respectively. Ropes of 4 cm nominal diameter with a maximum load of 175 kg; weight 0·91 kg per metre run, coefficient of friction between rope and pulley is 0·2 Angle of pulley groove is 45°.

Design the flywheel, which is to be cast in two halves, showing details of the joints at the rim and boss.

Width of key seat may be taken as 10 mm.
Draw a dimensioned sketch of a sectional elevation and end view.

(Bombay University, 1965)

5. The flywheel of a rolling-mill engine weighs 30 tonnes and has a mean radius of 335 cm and an average speed of 60 r.p.m. It is secured to a shaft of 30 cm diameter. During the rolling operation which lasts for 2 seconds, the speed drops down from 60 to 40 r.p.m. The flywheel has 10 arms of elliptical section, the minor axis being 0·5 of the major axis, and the tensile stress in arms as 120 kg/sq cm. Calculate (a) section of the rim if the radial depth is half the axial width if for cast iron the density is 7·25 kg/dm³, (b) the cross section of the arm.

Sketch the flywheel designed by you.

(Sardar Vallabhbhai Vidyapeeth, 1966)

6. The following particulars refer to a punching and shearing machine:
The diameter of the largest hole to be punched 2 cm
Thickness of steel plate 1·3 cm
Ultimate strength of steel plate in shear 3,500 kg/sq cm
Mechanical efficiency of the machine 80%
Mean speed of flywheel shaft 200 r.p.m.
Fluctuation of the flywheel speed is limited to 12% of the mean speed.
Design and prepare a dimensioned drawing of the flywheel required for the above machine.

The actual punching operation is completed during 45° rotation of the flywheel shaft. The mean diameter of the flywheel is limited to 1 metre by other design considerations and space requirements. The flywheel has six arms of elliptical cross section.

Select your own materials and stresses. (Gujarat University, 1966)

7. A 75 H.P. 400 r.p.m. engine has two cylinders with cranks at right angles. The coefficient of speed fluctuation is not to exceed 3% and the maximum variation of energy per revolution is found to be 20% of the mean energy of a revolution. If it is assumed that the arms and hub contribute 5% of the flywheel effect, determine the dimensions of the rim of the flywheel if the speed at the mean radius does not exceed 28 metre/sec. Assume a square cross section for the rim and the density of C.I. as 0·00725 kg/cu cm.

Sketch the flywheel with the assumption that the shafting is subjected to negligible bending moment and that the arms are about ¾ strong as the shaft to resist the maximum turning moment. Take allowable shear stress for shafting = 400 kg/sq cm and the allowable tensile stress for C.I. = 150 kg/sq cm.

(Bombay University, 1967)

8. Design a flywheel for a 4 cylinder 45 h.p. engine running at 1,200 r.p.m. The variation of speed to be within ± 2% of mean speed and the fluctuation of energy may be assumed as ¼ of that of one revolution. The cross section of flywheel rim is a rectangle with width equal to twice the depth. Density of steel 7·8 gm/cu cm and allowable shear stress in steel 350 kg/sq cm. The diameter of the flywheel should not exceed 25 cm. Sketch the flywheel.

(Bombay University, 1968)

Suggest the suitable cross sectional dimensions for the arms of the pulley, which has got six arms of elliptical cross section.

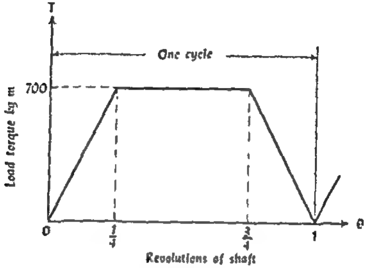


Fig 15-3

3. Fig 15-4 shows a rope drive between two shafts with axes parallel. The smaller driving pulley diameter has p.c.d of 60 cm and rotates at 500 r.p.m. The larger pulley is to rotate at 200 r.p.m. Ropes 3 cm diameter are to be used having a weight of 0.575 kg/metre, the pulley groove angle is 45° and the coefficient of friction between rope and pulley $\mu = 0.2$. Find the number of ropes required to transmit 120 h.p. if the maximum allowable tension in each rope is 90 kg.

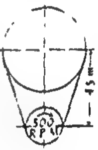


Fig 15-4

The larger pulley is to act as a flywheel to provide 30% excess power for a period of 6 seconds with a speed reduction not exceeding 10%. Calculate the minimum required weight of the larger pulley assuming a radius of gyration of 70 cm.

4. An automobile four cylinder engine develops 30 metric b.h.p. at 1,200 r.p.m. Fluctuation of energy is 30% of the energy developed in one revolution. Design a suitable flywheel of 200 mm diameter to keep the speed within ± 2% of the mean speed.

Given that:

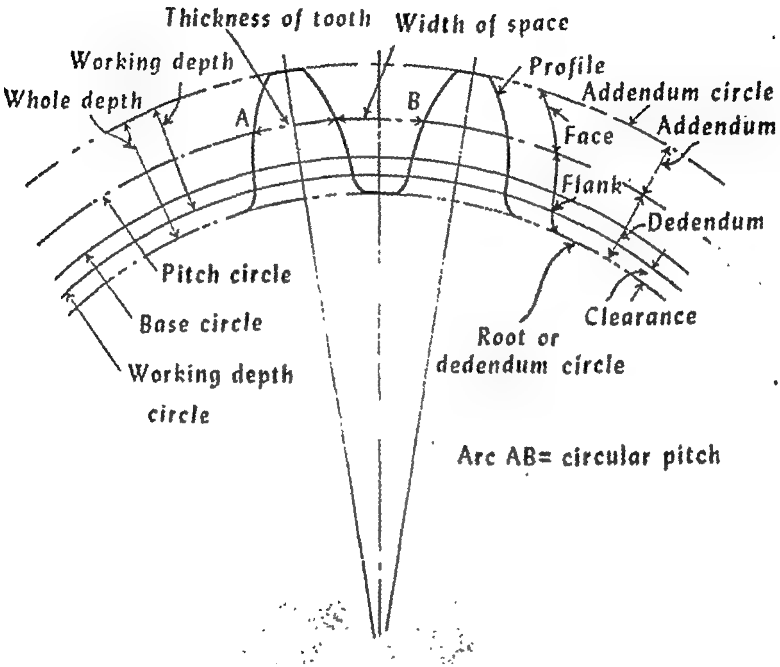
Width of steel rim of fly wheel	73 mm
Weight of steel	7,859 kg/cubic metre
Safe shear stress for shaft material	350 kg/sq cm

16-3. Spur gear terminology (fig. 16-1):

The following are the definitions of terms used with tooth of spur gears:

The addendum circle is the circle that limits the tops of the teeth.

The addendum is the radial distance from the pitch circle to the addendum circle.



CHAPTER XVI

16-1. Introduction:

Toothed gears are used in preference to belts where moderate or large amounts of power must be transmitted at a constant velocity as the drive is positive. Gears may transmit power from parallel, intersecting or skew shafts.

Three general types of toothed gearing are classified with respect to the relative position of the axes of the shafts on which the gears are mounted. The first type of gearing known as *spur gearing*, which is used for connecting shafts whose axes are parallel, includes external gearing, internal gearing, rack and pinion, helical gearing and herringbone gearing. The second type of gearing known as *bevel gearing*, which is used for connecting intersecting shafts, includes straight bevel gears, miter gears, crown gears and spiral gears. The third type of gearing includes gearing for shafts whose axes are *neither parallel nor intersecting* consisting of worm and worm wheels, hypoid gearing and spiral gearing.

(A) DESIGN OF SPUR GEARS:

16-2. General characteristics:

The following are the general characteristics of spur gearing·

(i) The drive transmits a constant velocity ratio

(ii) As the centre distance may be relatively short, the drive is a compact one

(iii) The provision can be made for interchanging them to change the speed of the driven members.

(iv) The provision can be made for shifting them along the axes to get the various speed changes for the driven members.

(v) As the loss of power transmitted is less than 1 per cent, the efficiency is very high.

(vi) The life of the drive is long.

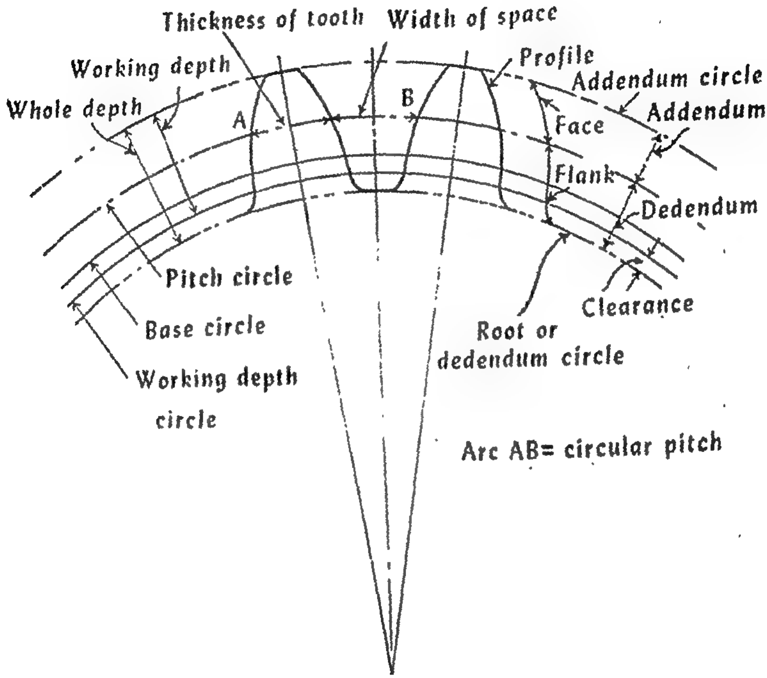
(vii) The maintenance cost of the drive is less.

16-3. Spur gear terminology (fig. 16-1):

The following are the definitions of terms used with tooth of spur gears:

The addendum circle is the circle that limits the tops of the teeth.

The addendum is the radial distance from the pitch circle to the addendum circle.



Spur gear tooth parts
Fig. 16-1

The pitch circle is the trace of the pitch cylinders on which the tooth curves are formed. The size of the gear is dictated by the diameter of the pitch circle.

The dedendum or root circle is the circle that limits the bottom of the tooth. The dedendum or root distance is the radial distance from the pitch circle to the dedendum or root circle.

The clearance is the distance between the top of the mating tooth and the bottom of the space of the other gear into which the tooth projects.

The working depth circle is a circle of radius equal to that of the dedendum circle plus the clearance.

The base circle is one that occurs only in the involute system of gears. It is the circle from which the involute curve of the tooth profile is generated. Base circle diameter = pitch circle diameter × cosine of the pressure angle.

The pressure angle is the angle which the line of action makes with the common tangent to the pitch circle. For involute system of gears, the pressure angle is constant and it may be either 14½ or 20 degrees

The face is that part of the tooth lying between the pitch and addendum circles. The face of the gear is its width measured along a tooth of a straight tooth gear.

The flank is that part of the tooth which lies between the pitch and dedendum circles.

The thickness of the tooth is its width *measured on the pitch circle. The* width of the space is the space between the teeth measured on the pitch circle.

The backlash is the difference between the thickness of a tooth and the space into which it meshes, measured on the pitch circles *Theoretically it should be zero. But from a practical view point some backlash is provided*

The circular pitch is the distance between similar points of adjacent teeth measured along the pitch circle It is denoted by a letter "*p*".

The diametral pitch is the number of teeth on the gear per unit of diameter of the pitch circle. The product of the circular pitch and the diametral pitch equals π. *The reciprocal of diametral pitch is known as module of gears* It is denoted by a letter "*m*".

The angles of action are the angles through which the gears turn while a pair of teeth are in action

The angle of approach is the angle through which a gear turns from the beginning of contact of a pair of teeth until the contact reaches the pitch point. The angle of recess is the angle through which a gear turns while the contact point of a pair of teeth moves from the pitch point to the point where the teeth pass out of contact.

Angle of action = angle of approach + angle of recess

16-4. Gear tooth forms:

The law of gearing states that *for a constant velocity ratio the profile of the teeth must be such that the common normal to the profiles at any point of contact always intersects the line of centres at the same point, which is called the pitch point, which is the point of tangency of their pitch circles* The number of curves can be employed for the teeth profile However, from a commercial stand point, *cycloidal and involute curves are employed giving rise to cycloidal and involute systems* Due to the number of advantages, the involute system is exclusively used in modern gear practice

The profiles of gear teeth as well as dimensions have been standardised. Gears having 14½ degree pressure angles are made either in 14½ degree full depth system or in the composite system. The latter type is stronger but is not interchangable In order to

get stronger teeth in involute system the pressure angle of 20 degree is adopted. However, the force tending to separate the mating gear is greater with 20 degree pressure angle. Still greater gear tooth strength may be obtained by using 20 degree involute stub tooth. As a general rule, stub tooth gears with less than 25 teeth are near about 25 per cent stronger than 20 degree full depth teeth and 40 per cent stronger than $14\frac{1}{2}$ degree involute teeth. The gain in strength decreases as the number of teeth increases.

The following proportions for tooth form have been recommended by American Standards Association.

Dimensions for $14\frac{1}{2}$ degree composite, $14\frac{1}{2}$ degree full depth and 20 degree full depth gears (involute system)

Part	In terms of circular pitch p
Addendum	$0{\cdot}3183\,p$
Minimum dedendum	$0{\cdot}3683\,p$
Working depth	$0{\cdot}6366\,p$
Minimum total depth	$0{\cdot}6866\,p$
Minimum clearance	$0{\cdot}0500\,p$
Pitch diameter	$0{\cdot}3183\,Np$
Outside diameter	$0{\cdot}3183\,(N+2)\,p$

Dimensions for 20 degree stub gears, helical gears and herringbone gears (involute system)

Addendum	$0{\cdot}2546\,p$
Minimum dedendum	$0{\cdot}3183\,p$
Working depth	$0{\cdot}5092\,p$
Minimum total depth	$0{\cdot}5729\,p$
Minimum clearance	$0{\cdot}0637\,p$
Pitch diameter	$0{\cdot}3183\,Np$
Outside diameter	$0{\cdot}3183\,(N+1{\cdot}6)\,p$

N = number of teeth in a gear

The following modules have been standardised by Indian Standards Institution:

1, (1·125), 1·25, (1·375), 1·5, (1·75), 2, 2·25, 2·5, (2·75), 3, [3·25], 3·5, [3·75], 4, (4·5), 5, 5·5, 6, [6·5], (7), 8, (9), 10, (11), 12, (14), 16, (18), 20.

The *pressure angle* is *the angle which the line of action makes with the common tangent to the pitch circle*. For involute system of gears, the pressure angle is constant and it may be either 14½ or 20 degrees.

The *face* is that part of the tooth lying between the pitch and addendum circles. The face of the gear is its width measured along a tooth of a straight tooth gear.

The *flank* is that part of the tooth which lies between the pitch and dedendum circles.

The *thickness* of the tooth is its width measured on the pitch circle. The width of the space is the space between the teeth measured on the pitch circle.

The *backlash* is the difference between the thickness of a tooth and the space into which it meshes, measured on the pitch circles *Theoretically it should be zero But from a practical view point some backlash is provided*

The *circular pitch* is the distance between similar points of adjacent teeth measured along the pitch circle. It is denoted by a letter "p".

The *diametral pitch* is the number of teeth on the gear per unit of diameter of the pitch circle. The product of the circular pitch and the diametral pitch equals π. The reciprocal of diametral pitch is known as module of gears. It is denoted by a letter "m"

The *angles of action* are the angles through which the gears turn while a pair of teeth are in action.

The *angle of approach* is the angle through which a gear turns from the *beginning of contact of a pair of teeth until the contact reaches the pitch point*. The angle of recess is the angle through which a gear turns while the contact point of a pair of teeth moves from the pitch point to the point where the teeth pass out of contact.

Angle of action = angle of approach + angle of recess

16-4. Gear tooth forms:

The law of gearing states that *for a constant velocity ratio the profile of the teeth must be such that the common normal to the profiles at any point of contact always intersects the line of centres at the same point, which is called the pitch point, which is the point of tangency of their pitch circles.* The number of curves can be employed for the teeth profile However, from a commercial stand point, *cycloidal and involute curves are employed giving rise to cycloidal and involute systems* Due to the number of advantages, the involute system is exclusively used in modern gear practice.

The profiles of gear teeth as well as dimensions have been standardised. Gears having 14½ degree pressure angles are made either in 14½ degree full depth system or in the composite system. The latter type is stronger but is not interchangable. In order to

surface finishing methods, operating conditions, peripheral velocities of wheels and efficiency.

The following table gives the peripheral speed of gears of different quality (accuracy):

Quality (accuracy)	Peripheral speed of gears metre/second
1—4	Above 15
5—7	Above 8 and upto 15
8—9	Above 1 and upto 8
10—12	upto 1

Three types of norm have been established for each degree of accuracy:

(i) Kinematic accuracy of wheel
(ii) Smooth operation
(iii) Contact between the teeth.

The following table gives the values of backlash required for gears:

BACKLASH FOR GEARS IN MILLIMETRES

Module mm	Pitch line velocity			
	Upto 8 metre/second		Above 8 metre/second	
	Minimum mm	Maximum mm	Module mm	Backlash mm
20	0·75	1·25	8	0·40
16	0·50	0·85	7	0·38
12	0·35	0·60	6	0·36
10	0·30	0·51	5	0·28
8	0·22	0·40	4	0·23
6	0·20	0.33	3.5	0·22
5	0·15	0·25	3	0·21
4	0·13	0·20	2·75	0·20
3	0·10	0·15	2·5	0·19
2·5	0·08	0·13	2	0·18
2	0·08	0·13		
1·5 and finer	0·00	0·10		

The modules given above are in mm. Modules given in round bracket should be given second choice, while those in square brackets should be the third choice. The rest of the values are *preferred modules, which should be normally used.*

The standards of many European countries provide for the *following modules:*

0·3, 0·4, 0·5, 0 6, 0·7, 0·8, 1, 1·25, 1·5, 1·75, 2, 2·25, 2·5, 3 3·5, 4, 4·5, 5, 5.5, 6, 6·5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 22, 24, 26, 28, 30, 33, 36, 39, 42, 45, 50 mm and further every 5 mm. These modules apply to straight and helical gears

A rack whose teeth are proportioned according to standard tooth form is called a basic rack It determines the form and proportions of the teeth of wheels as a result of rolling the gear blank relative to the cutter

Wheels cut with the shift of the basic rack are said to be corrected. The degree of correction is characterised by shift factors.

The proportions of the standard basic rack tooth profile are usually given in fractions of a module

Addendum $= f_0\, m$. . . . . (i)

Dedendum $= (f_0 + C_0)\, m$ . . . . (ii)

Tooth thickness $= \frac{\pi}{2} m$ . . . . (iii)

Circular pitch $= \pi m$ . . . . (iv)

f_0 is called addendum factor and its value = 1 for full depth system. C_0 is called clearance factor whose value is 0·25 Half the angle between the sides of a basic rack tooth is called the profile angle and its value is 20°.

16-5. Accuracy of Gears:

To ensure normal operation of a gear the elements of its wheel and housing should be manufactured with sufficient accuracy. The standards provide for 12 degrees of accuracy (quality): from 1 to 12, 1 being the highest.

The degrees of quality is specified depending on the kind of service of the gear and the demands it has to make The degree of quality (accuracy) also specifies the cutting method, tooth contact

stocked by manufacturers and suppliers. Where smooth action is not important, cast iron gears with cut teeth may be employed. The limiting pitch line velocity of commercially cut gears is about 5 metre/second. The velocities larger than this would cause vibration and noise which can be eliminated by non-metallic pinion as one member of the gear set. Non-metallic gears are made of various materials such as treated cotton pressed and moulded at high pressure, synthetic resins of the phenol type and raw hide. The raw hide pinions are affected by moisture. Gears made of phenolic resins are self supporting while other two types are supported by metal side plates at both ends of the plate. Some metallic shrouds are also used.

To save alloy steels large wheels are made with *fretting rings*. In this case the wheel centre is cast from cast iron or more rarely from steel. The ring is forged or roll expanded from the steel of the respective grade specified by the tooth design.

Material	Condition	Minimum tensile strength kg/sq mm	Brinell Hardness Number B.H.N.	Basic Surface Stress factor f_c	Basic Bending Stress factor f_b
White heart malleable iron castings Grade B		28	217 max	0·599	6·9
Blackheart malleable iron castings Grade B		32	149 max	0·599	7·72
Cast iron Grade 20	As cast	20	179 min	0·81	4·22
,, 25	,,	25	197 min	0·876	5·27
,, 35	,,	25	207 min	0·915	8·60
,, 35	Heat treated	35	300 min	1·00	8·60
Phosphor bronze casting	Sand cast	16	60 min	0·436	4·07
,,	chill cast	24	70 min	0.50	5·8
,,	centrifugally cast	26	90	0·69	6·92
Cast steel		55	145	1·125	13·38
Carbon steel					
Carbon 0·3% normalised		50	143	0·985	11·95
Carbon 0·3% hardened and tempered		60	152	1·125	14·80
Carbon 0·4% normalised		58	152	1·15	14·05
Carbon 0·4% hardened and tempered		60	179	1·44	14·80

The nature of admissible errors in ■ toothed gear depends on its purpose. Thus, for example, in a heavily loaded low speed gear the most important factor is length of contact along the teeth; in a high speed gear, main requirement is smoothness and for reversible gears the main consideration is the magnitude of backlash and the variation of this magnitude.

The value of composite error limits of different quality of gears have been standardised by ISI.

The following table gives the recommended values of quality of gears for various services

Kind of service	Quality (accuracy) of gears
Reduction gears for turbomachineries	3—6
Metal cutting machine tools	3—8
Automobiles	5—8
Air craft	6—9
General purpose reduction gears	6—9
Gears of rolling mills	6—10
Trucks	7—9
Hoisting mechanisms and cranes	7—10
Tractors	8—10
Mine winders	8—10
Farm machinery	8—11

Single reduction gears with spur wheels have a velocity ratio of 25; however under ordinary conditions, the velocity ratio should never exceed 8 for a single reduction gear Higher values of velocity ratio are permitted for helical and herringbone wheels

When the velocity ratio is high a multiple-reduction gear should be used If the velocity ratio ranges between 10 and 50, the gear should be designed as a double reduction gear At a gear reduction of 40 or more, a triple reduction gear can be used

16-6. Materials:

Metallic gears with cut teeth are commercially obtainable in cast iron, steel, brass and bronze in many sizes and they are

The table on pages 606 and 607 gives some of the properties and factors which will enable to determine the rating of machine cut spur and helical gears. This topic has been discussed fully at a later stage in this chapter. The procedure to determine the power transmitting capacity of a gear set is explained with illustrative examples.

The table also gives the properties of the materials (steel) employed in manufacture of gears:

16-7. Allowable stresses:

The static design stresses for the gearing can be obtained by dividing the ultimate strength of the material by the suitable value for the factor of safety. Buckingham suggests that the factors of safety given in table below should be used with the ultimate strength to obtain the *safe static stress, which is the allowable stress at zero pitch line velocity.*

Factors of safety for gear teeth:

For steady load on a single pair of gears 3

For suddenly applied loads on single pairs.........................4

For steady loads on gears of a train beyond the first mesh.............5

For suddenly applied loads on gears of a train beyond the first mesh.........6

The following table gives the safe static stresses for various materials:

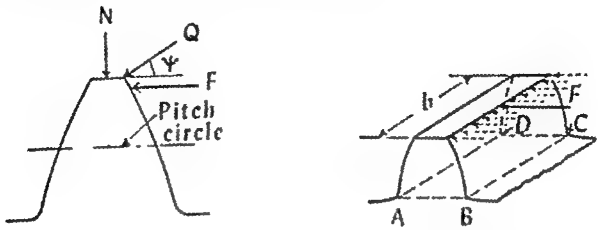
Materials	f_{static} kg/sq cm
Non-metallic materials	420
Cast iron	560
Semi-steel	840
Bronze	840
Cast steel	1,050
Forged steel	1,120
Hardened steel	2,100
Case hardened alloy steel	3,500

The actual allowable stresses or permissible stress due to load transmitted decrease with increase in velocity due to the increase in the effect of shock loads and inertia of parts at the higher speeds. Due to the uncertainty of the numerous variables involved, it

Material	Condition	Minimum tensile strength kg/sq mm	Brinell Hardness Number B.H.N.	Basic Surface Stress factor f_c	Basic Bending Stress factor f_b
Carbon 0 55% normalised		72	201	1 69	17 60
Carbon 0 55% hardened and tempered		70	223	1·83	16 90
0 55 carbon chromium steel hardened and tempered		90	225	2·105	22·2
Carbon manganese steel hardened and tempered		60	170	1·41	14 8
Manganese Molybdenum Steel 35 Mn 2 Mo 28 hardened and tempered		70	201	1·69	16 9
Chromium Molybdenum Steel 40 Cr 1 Mo 28 hardened and tempered		70	201	1·69	16 9
Nickel steel (40Ni 3) hardened and tempered		80	229	1·90	19·7
Nickel chromium steel (30Ni 4 Cr 1) hardened and tempered		154	444	3 87	35 9
Nickel chromium Molybdenum steel (40Ni 2 Cr 1 Mo 28) hardened and tempered		90	225	2 11	23 2
Surface hardened steels					
Carbon steel		55 12	145 core 460 case	1 97	8 35
0 4% carbon steel					
0 5% carbon steel		70 86	200 core 520 case	2 80	10 55
Carbon chromium steel 55% carbon chromium steel		86 61	250 core 500 case	3 58	12 91
1% chromium steel		70 86	500 case	2 80	10 55
Nickel steel 3%		"	200 core 300 case	3 57	12·91
Case hardened steels					
Carbon steel 0·12% [redacted] 0 22% carbon steel		50·39	650 case	7	28
Nickel steel 3% nickel steel		70.86	200 core 600 case	7 17	28.12
Fabric		—	—	0 39	3·16

16-9. Strength of gear teeth—Lewis equation:

In calculating the strength of gear teeth, a gear or pinion tooth is considered to be subjected to a load Q acting along the line of action as shown in fig. 16-2(*a*).



(a) Force on a tooth

(b) Action of a tooth as a beam

FIG. 16-2

This load Q may be resolved in two components: a rotational component F, tending to break the tooth by bending and a force $N = F \tan\Psi$ tending to separate the gears of the set and to produce a compressive stress in the tooth. The component F produces tensile stresses at B and compressive stresses at A. The uniform compression at B reduces the bending tensile stresses at A and increases the bending compressive stresses at B. If the material is stronger in compression than in tension, as is cast iron, the effect of the radial component is to strengthen the tooth. Since the uniform compressive stress is small compared to bending stress, it is usually, but not always, neglected in design. Thus, we assume that only the stress due to F need be considered.

In designing the tooth of a gear, it is assumed that it (tooth) acts as a cantilever beam which is subjected to an end load equal to the tooth pressure F. The dangerous section, therefore, becomes the root area $ABCD$, which must be large enough to withstand the load. Since the thickness AB depends upon the form of the tooth and upon the thickness of the tooth on the pitch circle (and therefore on circular pitch) and the length of the tooth or the face of gear AD can also be expressed in terms of circular pitch, it is evident that the creation of a safe root area depends upon the proper selection of a circular pitch that is large enough to withstand the forces that are involved in the design. An equa-

would be very difficult to obtain a purely rational expression for variation of allowable stress with pitch line velocity. Hence, an empirical expression known as a velocity factor is used, which was first of all suggested by Barth. The value of the velocity factor k is less than unity. When v is the pitch line velocity in metre/second, then

$k = \frac{3}{3+v}$ for ordinary industrial gears operating at velocities upto 10 metre/second

$= \frac{6}{6+v}$ for accurately cut gear operating at velocities upto 20 metre/second

$= \frac{0.75}{1+v} + 0.25$ for non-metallic gears

Permissible stress = safe static stress × velocity factor k.

It is obvious that for hand operated gears, the velocity factor is unity.

16-8. Design considerations:

The specifications for a gear drive include the horse power to be transmitted, the speed of the driver, the velocity ratio to be transmitted and the centre distance. As the usual sources of power i.e. turbines, high speed internal combustion engines and electric motors, run at higher speeds than those required by the driven units such as compressors, reciprocating pumps, machine tools, hoisting machinery, *etc.*, the usual drive will be a speed reduction. Occasionally, however, speed-increases are also required, the common example being centrifuges.

The following points should be taken into considerations while designing a gear drive for a particular service.

(i) The gear should have a sufficient strength so that it does not fail at starting torques or under dynamic loading during running conditions.

(ii) The teeth must have very good wear characteristics so that the life of the gear must be very satisfactory.

(iii) The suitable material combination must be chosen so that it gives good wear characteristics and the drive is silent.

(iv) The drive should be compact.

(v) The drive should be properly aligned.

(vi) The proper lubrication arrangements should be made.

strength of a tooth varies directly as the face width. Gear face width of considerable size are likely to be subjected to a non-uniform pressure distribution and the strength equation may give misleading results. Very narrow faces, however, are not economical. To minimise both these troubles, the face width should not be either too great or too short. For cast gears, the face width should not be wider than 2 times the circular pitch. For cut gears, the face width is generally between 3 and 3·5 times the circular pitch. For accurately cut gears, the width might be 5 times the circular pitch. The effective width of non-metallic pinions with metallic shrouds is usually 6 mm greater than the face width of a metallic gear of the same pitch to eliminate contacts with the shroud. The total length of a non-metallic pinion is 15 to 25 mm greater than the effective face width.

The load transmitting capacity of the speed reduction unit depends on the weakest tooth of the unit. **When both pinion and gear are made of the same material, the pinion is the determining factor.** When different materials are used, it becomes necessary to investigate a given case with care. In applying Lewis formula, it can be seen that the terms b and p are the same for both pinion and gear; hence it is only necessary to multiply the tooth factor y with allowable stress f to determine which of the two governs the design. *The product $f\,y$ is known as the strength factor of the gear.*

The strength of the tooth determined from Lewis equation is known as the *beam strength of the tooth* and we denote it as

$$F_b = f\,p\,y\,b \quad \text{..........} \quad \text{(vii)}$$

16-10. Dynamic Tooth Load:

In the preceding section the velocity factor was used to make the approximate allowance for the effect of the dynamic loading. The dynamic loading is due to the following reasons:

(i) The teeth are never perfectly spaced.

(ii) The elements of the face are not perfectly parallel to the axis.

(iii) The profiles are never perfect involutes.

(iv) When the tooth is under load, it deflects, which causes kinematic imperfections.

(v) The load is never distributed uniformly across the face.

(vi) The shaft and mountings deflect under load.

tion or formula which will provide such a pitch has been derived by Mr. Wilfred Lewis. This equation is known as *Lewis formula* and is accepted universally. Lewis formula is given as follows

$$F = f\,p\,y\,b \quad \ldots \text{(i)}$$

where F = tangential tooth load at the pitch line

p = circular pitch

f = permissible working stress

b = breadth of the face of the tooth

y = a constant which is based on the shape of the profile of the tooth and depends on the number of teeth of the gear. It is called a *tooth factor*.

The permissible stress $f = k\,f_{max}$. ... (ii)

where k is the velocity factor defined earlier and f_{max} the safe static stress.

The tooth factors for standard form of teeth can be obtained from the following formulas, where N represents the number of teeth.

For cycloidal, 14½ degree involute and generated gears,

$$y = 0.124 - \frac{0.684}{N} \quad \text{(iii)}$$

For 20 degree involute gears,

$$y = 0.154 - \frac{0.912}{N} \quad \text{(iv)}$$

For 20 degree stub involute gears

$$y = 0.175 - \frac{0.841}{N} \quad \text{(v)}$$

If V be the linear velocity in metre/minute, then

$$\text{h.p.} = \frac{FV}{4500} \quad \text{(vi)}$$

The maximum practical pitch line velocity is 10 to 12 metre per second for spur gears. Above 6 metre/second the operation of gears is apt to be noisy unless the teeth are very accurately aligned and run in a continuous bath of lubricant. The pitch line velocity of cast teeth is limited to 3 metre/second. In order to make the operation silent in cases where the load is moderate under high speed conditions, one of a pair of mating gears, generally, the pinion is made of softer material or of non-metallic material.

The derivation of Lewis equation is based on the assumption that the load is uniformly distributed across the face width. The

unable to resist rapid wear, when the unit is in continuous service. Gears may wear excessively because of improper or insufficient lubrication and because of foreign particles in the lubricant. In addition, there will be wear, causing pitting of the tooth surfaces due to compressive fatigue stresses. Properly applied lubrication may eliminate some of the troubles, but the teeth must be such that compressive fatigue failure of the material may not take place.

Tests have shown that in the order of increasing resistance to wear, the $14\frac{1}{2}°$ full depth involute tooth comes first, the 20° involute stub tooth comes next, while the best is 20° full depth involute teeth.

The limiting load on the basis of wear, is given by

$$F_w = D_p \; b \; Q W \quad \text{.......(i)}$$

where F_w is the limiting load for wear in kg, D_p the diameter of the pinion in cm, b the face width of gear in cm, Q the velocity ratio factor and W the material combination factor in kg/sq cm.

$$\left.\begin{aligned} Q &= \frac{2N_g}{N_g + N_p} \text{ for spur gears} \\ &= \frac{2N_g}{N_g - N_p} \text{ for internal gears} \end{aligned}\right\} \quad \text{.......(ii)}$$

where N_g and N_p are the number of teeth in the gear and pinion respectively. The material combination factor W depends on the maximum fatigue limit compressive stress, the pressure angle of the gear teeth and the moduli of elasticity of the materials of the pair. The value of W depends also on the degree of hardness of the tooth materials. The material combination factor is given by

$$W = \frac{f_{es}^2 \sin\phi}{1{\cdot}4}\left[\frac{1}{E_p} + \frac{1}{E_g}\right] \quad \text{.......(iii)}$$

where f_{es} = surface endurance limit of a gear pair in kg/sq cm
ϕ = pressure angle
E_p = modulus of elasticity of the pinion material in kg/sq cm
E_g = modulus of elasticity of the gear material in kg/sq cm.

The surface endurance limit may be estimated from

$$f_{es} = 28 \times \text{B.H.N.} - 700 \text{ kg/sq cm} \quad \text{.......(iv)}$$

The wear load F_w is an allowable load and must be greater than dynamic load F_d.

The following table gives the representative values of W for $14\frac{1}{2}°$ involute gearing:

Due to the above irregularities, there will be dynamic load due ■ shock and impact. The dynamic load on the gear tooth will be greater than the steady transmitted load.

The American Gear Manufacturers' Association recommends the use of the equation (i) for determining the dynamic tooth load.

$$F_d = \frac{0\cdot05V(bC+F)}{0\cdot05V+\sqrt{bC+F}} + F. \quad \ldots\ldots (i)$$

where F_d = dynamic tooth load in lb

V = pitch line velocity in f.p.m.

b = face width of the gear in inches

F = tangential transmitted steady load in lb

C = constant which depends on the material, tooth form and accuracy of construction of the gear.

Values of C as determined by Buckingham are given in table below:

Values of the constant C as suggested by Buckingham:

Material	Tooth form	Error in tooth action e in inch			
		0 0005	0 001	0 002	0 003
Grey iron and grey iron	14½° involute	400	800	1,600	2400,
Grey iron and grey iron	20°full depth involute	415	830	1,660	2,490
Grey iron and grey iron	20° stub involute	430	860	1,720	2,580
Grey iron and steel	14½° involute	550	1,100	2,200	3,300
Grey iron and steel	20°full depth involute	570	1,140	2,280	3,420
Grey iron and steel	20° stub involute	590	1,180	2,360	3,540
Steel and steel	14½° involute	800	1,600	3,200	4,800
Steel and steel	20°full depth involute	830	1,660	3,320	4,980
Steel and steel	20° stub involute	860	1,720	3,340	5,160

To ensure against tooth breakage, the beam strength of the tooth F_b as calculated by Lewis formula should be somewhat greater than the dynamic load F_d calculated by the equation (i). As an added precaution, Buckingham suggests the following relationship.

For steady loads $F_b \geq 1{\cdot}25F_d$.

For pulsating loads $F_b \geq 1{\cdot}35F_d$.

For shock loads $F_b \geq 1{\cdot}5F_d$.

16-11. Design for wear:

Reduction gear teeth may be strong enough to transmit the desired horse power and withstand the dynamic loading and yet be

Combination of materials			W
Cast iron pinion and gear			14
Semi steel pinion and gear			14
Non-metallic pinion and metallic gear			14
Machine steel pinion cast iron or semi steel gear			7
,,	,,	,, phosphor bronze gear	·6
,,	,,	,, machine steel gear	5·5
,,	,,	,, cast steel gear	3·5
Hardened steel pinion hardened steel gear			17·5
,,	,,	,, cast iron or semi steel gear	10
,,	,,	,, phosphor bronze gear	9 5
,,	,,	,, machine steel gear	7·7
,,	,,	,, cast steel gear	6·5

It should be noted that the units of *W* are kg/sq cm.

In order to get the values of *W* for 20° involute gearing, the values given in the following table should be increased by 33%.

If the number of teeth in a gear be a simple multiple of teeth in the pinion, the same pair of teeth will engage in every revolution of the gear. In order to distribute the wear more evenly, an additional tooth known as the *hunting tooth* is provided if the velocity ratio is permissible.

16-12. Gear wheel proportions:

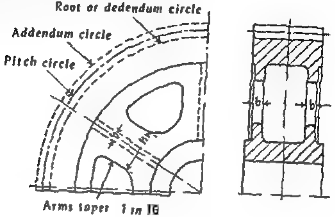
The important parts of the gear are (i) hub, (ii) web or arms and (iii) rim. An exact analysis of stresses is exceedingly difficult so we arrive at the various proportions by the methods employed in practice.

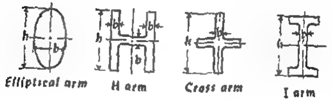
The following table represents the proportions as adopted by many manufacturers:

Dimensions of gear hubs

Type of service	Diameter of hub		Length of a hub
	cast iron	cast steel	
Light load without shock	$1\frac{5}{8}d + 3$ mm	$1\frac{1}{2}d + 6$ mm	$1{\cdot}75d$ to $2{\cdot}25d$
Medium load and medium shock	$1\frac{3}{4}d + 6$ mm	$1\frac{5}{8}d + 5$ mm	,,
Heavy load and great shock	$2d$	$1\frac{3}{4}d + 3$ mm	,,

The elliptical cross section is used for lighter loads, while remaining sections are used for large and heavy gears. With the elliptical arm, the major axis is twice the mior axis and the major axis lies in the plane of rotation.





Sections for gear arms

Fig 16-3

While determining the cross sectional dimensions of the arm, we assume that the rim is rigid so that the tooth load is equally shared by each arm, which is assumed to be a cantilever fixed at the hub and loaded at the pitch line. The bending moment acting on each arm is equated to the moment of resistance. When the permissible stress for the material of the arm is known, section modulus can be obtained When the section modulus is calculated, the dimensions of the arms can be specified. The dimensions of the arm at the pitch line are generally made approximately sevententh of those at the centre

Gears with wide faces are provided with either two rows of elliptical arms or H arms or cross arms.

Forged wheels are made solid or cored with round holes. The cored type is lighter but requires more machining. For a more convenient clamping of the wheels on the machine tool the web of the disc should be drilled between the rim and the hub. Sometimes large diameter holes are drilled to reduce weight.

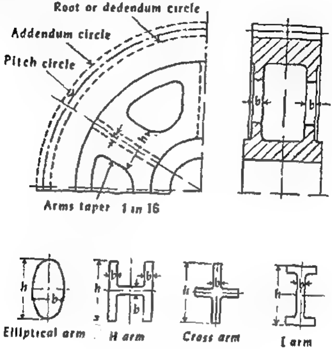
Empirical formulas are used to detrmine the rim thickness. The thickness of the rim should be such that the rigidity of the rim is insured so that the rim distributes the load on the teeth equally among the arms. The Nuttal Works recommend the following formula, for calculating the rim thickness:

$$\text{Rim thickness} = m\sqrt{\frac{\text{number of teeth}}{2 \times \text{number of arms}}} \quad \ldots\ldots\ldots\ldots\text{(i)}$$

In a good design, the rim should be provided with a central rib of thickness equal to half the circular pitch.

The diameter of the gear shaft can be calculated when the lay out of the shaft is known. When no such data are available, the diameter of the shaft is obtained by the following formula:

The elliptical cross section is used for lighter loads, while remaining sections are used for large and heavy gears. With the elliptical arm, the major axis is twice the mior axis and the major axis lies in the plane of rotation



Sections for gear arms

Fig 16-3

While determining the cross sectional dimensions of the arm, we assume that the rim is rigid so that the tooth load is equally shared by each arm, which is assumed to be a cantilever fixed at the hub and loaded at the pitch line. The bending moment acting on each arm is equated to the moment of resistance. When the permissible stress for the material of the arm is known, section modulus can be obtained When the section modulus is calculated, the dimensions of the arms can be specified. The dimensions of the arm at the pitch line are generally made approximately seven-tenth of those at the centre.

Gears with wide faces are provided with either two rows of elliptical arms or *H* arms or cross arms.

Forged wheels are made solid or cored with round holes. The cored type is lighter but requires more machining. For a more convenient clamping of the wheels on the machine tool the web of the disc should be drilled between the rim and the hub. Sometimes large diameter holes are drilled to reduce weight.

Empirical formulas are used to detrmine the rim thickness. The thickness of the rim should be such that the rigidity of the rim is insured so that the rim distributes the load on the teeth equally among the arms. The Nuttal Works recommend the following formula, for calculating the rim thickness:

$$\text{Rim thickness} = m\sqrt{\frac{\text{number of teeth}}{2 \times \text{number of arms}}} \quad \ldots\ldots\ldots\ldots\text{(i)}$$

In a good design, the rim should be provided with a central rib of thickness equal to half the circular pitch.

The diameter of the gear shaft can be calculated when the lay out of the shaft is known. When no such data are available, the diameter of the shaft is obtained by the following formula:

$$d = 13\sqrt[3]{\frac{\text{H.P.}}{\text{speed in r.p.m.}}} \text{ cm.} \quad \ldots\ldots\ldots\ldots\text{(ii)}$$

Toothed wheels fixed on the shaft are fitted by interference — for example, press or light press fit. Press fit is employed at impact load or speeds above 2,000 r.p.m. If the wheel is to be removed from the shaft (to replace the bearings or the wheel itself) medium fits are used. The fit is selected depending on the degree of accuracy of the toothed gear.

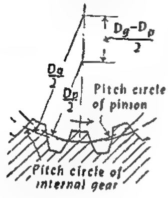
16-13. Internal gears:

A ring with the teeth on the inside is called an annular gear or internal gear. With such gears, a compact design is possible due to a shorter centre distance than with external gears of the same size. The centre distance in internal gearing is equal to the difference between the two pitch radii instead of their sum (fig. 16-4).

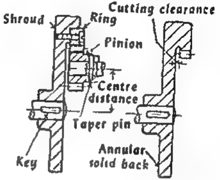
The internal gears have the following desirable characteristics:

(i) Much stronger teeth due to greater base width
(ii) More teeth in contact
(iii) Less sliding action, hence less wear
(iv) Higher efficiency
(v) Smoother operation.

For an internal gear of the same material as its pinion, it is unnecessary to calculate the strength of the tooth. As the tooth of the internal gear has a greater base width, the teeth of internal gears are much stronger than the pinion teeth.



Internal gear
Fig 16-4



Supports for internal gears
Fig 16-5

To avoid interference, the internal gear should have at least 12 teeth more than the pinion when $14\frac{1}{2}$ degree full height teeth are used. With 20 degree stub tooth gears, the internal gear should have at least 7 teeth more than the pinion

Internal gears are used extensively for increasing or decreasing speed in planetary gear combinations and for rear-axle drives of trucks and tractors

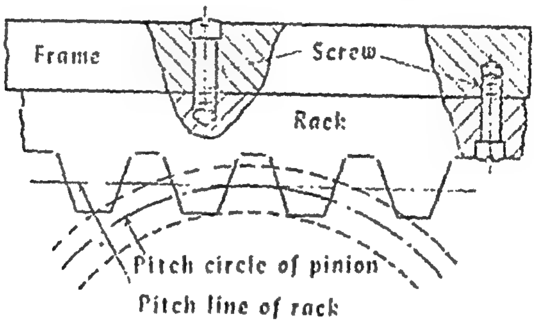
Internal gears may be obtained in either solid back type or ring type as shown in fig 16-5. The solid back type internal gear has a machined recess at least 8 mm deep to provide for cutting clearance for gear shaper cutters. The ring type gear should be seated in a machined recess in a shroud which may be keyed or screwed and pinned in place.

16-14. Racks:

They are used for converting rotary motion into reciprocating motion and vice versa. It is used for adjustable sliding members. A rack may be considered a spur gear with infinite number of teeth. Fig 16-6 shows the methods of connecting racks to the frames.

Examples:

1. *It is desired to transmit 15 h.p. from a motor shaft rotating at 1,440 r.p.m. to a low speed shaft with a speed reduction of 3:1. The teeth are 14½° involute with 25 teeth on pinion. The starting torque is 50% higher than the running torque. Both the pinion and gear are made of steel heat treated for which maximum allowable static stress may be taken as 2,100 kg/sq cm. The shaft on which the gear is to be mounted, and the key are of mild steel for which safe stress may be taken as 420 kg/sq cm. Design the suitable spur gear drive to satisfy the above conditions.*



Rack

FIG. 16-6

As the gear reduction ratio is 3, the number of teeth in the gear will be $3 \times 25 = 75$.

We assume module as 6 mm.

Pitch circle diameter of pinion $= 6 \times 25 = 150$ mm.

Pitch circle diameter of gear $= 150 \times 3 = 450$ mm.

As the starting torque is 50% higher than the running torque, spur gears should be designed for $15 \times 1{\cdot}5 = 22{\cdot}5$ h.p.

$$\text{Pitch line velocity } v = \frac{\pi \times 0{\cdot}15 \times 1440}{60} = 11{\cdot}3 \text{ metre/sec.}$$

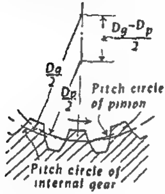
$$\text{Tangential tooth load } F = \frac{22{\cdot}5 \times 75}{11{\cdot}3} = 150 \text{ kg.}$$

$$\text{Velocity factor} = \frac{3}{3+v} = \frac{3}{3+11{\cdot}3} = 0{\cdot}21.$$

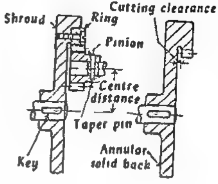
Allowable stress $= 2100 \times 0{\cdot}21 = 441$ kg/sq cm.

Circular pitch $m\pi = 0{\cdot}6 \times 3{\cdot}14 = 1{\cdot}884$ cm.

For an internal gear of the same material as its pinion, it is unnecessary to calculate the strength of the tooth. As the tooth of the internal gear has a greater base width, the teeth of internal gears are much stronger than the pinion teeth.



Internal gear
Fig. 16-4



Supports for internal gears
Fig. 16-5

To avoid interference, the internal gear should have at least 12 teeth more than the pinion when 14½ degree full height teeth are used. With 20 degree stub tooth gears, the internal gear should have at least 7 teeth more than the pinion.

Internal gears are used extensively for increasing or decreasing speed in planetary gear combinations and for rear-axle drives of trucks and tractors.

Internal gears may be obtained in either solid back type or ring type as shown in fig. 16-5. The solid back type internal gear has a machined recess at least 8 mm deep to provide for cutting clearance for gear shaper cutters. The ring type gear should be seated in a machined recess in a shroud which may be keyed or screwed and pinned in place.

16-14. Racks:

They are used for converting rotary motion into reciprocating motion and vice versa. It is used for adjustable sliding members. A rack may be considered a spur gear with infinite number of teeth. Fig. 16-6 shows the methods of connecting racks to the frames.

Bending moment at the bearing $= 165 \times 12 = 1{,}980$ kg cm.

Twisting moment $= 150 \times 22{\cdot}5 = 3{,}260$ kg cm.

The shaft will be under torsional shear stress and flexural stresses which will be alternating in character. We design the shaft for maximum shear stress. We have

equivalent twisting moment $= \sqrt{1980^2 + 3260^2} = 3{,}800$ kg cm.

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 420 = 3800$$

or $$d = \sqrt[3]{\frac{3800 \times 16}{420 \times \pi}} = 3{\cdot}58 \text{ cm; we adopt 4 cm.}$$

Similarly we calculate the diameter of the pinion shaft which will be 3 cm.

The thickness of the hub for the gear will be 18 mm and that for the pinion will be 15 mm.

Manytimes the pinion may be solid one and is bored out for the shaft as shown in fig. 16-7 and fig. 16-8.

We assume for the gears, four arms of elliptical section, the major axis being twice the minor axis. Here, we design the arms on driving torque. According to some authorities, they should be designed on stalling torque. *The stalling torque is the torque that will develop the maximum stress in the teeth at zero velocity.*

The modulus of section $= \frac{\pi}{64} a^3$ where a is the major axis of the ellipse.

Maximum bending moment on each arm $= \frac{3260}{4}$

$= 815$ kg cm.

We take the permissible bending stress as 420 kg/sq cm.

$$\therefore \quad 815 = \frac{\pi}{64} \times a^3 \times 420$$

or $$a = \sqrt[3]{\frac{64}{\pi} \times \frac{815}{420}} = 3{\cdot}42 \text{ cm; we adopt 35 mm.}$$

Thus, the arm section at the root will be 35 mm $\times$ 18 mm which would taper to 22 mm $\times$ 11 mm.

If we design the arms on stalling torque, the dimensions of the arm at the root will be 60 mm $\times$ 30 mm.

As both pinion and gear are made of the same material, the drive will be designed for the pinion.

Form factor $y = 0{\cdot}124 - \frac{0{\cdot}684}{25} = 0{\cdot}0966$.

According to Lewis formula, we have

$F = f p y b$.

On substitution of values, we get

$150 = 441 \times 1{\cdot}884 \times 0{\cdot}0966 \times b$

or $\quad b = \frac{150}{441 \times 1{\cdot}884 \times 0{\cdot}0966} = 1{\cdot}88$ cm.

We adopt width of tooth face as 40 mm which will be ample strong

The summarised particulars of the drive will be as follows:

Item	No. of teeth	m mm	PCD mm	Teeth width mm	Adden-dum mm	Blank diameter mm	Clear-ance mm	Deden-dum mm
Pinion	25	6	150	40	6	162	1·5	7·5
Gear	75	6	450	40	6	462	1·5	7·5

Gear shaft : The maximum thrust Q between a pair of teeth is directed along the path of contact and is such that $Q \cos \psi = F$.

$$\therefore \quad Q = \frac{F}{\cos \psi} = \frac{150}{\cos 14\frac{1}{2}^\circ} = \frac{150}{0{\cdot}9681} = 153 \text{ kg}$$

Let us make some estimation regarding the weight of the gear.

The weight of the gear can be approximated as knbp² kg, where

$k = 0{\cdot}012$ for spur gear and $0{\cdot}010$ for bevel gears

n = number of teeth

b = face width in cm

p = circular pitch in cm.

On substitution of values, we get

$W = 0{\cdot}012 \times 75 \times 4 \times 1{\cdot}884^2 = 12{\cdot}8$ kg.

The resultant of tooth thrust and weight of the gear is

$R = \sqrt{153^2 + 12{\cdot}8^2 + 2 \times 153 \times 12{\cdot}8 \times 0{\cdot}9681} = 165$ kg

We assume that the gear is overhung on the shaft. In order to reduce the bending effect, the overhung should be reduced to minimum. We assume the overhung to be 12 cm.

Allowable stress $= 0{\cdot}485 \times 560 = 270$ kg/sq cm.

Circular pitch $p = \pi \times 0{\cdot}4 = 1{\cdot}256$ cm.

Form factor y for a pressure angle of $14\frac{1}{2}°$ is

$$y = 0{\cdot}124 - \frac{0{\cdot}684}{16} = 0{\cdot}0813.$$

Safe load transmitted $= 270 \times 4 \times 1{\cdot}256 \times 0{\cdot}0813$
$= 110$ kg.

The h.p. transmitted $= \dfrac{110 \times 3{\cdot}2}{75} = 4{\cdot}68$ which will be satisfactory from the strength point. If the pump operates continuously the design should be checked for wear.

Tooth shape factor $Q = \dfrac{2N_g}{N_g + N_p} = \dfrac{2 \times 75}{75 + 16} = 1{\cdot}65.$

Material combination factor $W = 14.$

For cast iron pinon and gear, semi steel pinion and gear and non-metallic pinion and metallic gear, the material combination factor is taken as 14.

Limiting load for wear $=$ P.C.D. $\times b \times Q \times W$
$= 16 \times 0{\cdot}4 \times 4 \times 1{\cdot}65 \times 14 = 590$ kg.

The design is safe as far as wear is concerned, as the limiting load for wear is much greater than the tangential tooth load.

Let us consider the alternate design.

The static strength of the material for non-metallic pinion is 420 kg/sq cm. Let us assume the module as 6 mm and 5 cm face width. Pitch velocity $v = \dfrac{\pi \times 16 \times 0{\cdot}6 \times 960}{100 \times 60}$
$= 4{\cdot}85$ metre/sec.

Velocity factor $= 0{\cdot}25 + \dfrac{0{\cdot}75}{1 + 4{\cdot}85} = 0{\cdot}378.$

Allowable stress $= 420 \times 0{\cdot}378 = 160$ kg/sq cm.

Circular pitch $= \pi \times 0{\cdot}6 = 1{\cdot}884$ cm.

Form factor $= 0{\cdot}0813.$

Safe load transmitted $= 160 \times 5 \times 1{\cdot}884 \times 0{\cdot}0813$
$= 122$ kg.

The h.p. transmitted $= \dfrac{122 \times 4{\cdot}85}{75} = 7{\cdot}8.$

The alternate design is satisfactory from the stand point of strength. While the drive for non-metallic pinion has higher pitch line velocity than cast iron gears, it will probably operate much more calmly and reduce any shock tendencies.

Fig. 16-7 shows the final details of the gear under consideration

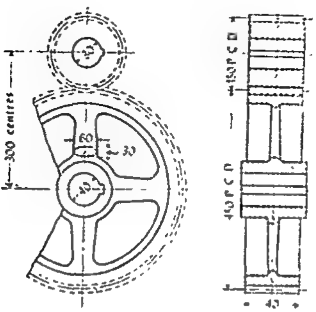


Fig. 16-7

2. *A pump rotates at about 200 r.p.m. and requires about 4 h.p. for operation. The unit is to be driven by 4 h.p. 960 r.p.m. motor through the medium of 14½° spur gearing. Design the suitable drive for the purpose.*

Velocity ratio $= \frac{960}{200} = 4\cdot8$. The suitable combination for a drive will be $\frac{63}{14}, \frac{75}{16}, \frac{85}{18}$. For avoiding interference the pinion must have 15 teeth for 14½° gears. Therefore, we adopt $\frac{75}{16}$ combination. If the pinion and gear are to be of the same material, the cast iron, the pinion will be weaker and it will serve as a basis for design. We assume module as 4 mm and width of face, b, as 4 cm.

The permissible static stress for cast iron is 560 kg/sq cm

The pitch line velocity $= \frac{\pi \times 16 \times 0\cdot4 \times 960}{100 \times 60} = 3\cdot2$ metre/sec

Velocity factor $= \frac{3}{3 + 3\cdot2} = 0\cdot485$

Module mm	Induced stress $\frac{71}{m^2}$ kg/sq cm	v metre/sec	Velocity factor	Allowable stress kg/sq cm
12	41·2	10·6	0·3145	132
10	71	8·85	0·326	137
8	139	7·1	0·344	145
6	330	5·3	0·369	155

With $m = 8$ mm, the allowable stress is 145 kg/sq cm and the induced stress is 139 kg/sq cm. This module will be used and face width b is given by $b = 3{\cdot}5 \times \pi \times 0{\cdot}8 = 8{\cdot}8$ cm.

4. *Fig. 16-8 shows a countershaft with a gear G mounted at one end. Gear G meshes with a pinion P mounted on the shaft of 5 h.p. motor. The pinion and gear have 18 teeth and 90 teeth respectively. The teeth are of involute form having a pressure angle of 20°. Minimum diameter of the pinion is to be 15 cm. The pinion shaft makes 500 r.p.m. and the belt tension ratio is 2:1. The other end of the shaft carries a belt pulley 50 cm in diameter from which the power is supplied to a machine directly below it.*

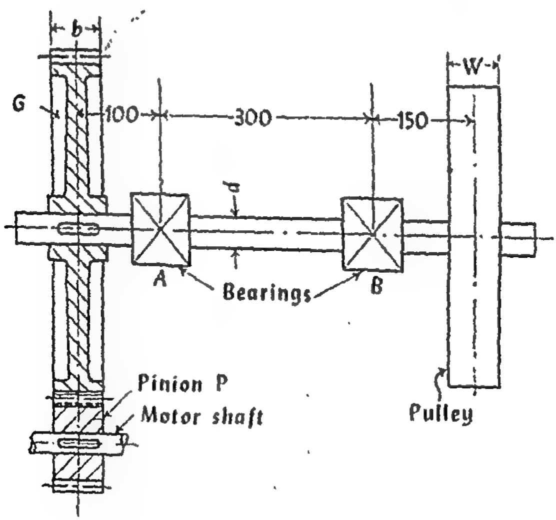


FIG. 16-8

Calculate the width of the belt for the pulley, the diameter of the gear shaft and the width of the gear wheels.

Choose your own materials for the parts and values for the stress.

3. A raw hide pinion is to transmit 30 h p. at 940 r p.m. Select a standard pitch for 20° full depth involute teeth. The safe static stress for non-metallic pinion may be taken as 420 kg/sq cm.

In order to avoid interference, we adopt 18 teeth The ratio of width of the face to circular pitch is taken as 3·5

The form factor for the pinion $= 0{\cdot}154 - \dfrac{0{\cdot}912}{18} = 0{\cdot}103.$

Torque to be transmitted $= \dfrac{71620 \times \text{h.p}}{\text{r p m.}} = \dfrac{71620 \times 30}{940}$

$= 2{,}280$ kg cm

Lewis formula $F = fpby$ is convenient to use when the pitch diameter of the gear is known as the tangential tooth load is readily obtainable. When the number of teeth is known, another expression is derived which will be useful in designing the pinion.

P.C.D of the pinion in cm $= N \times m$ where m is the module in cm.

Torque on the gear shaft $T = \dfrac{F \times N \times m}{2}$ . . . . (i)

Substituting for $F = fpby$, we get

$T = \dfrac{fNpbym}{2}$ . . . . (ii)

Let $k = \dfrac{b}{p}$ and as $p = \pi m$ we get

$T = \dfrac{fNky\pi^2 m^3}{2}$ or $f = \dfrac{2T}{k\pi^2 yNm^3}$ . . . . (iii)

Thus, the above expression may be used directly in determining the induced stress For this example, the relation between the induced stress and module will be

$$f = \frac{2 \times 2280}{3{\cdot}5 \times \pi^2 \times 0{\cdot}103 \times 18 \times m^3} = \frac{71}{m^3} \text{ kg/sq cm}$$

The velocity factor for non-metallic gears is given as $\left[\dfrac{0{\cdot}75}{1+v} + 0{\cdot}25\right]$ where v is the pitch line velocity in metre/second

Allowable stress $= 420 \left[\dfrac{0{\cdot}75}{1+v} + 0{\cdot}25\right]$ kg/sq cm

As module and pitch line velocity are unknown, it will be necessary to follow trial and error solution. We assume the values of standard modules and calculate the values of the induced and allowable stresses and adopt that value which gives induced stress less than the allowable stress.

The gear shaft is subjected to a constant twisting moment of 3,581 kg cm and as the bending moment at B is greater than that at A, the critical section is at the bearing B.

Equivalent twisting moment according to Rankine equals $6450 + \sqrt{6450^2 + 3581^2} = 12{,}000$ kg cm.

Equivalent twisting moment according to Guest equals

$$\sqrt{6450^2 + 3581^2} = 7{,}950 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, as calculated by Rankine theory (principal stress theory), we have

$$\frac{\pi}{16} d^3 \times 550 = 12000$$

$$\therefore \qquad d = \sqrt[3]{\frac{12000 \times 16}{550 \times \pi}} = 4{\cdot}87 \text{ cm; we adopt 5 cm.}$$

The diameter of the shaft calculated by Guest formula will come out to be 4·6 cm which is less than 5 cm. So we adopt 5 cm diameter shaft.

We assume that the spur wheels are made of cast steel with a safe static stress of 1,050 kg/sq cm. When both pinion and gear are made of the same material, pinion is weaker of the two.

$$\text{The pitch line velocity of the spur wheels} = \frac{\pi \times 100 \times 0{\cdot}9}{60}$$

$$= 4{\cdot}7 \text{ metre/second.}$$

$$\text{Velocity factor} = \frac{3}{3 + 4{\cdot}7} = 0{\cdot}39.$$

Permissible stress $f = 0{\cdot}39 \times 1050 = 410$ kg/sq cm.

The form factor for 20° full depth involute gears is given as $y = 0{\cdot}154 - \dfrac{0{\cdot}912}{N}$ where N is the number of teeth.

$$\text{For 18 teeth, } y = 0{\cdot}154 - \frac{0{\cdot}912}{18} = 0{\cdot}103.$$

Circular pitch $m\pi = 10 \times 3{\cdot}14 = 31{\cdot}4$ mm, i.e. 3·14 cm.

The face width, b, of the pinion can be obtained by Lewis formula $F = f \times p \times b \times y$.

On substitution of values, we get

$$79{\cdot}5 = 410 \times 3{\cdot}14 \times b \times 0{\cdot}103$$

or $$b = \frac{79{\cdot}5}{410 \times 3{\cdot}14 \times 0{\cdot}103} = 0{\cdot}61 \text{ cm.}$$

Speed reduction $= \frac{90}{18} = 5$

Speed of the gear shaft $= \frac{500}{5} = 100$ r.p.m.

Torque on the gear shaft $= \frac{71620 \times 5}{100} = 3{,}581$ kg cm.

If T_1 and T_2 be the tensions in tight and slack sides of the belt respectively, then

$$(T_1 - T_2)\frac{50}{2} = 3{,}581$$

or $$(T_1 - T_2) = \frac{3581 \times 2}{50} = 143 \text{ kg}$$

As the belt tension ratio 2:1, T_1 will be 286 kg and T_2, 143 kg. We adopt a leather belt of 1 cm thickness. We assume the safe stress to be 20 kg/sq cm

If W cm be the width of the leather belt, then

$$W \times 1 \times 20 = 286$$

or $$W = \frac{286}{20} = 14{\cdot}3 \text{ cm}$$ we adopt 15 cm

The pulley is overhung on the shaft by 15 cm

The gear shaft is subjected to bending and twisting. We assume that the shaft is made of mild steel having a safe stress of 550 kg/sq cm in tension and 420 kg/sq cm in shear. The critical sections are at two bearings A and B. We assume that the two belt ends are parallel. Belt pull $(T_1 + T_2)$ causes the bending moment at B of the magnitude $(286 + 143)\ 15 = 6{,}435$ kg cm

The minimum pitch circle diameter of the pinion is to be 15 cm. The number of teeth is 18. We assume module as 10 mm. Therefore the pitch circle diameter of the pinion will be $18 \times 10 = 180$ mm. As the speed reduction ratio is 5:1, the pitch circle diameter of the gear will be $5 \times 180 = 900$ mm i.e. 90 cm. Tangential tooth load on gears $= \frac{3581}{45} = 79{\cdot}5$ kg

The thrust between a pair of teeth is along the pressure line which is inclined at a pressure angle to the common tangent at pitch point.

The bending load on the shaft $= \frac{79{\cdot}5}{\cos 20°} = \frac{79{\cdot}5}{0{\cdot}9397} = 85$ kg

Bending moment at $A = 85 \times 10 = 850$ kg cm

Number of teeth in gear $C = \frac{225}{5} = 45.$

Number of teeth in internal gear $D = \frac{600}{5} = 120.$

Torque on the driving shaft $= \frac{71620 \times 30}{1200} = 1{,}800$ kg cm.

Tangential tooth load on the pinion $= \frac{1800}{7 \cdot 5} = 240$ kg.

The distance of pin C from the axis of the shaft is

$75 + \frac{225}{2} = 187 \cdot 5$ mm.

If W be the weight placed at the end of lever 750 mm long, then

$$W \times 750 = 240 \times 2 \times 187 \cdot 5$$

$$\therefore \quad W = 119 \text{ kg.}$$

The load on the pin on which the intermediate wheel is mounted $= 2F = 240 \times 2 = 480$ kg.

The maximum bending moment on the lever will be at a point where the pin for intermediate gear C is fixed.

Maximum bending moment on the lever $= 119\ (750 - 187 \cdot 5)$
$= 67{,}000$ kg mm.

The material of lever will be mild steel, for which we adopt 800 kg/sq cm as the permissible value of the tensile stress. We assume the lever to be of rectangular section having thickness to be $\frac{3}{8}$ of the depth.

If h cm be the depth of the lever, the modulus of section will be

$$\frac{1}{6} \times \frac{3}{8}h \times h^2 = \frac{h^3}{16} \text{ cm}^3.$$

$$\therefore \quad \frac{h^3}{16} \times 800 = 6700$$

or $$h = \sqrt[3]{\frac{6700 \times 16}{800}} = 5 \cdot 1 \text{ cm.}$$

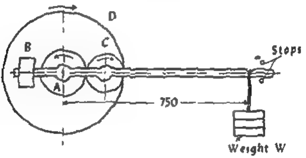
In order to account for the fixing of the pin, we adopt the depth of the lever to be 7 cm and the thickness of the lever as 2 cm.

The pin, on which the intermediate gear C revolves freely, will be designed from bearing considerations. We take the permissible stress at 70 kg/sq cm. The load on the pin is 480 kg. The

The width is too small; we adopt 5 cm for our purpose.

We could have taken a cheaper material cast iron and would have adopted 5 cm as the width of the gear. The theoretical width with cast iron would have been $= \frac{1050}{560} \times 0.61 = 1.14$ cm.

5 Fig 16-9 shows a gear dynamometer. The pinion A is keyed to the driving shaft and revolves in anti-clockwise direction. The internal gear wheel D which is keyed to the driven shaft and revolves in the clockwise direction. Power is transmitted through an intermediate gear wheel C, which is mounted on a pin fixed to the lever, which pivots freely on the common axis of the driving and driven shafts The counter-weight keeps the lever floating between stops when no power is being transmitted.



Epicyclic gear dynamometer
Fig 16-9

The weight W is adjustable The dynamometer transmits 30 h.p. The driving and driven shafts revolve at 1,200 and 300 r p m Gear wheel A has 30 teeth and a module of 5 mm. The wheels are made of carbon steel with a safe static stress of 1,540 kg/sq cm Design gear, lever, pins and the shafts (only for torsion)

The pitch circle diameter of pinion $A = 30 \times 5 = 150$ mm

The speed reduction ratio is $\frac{1200}{300} = 4$

The pitch circle diameter of the internal gear $D = 150 \times 4 = 600$ mm

If the pitch circle diameter of intermediate gear be x, then $150 + 2x = 600$

or $\quad x = \frac{600 - 150}{2} = 225$ mm.

Torque on the driver shaft = 1,800 kg cm.

We assume that the shaft is made of mild steel for which the safe shear stress is taken as 420 kg/sq cm.

If d cm be the diameter of the driver shaft, then

$$\frac{\pi}{16} d^3 \times 420 = 1800$$

or
$$d = \sqrt[3]{\frac{1800}{420} \times \frac{16}{\pi}} = 2{\cdot}8 \text{ cm; we adopt 3 cm.}$$

As the speed reduction is 4:1, the torque on the gear shaft will be four times that on the pinion shaft and as the diameter of the shaft is directly proportional to cube root of the torque. the diameter of the gear shaft will be $2{\cdot}8 \sqrt[3]{4} = 4{\cdot}45$ cm; we adopt 4·5 cm.

Exercises:

1. *It is desired to transmit 7·5 h.p. by a pair of spur gears at 1,450 r.p.m. of the pinion, for intermittent service. The pinion is made of cast iron and has 18 teeth, 14½° full depth. The gear runs at 580 r.p.m. Determine the diameter of the pinion shaft, module, face width and number of teeth for the gear.*

2. *The thrust on a drill press spindle is 1,600 kg. If a 16 teeth pinion driving a rack collared on the spindle is used to feed the drill in the work, determine the standard module for the semi steel pinion if the width of the face is 5 cm. Solve for both 14½° and 20° depth involute teeth and indicate which design is safe.*

3. *A motor develops 2,000 kg cm torque at 1,000 r.p.m. This torque is transmitted to a shaft running at 300 r.p.m. through a cast iron pinion and gear having 20° stub teeth. The centre distance must be maintained at 26 cm. Design the drive and check the design for dynamic loading and wear.*

4. *A 15 h.p. motor running at 940 r.p.m. drives a ventilating fan through a gear reduction having a velocity ratio of 3:1. In order to make the drive compact, 20° stub teeth are employed. The pinion is made of raw hide and the gear is to be made of cast iron. Assuming 20 teeth on pinion, determine the module, pitch diameters and the face widths of gears.*

5. *The eccentric shaft of a punch is driven by a cast iron gear having 105 teeth. The gear must transmit a torque of 60,000 kg cm when rotating at 35 r.p.m. It meshes with 21 teeth 14½° involute pinion.*

minimum bearing area of the pin will be $\frac{480}{70} = 6{\cdot}86$ sq cm.

We adopt pin of 2 cm dia. and length 3·5 cm. The minimum length of the pin will be equal to the width of the intermediate gear *C*.

As the bearing length of the pin in the intermediate gear is 3·5 cm while the thickness of the lever is 2 cm, the pin will be fixed in the lever and hence it will be treated as a cantilever. Assuming the load to be uniformly distributed the bending moment on the pin will be $\frac{480 \times 3{\cdot}5}{2} = 840$ kg cm and flexural stress will be $\frac{840}{\frac{\pi}{32} \times 2^3} = 1{,}070$ kg/sq cm which is within limits.

The load coming on each tooth of any gear will be 240 kg. All the gears are made of the same material, the carbon steel; the pinion *A* will be the weakest and it will serve as basis for design.

The safe static stress = 1,540 kg/sq cm.

The pitch line velocity $= \pi \times \frac{15 \times 1200}{100 \times 60} = 9{\cdot}42$ metre/sec.

We employ accurately cut gears for which the velocity factor may be taken as $\frac{6}{6+v}$.

The velocity factor $= \frac{6}{6+9{\cdot}42} = 0{\cdot}389$.

Allowable stress $= 0{\cdot}389 \times 1540 = 600$ kg/sq cm

We employ *20° full depth involute teeth.*

Form factor $y = 0{\cdot}154 - \frac{0{\cdot}912}{30} = 0{\cdot}124$.

Circular pitch $= \pi \times 0{\cdot}5 = 1{\cdot}57$ cm.

According to Lewis formula, we have

$$240 = 600 \times 1{\cdot}57 \times 0{\cdot}124 \times b$$

or $$b = \frac{240}{600 \times 1{\cdot}57 \times 0{\cdot}124} = 2{\cdot}05 \text{ cm.}$$

We adopt 3·5 cm as the face width for all the gears.

As the width of the gear comes out to be equal to the bearing length of the pin, the length of the pin may be retained as calculated earlier.

	Motor shaft to intermediate shaft		*Intermediate shaft to drum shaft*	
	Pinion	*Gear*	*Pinion*	*Gear*
Module				5 mm
Number of teeth		120		
Pitch diameter cm	10		12·5	

10. A 60 cm cast iron gear transmits 25 h.p. at 220 r.p.m. The module of the $14\frac{1}{2}°$ involute teeth is 6 mm and the face width is 6 cm. The gear is mounted on a shaft 5·5 cm diameter.

(*a*) *Determine dimensions for the hub diameter, rim thickness and bead.*

(*b*) *Determine the number of arms for the gear. Assuming elliptical shaped arms, determine the dimensions of the arm at the hub and at the pitch circle.*

(*c*) *Determine the dimensions of a cross shaped arm and of an I shaped arm.*

11. The gate of a sluice valve weighing 5 tonnes is raised by means of cast iron rack and pinion. Design a train of gears including the rack so that the gate may be raised by two men working on 40 cm crank handles and exerting a pressure of 13 kg each. Give also the linear speed of the rack motion, assuming that the crank makes 30 r.p.m.

12. Two shafts transmitting 24 h.p. and connected by a spur gear are to be run at 150 r.p.m. and 300 r.p.m. respectively. The axes of the shafts are approximately 45 cm apart. Calculate the diameters of the shafts allowing a safe stress in the shaft of 350 kg/sq cm and design suitable spur gear wheels for mounting on the shafts. Four arms of the wheel may be assumed to be of elliptical section and are to be designed for a safe stress of 210 kg/sq cm. The diameter of the nave may be taken to be double the shaft diameter, and its length as 1·8 times the shaft diameter.

13. An internal gear drive for intermittent service has a velocity ratio of 29:15 and the driving pinion has 30 teeth, 6 mm module, 7 cm face and is made of hardened steel. If the pinion rotates at 500 r.p.m., what is the horse power transmitted? What will be the gear material?

Assuming the face width to be three times the circular pitch, determine the standard module and the pitch diameter of the gear. Also, check the design for wear.

6. *A driving shaft P rotates at 300 r.p.m and is placed in alignment with a driven shaft Q rotating at 25 r.p.m. A spur gear on P drives a gear B on a back shaft S. Another gear C on shaft S drives a gear D on shaft Q. The centre distance between shafts P and S is to be not less than 225 mm and more than 150 mm. Gears C and D are of modules 6 mm and A and B are of module 5 mm. Find the teeth of the gears if no gear is to have more than 75 teeth and less than 12 teeth. The drive transmits 2 h.p. Choosing your own materials for the gear, design the gears.*

7. *A hotel restaurant belt conveyor system is to be driven by a 2 h.p 720 r.p.m motor through a pair of straight spur gear train having a velocity ratio of 3·5. Assuming a starting overload of 30 per cent, a Micarta pinion having 30 teeth, cast iron gear and 20° full depth teeth, determine the module, pitch diameters and face width.*

8. *In an automotive type of gear box, the second speed gear shaft is to be driven from a main shaft transmitting 50 h.p. at 2,500 r.p.m. with a velocity ratio 1·5.1. The shaft centre distance is not to exceed 100 mm. Determine the suitable module and number of teeth for 20° stub tooth gears of heat treated steel.*

9. *A hoist drum 60 cm in diameter is to lift a maximum load of 5 tonnes. The drive is through a double reduction spur gear using 950 r.p.m motor. The speed reduction between the motor shaft and the intermediate shaft is 5.1 and between the intermediate shaft and drum shaft is 6:1. The efficiency of each gear reduction is 96%.*

Calculate the horse of the driving unit and complete the tables.

Shaft	*Motor shaft*	*Intermediate shaft*	*Drum shaft*
Torque kg cm			
Speed r.p.m.			

If a brake is to be attached, to which shaft would you attach the brake? Suggest the suitable diameter for the shafts. Design any one gear of the system.

18. Design the gears, shafts and bearings for a double reduction unit. The first pinion shaft is to be attached to a standard 50 H.P. squirrel cage motor by means of a straight shaft coupling. The motor has no load speed of 1,500 r.p.m. The output shaft of the unit is to rotate at 25 r.p.m. and is to be connected to the driven machine through a straight shaft coupling. The load is steady and continuous. Only one unit is to be built. Size of the reduction unit is not important, but the noise level should be as low as possible, using spur gears.

19. In a standard design, two shafts are connected by gears having 30 and 133 teeth. The gears have 20° full depth teeth of module 1·5 mm. An order for a number of these machines can be obtained if the gear ratio can be changed to 4:1. Examination of the gear housing for clearances indicates that the smaller gear cannot have outside diameter greater than 54 mm. The shaft centre distance cannot be changed. Is it possible to replace the regular drive by two gears cut with standard gear cutters? If it is possible, make a dimensioned sketch of the drive and prove that it will operate satisfactorily.

20. Design any one set of gears of the four speed sliding gear box in a motor car having an engine of 30 H.P. running at 800 r.p.m. The speed ratios between the driving and driven shafts of 4:1, 2·5:1, 1·5:1 and 1:1 approximately are required in first, second, third and top gear respectively. The module of all the gears is the same and the distance between the axes of the mating gears is 150 mm. No pinion is to have less than 12 teeth. Sketch the arrangement of the unit including construction of the reversing gear. State the suitable number of teeth required on each gear wheel. Choose the suitable materials for the gears.

21. Draw the arrangement of a 3 stage spur gear reducer with important dimensions for the hoist of a crane which is to hoist a load of 5 tonnes at a speed of 5 metre/minute. The rope is so arranged that the speed at which the rope is wound on the barrel is twice the speed of the lift of the load. The barrel on which the rope is wound is 80 cm in diameter and the hoisting motor runs at 1450 r.p.m. Assume the modules of three stages as 5 mm, 8 mm and 10 mm respectively to find out the number of teeth on the gears. Assume that the largest gear is connected integrally to the barrel.

22. Design a spur gear drive for a heavy sugar mill crusher roller with the following particulars:—

Speed of driving pinion 716 r.p.m. (approximately); speed reduction 4:1, h.p. transmitted 100; centre to centre distance of gears 30 cm; service

14. A reduction gear is to transmit a maximum of 100 h p. at 60 r.p m. to the drum shaft of a winding drum. The gear ratio is 5:1 and the pinion is to have 21 teeth of involute form The wheel is to be of cast steel, with an elastic stress limit of 1,400 kg/sq cm The tooth factor may be taken as $y = 0{\cdot}154 - \frac{0{\cdot}912}{N}$ *for 20° pressure angle, where N is the number of teeth. The width of the face should not exceed four times the circular pitch. Take the speed factor* $C = \frac{3}{3+v}$, *where v is the pitch line peripheral speed in metre per second. Find a suitable module and face width and assuming 6 arms and a shaft diameter of 180 mm, determine suitable dimensions for the wheel boss, arms and rim. Make a neat sketch, showing base circle and tooth proportions. Give an approximate idea of the weight of the wheel Indicate a suitable material for the pinion*

15. A punch press running at 40 r.p m. is driven by 12·5 h p. motor running at 960 r.p m Using a double gear reduction, select all materials and design the gear train stating all the assumptions made Give a sketch of the gear train

16. A spur gear reduction unit is to transmit 75 H P. continuously with moderate shock load. The speeds of the shafts are to be 720 r.p.m and 144 r.p m. respectively. The centre distance is not restricted, but it should be small and a satisfactory contact ratio obtained. Fifty units are to be constructed Make a complete design for these two gears, including sketch showing all dimensions The cost should be kept low, but it is important that the gears be designed for a long and trouble free life Determine the shaft diameter on the basis of pure torsion

17. Assuming you are planning the design of a drive to connect the rotating drum of a cement kiln to a motor. The drum is to rotate at approximately 18 r.p m. and the speed of the motor is 960 r p m Assume that it is reasonable to consider V belt drive and/or spur gears in combinations of any one or two for the drive.

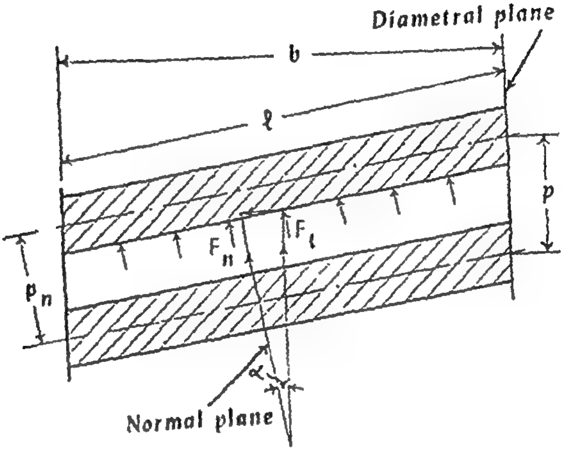
(a) *Make a neat sketch showing the layout of the drive you would propose.*

(b) *Specify the speed ratios, belt sheave diameters and number of gear teeth in combinations you propose in the drive. See fig. 16-21 for one solution of the problem pertaining to the drive.*

In order that contact may be maintained across the entire active face of the gear, the minimum value of b must be

$$b_{min} = \frac{p}{\tan\alpha} \quad \text{.......(ii)}$$

The American Gear Manufacturers' Association recommends minimum face width 15% higher than $\frac{p}{\tan\alpha}$.



Tooth of a helical gear

FIG. 16-10

If N_g be the number of teeth in a gear, then

$$p_n = p \cos\alpha = \frac{\pi D \cos\alpha}{N_g} \quad \text{.......(iii)}$$

where D is the pitch diameter.

If helical gears are manufactured with standard hobs, the normal diametral pitch is standard and so the diametral pitch in a diametral plane will be a decimal fraction and hence the pitch diameter also contains the decimal fraction.

The pressure angle Ψ_n in the normal plane is given by

$$\tan \Psi_n = \tan \Psi \cos\alpha \quad \text{.......(iv)}$$

where Ψ is the pressure angle in the diametral plane.

The centre distance is given as half the sum of the pitch circle diameters of the mating gears.

factor 0·8; pressure angle 20° for involute tooth system. The pinion is made of forged steel for which allowable stress is 1,100 kg/sq cm and the gear is of cast steel having allowable stress as 1,050 kg/sq cm. Allowable shear stress in mild steel shaft is 400 kg/sq cm. Draw the gear arrangement to a suitable scale. Also draw one tooth of the gear to full scale giving all dimensions. (*Bombay University, 1967*)

(B) DESIGN OF HELICAL GEARS

16-15. Introduction:

For high pitch line velocities and heavy loads, helical gears are used. In helical gears tooth elements are helices instead of being right lines parallel to the axes of the gears as we find in spur gears. Two important types of gearing are helical gears and double helical or herringbone gears When straight tooth spur gears begin to engage, the contact theoretically extends across the entire tooth on a line parallel to the axis of the gear This sudden application of load results in noisy operation and high impact stress In helical gearing, contact begins at one end of the entering tooth and gradually extends along a diagonal line across the tooth face as the gear rotates The gradual engagement and load application reduces the noise and the dynamic load. Therefore, the operation is silent and hence higher pitch line velocites can be employed. Pitch line speeds of 20 to 35 metre/sec are common with turbine and automobile gears Herringbone gear sets of special design have been successfully operated at pitch line speeds of 60 metre/second

As the contact extends along the diagonal line, the average lever arm is much less than the tooth height and therefore it can sustain greater tangential loads than straight tooth spur gears of the same size

16-16. Proportions for helical gears:

Fig. 16-10 shows a tooth of a helical gear Here we are concerned with two pitches. the normal pitches and the real pitches. The relation between the normal pitches, p_n, and real, pitches, p, are:

$$p_n = p \cos\alpha \qquad \ldots(1)$$

where α is the helix angle.

p_n = normal circular pitch
f = allowable stress for the material
b = width of the face measured parallel to the axis
y' = tooth factor corresponding to the formative number of teeth given by

$$N' = \frac{N}{\cos^3\alpha} \qquad \text{(ii)}$$

The face width b is made from $2p$ to $4p$.

(b) Design for dynamic load:

With the notations of art. 16-10, the dynamic load on the helical gear is given by

$$F_d = \frac{0{\cdot}05V(bC\cos^2\alpha + F)\cos\alpha}{0{\cdot}05V + \sqrt{bC\cos^2\alpha + F}} + F \qquad \text{(iii)}$$

The following relationships should be satisfied:

For steady loads $F_b \geq 1{\cdot}25\ F_d$.
For pulsating loads $F_b \geq 1{\cdot}35\ F_d$.
For shock loads $F_b \geq 1{\cdot}5\ F_d$.

(c) Design for wear:

With the notations of art. 16-11, the limiting load that can be carried without undue wear is given by

$$F_w = \frac{D_p\, b\, Q\, W}{\cos^2\alpha} \qquad \text{(iv)}$$

The value of wear load F_w should be greater than F_d.

16-18. Herringbone gears:

These gears are used to eliminate the axial thrust. There are many advantages of these gears such as silent operation, absence of vibration, higher efficiencies, higher velocity ratios, high pitch line velocities, etc. These gears find many applications in engineering such as drives for rolling mills, drives for reciprocating machineries, drives for hoisting machineries, drives for machine tools, drives for alternators from steam turbines, etc.

Speed reducers employing helical and herringbone gears are employed to get speed reduction ratios upto 800. For speed reduction upto 11, single pair units are employed. For higher ratios double pair or triple pair reducers are provided.

The velocity ratio is given by

$$\text{velocity ratio} = \frac{N_g}{N_p} = \frac{D_g \cos\alpha_g}{D_p \cos\alpha_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots (v)$$

If the tooth pressure is F_n, its component F_t, along the axis of the gear is called the end thrust. If α be the helix angle, this end thrust has the value $F_n \sin\alpha$ or $F \tan\alpha$, where F is the tangential force at the pitch line The end thrust produced by the component F_t is objectionable but it can be neutralised by using two helical gears with teeth sloping in opposite directions. These gears are placed side by side as a result end thrust of one is opposite to that of the other and thus the thrust is eliminated. Sometimes both sets of teeth are formed on one blank and the resulting gear is called a herringbone gear.

If the helix angle is small say less than 20°, the end thrust may be taken up by a thrust bearing either of roller type or ball type or even by the end of a plane bearing.

Though helical gears are not interchangable, however, the following table gives the American Gear Manufacturers' Association's recommended practice

Helical gear proportions:

Item	Maximum	Minimum
Pressure angle in the plane of rotation	25°	15°—23′
Helix angle α	45°	20°
Addendum	m	[illegible]·7 m
Clearance	0 3 m	0 157 m

Helix angles of 15°-15′ and 23° are in common use for single helical gears and 30° and 45° for herringbone gears The larger angles are used with the higher pitch line velocities.

16-17. *Design of helical gear teeth:*

(a) Design for strength:

As the contact extends along the diagonal line, according to Buckingham the modified form of Lewis equation is applied to helical gears. So, we get

$$F_b = 0{\cdot}75\, p_n f\, b\, y' \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

where F_b = the beam strength of the tooth in plane of rotation

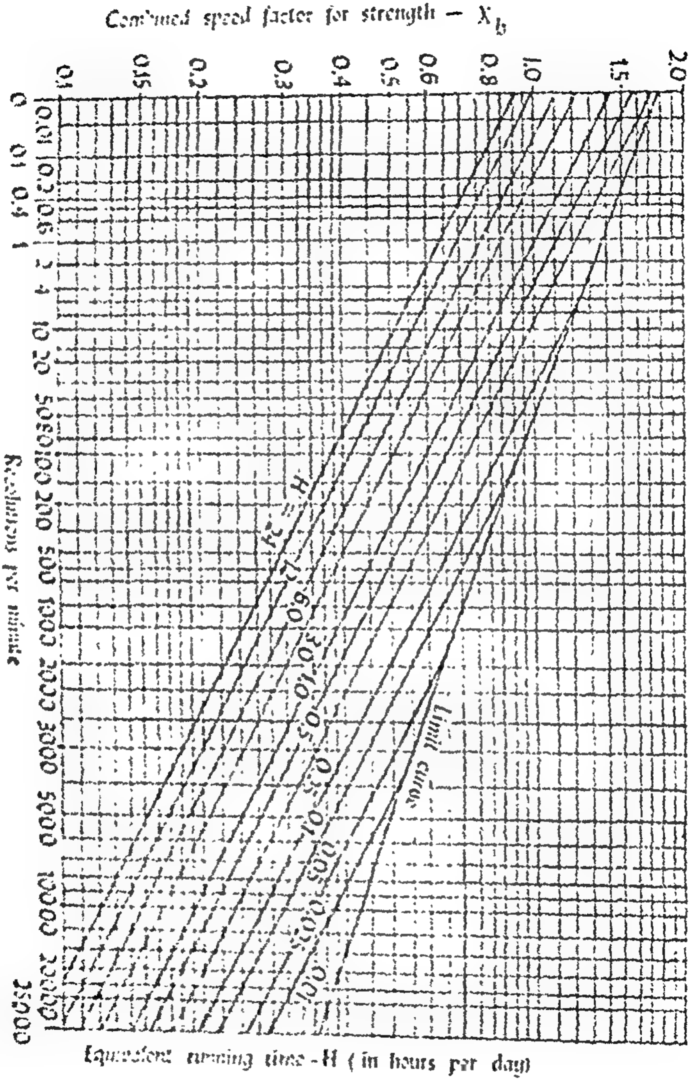


Fig. 16-11 Combined speed and running time factors for Spur and Helical gears for strength

Speed factor for strength
Spur gears with 20° pressure angle and helical gears)

Fig. 16-11

16-19. Rating of machine cut Spur and Helical gears:

Generally the cut gears may be purchased from ready stock and before it can be used whether the given pair of gears will be able to transmit the required horse power without fatigue failure or surface failure may be ascertained. Thus we should calculate the rating of the given pair of gears from

(i) Strength consideration

(ii) Wear consideration

Let m = module of wheel pinion or gear in mm

N = speed of wheel in r p m

b = face width of wheel in mm

T = number of teeth in wheel

The horse power rating of a spur gear with 20° pressure angle and helical gears for *strength* is given by

$$\text{H.P. (strength)} = \frac{X_b\, Y_b\, f_b\ b m^2\ NT}{25{\cdot}4} \times \frac{1774}{10^6} \quad \ldots \quad (i)$$

where X_b = speed factor for strength for appropriate running time

Y_b = strength factor for gears

f_b = bending stress factor.

Speed factor X_b depends upon

(i) running time in hour per day and

(ii) speed in r.p.m of the wheel

For a given running time, the speed factor decreases with the increase in speed *Speed factor for strength* for various speeds of wheels and running time in hours per day can be read from curves of fig. 16-11. The strength factor for spur external gears with 20° pressure angle and for helical gears with 30° helix angle can be read from curves of fig. 16-12. For other helix angle the strength factor obtained from this figure is to be multiplied by a factor $\left(\frac{\cos\beta}{\cos 30°}\right)^2$, where

β is the helix angle of the gear.

The above factor (strength factor) for helical gear is to be used if the face width is sufficient to give overlap In other words, axial pitch of the gears is less than face width

$$\text{Axial pitch} = m\pi \cot\beta \quad \ldots \quad (ii)$$

When the face width is less than axial pitch, then the strength factor obtained from curves given in fig. 16-12 should be multiplied by the factor

$$\frac{1}{3}\left[2+\frac{\text{face width}}{\text{axial pitch}}\right]^2.$$

The bending stress factor f_b depends on the tensile strength of the material of the wheels. The values of the bending stress factor are given in a table on page 607.

The allowable tangential load in kg/mm of face width for *strength* is given by

$$F=\frac{X_b\ Y_b\ f_b\ m}{25{\cdot}4}\ \text{kg/mm of face width}\ldots\ldots\ldots\ldots\ldots \quad \text{(iii)}$$

We shall have two horse power ratings from equation (i); one for pinion and other for gear.

The horse power rating of a spur gear with 20° pressure angle and helical gears for *wear* is given by

$$\text{H.P. (wear)}=\frac{X_c\ Y_c\ f_c\ b\ m\ NT}{K}\times\frac{1774}{10^8}\ \ldots\ldots\ldots\ldots\ldots \quad \text{(iv)}$$

where X_c = speed factor for wear for appropriate running time
Y_c = zone factor
f_c = surface stress factor
K = pitch factor.

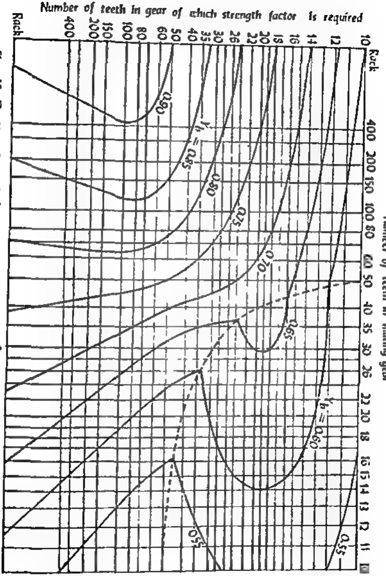
Speed factor X_c depends on

(i) running time in hour per day and
(ii) speed in r.p.m. of the wheel.

For a given running time, the speed factor decreases with the increase in speed. *Speed factor for wear* for various speeds of wheels and running times in hours per day can be read from curves of fig. 16-13.

The speed factors X_b and X_c for 12 hours running time per day are equal, except for speeds below 40 r.p.m.

The zone factor Y_c is different for spur gears with 20° pressure angle and helical gears. The zone factor for external spur gears with 20° pressure angle can be read from curves of fig. 16-14, while the zone factor for helical gears with 30° helix angle can be read from curves of fig. 16-15. For other helix angle, the zone factor can be obtained from the curves of fig. 16-15 by multiply-



Strength factor (Spur gears with 20° pressure angle and helical gears with helix angle 30°)

Fig. 16-12

When the face width is less than axial pitch, then the strength factor obtained from curves given in fig. 16-12 should be multiplied by the factor

$$\tfrac{1}{3}\left[2+\frac{\text{face width}}{\text{axial pitch}}\right]^2.$$

The bending stress factor f_b depends on the tensile strength of the material of the wheels. The values of the bending stress factor are given in a table on page 607.

The allowable tangential load in kg/mm of face width for *strength* is given by

$$F=\frac{X_b\ Y_b\ f_b\ m}{25{\cdot}4}\ \text{kg/mm of face width}\ldots\ldots\ldots\ldots\ldots \quad \text{(iii)}$$

We shall have two horse power ratings from equation (i); one for pinion and other for gear.

The horse power rating of a spur gear with 20° pressure angle and helical gears for *wear* is given by

$$\text{H.P. (wear)}=\frac{X_c\,Y_c\,f_c\,b\,m\,NT}{K}\times\frac{1774}{10^8}\ldots\ldots\ldots\ldots\ldots \quad \text{(iv)}$$

where X_c = speed factor for wear for appropriate running time

Y_c = zone factor

f_c = surface stress factor

K = pitch factor.

Speed factor X_c depends on

(i) running time in hour per day and

(ii) speed in r.p.m. of the wheel.

For a given running time, the speed factor decreases with the increase in speed. *Speed factor for wear* for various speeds of wheels and running times in hours per day can be read from curves of fig. 16-13.

The speed factors X_b and X_c for 12 hours running time per day are equal, except for speeds below 40 r.p.m.

The zone factor Y_c is different for spur gears with 20° pressure angle and helical gears. The zone factor for external spur gears with 20° pressure angle can be read from curves of fig. 16-14, while the zone factor for helical gears with 30° helix angle can be read from curves of fig. 16-15. For other helix angle, the zone factor can be obtained from the curves of fig. 16-15 by multiply-

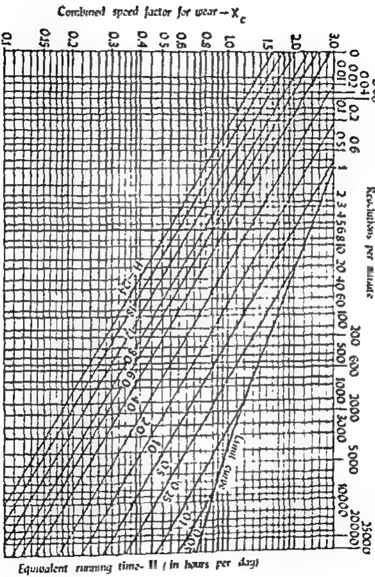


Fig 16-13 Combined speed and running time factors for Spur and Helical gears for wear

Speed factors for wear
(Spur gear with 20° pressure angle and helical gears)
Fig. 16-13

When the face width is less than axial pitch, then the strength factor obtained from curves given in fig. 16-12 should be multiplied by the factor

$$\tfrac{1}{3}\left[2 + \frac{\text{face width}}{\text{axial pitch}}\right]^2.$$

The bending stress factor f_b depends on the tensile strength of the material of the wheels. The values of the bending stress factor are given in a table on page 607.

The allowable tangential load in kg/mm of face width for *strength* is given by

$$F = \frac{X_b\ Y_b\ f_b\ m}{25{\cdot}4} \text{ kg/mm of face width} \qquad \text{(iii)}$$

We shall have two horse power ratings from equation (i); one for pinion and other for gear.

The horse power rating of a spur gear with 20° pressure angle and helical gears for *wear* is given by

$$\text{H.P. (wear)} = \frac{X_c\, Y_c\, f_c\, b\, m\, NT}{K} \times \frac{1774}{10^8} \qquad \text{(iv)}$$

where X_c = speed factor for wear for appropriate running time
Y_c = zone factor
f_c = surface stress factor
K = pitch factor.

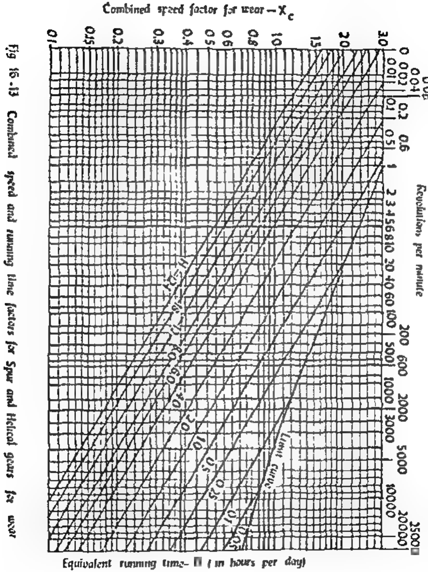
Speed factor X_c depends on

(i) running time in hour per day and
(ii) speed in r.p.m. of the wheel.

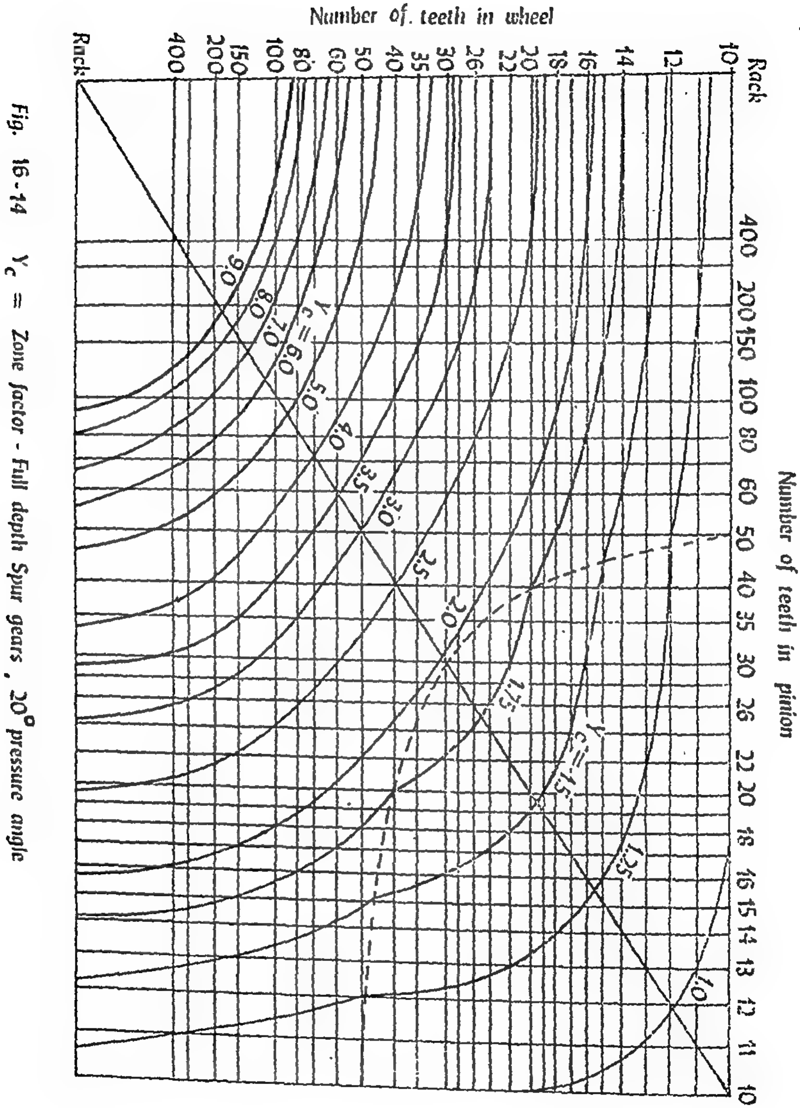
For a given running time, the speed factor decreases with the increase in speed. *Speed factor for wear* for various speeds of wheels and running times in hours per day can be read from curves of fig. 16-13.

The speed factors X_b and X_c for 12 hours running time per day are equal, except for speeds below 40 r.p.m.

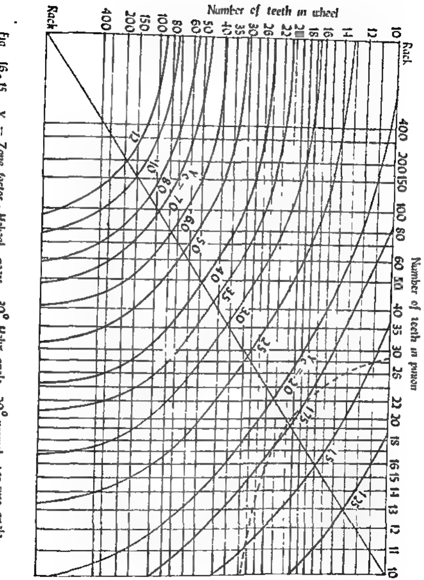
The zone factor Y_c is different for spur gears with 20° pressure angle and helical gears. The zone factor for external spur gears with 20° pressure angle can be read from curves of fig. 16-14, while the zone factor for helical gears with 30° helix angle can be read from curves of fig. 16-15. For other helix angle, the zone factor can be obtained from the curves of fig. 16-15 by multiply-



Speed factors for wear
(Spur gear with 20° pressure angle and helical gears)
Fig. 16-13



Zone factor
(External spur gears with 20° pressure angle)
FIG. 16-14



Zone factor
(Helical gears with 30° helix angle)
FIG. 16-15

ing with a factor $\left(\frac{\cos 30^\circ}{\cos\beta}\right)^2$, where β is the helix angle of the gear.

The zone factor for internal gears is given by

$$Y_c \text{ (internal gear)} = Y_c \text{ (external gear)} \left[\frac{G+1}{G-1}\right]^{0.8} \quad \ldots\ldots \text{(v)}$$

where G = speed reduction ratio.

The surface stress factor depends upon the hardness of the material. The values of this factor for different materials is given in a table on page 607.

The pitch factor K depends on the circular pitch on a section at right angles to the axis. The values of the pitch factors can be read from fig. 16-16.

We shall have further two values of horse power ratings from equation (iv): one for pinion and other for gear.

The horse power rating of any pair of gears is the *least of the four values:* two for pinion and two for gear.

The allowable tangential load in kg/mm face width for wear is given by

$$F_w = \frac{X_c Y_c f_c}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

We shall have four values of allowable tangential tooth load: two from strength consideration and other two from wear consideration. The *least of the four values* is the allowable tangential tooth load for any pair of gears.

Examples:

1. A pair of straight spur gears is required to reduce speed from 500 to 100 r.p.m. per minute for 12 hours running time per day continuously. The gears are of 8 module, 80 mm face width and 20° pressure angle. The pinion has 20 teeth.

The properties of materials of wheels are given below:

Material	*Pinion* 0·40% *carbon steel*	*Gear cast iron*
Bending stress factor f_b	*14·05*	*4·22*
Surface stress factor f_c	*1·125*	*0·81*

Determine the horse power rating of the gear set.

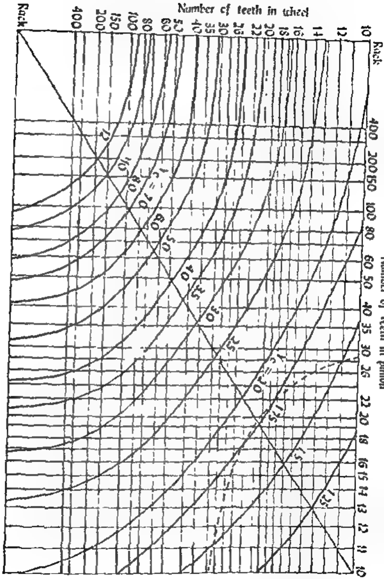


Fig. 16·15 Y_c = Zone factor - Helical gears - 30° Helix angle, 20° normal pressure angle

Zone factor
(Helical gears with 30° helix angle)
Fig. 16-15

ing with a factor $\left(\frac{\cos 30°}{\cos\beta}\right)^2$, where β is the helix angle of the gear.

The zone factor for internal gears is given by

$$Y_c \text{ (internal gear)} = Y_c \text{ (external gear)} \left[\frac{G+1}{G-1}\right]^{0.8} \quad \ldots\ldots(v)$$

where G = speed reduction ratio.

The surface stress factor depends upon the hardness of the material. The values of this factor for different materials is given in a table on page 607.

The pitch factor K depends on the circular pitch on a section at right angles to the axis. The values of the pitch factors can be read from fig. 16-16.

We shall have further two values of horse power ratings from equation (iv): one for pinion and other for gear.

The horse power rating of any pair of gears is the *least of the four values:* two for pinion and two for gear.

The allowable tangential load in kg/mm face width for wear is given by

$$F_w = \frac{X_c Y_c f_c}{K} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(vi)$$

We shall have four values of allowable tangential tooth load: two from strength consideration and other two from wear consideration. The *least of the four values* is the allowable tangential tooth load for any pair of gears.

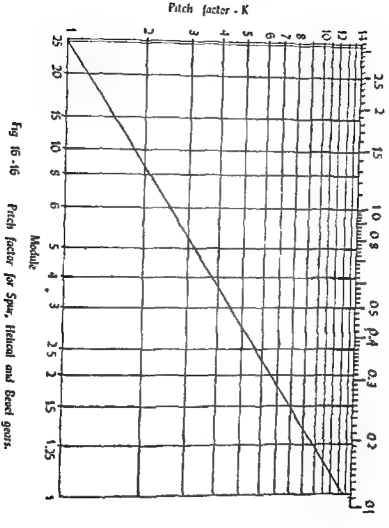
Examples:

1. A pair of straight spur gears is required to reduce speed from 500 to 100 r.p.m. per minute for 12 hours running time per day continuously. The gears are of 8 module, 80 mm face width and 20° pressure angle. The pinion has 20 teeth.

The properties of materials of wheels are given below:

Material	*Pinion* 0·40% *carbon steel*	*Gear cast iron*
Bending stress factor f_b	*14·05*	*4·22*
Surface stress factor f_c	*1·125*	*0·81*

Determine the horse power rating of the gear set.



Pitch factor
Fig. 16-16

Horse power for strength:

$$\text{H.P. (strength)} = \frac{X_b\, Y_b\, f_b\, bm^2\, NT}{25{\cdot}4} \times \frac{1774}{10^8}$$

After reading the values of X_b and Y_b from the curves and substituting in the equation, we get

$$\text{Pinion} = \frac{0{\cdot}3175 \times 0{\cdot}72 \times 14{\cdot}05 \times 80 \times 8^2 \times 500 \times 20}{25{\cdot}4} \times \frac{1774}{10^8}$$

$$= 114{\cdot}4 \text{ H.P.}$$

$$\text{Gear} = \frac{0{\cdot}42 \times 0{\cdot}615 \times 4{\cdot}22 \times 80 \times 8^2 \times 100 \times 100}{25{\cdot}4} \times \frac{1774}{10^8}$$

$$= 39 \text{ H.P.}$$

Horse power for wear:

$$\text{H.P. (wear)} = \frac{X_c\, Y_c\, f_c\, b\, m\, N\, T}{K} \times \frac{1774}{10^8}.$$

After reading the values of X_c, Y_c and K from the curves and substituting in the equation, we get

$$\text{Pinion} = \frac{0{\cdot}305 \times 2{\cdot}2 \times 1{\cdot}125 \times 80 \times 8 \times 500 \times 20}{2{\cdot}5} \times \frac{1774}{10^8}$$

$$= 34{\cdot}3.$$

$$\text{Gear} = \frac{0{\cdot}4 \times 2{\cdot}2 \times 0{\cdot}81 \times 80 \times 8 \times 100 \times 100}{2{\cdot}5} \times \frac{1774}{10^8}$$

$$= 32{\cdot}4.$$

The horse power rating of the gear set is the least of the four values. The least value is 32·4 H.P. Hence the rating of the speed reducer is 32·4 H.P.

2. *Determine the horse power capacity of a pair of helical turbine gears having a transmission ratio of 8:1. The lower speed is 500 r.p.m. The teeth are 20° full depth, 30° helix angle, 6 module and 100 mm face width. The material is 0·4% carbon steel untreated. The pinion has 30 teeth. You consider only the strength view point. The gear is to run continuously. The bending stress factor is 14·05.*

The horse power for strength of a wheel, with usual notations, is given by the equation

$$\text{H.P. (strength)} = \frac{X_b\, Y_b\, f_b\; b\, m^2\, NT}{25{\cdot}4} \times \frac{1774}{10^8}$$

After reading the values of X_b and Y_b from the curves and substituting in the equation, we get

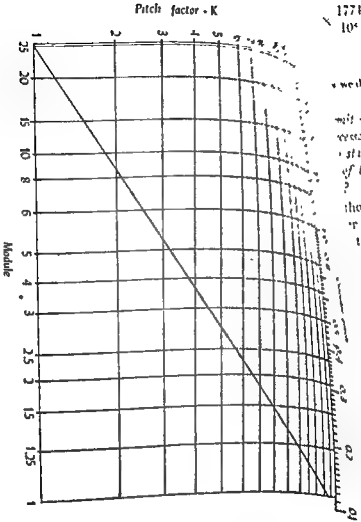


Fig 16-16 Pitch factor for Spur, Helical and Bevel gears.

Pitch factor

FIG. 16-16

$$\text{Velocity } V = \frac{\pi}{100} \times \frac{20{\cdot}7}{p_d} \times \frac{1500}{60} = \frac{16{\cdot}3}{p_d} \text{ metre/sec.}$$

$$\therefore \quad \text{Velocity factor} = \frac{6}{6 + \frac{16{\cdot}3}{p_d}} = \frac{p_d}{p_d + 2{\cdot}72}.$$

$$\therefore \quad \text{Allowable stress} = 560\left[\frac{p_d}{p_d + 2{\cdot}72}\right] \text{kg/sq cm.}$$

We choose the diametral pitch or the module so that the induced stress is less than the allowable stress.

Induced stress $= 65{\cdot}7\, p_d^3 = \dfrac{65{\cdot}7}{m^3}$ when m is measured in cm.

Allowable stress $= 560\left[\dfrac{p_d}{p_d + 2{\cdot}72}\right] = 560\left[\dfrac{1}{1 + 2{\cdot}72\, m}\right]$ kg/sq cm; here m is also measured in cm.

HELICAL GEAR PROPORTIONS

Module mm	Module cm	Induced stress $\frac{65{\cdot}7}{m^3}$ kg/sq cm	Allowable stress $560\left[\frac{1}{1+2{\cdot}72\ m}\right]$ kg/sq cm
6	0·6	304	213
8	0·8	129	177
10	1·0	65·7	151

We adopt 8 mm module.

The pitch diameter $= 20{\cdot}7 \times 0{\cdot}8 = 16{\cdot}56$ cm.

Width $= 9{\cdot}42 \times 0{\cdot}8 = 7{\cdot}536$ cm; we adopt 8 cm.

Tangential tooth load $= \dfrac{185}{0{\cdot}8} = 232$ kg.

The axial thrust $= 232 \times \tan 30° = 134$ kg.

Exercises.

1. State the comparative advantages and disadvantages of helical gears and straight spur gears.

Using a 3 dimensional sketch, derive an equation relating the pressure angle in a plane normal to the teeth, the pressure angle in the plane of rotation and the helix angle.

2. A pair of helical gears is to transmit 50 h.p. at 3,000 r.p.m. of the pinion. The velocity ratio is to be 5 and the helix angle is to be 23°.

$$\text{Pinion} = \frac{0.165 \times 0.8 \times 14.05 \times 100 \times 6^2 \times 4000 \times 30}{25.4} \times \frac{1774}{10^6}$$
$$= 56.$$

$$\text{Gear} = \frac{0.292 \times 0.705 \times 14.05 \times 100 \times 6^2 \times 500 \times 240}{25.4} \times \frac{1774}{10^6}$$
$$= 87.4.$$

Thus we see that the rating of the reducer is 56 H.P.

Note: If pinion and gear are of the same material, the pinion is weaker and the power transmitting capacity is governed by the pinion.

3. A helical cast steel gear with 30° helix angle has to transmit 40 h.p. at 1,500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and the width for 20° full depth teeth. The static stress for cast steel may be taken as 560 kg/sq cm. The width of the face may be taken as 3 p_n. What will be the end thrust on the gear?

Let us consider this example by diametral pitch method. The diametral pitch is the number of teeth per unit diameter of pitch circle. The product of the normal circular pitch and the normal diametral pitch equals π.

$$\text{Torque transmitted} = \frac{71620 \times 40}{1500} = 1{,}915 \text{ kg cm}$$

$$\text{Formative number of teeth} = \frac{24}{(\cos 30^\circ)^3} = 37$$

$$y' = 0.154 - \frac{0.912}{37} = 0.127.$$

Let p_d be the normal diametral pitch. The normal circular pitch p_n equals $\frac{\pi}{p_d}$. Here p_d is taken in MKS system. Its unit being cm^{-1}.

$$\text{The width of the gear} = 3p_n = \frac{\pi \times 3}{p_d} = \frac{9.42}{p_d}.$$

$$\text{The diametral pitch in the plane of rotation} = \frac{p_d}{\cos 30^\circ} = 1.16 p_d$$

$$\text{The pitch circle diameter of the gear} = \frac{24}{1.16 p_d} = \frac{20.7}{p_d}.$$

$$\text{Tangential tooth load} = \frac{1915}{20.7} \times 2\ p_d = 185\ p_d.$$

$$\text{Induced stress} = \frac{F_b}{0.75\ p_n\ b\ y'}$$
$$= \frac{185\ p_d \times p_d \times p_d}{0.75 \times \pi \times 9.42 \times 0.127} = 65.7\ p_d^3 \text{ kg/sq cm.}$$

$$\text{Velocity } V = \frac{\pi}{100} \times \frac{20\cdot7}{pd} \times \frac{1500}{60} = \frac{16\cdot3}{pd} \text{ metre/sec.}$$

$$\therefore \quad \text{Velocity factor} = \frac{6}{6 + \dfrac{16\cdot3}{pd}} = \frac{pd}{pd + 2\cdot72}.$$

$$\therefore \quad \text{Allowable stress} = 560\left[\frac{pd}{pd + 2\cdot72}\right] \text{kg/sq cm.}$$

We choose the diametral pitch or the module so that the induced stress is less than the allowable stress.

$$\text{Induced stress} = 65\cdot7\, pd^3 = \frac{65\cdot7}{m^3} \text{ when } m \text{ is measured in cm.}$$

$$\text{Allowable stress} = 560\left[\frac{pd}{pd + 2\cdot72}\right] = 560\left[\frac{1}{1 + 2\cdot72\, m}\right]$$

kg/sq cm; here m is also measured in cm.

HELICAL GEAR PROPORTIONS

Module		Induced stress $\frac{65\cdot7}{m^3}$ kg/sq cm	Allowable stress $560\left[\frac{1}{1+2\cdot72\ m}\right]$ kg/sq cm
mm	cm		
6	0·6	304	213
8	0·8	129	177
10	1·0	65·7	151

We adopt 8 mm module.

The pitch diameter $= 20\cdot7 \times 0\cdot8 = 16\cdot56$ cm.

Width $= 9\cdot42 \times 0\cdot8 = 7\cdot536$ cm; we adopt 8 cm.

$$\text{Tangential tooth load} = \frac{185}{0\cdot8} = 232 \text{ kg.}$$

The axial thrust $= 232 \times \tan 30° = 134$ kg.

Exercises.

1. *State the comparative advantages and disadvantages of helical gears and straight spur gears.*

Using a 3 dimensional sketch, derive an equation relating the pressure angle in a plane normal to the teeth, the pressure angle in the plane of rotation and the helix angle.

2. *A pair of helical gears is to transmit 50 h.p. at 3,000 r.p.m. of the pinion. The velocity ratio is to be 5 and the helix angle is to be 23°.*

If the minimum pitch diameter of the pinion is 12 cm, choosing your own material for the gears determine the pitch, face width and number of teeth in gears. Check the design for dynamic load and wear. The teeth are 20° full depth involute.

3 *Determine the horse power capacity of the pair of helical gears if the pair has a module of 5 mm, a normal pressure angle of 20°, a 7·5 cm face width and a 12° helix angle. The 18 teeth pinion operates at 1,450 r.p.m. and is made of cast steel. The gear has 108 teeth.*

4 *A pair of equal diameter herringbone gears of cast iron have a centre distance of 7·5 cm and are to transmit 6 h.p. at 1,000 r.p.m. Select a proper pitch, pitch diameter, helix angle and width of a face for 20° full depth teeth.*

5. *A certain machine has a 2·1 ratio, 3 mm module spur gear drive operating at a centre distance of 15 cm. It is desired to change this ratio to 2·5:1, maintaining the original centre distance, and substituting a drive at least as strong as the original. Give complete data as to the substitute drive.*

6. *Determine the horse power capacity of a pair of helical turbine gear having a transmission ratio 8.3. The lower speed is 3,000 r.p.m. The teeth are 20° full depth, 30° helix angle, 6 module and 100 mm face width. The material is 0.4% carbon steel untreated. The pinion has 27 teeth. Determine the rating of the speed reducer.*

(C) DESIGN OF BEVEL GEARS

16-20. Introduction:

Bevel gears are used for connecting two shafts which are at right angles. They may be connected at any desired angle. Some of the applications of bevel gears are found in rear-axle drives of automobiles, the vertical spindle of a drilling machine, the elevating screws for the cross rail of a planer, the crushing head or cone of a gyratory rock crusher, etc.

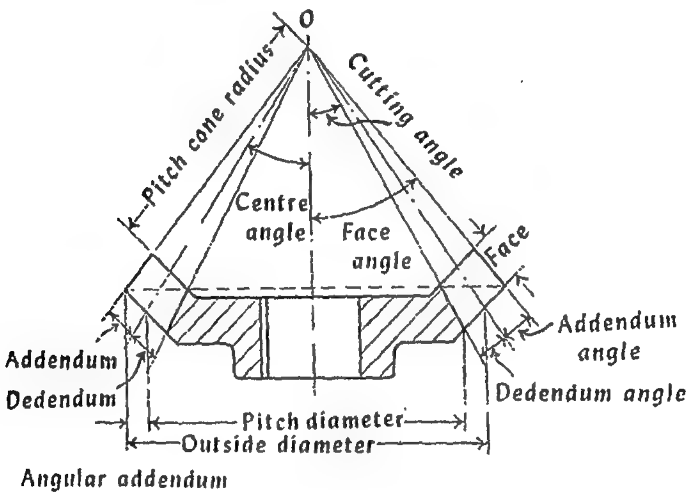
Two types of bevel gears are used—straight tooth gears and spiral tooth gears. Here, we shall limit ourselves to straight tooth gears, in which case the elements of the teeth converge to a common point *O* called the apex (fig. 16-17).

There are various types of bevel gears. When a pair of bevel gears of the same size are on shafts intersecting at right angles, they are called miter gears. The purpose of the miter gears is to change the direction of the shaft through a right angle without attempting any change in speed. Angular bevel gears are mounted

on intersecting shafts which make an angle of other than 90° with one another. In a crown gear, the pitch angle is 90°. In internal bevel gear, the teeth are cut on the inside of the pitch cone.

16-21. Definitions:

With bevel gears any combinations of numbers of teeth may be used; however, it should be remembered that bevel gears are not interchangeable and must be made in pairs.



Bevel gear tooth parts
FIG. 16-17

The definitions relating to bevel gears are given below and are shown graphically in fig. 16-17.

The pitch line is a straight line passing through the apex of the cone and lying on the slant surface.

The pitch angle is the angle between the pitch line and the axis of the cone. The length of the pitch line from the cone apex to the base is known as the pitch cone radius.

The addendum is the distance by which tooth extends outside the pitch line at the outer edge while the dedendum is the depth of the tooth below the pitch line at the outer edge.

The apex of the cone, O, is always shown on the working drawing. The formulas for calculating proportions of bevel gears are given in table below:

If the minimum pitch diameter of the pinion is 12 cm, choosing your own material for the gears determine the pitch, face width and number of teeth in gears. Check the design for dynamic load and wear. The teeth are 20° full depth involute.

3. *Determine the horse power capacity of the pair of helical gears if the pair has a module of 5 mm, a normal pressure angle of 20°, a 7·5 cm face width and a 12° helix angle. The 18 teeth pinion operates at 1,450 r.p m and is made of cast steel The gear has 108 teeth*

4. *A pair of equal diameter herringbone gears of cast iron have a centre distance of 7·5 cm and are to transmit 6 h p. at 1,000 r.p.m Select a proper pitch, pitch diameter, helix angle and width of a face for 20° full depth teeth.*

5. *A certain machine has a 2:1 ratio, 3 mm module spur gear drive operating at a centre distance of 15 cm. It is desired to change this ratio to 2·5:1, maintaining the original centre distance, and substituting a drive at least as strong as the original . Give complete data as to the substitute drive.*

6. *Determine the horse power capacity of a pair of helical turbine gear having a transmission ratio 8.3 The lower speed is 3,000 r p m The teeth are 20° full depth, 30° helix angle, 6 module and 100 mm face width. The material is 0 4% carbon steel untreated The pinion has 27 teeth Determine the rating of the speed reducer*

(C) DESIGN OF BEVEL GEARS

16-20. Introduction:

Bevel gears are used for connecting two shafts which are at right angles. They may be connected at any desired angle Some of the applications of bevel gears are found in rear-axle drives of automobiles, the vertical spindle of a drilling machine, the elevating screws for the cross rail of a planer, the crushing head or cone of a gyratory rock crusher, etc

Two types of bevel gears are used—straight tooth gears and spiral tooth gears. Here, we shall limit ourselves to straight tooth gears, in which case the elements of the teeth converge to a common point O called the apex (fig. 16-17).

There are various types of bevel gears When a pair of bevel gears of the same size are on shafts intersecting at right angles, they are called miter gears. The purpose of the miter gears is to change the direction of the shaft through a right angle without attempting any change in speed. Angular bevel gears are mounted

accurate results can be obtained by taking the pitch and the linear velocity of the tooth at the mid face and then calculating the strength by the same method as for spur gears.

The modified formula for the strength of teeth for bevel gears is given as

$$F_b = f\,b\,p\,y\left(1 - \frac{b}{l}\right) \quad \text{.......(i)}$$

where f = permissible stress for the gear

b = the face width of the gear

p = circular pitch

y = the form factor based on the formative number of teeth, and not on the actual number of teeth

l = the cone distance.

As the terms in the bracket $\left(1 - \frac{b}{l}\right)$ takes care for the bevel gears, that factor may be termed as the bevel factor.

For satisfactory operation of the bevel gears the ratio $\frac{b}{p}$ should be between 2 and 3 and the ratio $\frac{b}{l}$ should not exceed $\frac{1}{3}$. To secure the ratio $\frac{b}{l}$ less than $\frac{1}{3}$, the number of teeth in the pinion must not be less than $\frac{48}{\sqrt{1+R^2}}$, where R is the required reduction ratio. The above formula (i) should be used when $\frac{b}{l}$ does not exceed $\frac{1}{3}$.

The dynamic and wear loads may be checked by formulas for spur gears using the virtual number of teeth for the pinion and gear and the pitch line velocity at the large diameter and F_t the equivalent tangential load at this pitch line velocity.

The cone distance l is given by

$$l = \sqrt{r_p^2 + r_g^2} \quad \text{.......(ii)}$$

where r_p = pitch radius of pinion

r_g = pitch radius of gear.

When the bevel gears connect two shafts at right angles, the pitch cone angle θ_p for the pinion is given by

$$\theta_p = \tan^{-1}\frac{1}{R} \quad \text{.......(iii)}$$

where R is the required reduction ratio.

$$\theta_g = \tan^{-1}R \quad \text{.......(iv)}$$

FORMULAS FOR CALCULATING PROPORTIONS OF BEVEL GEARS

Item	Formula
Pitch angle or the centre angle	$\theta p = \tan^{-1} \dfrac{\sin\alpha}{\dfrac{Ng}{Np} + \cos\alpha}$ $\theta g = \tan^{-1} \dfrac{\sin\alpha}{\dfrac{Np}{Ng} + \cos\alpha}$
Shaft angle	$\alpha = \theta p + \theta g$
Pitch diameter	$Dp = Np \times m,\ Dg = Ng \times m$
Apex distance, cone distance	$\dfrac{D}{2\sin\theta}$
Formative number of teeth	$N' = \dfrac{N}{\cos\theta}$
Cutting angle or root angle	$\theta - \tan^{-1} \dfrac{\text{dedendum}}{\text{apex distance}}$
Face angle	$\theta + \tan^{-1} \dfrac{\text{addendum}}{\text{appex distance}}$
Outside diameter	$D + 2 \times \text{addendum} \times \cos\theta$
Velocity ratio	$\dfrac{\text{speed of pinion}}{\text{speed of gear}} = \dfrac{Ng}{Np} = \dfrac{Dg}{Dp}$

The addendum angle is the angle between the pitch line and the tooth top line, which intersect at the apex while the dedendum angle is the angle between the pitch line and tooth base line, which intersect at the apex

The root angle or the cutting angle is the angle between the tooth base line and the cone axis which intersect at the apex.

The face angle is the pitch cone angle plus the addendum angle. The diameter of the base cone is known as the pitch diameter. The outside diameter is the pitch diameter plus twice the angular addendum, which is defined as the distance of the outer edge of tooth from cone axis minus one-half the pitch diameter.

16-22. Strength of bevel gear teeth:

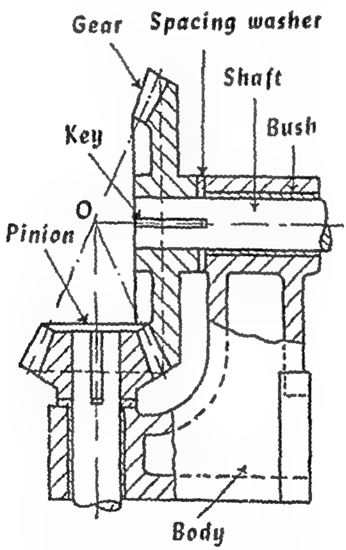
Various formulas have been suggested for calculating the safe working load on a bevel gear tooth, taking into account the variation in size between small and large ends. The sufficient

The backing of the gear is given by the relation

$$B_g = 0{\cdot}25D_g - B_p \quad \text{(ii)}$$

From the above relation it can be seen that the backing distance for the miter gears will be $\frac{D_p}{8} = \frac{D_g}{8}$.

Since the bevel gears are cut on cones, the clearance between the teeth is affected by any variation in the location of the bevel gears on their shafts. Since manufacturing tolerances affect the location of the gears when the machine is assembled, means must always be provided to allow adjustment of the position of the bevel gears at assembly. This adjustment is best accomplished by the use of shims to provide a positive location once the adjustment has been completed. The proper lubrication should be provided. If the gears are not enclosed, protective guards should be provided. Fig. 16-19 shows both pinion and gear mounted overhung. The bearing supports are rigid.



Bevel gear mounting
FIG. 16-19

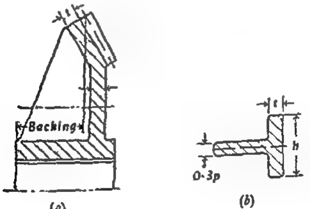
16-24. Bearing loads:

The resultant tooth load on either wheel or pinion of a pair of straight bevel gears is compounded of three forces acting at the mid-point of the face width.

16-23. Constructional details:

The bevel gear cast blanks have either solid webs or T shaped arms. The T section is generally adopted because it is cheap and the stem of the T provides an ideal strengthening rib against the thrust load The strengthening rib is not taken into account when considering the resistance of the section to bending in the plane of rotation. Sometimes the gear is a steel ring attached to a separate hub

The same general plan as used for the design of arms for spur gears may be applied to the case of bevel gears. We imagine that the arms extend upto the centre of the shaft. Therefore, the bending moment on the arms will be equal to the torque on the gear. Because of an eccentric force application, there is possibility of twisting on the arm, so it is advisable to assume that only half of the arms carry the tangential load The resisting moment of each arm will be $\frac{th^2}{6}f$. The value of t may be taken equal to the rim thickness and the thickness of the stem as $0{\cdot}3p$.



(a) (b)

Arms for bevel gears

Fig. 16-18

The rim, bead and hub dimensions may be made according to instructions given for spur gears except that the hub should be long enough to give positive backing as shown in fig 16-18(a). The practical rule for the backing length of the pinion is given as

$$B_p = \frac{0{\cdot}25 D_p D_g}{D_p + D_g} \qquad \text{...(i)}$$

(c) *Shaft angle 135°:*

$$\theta_p = \tan^{-1}\frac{\sin 135°}{6 + \cos 135°} = \tan^{-1}\frac{0.7071}{6 - 0.7071} = 7° - 36'.$$

$$\theta_g = \tan^{-1}\frac{\sin 135°}{\frac{1}{6} + \cos 135°} = \tan^{-1}\frac{0.7071}{0.166 - 0.7071} = 127° - 24'.$$

2. *A pair of bevel gears is required to transmit 25 h.p. at 600 r.p.m. The output shaft speed is 300 r.p.m. and is at right angles to the input shaft. Both gears are carried on overhanging shafts supported in the housing very close to the gears.*

The torque on the pinion shaft $= \dfrac{71620 \times 25}{600} = 3{,}000$ kg cm.

The velocity ratio is 2.

The semi-pitch cone angle of the pinion $\theta_p = \tan^{-1}\dfrac{1}{R}$

$$= \tan^{-1} \tfrac{1}{2} = 26° - 34'.$$

Let us adopt 28 teeth on pinion. The formative number of teeth

$$= \frac{28}{\cos 26° - 34'} = \frac{28}{0.8954} = 31.3, \text{ say } 32.$$

We adopt pressure angle as 20° and adopt full depth teeth.

$$y = 0.154 - \frac{0.912}{32} = 0.125.$$

Let us take the width of the gear as $\frac{1}{3}$rd the cone distance.

$\therefore$ Bevel factor $= \dfrac{l - b}{l} = \dfrac{l - 0.33l}{l} = 0.66.$

Let p cm be the circular pitch. Therefore, the pitch circle radius of the pinion $= \dfrac{28 \times p}{2\pi} = 4.46p$. The cone distance is equal to $\dfrac{4.46p}{\sin 26° - 34'} = 10p$. The face width will be equal to $\dfrac{10p}{3} = 3.33p$.

The tangential effort on the pinion $= \dfrac{3000}{4.46p} = \dfrac{672}{p}$ kg.

Let us assume the material for the pinion and gear to be cast steel for which the safe static stress to be taken as 1,050 kg/sq cm.

The pitch line velocity $= \dfrac{28 \times p}{100} \times \dfrac{600}{60} = 2.8p$ metre/second.

The velocity factor $= \dfrac{3}{3 + 2.8p}$.

(i) Force, acting perpendicular to the axis and of magnitude

$$\frac{\text{torque}}{\text{pitch diameter}} \times \frac{4l}{2l-b} = Q$$

where l = cone distance and

b = face width

(ii) Force, perpendicular to (i) and perpendicular to axis, of magnitude $Q \tan \Psi \cos\theta$

(iii) Axial force, acting from apex, of magnitude $Q \tan \Psi \sin\theta$.

When the components of the forces are known, by knowing the locations of gears on the shaft, the components of bearing loads can be easily calculated

Owing to limitations in gear cutting machines, bevel gears cannot conveniently be made of higher ratio than about 8 to 1; in general it is preferable not to exceed 6 to 1.

Examples:

1. A bevel gear is required to transmit 40 h p with a velocity ratio of 6:1 from a bevel pinion with the following shaft angles:

(a) 75° (b) 90° (c) 135°

Determine the semi pitch cone angles for pinion and gear.

(*a*) *Shaft angle 75°.*

$$\theta_p = \tan^{-1}\frac{\sin\alpha}{\frac{N_g}{N_p} + \cos\alpha} = \tan^{-1}\frac{\sin 75^\circ}{6 + \cos 75^\circ}$$

$$= \tan^{-1}\frac{0\ 9659}{6 + 0\ 2588} = 8^\circ - 42'$$

$$\theta_g = \tan^{-1}\frac{\sin\alpha}{\frac{N_p}{N_g} + \cos\alpha} = \tan^{-1}\frac{\sin 75^\circ}{\frac{1}{6} + \cos 75^\circ}$$

$$= \tan^{-1}\frac{0\cdot 9659}{0\cdot 166 + 0\cdot 2588} = 66^\circ - 18'$$

(*b*) *Shaft angle 90°:*

$$\theta_p = \tan^{-1}\frac{\sin 90^\circ}{6 + \cos 90^\circ} = \tan^{-1}\tfrac{1}{6} = 9^\circ - 32'.$$

$$\theta_g = \tan^{-1}\frac{\sin 90^\circ}{\frac{1}{6} + \cos 90^\circ} = \tan^{-1} 6 = 80^\circ - 28'.$$

Torque on the pinion shaft $= \dfrac{71620 \times 7 \cdot 5}{500} = 1{,}080$ kg cm.

As the speed reduction is 2:1, the torque on the gear shaft will be $1{,}080 \times 2 = 2{,}160$ kg cm. The pitch circle radius of the gear $= \dfrac{20 \times 2}{2} = 20$ cm.

For determining forces at the mid point of the face width of a gear, we adopt the methods described in art. 16-24.

We assume the ratio $= \dfrac{b}{l} = \frac{1}{3}$.

$\therefore$ Q, *the force acting perpendicular to the axis* $= \dfrac{2160}{40}\left[\dfrac{4}{2 - \frac{1}{3}}\right]$

$= 130$ kg.

Force perpendicular to Q and perpendicular to axis

$= 130 \tan 14\frac{1}{2}^\circ \times \cos 63^\circ - 24'$

$= 130 \times 0 \cdot 2586 \times 0 \cdot 4478 = 15 \cdot 1$ kg.

Axial force acting from the apex $= 130 \tan 14\frac{1}{2}^\circ \times \sin 63^\circ - 24'$

$= 130 \times 0 \cdot 2856 \times 0 \cdot 8942 = 30 \cdot 2$ kg.

Due to tangential load of 130 kg, the horizontal reaction at the right hand bearing will be $\frac{4}{10} \times 130$ kg $= 52$ kg and for left hand bearing the horizontal reaction will be $\frac{6}{10} \times 130$ kg $= 78$ kg.

Vertical reaction due to axial force at each bearing will be equal to $\dfrac{30 \cdot 2 \times 20}{25} = 25 \cdot 6$ kg. The magnitude of the reaction at each bearing will be the same but their directions will be opposite.

Vertical reactions due to third force will be $\frac{6}{10} \times 15 \cdot 1 = 9 \cdot 06$ kg at left hand bearing and $\frac{4}{10} \times 15 \cdot 1 = 6 \cdot 04$ kg at right hand bearing.

The maximum load will be on the left hand bearing and will be equal $\sqrt{78^2 + (25 \cdot 6 + 9 \cdot 06)^2} = 85$ kg.

Maximum bending moment on the gear will be $85 \times 10 = 850$ kg cm and twisting moment 2,160 kg cm.

We assume the permissible shear stress to be 400 kg/sq cm.

If d cm be the diameter of the solid shaft, then

According to modified Lewis formula, we have

$$\frac{672}{p} = 1050\left[\frac{3}{3+2{\cdot}8p}\right]3{\cdot}33p \times 0{\cdot}125 \times 0{\cdot}66p.$$

From which, we get $p^3 - 1{\cdot}63p - 1{\cdot}74 = 0$.

The value of p will be obtained by trial and error method. The value of p lies between 1·6 and 1·7 cm. The corresponding value of the module will be 5·1 and 5·42. The next standard module will be 6 mm. Therefore, the circular pitch will be $6 \times \pi = 18{\cdot}8$ mm, i.e. 1·88 cm

This will give a pitch circle diameter for pinion as $28 \times 6 = 168$ mm and that for gear $168 \times 2 = 336$ mm.

After module has been decided upon, other dimensions for the tooth can be obtained

The diameter of the pinion shaft can be calculated, considering the load to be uniform We design the shaft from torque consideration only taking the lower value of permissible shear stress.

We assume the permissible shear stress as 350 kg/sq cm. If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16}\, d^3 \times 350 = 3000$$

or $$d = \sqrt[3]{\frac{3000}{350} \times \frac{16}{\pi}} = 3{\cdot}52 \text{ cm, we adopt 4 cm.}$$

Since the speed reduction is 2.1, the wheel shaft will transmit a torque twice that of the pinion shaft; hence the diameter of the gear shaft will be $3{\cdot}52\sqrt[3]{2} = 4{\cdot}45$ cm; we adopt 4·5 cm The standard keyways should be provided in the pinion and gear hubs. The gear should be provided with four arms.

3. *Two shafts intersecting at an angle of 90° are connected by a straight tooth bevel gears to give a velocity ratio of 2:1. The pinion has a mean diameter of 20 cm and delivers 7·5 h p. at 500 r.p.m. The teeth have a pressure angle of 14½°. The mean plane of the gear is 15 cm from the right hand bearing and 10 cm from the left hand bearing. Determine the pitch angles of the gears, the forces on the gear bearings, the bending moment on the gear shaft and the necessary diameter of the shaft.*

The semi pitch cone angle for the pinion $\theta_p = \tan^{-1}\frac{1}{2} = 26° - 34'$.

$$\theta_g = \tan^{-1} 2 = 63° - 24'.$$

$$\frac{\pi}{16} d^3 \times 400 = \sqrt{850^2 + 2160^2} = 2{,}321 \text{ kg cm}$$

or $$d = \sqrt[3]{\frac{2321}{400} \times \frac{16}{\pi}} = 3\cdot1 \text{ cm; we adopt } 3\cdot5 \text{ cm.}$$

4. The bevel gear wheel of 4 arms of T section is subjected to a twisting moment of 6,000 kg cm. If the permissible stress for the gear material is 650 kg/sq cm, suggest the suitable dimensions for the arms.

In designing the arms for the gear, the flange of the T section will be assumed to resist the bending moment produced due to the tangential load. In order to account for the possibility of the twisting on the arm, we assume that only half of the arms carry the tangential load. Therefore, the bending moment to be resisted by T section $= \frac{6000}{2} = 3{,}000$ kg cm. We assume the width of the flange to be six times the thickness of the flange.

$\therefore$ Modulus of section $= \frac{1}{6}tb^2 = \frac{1}{6}t\,(6t)^2 = 6t^3$.

$\therefore$ $3000 = 6t^3 \times 650$

or $$t = \sqrt[3]{\frac{3000}{6 \times 650}} = 0\cdot915 \text{ cm, we adopt 1 cm.}$$

The width of the flange will be $1 \times 6 = 6$ cm. The vertical leg may be made 1 cm thick and the height of the leg should be about 2·5 cm less than the length of the hub.

Exercises:

1. A straight tooth bevel gear and pinion provides a 3 : 1 ratio and have perpendicular axes. Could this same pinion be used with a smaller gear in another installation with perpendicular axes to provide a 2 : 1 ratio? Explain briefly.

2. Why must bevel gear assemblies be constructed so that the position of the gear and pinion can be adjusted lengthwise along their shafts at assembly?

3. Explain how the position of the gear and pinion are adjusted axially and why this is necessary.

4. A bevel pinion and gear are to be designed to operate with a shaft angle of 75°, 90° and 120°. The speed ratio is 5:1.

Calculate the centre angles for each drive.

Ans. 10°-24′ and 64°-36′; 18°-53′ and 77°-41′.

5. The vertical spindle of a drilling machine is to be driven by means of a pair of straight bevel gears with 20° full depth teeth of module 5 mm. The pinion is to be mounted on the horizontal shaft. The reduction ratio is 2:1. If the drill requires a torque of 30,000 kg cm at 400 r.p.m., determine the proportions for bevel gears if they are made of steel with a static stress of 1,400 kg/sq cm.

6. A pair of bevel wheel transmits 50 h.p. One of the wheels has 60 teeth and 50 mm pitch cone diameter the other wheel has 45 teeth. Find the driving force between the teeth and determine the diameter of the shaft (pure torsion only). The shaft of the 60 teeth bevel wheel makes 120 r.p.m. Calculate the dimensions of the arms of the wheel (assuming 4 arms) and of the nave. Draw sectional elevation and plan.

7. A gear box is required for speed reduction ratio of 3:1. The shafts are at right angles and the high speed shaft transmits 30 h.p. at 900 r.p.m. Gears are to be straight bevel type and are made of steel having a strength of 2,100 kg/sq cm. The shafts are of carbon steel. The gear shaft is supported in two bearings, while that for the pinion is overhanging. Provision should be made for preventing dust. Lubrication by splash from the reservoir on the housing should be included in the design. Housing should be made from good grade of cast iron. You may take 96 as value of factor in calculating limiting load for wear and profile error less than 0·05 mm. Calculate the gear and pinion dimensions and bearing loads.

Draw an assembly drawing in section of the gear box.

8. A pair of bevel gears is required to transmit 25 h.p. at 600 r.p.m. The output shaft speed is 300 r.p.m. and is at right angles to the input shaft. Both gears are carried on overhanging shafts supported in the housing very close to the gears. The gear is of cast iron and the pinion is of steel.

Design the gear wheel and prepare its dimensioned drawing. The static strength of iron is 560 kg/sq cm and the pitch line velocity should not exceed 500 metre/minute. The wear factor is 44.

(D) DESIGN OF WORM GEARS

16-25. Introduction:

Worm gearing is used to transmit power between shafts with perpendicular, non-intersecting axes. The essential elements of worm gearing are a worm and a wheel. The worm is usually of

cylindrical form and resembles a screw; a section through the worm wheel will show that the teeth are straight sided and similar to those of an involute rack. The worm may be left handed or right handed and single threaded or multiple threaded. The worm wheel is essentially a helical gear with a face curved to fit a portion of the worm periphery.

The worm gearing is classified as non-interchangable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of different diameter, even if the thread pitch is the same

The principal advantages of the worm and worm wheel drive are

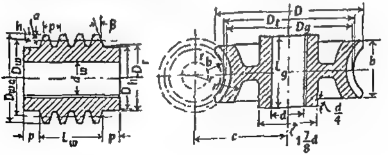
(i) The suitability for transmission of power at high velocity ratio

(ii) The self-locking property of the drive

(iii) The silent operation

16-26. Worm gear nomenclature:

Fig. 16-20 shows the principal elements and parts of the drive. The same terminology is applied to worm gear teeth as to all other forms of toothed gearing. The circular pitch p of the worm wheel becomes the linear pitch of the worm. The lead of the worm is the distance that a point on the pitch circle of the worm wheel will advance during one revolution of the worm. A double threaded worm has a lead equal to two times the pitch, in a single threaded worm the lead and pitch are alike



Worm and worm wheel
Fig. 16-20

The velocity ratio depends upon the lead of the worm and the pitch diameter of the wheel. The velocity ratio R is given by

$$R = \frac{n_w}{n_g} = \frac{N_g}{N_w} = \frac{D_g}{D_w \tan\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

where n_w is the r.p.m. of the worm, n_g r.p.m. of the gear, N_g the number of teeth on the gear, N_w the number of threads in parallel on the worm, D_g the gear pitch diameter, D_w the worm pitch diameter and α the helix angle of the worm. The helix angle α of the worm threads is the angle between a line tangent to the thread helix at the pitch line and a plane perpendicular to the axis of the worm.

$$\alpha = \tan^{-1}\frac{\text{lead}}{\pi D_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Unlike the most gearing the velocity ratio is independent of the pitch diameter of one of the elements—the worm.

The tooth pressure angle is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle. (See fig. 16-20.) For single and double threaded worms, the pressure angles of $14\frac{1}{2}°$ are used and for triple threaded and for quadruple threaded worms the pressure angles of 20° are used. For automotive applications the pressure angle of 30° is recommended to obtain a high efficiency and to permit overhauling.

The following are the approximate proportions which can be recommended for worms:

PROPORTIONS OF WORMS

p = circular pitch in cm

	Symbol	Single and double threads	Triple and quadruple threads
Normal pressure angle	β	$14\frac{1}{2}°$	20°
Pitch diameter, bored for shaft, cm	Dw	$2{\cdot}4p + 2{\cdot}8$	$2{\cdot}4p + 2{\cdot}8$
Pitch diameter, integral with shaft, cm	Dw	$2{\cdot}35p + 1$	$2{\cdot}35p + 1$
Face length, cm	w	$(4{\cdot}5 + 0{\cdot}02Nw)\,p$	$(4{\cdot}5 + 0{\cdot}02Nw)\,p$
Depth of tooth, cm	h	$0{\cdot}686p$	$0{\cdot}623p$
Addendum, cm	a	$0{\cdot}318p$	$0{\cdot}286p$
Hub diameter, cm	Dh	$1{\cdot}66p + 2{\cdot}5$	$1{\cdot}726p + 2{\cdot}5$
Maximum bore for shaft, cm	dw	$p + 1{\cdot}35$	$p + 1{\cdot}35$

cylindrical form and resembles a screw; a section through the worm wheel will show that the teeth are straight sided and similar to those of an involute rack The worm may be left handed or right handed and single threaded or multiple threaded The worm wheel is essentially a helical gear with a face curved to fit a portion of the worm periphery.

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The principal advantages of the worm and worm wheel drive are:

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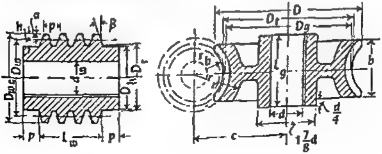

Worm and worm wheel
FIG. 16-20

The velocity ratio depends upon the lead of the worm and the pitch diameter of the wheel. The velocity ratio R is given by

$$R = \frac{n_w}{n_g} = \frac{N_g}{N_w} = \frac{D_g}{D_w \tan\alpha} \quad \text{.......................(i)}$$

where n_w is the r.p.m. of the worm, n_g r.p.m. of the gear, N_g the number of teeth on the gear, N_w the number of threads in parallel on the worm, D_g the gear pitch diameter, D_w the worm pitch diameter and α the helix angle of the worm. The helix angle α of the worm threads is the angle between a line tangent to the thread helix at the pitch line and a plane perpendicular to the axis of the worm.

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Pitch diameter, integral with shaft, cm	Dw	$2{\cdot}35p + 1$	$2{\cdot}35p + 1$
Face length, cm	w	$(4{\cdot}5 + 0{\cdot}02Nw)\,p$	$(4{\cdot}5 + 0{\cdot}02Nw)\,p$
Depth of tooth, cm	h	$0{\cdot}686p$	$0{\cdot}623p$
Addendum, cm	a	$0{\cdot}318p$	$0{\cdot}286p$
Hub diameter, cm	Dh	$1{\cdot}66p + 2{\cdot}5$	$1{\cdot}726p + 2{\cdot}5$
Maximum bore for shaft, cm	dw	$p + 1{\cdot}35$	$p + 1{\cdot}35$

The worm gear efficiency depends on the material of the worm and the worm wheel, the amount and character of the lubricant, the velocity of rubbing and the size of the helix angle of the worm. Single threaded worms have low helix angles and efficiencies. Such worms are used for getting large mechanical advantage. Due to their low efficiencies they are employed where self-locking property is desirable as in hoisting machinery. For such worms the helix angles must be generally less than 5°. To get maximum efficiency multiple threaded worms are employed, however, the use of such worms increases the size of the unit. For a compact design, the helix angle is given as

$$\tan\alpha = \xi R \quad \ldots\ldots(iii)$$

According to F. A. Halsey, the angle α less than 9° results in a rapid wear and the safe value is 12½°.

The following expressions for efficiency are based on velocity ratio:

$$\eta = 100 - R \quad \ldots\ldots(iv)$$

$$\eta = 100 - \frac{R}{2} \quad \ldots\ldots(v)$$

Equation (iv) is applicable to worm gear sets without cases mounted or installed by the purchaser, while equation (v) to commercial worm gear reducers.

PROPORTIONS OF WORM GEARS

p = circular pitch in cm

	Symbol	Single and double threads	Triple and quadruple threads
Normal pressure angle	ϕ	14½°	20°
Outside diameter, cm	D_o	$D_g + 1{\cdot}0135p$	D_g [illegible]
Throat diameter, cm	D_t	$D_g + 0{\cdot}636p$	$D_g + 0{\cdot}5$[illegible]p
Face width, cm	b	$2{\cdot}38p + 0{\cdot}6$[illegible]	$2{\cdot}1$[illegible]$p + 0{\cdot}5$
Radius of gear face, cm	r_f	$0{\cdot}882p - 1{\cdot}4$	$0{\cdot}914p + 1{\cdot}4$
Radius of gear rim, cm	r_r	$2{\cdot}2p + 1{\cdot}4$	[illegible]$p + 1{\cdot}4$
Radius of edge, cm	r_e	$0{\cdot}25p$	$0{\cdot}2$[illegible]p

16-27. Strength of worm gear teeth:

The power transmitting capacity of a worm gearing is decided by the strength, the ability to resist wear and abrasion and the

heat radiating capacity. In determination of tooth size and strength, it is safe to assume that the worm gear wheel should govern the design rather than the worm. The worm is made of steel while gear is made of cast iron and bronze. The continuous section of the worm thread offers greater resistance to bending than the gear tooth.

In worm gearing, two or more teeth are usually in contact, but due to uncertainty of load distribution among themselves, we assume that the load is transmitted by one tooth only.

According to Lewis formula

$$\text{permissible tooth load} = f\left[\frac{3}{3+v}\right] b\,p\,y \quad \ldots\ldots\ldots\ldots\ldots \text{(i)}$$

The static stress of 370 kg/sq cm for cast iron and 550 for bronze are the usual values. The permissible tooth load found out by equation (i) must be greater than actual tooth load which is determined from the torque transmitted and the worm wheel pitch diameter.

The limiting load F_w for wear in a worm gear set is found by

$$F_w = D_g\, b\, W \quad \ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

where D_g is the pitch diameter of the gear in cm, b the face of gear in cm and W a constant dependent upon the materials of the worm and gear. For continuous service, the worm should be hardened and have a Brinell Hardness Number of 250 or more.

The following table gives the value of W with hardened worms:

WEAR CONSTANT W FOR WORM GEAR

Material of worm wheel	W
Cast iron or semi steel	3·5
Manganese bronze	5·6
Phosphor bronze	7
Non-metallic materials	8·75

The limiting input horse power rating of a plain worm gear unit from stand point of heat dissipation, for worm gear speeds upto 2,000 r.p.m. may be estimated by

$$\text{H.P.} = \frac{2C^{1.7}}{R+5} \quad \ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

where C = centre distance in cm and
R = speed reduction ratio.

When the worm is underneath the wheel, it should be *run in oil bath* to insure adequate lubrication. The gears should be entirely *enclosed* to prevent oil leakage and to protect them from *dust or foreign* material. In order to increase the *efficiency* and to maintain proper alignment, ball bearings or *roller bearings* should be used.

16-28. Bearing loads on the shafts:

The worm shaft is usually mounted on ball bearings *and there is often a double* row radial ball bearings on the side where an axial *thrust load* exists. The gear shaft is supported either by ball bearings or Timken adjustable roller bearings.

$$\text{End thrust on worm} = P = \frac{2 \times \text{torque on gear wheel}}{\text{pitch diameter of wheel}} \quad \ldots (i)$$

$$\text{End thrust on wheel} = Q = \frac{2 \times \text{torque on worm}}{\text{pitch diameter of worm}} \quad \ldots (ii)$$

Journal loads on worm bearings

$$= \tfrac{1}{2} \sqrt{\left[P^2 \left(\tan\beta \sec\alpha \pm \frac{d_w}{l}\right)^2 + Q^2 \right]} \quad \ldots \ldots (iii)$$

Journal loads on worm wheel bearings

$$= \tfrac{1}{2} \sqrt{\left[Q^2 \left(\tan\beta \operatorname{cosec}\alpha \pm \frac{D_g}{L}\right)^2 + P^2 \right]}. \quad \ldots \ldots (iv)$$

where l and L are the respective journal bearing spans on worm shaft and wheel shaft respectively. The two bearings on each shaft are assumed to be equidistant from the common perpendicular to the shafts.

Because the teeth of the gear are cut in a concave surface so as to pertially wrap around the worm, as shown in fig 16-20, it is necessary that the gear be so located that the centre line of its face lines up with the centre line of the worm axis which condition is shown in fig. 16-20, but to obtain it in a machine means must be provided to adjust the gear position axially at assembly. One method of getting this adjustment is a shimmed cap as is the case with bevel gears. No adjustment is needed on the worm location, since the operation of the worm and gear is not affected by axial variation in the position of the worm

The illustrative examples given below give some ideas regarding the design procedure adopted for worm and worm wheel.

Examples:

1. *A triple threaded worm has a pitch diameter of 10 cm and an axial pitch of 2 cm. Determine the helix angle.*

$$\text{Helix angle} = \frac{\text{lead}}{\pi d_w}$$

where d_w = mean pitch diameter of the worm

α = helix angle.

$$\therefore \tan \alpha = \frac{3 \times 2}{\pi \times 10} = 0{\cdot}191.$$

$$\therefore \quad \alpha = 10° — 49'.$$

2. *A worm gear reducer unit is to have 40 cm centre distance. What should be the approximate worm diameter and axial pitch of the worm?*

The following design equations are employed:

$$d_w = \frac{C^{0{\cdot}875}}{1{\cdot}89} = 3p_c \qquad \text{(i)}$$

where d_w = mean pitch diameter of the worm

C = centre distance between axis of worm and axis of gear

p_c = circular pitch of gear.

$$b = 0{\cdot}37\ d_w \qquad \text{(ii)}$$

where b = face width of gear.

$$l = p_c\left[4{\cdot}5 + \frac{N_g}{50}\right] \text{cm} \qquad \text{(iii)}$$

where l = axial length of worm in cm

N_g = number of teeth in gear.

$$d_w = \frac{C^{0{\cdot}875}}{1{\cdot}89} = \frac{40^{0{\cdot}875}}{1{\cdot}89} = 13{\cdot}3 \text{ cm.}$$

Further $d_w = 3\ p_c$.

$$\therefore \quad p_c = \frac{13{\cdot}3}{3} = 4{\cdot}43 \text{ cm.}$$

Further $p_c = m\ \pi$.

$$\therefore \quad m = \frac{p_c}{\pi} = \frac{4{\cdot}43}{\pi} = 1{\cdot}41 \text{ cm.}$$

We adopt 16 mm module.

3. *A speed reducer unit is to be designed for an input 1 h.p. with a transmission ratio of 27. The speed of the hardend steel worm is 1,750*

where C = centre distance in cm and

R = speed reduction ratio.

When the worm is underneath the wheel, it should be run in oil bath to insure adequate lubrication. The gears should be entirely enclosed to prevent oil leakage and to protect them from dust or foreign material. In order to increase the efficiency and to maintain proper alignment, ball bearings or roller bearings should be used.

16-28. Bearing loads on the shafts:

The worm shaft is usually mounted on ball bearings and there is often a double row radial ball bearings on the side where an axial thrust load exists The gear shaft is supported either by ball bearings or Timken adjustable roller bearings

$$\text{End thrust on worm} = P = \frac{2 \times \text{torque on gear wheel}}{\text{pitch diameter of wheel}} \quad \ldots \text{(i)}$$

$$\text{End thrust on wheel} = Q = \frac{2 \times \text{torque on worm}}{\text{pitch diameter of worm}} \quad \ldots \text{(ii)}$$

Journal loads on worm bearings

$$= \tfrac{1}{2} \sqrt{\left[P^2 \left(\tan\beta \sec\alpha \pm \frac{d_w}{l}\right)^2 + Q^2\right]} \quad \ldots \text{(iii)}$$

Journal loads on worm wheel bearings

$$= \tfrac{1}{2} \sqrt{\left[Q^2 \left(\tan\beta \operatorname{cosec}\alpha \pm \frac{D_g}{L}\right)^2 + P^2\right]} \quad \ldots \text{(iv)}$$

where l and L are the respective journal bearing spans on worm shaft and wheel shaft respectively. The two bearings on each shaft are assumed to be equidistant from the common perpendicular to the shafts

Because the teeth of the gear are cut in a concave surface so as to pertially wrap around the worm, as shown in fig. 16-20, it is necessary that the gear be so located that the centre line of its face lines up with the centre line of the worm axis which condition is shown in fig. 16-20, but to obtain it in a machine means must be provided to adjust the gear position axially at assembly. One method of getting this adjustment is a shimmed cap as is the case with bevel gears. No adjustment is needed on the worm location, since the operation of the worm and gear is not affected by axial variation in the position of the worm

$$\text{Velocity factor} = \frac{3}{3+v} = \frac{3}{3 \cdot 55} = 0 \cdot 845.$$

Form factor for $14\frac{1}{2}°$ involute profile having 54 teeth is 0·111.

$p = 0{\cdot}942$ cm. $b = 2{\cdot}8$ cm.

$F = 560 \times 0{\cdot}845 \times 0{\cdot}111 \times 0{\cdot}942 \times 2{\cdot}8$

$= 138$ kg.

$$\text{H.P. transmitted} = \frac{138 \times 0{\cdot}55}{75} = 1{\cdot}02.$$

Thus given reducing unit can transmit 1·02 h.p. safely.

Wear load $F = D_g \times b \times W$ kg

where D_g = pitch circle diameter of the gear in cm

b = face width of gear in cm

W = material combination factor in kg/sq cm.

For hardened steel worm and phosphor bronze worm gear

$W = 7$ kg/sq cm.

On substitution of values, we get

$F_w = 16{\cdot}2 \times 2{\cdot}8 \times 7 = 318$ kg.

$$\text{H.P. (wear)} = \frac{318 \times 0{\cdot}55}{75} = 2{\cdot}33.$$

Permissible input power from heat dissipation view point

$$= \frac{2C^{1.7}}{R+5}$$

where C = centre distance in cm, and

R = speed reduction ratio.

$$\therefore \quad \text{H.P.} = \frac{2 \times 10^{1.7}}{27+5} = 1{\cdot}57.$$

Thus, the entire calculations can be summarised as follows:

Safe power	*Based on*
1·02	Strength
1·57	Heat dissipation capacity
2·33	Wear.

The rating of the given unit is 1 H.P.

4. *Design a wrom gear drive for a hoist with a drum diameter of 75 cm to lift a load of 250 kg through a distance of 20 metre in 15 seconds, driven by 950 r.p.m. motor.*

$$\text{Speed of the drum shaft} = \frac{20 \times 4}{0{\cdot}75 \times \pi} = 34 \text{ r.p.m.}$$

$$\text{Velocity ratio} = \frac{\text{motor speed}}{\text{drum speed}} = \frac{950}{34} = 28.$$

r.p.m. The worm wheel is to be made of phosphor bronze. The tooth form is to be $14\frac{1}{2}°$.

It is necessary to choose a centre distance for trial, which we assume to be 10 cm. The mean diameter of the worm is given by the equation $d_w = \frac{C^{0.875}}{1.89}$ cm

On substitution of values, we get

$$d_w = \frac{10^{0.875}}{1.89} = 4 \text{ cm}$$

We have $d_w = 3\ p_c$. Therefore, circular pitch will be approximately $\frac{4}{3} = 1.33$ cm

The pitch circle diameter of the gear will be $20 - 4 = 16$ cm

The number of teeth in the gear must be some multiple of 27; i.e. 27,54,81, etc

The pitch must also be equal to $\frac{\pi \times 16}{p_c} = 27,54,81$, etc. and it must be near about 1·33 cm

We assume double start worm. Therefore, the number of teeth in worm gear will be $2 \times 27 = 54$ teeth

$\therefore$ Nearest pitch $= \frac{\pi \times 16}{54} = 0.93$ cm

If m is the module, then circular pitch will be equal to $m\pi$

Therefore, probable m will be $\frac{0.93}{\pi} = 0.295$ cm. We adopt 3 mm. as the module. Therefore, circular pitch will be $\pi \times 3 = 9.42$ mm i.e. 0·942 cm. *The axial pitch of the worm is equal to the circular pitch of the gear.* The pitch circle diameter of the worm gear will be $= \frac{0.942 \times 54}{\pi} = 16.2$ cm

Pitch circle diameter of the worm $= 20 - 16.2 = 3.8$ cm.

Face width $b = 0.73 \times 3.8 = 2.78$ cm; we adopt 2·8 cm

Let us check the design from strength, wear and heat dissipation considerations

Tangential tooth load $= f\ b\ p\ y$.

For phosphor bronze, the static stress may be taken as 560 kg/sq cm.

$$\text{Pitch line velocity of gear} = \pi \times \frac{16.2}{100} \times \frac{1750}{27} \times \frac{1}{60}$$

$$= 0.55 \text{ metre/sec.}$$

For hardened steel worm and phosphor bronze worm gear,

$W = 7$ kg/sq cm.

On substitution of values, we get

$F_w = 28 \times 6{\cdot}91 \times 7 = 1{,}350$ kg.

As the wear load is more than the beam strength, which is 1,030 kg, re-design is unnecessary. If the wear load were to be less than the active load, the re-design would be necessary. We could have adopted double threaded worm with finer circular pitch.

The efficiency of the gear set $= 100 - \frac{R}{2} = 100 - \frac{28}{2} = 86\%.$

H.P. absorbed at the load $= \frac{250 \times 20}{15 \times 75} = 4{\cdot}45.$

As the efficiency is 86%, the input horse power $= \frac{4{\cdot}45}{0{\cdot}86} = 5{\cdot}17.$

Let us consider the heat dissipation capacity of the gear reducer unit designed.

The pitch diameter of the worm is approximately equal to three times the circular pitch of the gear. As the circular pitch is 3·14 cm, the pitch diameter of the worm is equal to $3 \times 3{\cdot}14 = 9{\cdot}14$ cm. The centre distance $C = \frac{28 + 9{\cdot}14}{2} = 18{\cdot}57$ cm.

$$\text{H.P.} = \frac{2C^{1.7}}{R+5} = \frac{2 \times 18{\cdot}57^{1.7}}{28+5} = 9{\cdot}15.$$

Thus, the design is satisfactory from heat dissipation view point.

Note: The artificial cooling is obtained by circulating the lubricant and cooling it outside the housing, by circulating water in cooling coils inside of the housing or blowing air across a finned part of the housing.

Two types of worm gear construction similar to one shown in fig. 16-5 are common. In one construction, the worm gear is an integral casting and must have a hub, web and rim of the same material as the teeth, while in other construction the bronze rim is mounted on a cast iron spider. This construction requires additional machining than the solid type of wheel, but the additional cost of machining is usually more than compensated for by saving in material particularly in large gears.

5. *Fig. 16-21 shows a layout drawing giving details of a drive to a rotary cement kilm. Using the data given in fig. 16-21, calculate*

In order to make the drive compact, we adopt 28 teeth bronze worm wheel and a single threaded worm

Torque on hoisting drum $= \dfrac{250 \times 75}{2} = 9{,}300$ kg cm

Tangential tooth load on wheel $= \dfrac{9300 \times 2\pi}{28p} = \dfrac{2090}{p}$ kg

where p is the probable circular pitch in cm. This load should be less than or equal to the strength of the tooth calculated from Lewis formula

The permissible static stress for phosphor bronze is 500 kg/sq cm For a pressure angle of $14\frac{1}{2}°$, for a 28 teeth wheel,

$$y = 0.124 - \frac{0.684}{28} = 0.100$$

The face width b of a gear is given by

$$b = 0.73\, d_w = 0.73 \times 3p = 2.19p$$

Let us take $b = 2.2p$

$$\text{Pitch line velocity} = \frac{28 \times p}{100} \times \frac{34}{60}$$

$$= 0.159p \text{ metre/sec.}$$

Let us solve the problem by trial and error method.

Module m mm	Circular pitch p cm	Face width $b = 2.2p$ cm	Velocity factor $\frac{3}{3+0.159p}$	Tangential tooth load $\frac{2090}{p}$ kg	Beam strength $fbpy\left[\frac{3}{3+0.159p}\right]$ kg
6	1.88	4.15	0.91	1,110	430
8	2.51	5.52	0.882	833	672
10	3.14	6.91	0.858	664	1,030
12	3.77	8.3	0.833	554	1,430

We adopt 10 mm module, which will provide a circular pitch of 3.14 cm.

Note: Hoist service is usually intermittent and the question of wear is not always considered. In a design of this character, however, and particularly in view of the comparatively small size of the worm gear, it may be advisable to investigate the possibilities of failure or unsatisfactory service from such cases.

$$\text{Pitch diameter of gear wheel} = \frac{28 \times 3.14}{\pi} = 28 \text{ cm.}$$

Wear load $F_w = D_g \times b \times W$ kg.

where D_g = pitch circle diameter of the gear in cm

b = face width of gear in cm

W = material combination factor in kg/sq cm.

(v) *Diameter of the worm shaft using a shear stress of 250 kg/sq cm in conjunction with the equivalent twisting moment formula for a steady torque and bending moment. Consider the worm shaft as being simply supported in its bearings.*

The load on the journal bearings of a kiln is 24,000 kg and the diameter of the journal is 90 cm. As the coefficient of friction at journal bearing is 0·12, the torque required to overcome the friction at the journal bearings is $24000 \times 0{\cdot}12 \times \frac{90}{2} = 130{,}000$ kg cm. Hence the torque required to turn the kiln is 130,000 kg cm.

$$\text{H.P. required at kiln} = \frac{130000}{100} \times \frac{2\pi \times 5}{4500} = 10{\cdot}1.$$

As the efficiency of the worm gear box is 85%. and that of spur gear drive is 60%, the horse power of the electric motor will be $\frac{10{\cdot}1}{0{\cdot}85 \times 0{\cdot}60} = 20.$

Speed reduction at spur gear is $\frac{240}{24} = 10$. Hence the speed of the spur gear pinion will be $5 \times 10 = 50$ r.p.m. Worm gear-box speed reduction will be $\frac{600}{50} = 12$.

$$\text{Torque on the motor shaft} = \frac{71620 \times 20}{600} = 2{,}400 \text{ kg cm.}$$

Pitch circle diameter of worm = 10 cm.

Tangential force on the worm = axial force on the wheel

$$= \frac{2400}{5} = 480 \text{ kg.}$$

Axial force on the worm = tangential force on the wheel.

$\therefore$ $F_y = 480$ kg.

$\therefore$ $480 = F\cos 12^\circ \times \sin 12^\circ + 0{\cdot}03 \times \cos 12^\circ F.$

From the above equation we get $F = 2{,}140$ kg.

F = turning force on the gear = axial force on the worm
$= 2140 \cos 20^\circ \times \cos 12^\circ - 0{\cdot}03 \times 2140 \times 0{\cdot}208$
$= 1{,}956$ kg.

Journal load on worm shaft = 480 kg.

Thrust load on the worm shaft bearings = 1,956 kg.

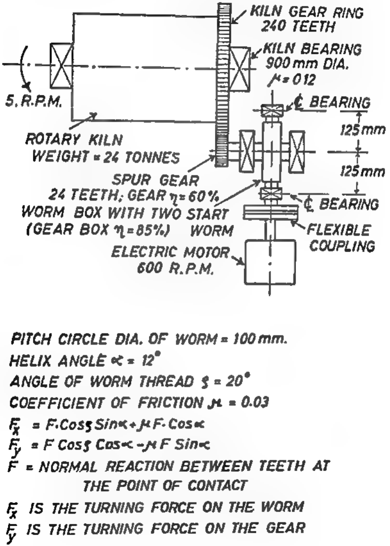
In the initial stage we neglect the effect of axial load on the shaft.

(*i*) *Torque to turn kiln*

(*ii*) *Electric motor horse power*

(*iii*) *Worm gear-box ratio*

(*iv*) *Journal and thrust loads on the worm shaft bearings*



Drive for a rotary kiln

Fig. 16-21

horse power input ratings from the standpoint of strength, wear and heat dissipation. The teeth are of $14\frac{1}{2}°$ involute, full depth form.

8. *A worm gear reducer unit is to have 25 cm centre distance. What should be the worm diameter and axial pitch of the worm?*

Ans. 8 cm; 2·492 cm.

9. *Design a worm and worm gear to transmit 30 h.p. at a speed of 480 r.p.m. The desired velocity ratio is 15:1. The efficiency must not be less than 90%. Use a worm with three threads.*

10. *It is required to design a suitable worm gearing with the following data:*

H.P. to be transmitted	5
Speed of worm	960 r.p.m.
Speed reduction	50
Duty	12 hours per day.

The drive should be as compact as possible.

11. *Design a set of worm gearing for use in the apron of a centre lathe. The worm is to slide on and revolve with the lead screw, which is to rotate at a minimum speed of 10 r.p.m. The speed reduction of worm gearing is to be 30. The outside diameter of the lead screw is 50 mm, having 6 mm pitch single start threads. The gear should be designed for the maximum possible efficiency and to work at a maximum centre distance of 10 cm.*

Materials:

Worm:	*Hardened steel*
Worm wheel:	*Phosphor bronze*
Duty:	*8 hours/day.*

12. *Design and prepare an assembly drawing of a worm gear reduction having the following data:*

H.P. at input shaft	15
Speed of input shaft	1,200 r.p.m.
Speed of output shaft	150 r.p.m.
Lewis factor based on DP	12 teeth — 0·245
	60 teeth — 0·421

(*University of Bombay, 1953*)

Bending moment on the worm shaft $= \frac{480 \times 25}{4}$

$= 3{,}000$ kg cm.

Twisting moment $= 2{,}400$ kg cm.

If d cm be the diameter of the solid shaft, then according to maximum shear stress theory, we have

$$\frac{\pi}{16} d^3 \times 250 = \sqrt{2400^2 + 3000^2}.$$

From above equation we get $d = 4{\cdot}35$ cm.

In order to account for the thrust, which has been neglected for finding out the diameter of the worm shaft, we increase the diameter to 55 mm.

The students are advised to check this diameter as an exercise.

Exercises:

1. State the most important advantages and disadvantages of worm gearing compared to other types of gearing.

2. Make a force analysis of a worm and gear with the gear driving the worm. Derive an equation for the efficiency of the drive. Derive an equation that expresses the relationship between the lead angle, pressure angle and coefficient of friction if the drive is self locking. Show that when this condition of worm driving exists, the efficiency is less than 50%.

Ans $\mu \geq \cos\phi_n \tan\alpha$.

3. Explain how the position of the gear is adjusted axially in the worm gear speed reducer. Why such an adjustment is necessary?

4. Explain why bronze gears are commonly used with hardened steel worms, rather than having both the worm and gear made of hardened steel.

5. A double threaded worm has a lead angle of 23°. For an axial pitch of 3·14 cm, what is the diameter of the worm?

Ans 4·73 *cm.*

6. A worm gear reducer unit has a centre distance of 22 cm and a transmission ratio of 18. What is its approximate horse power input rating in order to prevent over heating? *Ans.* 11·7 *h.p.*

7. A hardened steel worm rotating at 960 r.p.m. transmits power to a phosphor bronze gear with a transmission ratio of 15:1. The centre distance is 23 cm. Determine the remaining design and give estimated

State the actual speed ratios achieved. If the input is 25 h.p. at 2,000 r.p.m., determine:

(a) the greatest horizontal force exerted on the layshaft by the gears;

(b) the greatest torque reaction on the casing.

Neglect friction.

3. A pair of spur gears is required to transmit 30 h.p. with a speed reduction of exactly $3\frac{1}{2}$ to 1, the pinion being driven at 450 r.p.m. The centre distance is to be approximately 25 cm. The teeth are to be of Standard involute form having module of 5 mm. Design the gears using Lewis formula.

Adopt working stresses of 1,400 kg/sq cm for cast steel and 1,820 kg/sq cm for forged steel.

Use elliptical section arms for the wheel and take the permissible maximum bending stress to be 600 kg/sq cm.

In designing each shaft, assume a maximum bending moment of 12,500 kg cm to act in addition to the torque. Use a working shear stress of 500 kg/sq cm.

Ans. Tooth breadth 50 mm. Pinion shaft 35 mm. Wheel shaft 65 mm.

4. A pair of straight bevel gears are used to drive the vertical spindle of a drilling machine. The pinion is mounted on the horizontal drive shaft which rotates at 950 r.p.m. by a motor which delivers 22 h.p. The pinion is to have 24 machine cut stub teeth, pressure angle 20°, 4 mm module, speed ratio approximately 2:1. Determine a suitable face width for the gears and obtain the available drilling torque, neglecting gearing and bearing friction. Permissible static stress at low speed = 1,400 kg/sq cm. Tooth form factor y for 20° stub tooth spur gear is $y = 0{\cdot}175 - \dfrac{0{\cdot}90}{n}$.

5. A shaft 55 mm diameter has a gear-wheel keyed to it by a key. The gear is of cast steel and is to have 125 teeth, 5 mm module and transmits 18 h.p. at 90 r.p.m. The teeth are of standard involute form and machine cut with $14\frac{1}{2}°$ pressure angle. Form factor $y = 0{\cdot}119$. Obtain the length of hub, width of tooth, suitable section for the arms and for the rim. The wheel has five arms.

Permissible shear stress in key 420 kg/sq cm

Permissible stress in arm and rim 630 kg/sq cm

Permissible stress in teeth at low speed 1,200 kg/sq cm.

6. Design a worm reduction gear to transmit 35 h.p. with speeds of worm and gear wheel at 1,600 r.p.m. and 80 r.p.m. respectively. The distance between the shaft centres is to be nearly 12 inches (30 cm).

Safe design stresses under static conditions, i.e. at zero velocity, are for bronze 1,500 psi (315 kg/sq cm) and for steel 6,000 psi (420 kg/sq cm).

Prepare a fully dimensioned sketch of the unit.

(Gujarat University, 1958)

7. The shaft of a wire pulling machine rotates at 1,450 r.p.m. and is required to transmit a torque of 1,000 kg cm from the gear located at one end of a shaft to a flexible coupling at the other end. The bearings that support shaft are 80 cm

EXAMPLES XVI

1. A pinion, A (fig. 16-22), having 40 teeth transmits 50 h p. through a gear B to a pinion C, the speed of A being 200 r p m. The gear B is to be of cast steel working stress of 700 kg/sq cm. The weight of pinions A and C is 6 85 kg and that of gear B, 72·6 kg The pressure angle is 20°.

Determine: (a) The magnitude and direction of the *resultant thrusts* on the three bearings. (Give a diagram).

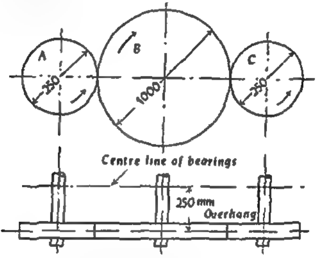


FIG. 16-22

(b) The maximum bending and twisting moments on the shaft B and its diameter if it is to be capable of transmitting the full horse-power and the shear stress is not to exceed 500 kg/sq cm

(c) The face width of gear B if the tooth factor $y = 0\ 15$.

Sketch and dimension a few teeth for the gear B.

Ans. (a) A 1,530 kg at 20°. B 2,780 kg vertical. C 1,530 kg at 20°.
(b) 70,600 kg cm, 72,500 kg cm, 10 cm dia. (c) 7 cm.

2. A three-speed gear box of sliding gear type is to have the following forward *speed ratios* as *nearly* as possible

Bottom gear 5 to 1
Second gear 3 to 1
Top gear $1\frac{3}{4}$ to 1.

The input and output shafts are to be in line and the centre distance (which is horizontal) between them and the layshaft is to be 15 cm exactly. The module is to be 5 mm for all wheels and no wheel must have less than 20 teeth. The pressure angle is to be 20°. Determine suitable number of teeth and pitch circle diameters for all wheels and sketch diagrammatically the arrangement of the box, describing its operation.

A pair of 20° involute full depth straight bevel gears are to transmit 20 h.p. from a 900 r.p.m. electric motor to another shaft at 90° running at 300 r.p.m. the pitch diameter of the pinion is 90 mm. Design the pinion and draw a neat dimensioned sketch. Lewis form factor $y = 0{\cdot}154 - \frac{0{\cdot}912}{n}$ where n is the formative number of teeth.

11. Design a back gear for a lathe to give a reduction of 9:1.

Distance between shaft centres 15 cm

H.P. transmitted 2·5

Speed of cone pulley 600 r.p.m.

Length of cone pulley 20 cm (exclusive of gears)

Safe static stress for materials of gears 2,500 kg/sq cm

Safe shear stress for spindle 500 kg/sq cm

Minimum no of teeth 20 with 20° pressure angle

Form factor	20	60	90	180	teeth
y	0·1	0·13	0·14	0·16	
q	3·3	2·75	2·6	2·5	

Sketch neatly the arrangement.

(Bombay University, 1969)

12. (a) Write explanatory notes on the following illustrating your answers with sketches:

(i) Load variation on a gear tooth as it passes through engagement.

(ii) Undercutting in gears

(iii) Gear tooth failures

(iv) Equivalent spur gear.

(b) In a gear box the centre distance between the bearings on the output shaft is 50 cm and its diameter at the bearings is 8 cm. The helical output gear is mid way between the bearings. The diameter of the gear hub is 18 cm and its axial length is 15 cm. The bearings are self aligning roller bearings with an outside diameter of 16 cm and an axial width of 5 cm.

Make a well proportioned sketch of the assembly of the gear, the shaft and the bearing mounted in the body of the gear box. The sketch should show explicitly how all the components are fixed and only the relevant parts of the gear and gear box need be shown.

You are free to modify the shaft and use any additional components you wish, provided you label them on the assembly.

(Bombay University, 1969)

13. A gear drive is to be designed to transmit 25 h.p. The velocity ratio is 1·2 and the r.p.m. of the smaller spur wheel is 200. The centre distance shall be equal to 45 cm. Pinion and gear are made of the same material having static stress of 550 kg/sq cm. Lewis factor for 20° full depth involute gears is given by

$$Y = 0{\cdot}154 - \frac{0{\cdot}912}{N}$$

where N is the number of teeth on gears. Sketch the gear wheel showing arms and the position of the key way.

(Sardar Patel University, 1969)

apart and the gear is located 10 cm outside its adjacent bearing centre to centre The gear is driven by a driving gear from a motor with a velocity ratio 1:1. Specify the horse power of the driving motor and design the suitable spur gear for the shaft.

8. What is the visual evidence of gear teeth failure by bending fatigue and by surface fatigue? What surface treatments could be used to increase the bending fatigue life and surface fatigue life of gear teeth?

An overhead crane has a rated capacity of 20 tonnes while it hoists at a speed of [illegible] metre/minute when the direct current motor [illegible] developing its rated horse power. A current relay limits the maximum motor torque to 125% of its rated torque. The rope drum diameter is 750 mm and the sheave diameters are 500 mm. The rope drum is driven by the direct current motor through a closed speed reduction gear. All gear wheels are of spur type. The final pair of spur gears, which is to be designed, are to provide a ratio of 3 1, with the pinion keyed to a 85 mm diameter shaft and the gear keyed to the 125 mm diameter shaft of the rope drum

Design the final pair of spur gears and make a neat dimensioned sketch, to scale, showing the cross section of your gear and pinion

9 Design a gear drive for a small hand driven blower for a blacksmith hearth. Blower speed is 800 r.p.m and approximate speed of hand lever is 70 r.p.m Force at the hand crank is 5 kg. Length of the crank is 30 cm The gear must be completely enclosed in a gear case and should incorporate cheap production methods Material used · Cast iron for gears, lever and gear case

Mild steel for shaft and hand crank

Take allowable static stress for cast iron as 560 kg/sq cm and the form factor for 20° gears as $y = 0\ 154 - \dfrac{0\ 912}{\text{number of teeth}}$

For mild steel take the allowable tensile stress as 800 kg/sq cm, (for shear 400 kg/sq cm and 70 kg/sq cm for bearing pressure

(Bombay University, 1968)

10. Discuss the various types of gear teeth failures and state the methods used to reduce or prevent them

Pinion		Gear		Bending stress based on endurance limits kg/sq mm	Load stress factor for 20° teeth kg/sq cm
Material	BHN	Material	BHN		
Steel	150	Steel	150	25	2 9
	200		200	42	5 5
	250		250	50	9 1
	300		300	35	13·6
	350		350	62	19
	400		400	70	33

17-2. Welding Processes:

In forge working, the parts which are to be connected are heated to the plastic state at the regions, where they are to be joined and by application of external mechanical pressure they are forged together by a hand hammer or press. This kind of welding is confined to odd shaped pieces made for millwrights and the maintenance staff of an engineering works. Wrought iron and low carbon steel may be forge welded. The machine forging finds its extensive use in the manufacture of wrought iron pipes.

In electric resistance welding, the parts to be joined are pressed together and current is passed from one part to other until the metal is heated to the fusion temperature at the joint. When welding temperature is reached, mechanical pressure is applied for the purpose of consolidating the metal and forming a sound weld. The low voltage is usually obtained by a transformer. For relatively thick plates, butt joint may be used. In case of lap joint for thin plates, if the pressure is applied by two electrodes on each side of the over lapped plates, a spot weld is obtained. If rollers are used instead of point electrodes and if they (plates to be joined) are pulled between the rollers, we get a uniform continuous strip of welded surfaces. Such a joint is known as seam weld.

Fusion welding is the process of joining two pieces of metals in the molten state without application of mechanical pressure. Generally, this method of welding is preferable to pressure welding. Internal stresses set up during the welding process may be removed by the heat treatment process, which consists of heating the entire joint or a member to a temperature somewhat below the critical temperature of the metal and then cooling it slowly. Gas welding process uses oxy-hydrogen or oxy-acetylene gas which is burnt in a welding torch, which provides a pointed flame which is allowed to play upon the joint as a result the welding metal or filler rod melts which on cooling results in a strong joint. Long seams are gas welded by heating the edges of the plates which are butted together and passed between the rollers thereby spreading the metal as the weld develops. This method gives a very good almost undetectable joint. For welding non-ferrous metals of low melting points, oxy-hydrogen process is used; while for welding ferrous metals and some non-ferrous metals, oxy-acetylene process is used. For a repair medium oxy-acetylene process is employed. Gas welding equipment may be modified for use in flame cutting of plates.

Electric arc welding:

The welding temperature is developed by an electric arc which is struck between the work to be welded and the electrode which is held by the operator or guided automatically in a specially designed machine. This method employs either a carbon rod electrode with a separate filler rod used for the source of weld metal or metallic rod electrode in which the electrode serves also as a source of welding metal.

In thermit welding, a mixture of iron oxide and aluminium called thermit is ignited and the iron oxide is reduced to molten metal. The molten metal is poured into a mould constructed around the joint and fuses with the parts which are to be connected. A major advantage of this type of welding is that all parts

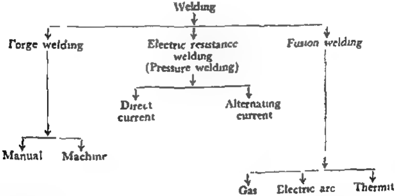
CHAPTER **XVII**

WELDED CONNECTIONS

17-1. Introduction:

Welding, rather than being a machine element is a manufacturing process, which reminds us that there are many facets of design in addition to stress analysis. Of the total design time stress analysis and proportioning parts consume only a minor part of the time. In majority of instances designs are affected by the manufacturing processes, which must be learnt. The effect of welding on design is very great and hence the designer must exercise his ingenuity in applying welding advantageously in his own design.

Welding may be defined as the joining of two pieces of metal by application of heat.



Gene ... of welding are fabrication as an alternate ... cation as a substitute for rivetir ...

Whether to weld or cast or forge etc. is an economic problem that may be answered correctly in different ways, depending upon local circumstances. Welding may be the least expensive process where the pattern cost for castings would be a large percentage of the total cost or where there are unusual machining or casting difficulties. Special rolled shapes, special screws and special studs are designed to be welded in place.

17-2. Welding Processes:

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Electric arc welding:

The welding temperature is developed by an electric arc which is struck between the work to be welded and the electrode which is held by the operator or guided automatically in a specially designed machine. This method employs either a carbon rod electrode with a separate filler rod used for the source of weld metal or metallic rod electrode in which the electrode serves also as a source of welding metal.

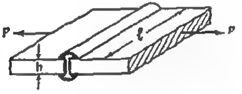
In thermit welding, a mixture of iron oxide and aluminium called thermit is ignited and the iron oxide is reduced to molten metal. The molten metal is poured into a mould constructed around the joint and fuses with the parts which are to be connected. A major advantage of this type of welding is that all parts

of the weld section are molten at the same time and the cooling of the weld is almost uniform. As a result residual stresses are minimum and the heat treatment of the joint will be eliminated. The thermit welding is particularly useful in joining together parts of large castings or forgings that are too complicated to manufacture in one piece, in welding rail joints and in repair of heavy parts such as locomotive frames, ship rudder posts, etc.

17-3. Types of Welded Joints:

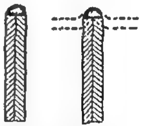
The most important of the various types of welded joints in common use are:

(a) Butt welds
(b) Fillet welds
(c) Edge welds
(d) Tack welds



Butt weld
Fig 17-1

Fillet weld
Fig 17-2



Edge weld
Fig. 17-3

A butt weld is obtained by butting together the edges of two pieces having practically the same cross section and heating until fused together Fig 17-1 shows a certain butt weld, in which the metal is deposited between the edges to be joined. A fillet weld is one which is placed in a corner made by two adjoining members. They are made with equal legs so that the length of these legs is used to represent the size of the weld. Fig. 17-2 shows a fillet, in which the weld metal is deposited in the corner between two surfaces. *Fig.* 17-3 shows the edge weld where the weld metal is deposited on the edges of thin plates. Tack welds are used for holding metal parts in positions while the major welding operation is in process

17-4. Working Stresses in Welds:

The design stress values recommended for welded joints vary with the type of loading. In machine design applications, repeated loads, reversal of loads and suddenly applied loads lower the value of permissible stresses. For tension (usually butt welds), the values of permissible stresses vary from 420 to 840 kg/sq cm depending on type of loading. Compression values are higher. For shear (fillet welds) design, stress values from 280 to 560 kg/sq cm are common. For fillet welds, (fig. 17-6 and fig. 17-7) the working stress may be expressed in another form as allowable load per unit length of a weld of given length. Suppose that the fillet weld of h cm is subjected to a shear loading. Suppose that the shearing stress of 800 kg/sq cm is permissible, then if we consider 1 cm length of weld, the area of throat will be $h \sin 45° \times 1 = 0{\cdot}707h$ sq cm.

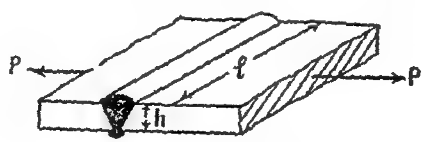
Allowable shearing load per cm length of weld will be equal to $0{\cdot}707h \times 800 = 565{\cdot}6h$ kg.

Thus, we specify the permissible shear load for a parallel fillet weld as $565{\cdot}6 \times 1{\cdot}5 = 845$ kg for a 15 mm fillet. This form is more convenient for design purposes.

17-5. Strength of Welds:

(*a*) *Strength of butt welds:*

These welds are sometimes called V welds which may act only in tension or compression, whereas fillet welds undergo shear as well as tension or compression and frequently bending in addition. Fig. 17-4 shows a single V groove weld loaded by a tensile



Single V groove butt weld
FIG. 17-4

force P. The strength of the above butt weld is assumed to be equal to net cross sectional area through the weld multiplied by the allowable stress.

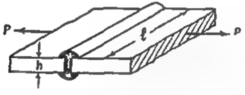
Load $= f_t \times$ area of the weld.

of the weld section are molten at the same time and the cooling of the weld is almost uniform. As a result residual stresses are minimum and the heat treatment of the joint will be eliminated. The thermit welding is particularly useful in joining together parts of large castings or forgings that are too complicated to manufacture in one piece, in welding rail joints and in repair of heavy parts such as locomotive frames, ship rudder posts, etc.

17-3. Types of Welded Joints:

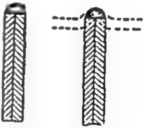
The most important of the various types of welded joints in common use are:

(a) Butt welds
(b) Fillet welds
(c) Edge welds
(d) Tack welds.



Butt weld
Fig 17-1

Fillet weld
Fig. 17-2



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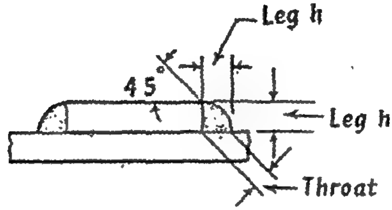
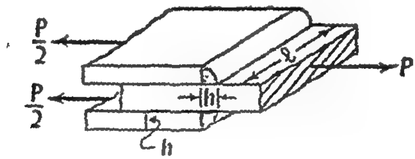
A butt weld is obtained by butting together the edges of two pieces having practically the same cross section and heating until fused together Fig 17-1 shows a certain butt weld, in which the metal is deposited between the edges to be joined. A fillet weld is one which is placed in a corner made by two adjoining members. They are made with equal legs so that the length of these legs is used to represent the size of the weld. Fig 17-2 shows a fillet, in which the weld metal is deposited in the corner between two surfaces. Fig. 17-3 shows the edge weld where the weld metal is deposited on the edges of thin plates Tack welds are used for holding metal parts in positions while the major welding operation is in process

as shown in fig. 17-6 or the leg lengths of the largest inscribed right angled triangle in case of fillet weld with unequal legs. For the cross section appearing in fig. 17-6 the weld has equal legs h and the minimum cross sectional dimension of the weld is termed the throat distance.

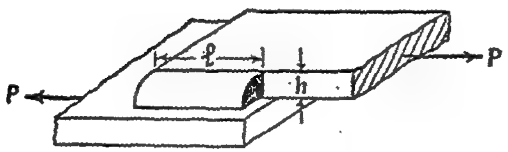


Fillet weld with equal legs
Fig. 17-6

Throat distance $= h \times \sin 45° = 0{\cdot}707h$.



Transverse fillet weld
Fig. 17-7



Parallel fillet weld
Fig. 17-8

The area assumed to be resisting the load is always the *throat area*, because weld failures are more often across the throat, but the size of the weld is its *leg dimension h*.

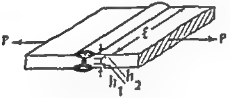
Fig. 17-7 shows a transverse fillet weld subjected to a load P.

Tensile stress intensity induced in the weld $= \dfrac{P}{2 \times l \times h \sin 45°}$

$$= \frac{0{\cdot}707P}{hl}$$

Fig. 17-8 shows a parallel fillet weld which is subjected to a

$P = f_t \times h \times l$ where h is the weld throat, l the length of the weld and f_t the permissible stress intensity in the weld material. Length of the weld will be equal to the width of the plate. In calculating the value of the throat, we do not consider the reinforcement of welds The reinforcement is desirable in order to compensate for flaws, but the amount of reinforcement varies throughout the length of the weld and it produces stress concentration at end points Fig. 17-5 shows a double V groove weld subjected to a load P. It is easy to see that the tensile stress induced in the weld is given by $f_t = \frac{P}{(h_1 + h_2)\, l}$.



Double V groove butt weld
Fig. 17-5

Note: According to Lincoln Electric Co. a properly made butt weld has equal or better strength than the plate and there is no need for calculating the stress in the weld.

(b) *Strength of fillet welds*

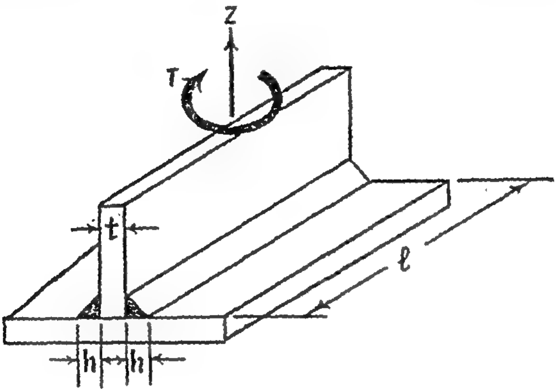
The fillet welds are of two types. (i) transverse fillet welds, which are assumed to fail in tension and (ii) parallel fillet welds which are assumed to fail in shear *It can be proved mathematically that the plane of maximum shear stress in the conventional 45° fillet weld is the 45° throat when subjected to parallel load and the 67½° throat when subjected to a transverse load. This results in greater strength for a transverse load.* According to experiments carried out also, the transverse fillet welds are found somewhat stronger than the parallel fillet welds. *However, it is a common practice to treat both of these types of fillet welds as being of equal strength.*

Before, we calculate the strength of the joint, the definitions of several characteristic dimensions are necessary. The size of a fillet weld is specified by the leg length of the largest inscribed isosceles right angled triangle in case of fillet weld with equal legs

$$f_s \max = \frac{2T}{\pi h d^2 (0{\cdot}707)} = \frac{2{\cdot}83\ T}{\pi h d^2} \quad \text{......................(ii)}$$

(*b*) *Torsion resisted by long adjacent fillet welds:*

Let T be the torque acting about the vertical plate which is attached to a horizontal plate by two identical fitted welds as shown in fig. 17-10. Let l and h be the length and leg of the



Long adjacent fillet weld
FIG. 17-10

fillet weld respectively. The effect of the applied torque is to rotate the vertical plate about the Z axis through its mid point. This rotation is resisted by shearing stresses developed between two fillet welds and the horizontal plate. We assume that intensities of these horizontal shearing stresses vary from zero at the Z axis to a maximum at the ends of the plate. Let us denote the shearing stress at the ends of the plate by f_s. This variation of shearing stress is analogus to the variation of normal stress over the depth of a beam subjected to pure bending.

$$f_s = \frac{T \times l/2}{2h \cdot \frac{l^3}{12}} = \frac{3T}{hl^2} \quad \text{..............................(iii)}$$

This is not the maximum shear stress in the fillet weld. The maximum value of the shear stress will be in the throat and is given by

$$f_s = \frac{3T}{hl^2 \times 0{\cdot}707} = \frac{4{\cdot}24T}{hl^2} \quad \text{......................(iv)}$$

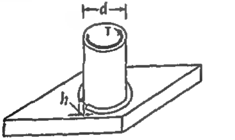
load P. Shear stress intensity in the weld $= \frac{0.707P}{hl}$. If the welds are long, the loading is not distributed uniformly. Values of allowable load per cm of weld should be reduced to about 90% that for short welds.

17-6. Special cases of fillet welds:

(*a*) *Torsion of a circular fillet weld:*

Fig. 17-9 shows a circular shaft, subjected to a torque T, which is connected to a rigid plate by a fillet weld of leg h. The shearing stress in the fillet weld in a horizontal plane, which concides with the upper face of the rigid plate can be determined from simple torsion formula. The intensity of stress is given by

$f_s = \frac{T\,d/2}{J}$ where J is the polar second moment of area of the weld section and d the diameter of the solid shaft.



Circular fillet weld subjected to torsion
Fig 17-9

The value of J can be taken as $\pi hd\,(d/2)^2$

$$\therefore \quad f_s = \frac{T\,d/2}{\pi hd\,(d/2)^2} = \frac{2T}{\pi hd^2}. \qquad \ldots (i)$$

This shearing stress occurs in a horizontal plane along a leg of the fillet weld. This is not the maximum shearing stress. The maximum value of the shearing stress occurs on the throat of the weld, which is inclined at 45° to this horizontal plane. The length of the throat is equal to $h \sin 45° = 0.707h$. Since the throat distance is smaller than the leg h, the shearing stress across the throat is greater than in a plane coinciding with the leg. Thus along the 45° throat section, we have a maximum shearing stress

Fig. 17-11 gives in tabular form the values of section modulus Z_W in bending and polar moment of inertia J_W in torsion of typical welded connections with the weld treated as a line. In fig. 17-11, N_x and N_y are respectively the distances of x and y axis from the faces as shown.

Section moduli in the above formulas are for maximum force at the top as well as at the bottom portions of the welded connections. For the unsymmetrical connections, the maximum bending force is at the bottom.

If there is more than one force applied to the weld, these are found and combined. All the forces, which are combined, must occur at the same point in the welded joints.

17-8. Fillet Welds under Varying Loads:

When the fillet welds are under varying load, the following design values should be used which are based on AWS code:

Let k be the ratio of minimum load to maximum load.

Allowable fatigue strength:

For 2×10^6 cycles: $\dfrac{900}{1-k/2}$ kg/cm

For $0{\cdot}6 \times 10^6$ cycles: $\dfrac{1260}{1-k/2}$ kg/cm

For $0{\cdot}1 \times 10^6$ cycles: $\dfrac{1500}{1-k/2}$ kg/cm

The above values should be used if they are less than 1,570 kg/cm; if the computed value is greater than 1,570 kg/cm, the latter value should be used.

The following values of k should be used:

$k = +1$, Steady load
$= 0$ Released load
$= -1$ Completely reversed load

The fatigue strength related to the number of cycles can be expressed by the relation

$$F_A = F_B \left[\frac{N_B}{N_A}\right]^C$$

where F_A = the fatigue strength for N_A cycles
F_B = the fatigue strength for N_B cycles

17-7. Design procedure recommended by American Welding Society:

Stress in a fillet weld shall be considered as shear stress on the throat for any direction of applied load.

The allowable parallel load per cm of weld in a statically loaded fillet weld is

$P = f_s \times$ area of the throat of the weld

$= 0{\cdot}707\, h\, f_s$ kg/cm ....(i)

where f_s = allowable shear stress according to AWS code.

The throat area of 1 cm of weld at 45° is $0{\cdot}707h$ sq cm.

The allowable transverse load per cm of weld in a statically loaded fillet weld is

$p = f_s \times$ area of the throat of one cm weld at $67\tfrac{1}{2}°$

$$= \frac{f_s \times 0{\cdot}765h}{\cos 22\tfrac{1}{2}°}$$

$= 0{\cdot}832h\, f_s$ kg/cm ....(ii)

If part of the load is applied parallel and part transverse, the allowable parallel load must be used.

Where bending or torsion is encountered, the procedure for the design is to *treat the weld as a line with no cross sectional area*. It can be shown that the property such as section modulus of any thin area is equal to the property of the section when treated as line multiplied by its thickness with negligible error.

If a weld is subjected to bending moment M, then

$$f = \frac{M}{Z_w} \qquad \text{...(iii)}$$

where

Z_w = section modulus of the weld treated as a line, for bending the units being cm²

f = load in kg per cm

If a weld is subjected to twisting moment T, then

$$f = \frac{T \times c}{J_w} \qquad \text{...(iv)}$$

where

J_w = polar moment of inertia of the weld treated as a line, the units being cm³

c = distance to the outer fibre in cm.

C = constant which varies slightly with the specimen. The value 0·13 has been used for butt welds and 0·18 for plates in axial loading tension and/or compression.

Any abrupt change in the section along the path of stress flow will reduce the fatigue strength.

Examples:

1. A spherical gas tank is made of 1 cm steel plate hemispheres butt welded together. The tank is 1,500 cm in diameter. Determine the allowable internal pressure to which the tank may be subjected if the permissible stress be limited to 840 kg/sq cm.

The strength of butt weld is usually taken to be equal to the product of the tensile working stress of the weld material by the net cross sectional area of the weld. The net cross sectional area of the weld = πDt where D is the inner diameter of the spherical tank and t the thickness of the spherical tank.

Area of the weld = $\pi \times 1500 \times 1 = 4{,}720$ sq cm.

The permissible tensile stress intensity in the weld is 840 kg/sq cm.

$\therefore$ Bursting load that can be resisted by the weld $= 4720 \times 840$
$= 3{\cdot}95 \times 10^6$ kg.

Let p be the pressure of the gas.

$\therefore$ Bursting load on the weld $= p \times \dfrac{\pi}{4} \times 1500^2$ kg.

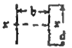
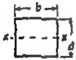
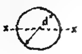
$$\therefore \quad \frac{\pi}{4} \times 1500^2 \times p = 3{\cdot}95 \times 10^6$$

$$\text{or} \quad p = \frac{3{\cdot}95 \times 4}{\pi} \times \frac{10^6}{(1500)^2}$$
$$= 2{\cdot}24 \text{ kg/sq cm.}$$

2. Two plates are joined by fillet welds as shown in fig. 17-12 and subjected to a tensile load of 40,000 kg. What length of 1 cm weld is required to resist the load? The allowable working stress in shear for the weld material is 800 kg/sq cm.

If l be the length of the parallel fillet weld and h the leg of the fillet weld, the throat area will be equal to $2\ l\ h \sin 45°$.

Area of throat $= 1{\cdot}414l \times h = 1{\cdot}414 \times l \times 1$
$= 1{\cdot}414 \times l$ sq cm.

PROPERTIES OF WELD TREATED AS A LINE		
OUTLINE OF WELDED JOINT	BENDING	TWISTING
	$z_w = \frac{d^2}{6}$	$J_w = \frac{d^3}{12}$
	$z_w = \frac{d^2}{3}$	$J_w = \frac{d(3b^2+d^2)}{6}$
	$z_w = bd$	$J_w = \frac{b^3+3bd^2}{6}$
	$z_w = \frac{4bd+d^2}{6} = \frac{d^2(4bd+d)}{6(2b+d)}$ TOP BOTTOM	$J_w = \frac{(b+d)^4-6b^2d^2}{12(b+d)}$
	$z_w = bd + \frac{d^2}{6}$	$J_w = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{(2b+d)}$
	$z_w = bd + \frac{d^2}{3}$	$J_w = \frac{(b+d)^3}{6}$
	$z_w = \frac{2bd+d^2}{3} = \frac{d^2(2b+d)}{3(b+d)}$ TOP BOTTOM	$J_w = \frac{(b+2d)^3}{12} - \frac{d^2(b+d)^2}{(b+2d)}$
	$z_w = bd + \frac{d^2}{3}$	$J_w = \frac{b^3+3bd^2+d^3}{6}$
	$z_w = \frac{\pi d^2}{4}$	$J_w = \frac{\pi d^3}{4}$

b - width d - depth

Properties of weld treated as a line
FIG. 17-11

Total length of welds required $= \frac{18000}{400} = 45$ cm.

The length of transverse weld is 10 cm. Therefore, the remaining length of weld i.e. 35 cm should be so arranged that the centre of gravity of length of welds lies along the line of action of load. The length of welds A and B will be 18 cm each.

5. *How long of a 1 cm fillet weld is required to weld the long side of an angle 150 × 100 × 15 to a steel plate with side welds only if the load is 12,000 kg and the allowable shear per cm of fillet weld is 700 kg? The centre of gravity of a 150 × 100 × 15 angle is at a distance of 5 cm.* (Refer fig. 17-14.)

The length of fillet weld required $= \frac{12000}{700} = 18$ cm.

Since the side welds only are to be used, then 18 cm required length must be divided into lengths for A and B so that their centre of gravity should coincide with the line of action of load. If x be the length of weld for side A, then the length of weld for side B will be $(18 - x)$ cm. Taking moment about the load line, we get $x \times 10 = 5(18 - x.)$

or $\qquad x = 6$ cm.

The length of weld for B will be 12 cm.

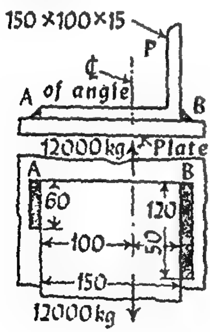


Fig. 17-14

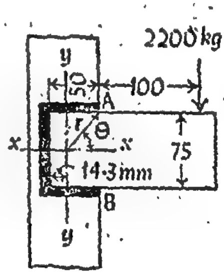


Fig. 17-15

6. *A bracket carrying a load of 2,200 kg is to be welded on to a stanchion by three fillets as shown in fig. 17-15. Caclulate the size of the welds if the working stress in the welds is 800 kg/sq cm.*

First of all, we calculate the centroid G of the weld by taking moment about the left hand edge of the plate. *We assume welds of unit throat thickness.*

Permissible stress intensity in the weld material is 800 kg/sq cm.

$\therefore \quad 40000 = 1{\cdot}414 \times l \times 800$

or $\quad l = \dfrac{40000}{1{\cdot}414 \times 800} = 35{\cdot}4$ cm.

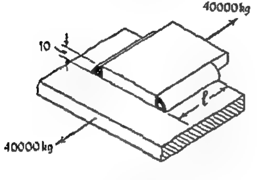


FIG. 17-12

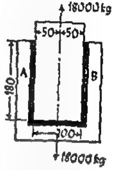


FIG. 17-13

3. *A 6 cm diameter solid shaft is welded to a flat plate by 1 cm fillet weld. Determine the maximum torque that the welded joint can sustain if the permissible shear stress intensity in the weld material is not to exceed 700 kg/sq cm.*

The maximum shear stress intensity in the weld is given by the formula

$f = \dfrac{2{\cdot}83T}{\pi h d^2}$ where T is the torque acting on the shaft, h the leg of the weld and f the diameter of the shaft

On substitution of the values, we get

$$700 = \frac{2{\cdot}83T}{\pi \times 1 \times 6^2}$$

$$\therefore \quad T = \frac{700 \times \pi \times 36}{2{\cdot}83} = 28{,}000 \text{ kg cm}$$

4. *Design a welded connection, as shown in fig 17-13, consisting of two 1 cm plates which are to be welded with 5 mm filled welds. The load acting on the plates is 18,000 kg. The allowable shear load per linear cm of weld is 400 kg.*

Note: The fillet welds should be considered under shear if the welds are placed either parallel or transverse to the direction of the load. The important point to be observed in designing welds, which transmit or resist forces, is to arrange them so that the line of action of the forces should coincide with the centre of the lengths of welds.

Determine the size of the weld required for 4,000,000 cycles. Assume the shear load is distributed over the entire weld.

The weld is subjected to a fatigue loading. The bending moment varies from a maximum of $1000 \times 20 = 20{,}000$ kg cm in one direction to maximum value of the same magnitude in opposite direction. The shear force varies from 1,000 kg up to 1,000 kg. as shown.

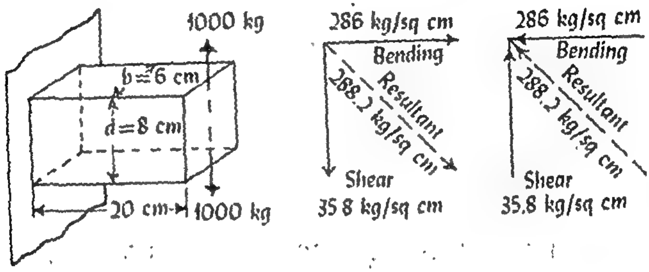


FIG. 17-16

The bending stress is maximum in the horizontal welds. The top and bottom welds are stressed the same.

The section modulus $= bd + \frac{d^2}{3}$ where b is the width of the rectangular section and d is the depth of the section.

$$\therefore \quad Z_w = 6 \times 8 + \frac{8^2}{3} = 69{\cdot}4 \text{ cm.}^2$$

The load in kg/cm due to bending $= \dfrac{20000}{69{\cdot}4}$

$= 286$ kg/cm.

Average shear force, assuming that the shear force is uniformly distributed over the entire weld $= \dfrac{1000}{2 \times 6 + 2 \times 8} = 35{\cdot}8$ kg/cm.

Resultant force $= \sqrt{286^2 + 35{\cdot}8^2} = 290$ kg/cm.

The maximum force varies from 290 kg/cm in one direction to 290 kg/cm in the opposite direction. This is true for both top and bottom welds.

Allowable force/cm for 2×10^6 cycles is $\dfrac{900}{1 - \frac{k}{2}} = \dfrac{900}{1 - \frac{1}{2}(-1)}$

$= 600$ kg/cm,

$$x = \frac{5 \times 2 \times 2{\cdot}5 + 7{\cdot}5 \times 0}{5 \times 2 + 7{\cdot}5} = 1{\cdot}43 \text{ cm.}$$

The eccentricity of the load $= 10 + 5 - 1{\cdot}43 = 13{\cdot}57$ cm.

$I_{xx} = 2 \times 5 \times 3{\cdot}75^2 + \frac{1}{12} \times 7{\cdot}5^3 = 176{\cdot}5$ cm^4.

$I_{yy} = 2 \times \frac{5^3}{12} + 2 \times 5 \times 1{\cdot}07^2 + 7{\cdot}5 \times 1{\cdot}43^2 = 47{\cdot}5$ cm^4.

$I = 176{\cdot}5 + 47{\cdot}5 = 224$ cm^4.

Moment of the load $= 2200 \times 13{\cdot}57 = 30{,}000$ kg cm

Maximum radius $r = \sqrt{3{\cdot}75^2 + 3{\cdot}57^2} = 5{\cdot}07$ cm

$$\cos\theta = \frac{3{\cdot}57}{5{\cdot}07} = 0{\cdot}704$$

$$F_s = \frac{2200}{17{\cdot}5} = 125 \text{ kg/sq cm}$$

$$F_b = \frac{30000}{224} = 134 \text{ kg/sq cm}$$

$F_r = \sqrt{125^2 + 134^2 + 2 \times 125 \times 134 \times 0{\cdot}704} = 240$ kg per cm length. Assuming permissible shear stress intensity to be 800 kg/sq cm, we can verify that 5 mm weld will suffice

7. *A circular bar of 6 cm diameter is welded to a steel plate, and the bar acts as a cantilever of length 20 cm, the load being 1,000 kg Determine the size of the weld if the allowable load is 1,700 kg/cm*

As the bar acts as a cantilever, the weld is subjected to a vertical shear force of 1,000 kg and a bending moment $1000 \times 20 = 20{,}000$ kg cm

We treat the weld as a line From table of fig 17-11, the modulus of section $= \frac{1}{4} \times \pi \times 6^2 = 28{\cdot}2$ cm^2.

Force per unit cm of weld at top and bottom $= \dfrac{20000}{28{\cdot}2}$

$= 710$ kg/cm

Vertical shear, assuming uniform distribution of shear force

$$= \frac{2000}{6 \times \pi} = 53 \text{ kg/cm}$$

Resultant load $= \sqrt{710^2 + 53^2} = 715$ kg/cm

Size of weld $= \dfrac{715}{1700} = 0{\cdot}42$ cm

We adopt 10 mm weld.

8. *A rectangular beam as shown in fig. 17-16 is to be welded to a side member. The maximum load of 1,000 kg is applied repetitiously.*

12. A 60 cm long plates are joined by two 1 cm fillet welds as shown in fig. 17-10. Determine the maximum torque the welded joint can resist if the permissible shear stress intensity in the weld material is limited to 800 kg/sq cm. *Ans. 680,000 kg cm.*

EXAMPLES XVII

1. Calculate the allowable load per linear cm of the fillet welds for the following sizes assuming shear stress intensity to be 800 kg/sq cm:

5 mm, 10 mm, 15 mm, 20 mm and 25 mm.

Ans. 280 kg, 560 kg, 840 kg, 1,120 kg and 1,400 kg.

2. Two 15 mm plates 75 mm wide are to be welded with 10 mm fillet welds. The load acting on the welded plates is 15,000 kg. The allowable shearing stress for the weld is 800 kg/sq cm.

Ans. 14 cm on each side.

3. How long of a 15 mm fillet weld is required to weld the long side of an angle $150 \times 80 \times 15$ to a steel plate, using the side welds only, if the load is 10,000 kg and the allowable shear load per cm of 15 mm fillet weld is 800 kg?

Ans. 12·5 cm; the length should be so divided that the centre of gravity should coincide with the line of action of load.

4. Show that the plane of maximum shear occurs at 45° for a parallel load on a fillet weld of equal legs. Neglect bending. Determine the allowable force P per cm of weld length if the allowable shear stress is 950 kg/sq cm.

Ans. 670 h, where h is the leg size in cm.

5. Show that the plane of maximum shear force occurs at $67\frac{1}{2}°$ for a transverse load on a fillet weld of equal legs. Neglect bending. Determine the allowable force P per cm of weld length if the allowable shear stress is 1,000 kg/sq cm.

Ans. 827 h, where h is the leg size in cm.

6. A 4 cm by 2 cm bar is welded to a 10 cm diameter shaft. The bar acts as a cantilever of length 30 cm and carries a static loading of 200 kg. Determine the size of a 45° fillet weld assuming that shear is uniformly distributed in the weld for an allowable load of 1800 kg/cm.

Ans. 5 mm weld size.

where $k = \frac{\text{minimum stress}}{\text{maximum stress}} = -1$ since the load is completely reversed.

The allowable force for 4×10^6 cycles will be $600\left[\frac{2 \times 10^6}{4 \times 10^6}\right]^{0.12}$
$= 550$ kg/cm

The weld size will be $\frac{290}{550} = 0{\cdot}55$ cm; we adopt 10 mm size weld.

Exercises:

1. *What general methods are used in making welded connections?*
2. *Name the principal types of welds used.*
3. *How are the stressed determined in welded connections?*
4. *What permissible stresses are used in welded connections?*
5. *Explain how you would determine the allowable loads per linear cm of triangular fillet welds*
6. *What are the advantages of welded joints?*

7. *The end reactions of a beam connected to two columns are 8,000 kg The allowable shear load per cm of fillet weld is 500 kg. What length of fillet weld is required to carry this load?* *Ans.* 16 *cm.*

8. *Two plates are joined by a parallel fillet welds. The plate is subjected to a tensile load of 35,000 kg. What length of 15 mm weld is required to resist this load? The allowable working stress in shear for the weld material is 800 kg/sq cm*

Ans 41 *cm length of weld adjusted on both sides.*

9. *Treating the weld as a line, determine the section modulus Z_w in bending of a weld d cm high.*

Ans $\frac{d^2}{6}$ cm^2.

10. *Treating the weld as a line, determine the moment of inertia J_w about a centre of gravity of a circular weld of diameter d.*

Ans. $\frac{\pi d^3}{4}$ cm^3

11. *A 5 cm diameter solid shaft is welded to a flat plate by 1 cm fillet weld. What will be the maximum torque that the welded joint can sustain if the permissible shear stress intensity in the weld material is not to exceed 800 kg/sq cm?* *Ans.* 22,200 *kg cm.*

CHAPTER **XVIII**

DESIGN OF MISCELLANEOUS MACHINE PARTS—I ENGINES AND BOILERS

18·1. Design of flat plates:

Cylinder heads, steam chest covers, piston faces, valve faces and similar machine parts subjected to pressure may be considered as flat plates. Generally, the shape is either circular or rectangular and for each of these the loading may be either concentrated or distributed. The plate may be rigidly fixed at the edges or may be merely supported there. The exact theory of flat plates is complicated and requires the knowledge of theory of elasticity, which is beyond the scope of this book. Here, we derive in a simple manner the formulas for circular and rectangular plates, subjected to a uniformly distributed load and simply supported at the outside edge. The results obtained can be successfully applied to blank flanges, cylinder heads, piston tops and steam chest covers.

In the use of these results, it should be remembered that the results give the average skin stress across what corresponds to the breadth of the plate, but they do not give the absolute maximum. The absolute maximum stress occurs at the centre of the plate and its value is about 25% more than the average

(a) Circular plate subjected to a uniformly distributed load of p and supported on the outside edge (Fig 18·1):

Let a be the radius of the circular plate Let us consider the equilibrium of half the plate The resultant of fluid pressure is of magnitude $\frac{\pi}{2} a^2 p$ and acts at a load centre G_l or centroid of the semi-circular area The load centre is at a distance of $\frac{4a}{3\pi}$ from the diameter and lies on the line of symmetry of the area. The resultant reaction $= \frac{\pi}{2} a^2 p$ acts at the reaction centre G_r or the centroid of the semi-circular arc The reaction centre is at a distance of $\frac{2a}{\pi}$ from the diameter and lies on the radius of symmetry. As two forces are equal in magnitude and opposite in direction

and manipulating the nuts and must provide a good holding for bolts and studs. The bolts and studs are required to compress the packing material with sufficient force to make the joint pressure tight. This force is indeterminate. In order to allow for this force, we design the cover and bolts by assuming that the area, over which the fluid pressure acts, extends upto the inner edge of the bolt.

The number of bolts are such that the pitch of the bolts for steam tight joints do not exceed five times the diameter of bolts. Generally, for such work bolts of less than 18 mm diameter are avoided.

Studs are preferred to bolts, for the absence of bolt-heads allows the flanges to be made narrower.

The thickness of the flange should not be less than the bolt diameter. It may be from $1 \cdot 2t$ to $1 \cdot 5t$,where t is the thickness of the cylinder cover. It is better, however, to calculate the thickness based on the assumption that the flange is a cantilever built in at the skirt with a load equal to the pull on the bolts. The arm of the cantilever is the distance of the bolt axis from the outer edge of the skirt.

If there be n bolts of root diameter d_c, subjected to a tensile stress intensity f_1, and a flange of thickness t_1, having a distance x from the bolt circle to the root of the flange, then bending moment M on the flange is given by

$$M = \frac{\pi}{4} d^2_c n f_1 x \quad \text{.......... (x)}$$

If f_2 be the bending stress allowable for the flange and D_1 the outer diameter of the skirt, then the resisting moment M_1 is given by

$$M_1 = \frac{\pi D_1 t_1^2 f_2}{6} \quad \text{.......... (xi)}$$

By equating the bending moment to resisting moment, we get

$$\frac{\pi}{4} d_c^2 n f_1 x = \frac{\pi D_1}{6} t_1^2 f_2$$

or

$$t_1 = d_c \sqrt{\frac{3 n x f_1}{2 D_1 f_2}} \quad \text{.......... (xii)}$$

Examples:

1. *The piston of a gas engine is 150 mm diameter and is made of cast iron. The maximum pressure acting on it is 30 kg/sq cm. Since the*

The resultant of the fluid pressure $= \frac{pab}{2}$ acts at the load centre, which is at a distance $\frac{H}{3}$ from the diagonal, while the reaction force acts at the reaction centre, which is at a distance $\frac{H}{2}$ from the diagonal. As two equal and opposite forces do not act along the same line of action, they give rise to a couple, the arm of the couple being $\left[\frac{H}{2} - \frac{H}{3}\right] = \frac{H}{6} = \frac{ab}{6\sqrt{a^2+b^2}}$.

The moment of the couple $= \frac{pab}{2} \times \frac{ab}{6\sqrt{a^2+b^2}}$

$$= \frac{p}{12} \frac{a^2b^2}{\sqrt{a^2+b^2}} \qquad \text{(v)}$$

If t be the thickness of the plate and f the permissible stress, then by equating the resisting moment to the bending moment, we get $\frac{1}{6} f t^2 \sqrt{a^2+b^2} = \frac{p}{12} \frac{a^2b^2}{\sqrt{a^2+b^2}}$

or
$$t = ab\sqrt{\frac{p}{2f} \times \frac{1}{(a^2+b^2)}} \qquad \text{(vi)}$$

(c) Circular plate subjected to a uniformly distributed load p and clamped on the outside edge:

It can be proved from classical theory of plates that the maximum stress occurs at the edge of the plate and is equal to

$$f = \frac{3}{4} p \frac{a^2}{t^2} \qquad \text{(vii)}$$

or
$$t = a\sqrt{\frac{3}{4} \times \frac{p}{f}} \qquad \text{(viii)}$$

If D be the diameter of the plate, the above formula will be

$$t = D\sqrt{\frac{3}{16} \times \frac{p}{f}} \qquad \text{(ix)}$$

The above equations do not apply to flat plates which are ribbed.

The above theory can be applied for the plate, which forms the flanged joint. Steam engine chests and pipe flanges must provide a true or rigid surface for connection to mating parts, must be strong enough to resist the bending stresses set up when the bolts are tightened up, must be large enough to allow room for seating

9. How do you determine the height of the cylinder?
10. What are the arrangements to be provided for draining the cylinder?
11. How do you propose to relieve the pressure from the cylinder?
12. How do you propose to fix the cylinder to the foundation?
13. What will be the thickness of the cylinder flange?
14. What will be the shape of the flange? Why do you prefer such a shape?
15. What are the machining operations to be carried out on the cylinder of the press? How do you propose to carry them out?
16. What will be the suitable material for the ram? What are your specifications for the material selected?
17. How do you fix the size of the ram? Do you propose to make it solid or hollow?
18. What will be the maximum stress in the ram? Which are the points where these stresses occur?
19. How do you propose to prevent the leakage of the fluid from the cylinder?
20. Which kind of sealing material is generally used for systems operating at higher pressures?
21. Make a sketch of the various shapes of sealing devices which are normally used with hydraulic systems.
22. What will be the working principles of these sealing devices?
23. How are the sealing devices manufactured?
24. Will the seal be a static seal or a dynamical seal?
25. How will you select a proper hydraulic seal?
26. What are your recommendations for the material of the gland?
27. What will be the shape of the gland? What are the machining operations to be carried out on the gland? How do you propose to carry them out?
28. What do you understand by the term friction factor? How will you calculate the total load on the gland studs?
29. How will you fix the number and size of the studs?
30. What are your recommendations for the material of the studs?
31. How will you fix the length of the studs?

skirt of the piston is quite thin and therefore flexible, the piston face may be considered as a flat plate merely supported at the edges. Determine the thickness for the face if maximum tensile stress of 300 kg/sq cm be permitted.

With usual notations, the thickness of the flat circular plate supported at the edges and subjected to uniformly distributed load is given by

$$t = a\sqrt{\frac{p}{f}}.$$

On substitution of values, we get

$$t = \frac{15}{2}\sqrt{\frac{30}{300}} = 2.37 \text{ cm, we adopt } 2{\cdot}4 \text{ cm.}$$

2. *A steam chest cover 30 cm long by 22 cm wide sustains a steam pressure of 9 kg/sq cm. Assume the cover to be merely supported at the edges, determine the thickness for a maximum stress of 350 kg/sq cm in tension.*

With usual notations, the thickness of the steam chest cover is given by

$$t = ab\sqrt{\frac{p}{2f} \times \frac{1}{a^2+b^2}}.$$

On substitution of values, we get

$$t = 30 \times 22\sqrt{\frac{9}{2\times350} \times \frac{1}{30^2+22^2}} = 2{\cdot}1 \text{ cm; we adopt } 2{\cdot}2 \text{ cm.}$$

3. *A cast iron blank flange is required for a pipe 30 cm diameter carrying steam at a pressure of 7 kg/sq cm. Assuming the flange to be fixed by studs and nuts, design*

(i) *the diameter of the steam pipe,*

(ii) *the size and number of studs required,*

(iii) *the size and thickness of blank cover flange,*

(iv) *the thickness of the flange at the opening of the pipe.*

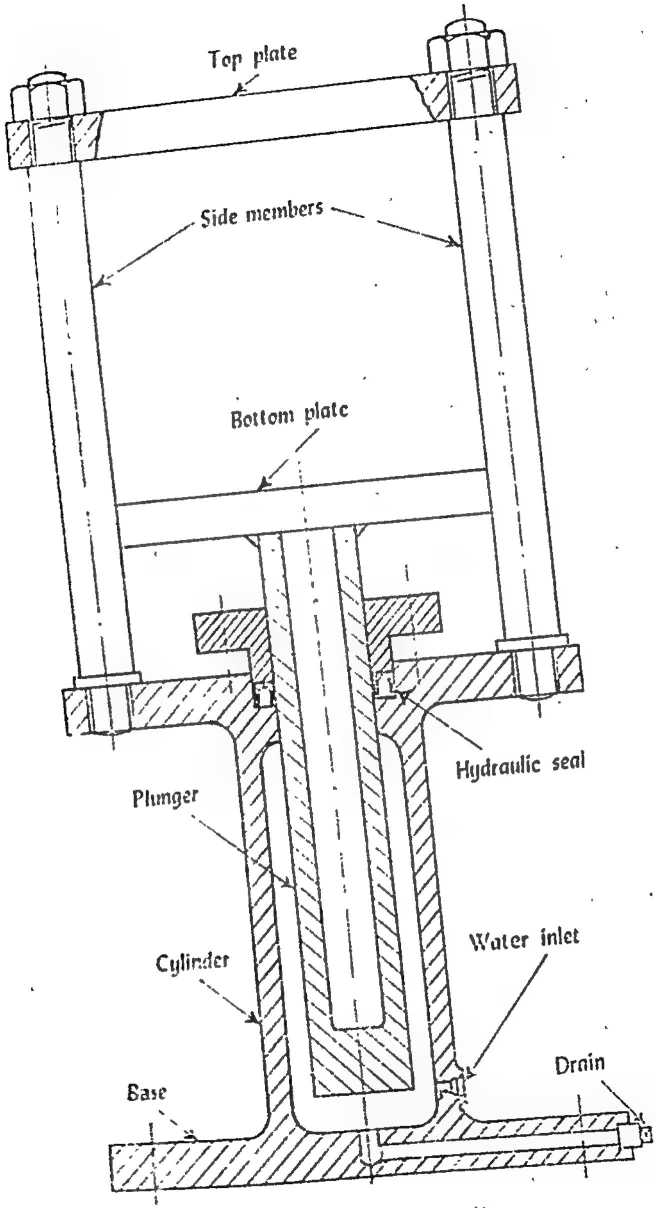
Use your own values for the stresses.

Let us assume permissible stress for cast iron to be 150 kg/sq cm. The thickness of the cast iron steam pipe is given by

$t = \frac{pD}{2f} + c$ cm, where the value of "c" can be taken as 0·9 cm. (Please refer page 125.)

On substitution of values, we get

$$t = \frac{7 \times 30}{2 \times 150} + 0{\cdot}9 = 1{\cdot}6 \text{ cm.}$$



Hydraulic Press

FIG. 18-3

32. How do you propose to fix the platform to the ram?
33. What will be the suitable material for the platform?
34. How will you fix the size of the platform?
35. How do you propose to guide the platform?
36. What will be the suitable material for the side members?
37. How will you fix the size and the length of the side members and their numbers?
38. What are the nature of threads on the gland studs and the side members?
39. What is the nature of load coming on side members when the press is working? When the press is not working?
40. How will you keep the top plate in proper position on the side members?
41. What are the operations to be carried out on the side members?
42. What are your recommendations for the material of the top plate?
43. How will you fix the proportions for the top plate?
44. What will be the approximate weight of each component of the press?
45. What will be the approximate weight of the press?
46. Write short notes describing the procedure for assembly of the press.

The above list is not exhaustive. It will serve as a guide for preparing a list for designing various components

Material Selection

The selection of the material specifically suited for a machine component is a most important task of a designer. The following group of properties are to be considered by the designer:

(i) Mechanical properties such as yield strength, modulus of elasticity, percentage elongation, endurance strength, and ultimate strength.

(ii) Physical properties such as thermal expansion, creep rate, thermal conductivity, shrinkage rate and age hardening.

(iii) Technological properties such as castability, forgeability, machinability, weldability, formability, etc.

$$\text{Thickness of the top of the ram} = 10\sqrt{\frac{160}{900}}$$
$$= 4{\cdot}2 \text{ cm.}$$

The ram is subjected to three principal stresses.

(i) Direct compressive stress due to fluid pressure

(ii) Tangential or hoop stress which is due to fluid pressure and is maximum at the inner radius and the fluid pressure intensity of $p = 160$ kg/sq cm is acting on the outer periphery

(iii) Radial stress which is compressive in nature and is 160 kg/sq cm at the outer periphery and zero at the inner periphery.

We determine the inner diameter of the ram, considering it to be a thick cylinder subjected to an external fluid pressure. It is assumed that the cylinder is made of ductile material for which theory of maximum shear stress is the design criterion. The design equation for a single thick cylinder will be

$$\frac{2p}{f} + \frac{1}{k^2} = 1 \quad \text{(Refer art. 3-7 on page 137.)}$$

In the above equation p is the pressure of the fluid acting on the side of the ram. f is the permissible stress intensity and k is the ratio of the outer diameter of the ram to the inner diameter of the ram. We assume that the permissible compressive stress intensity for the material of the ram is 900 kg/sq cm. We neglect the column action.

$$\therefore \qquad \frac{2 \times 160}{900} + \frac{1}{k^2} = 1$$

or

$$k = \sqrt{\frac{1}{1 - \frac{2 \times 160}{900}}} = 1{\cdot}25.$$

$$\therefore \quad \text{Inner diameter of the ram} = \frac{200}{1{\cdot}25} = 160 \text{ mm.}$$

Packing for Hydraulic rams:

The success of applying fluid power to any application depends largely on the ability of the sealing devices to prevent both internal and external leakages in the system. Losses of fluid from hydraulic machinery caused by leakage can be very costly not only in terms of the cost of the fluid but also in production.

(iv) Chemical properties such as material's microstructure, the practicability of surface treatment and paint adhesion.

The designer must select a material strong enough to transmit the loads economically, tough enough to withstand environmental changes, soft enough to be machinable and inert enough to resist corrosion.

If the designer can answer the following questions affirmatively he has probably made the appropriate material selection:

1. Will the material satisfy the strength and rigidity requirements?
2. Can the material be made into parts?
3. Will the part, made of this material, be able to perform its task during the expected life in the environment in which it may have to function?
4. Will the material be available in sufficient quantity when needed?
5. Is the cost of making the part from this material as low as that of any other material which also meets the requirements?

Design of a ram

The ram has to develop an effective load of 40,000 kg on the bale. The ram reciprocates in the cylinder of a press. Hence there is a sliding friction. In order to overcome the frictional resistance, more force must be exerted on the top of the ram by the fluid. We assume the friction factor to be 1·2. Hence the fluid pressure load on the top of the ram will be $1{\cdot}2 \times 40000 = 48{,}000$ kg

Let D cm be the outer diameter of the ram or plunger which we propose to make hollow

$$\therefore \quad \frac{\pi}{4} D^2 \times 160 = 48000$$

or
$$D = \sqrt{\frac{48000}{160} \times \frac{4}{\pi}}$$

$$= 19{\cdot}55 \text{ cm; we adopt } 20 \text{ cm.}$$

The ram is made hollow to reduce the weight. The material of the ram is taken as steel for which permissible tensile stress intensity may be taken as 900 kg/sq cm

Let us determine the thickness of the top of the plunger. We consider it as a flat plate supported at the periphery and loaded with uniformly distributed load of pressure intensity 160 kg/sq cm.

$$\text{Thickness of the top of the ram} = 10\sqrt{\frac{160}{900}}$$
$$= 4{\cdot}2 \text{ cm.}$$

The ram is subjected to three principal stresses.

(i) Direct compressive stress due to fluid pressure

(ii) Tangential or hoop stress which is due to fluid pressure and is maximum at the inner radius and the fluid pressure intensity of $p = 160$ kg/sq cm is acting on the outer periphery

(iii) Radial stress which is compressive in nature and is 160 kg/sq cm at the outer periphery and zero at the inner periphery.

We determine the inner diameter of the ram, considering it to be a thick cylinder subjected to an external fluid pressure. It is assumed that the cylinder is made of ductile material for which theory of maximum shear stress is the design criterion. The design equation for a single thick cylinder will be

$$\frac{2p}{f} + \frac{1}{k^2} = 1 \text{ (Refer art. 3-7 on page 137.)}$$

In the above equation p is the pressure of the fluid acting on the side of the ram, f is the permissible stress intensity and k is the ratio of the outer diameter of the ram to the inner diameter of the ram. We assume that the permissible compressive stress intensity for the material of the ram is 900 kg/sq cm. We neglect the column action.

$$\therefore \qquad \frac{2 \times 160}{900} + \frac{1}{k^2} = 1$$

or
$$k = \sqrt{\frac{1}{1 - \frac{2 \times 160}{900}}} = 1{\cdot}25.$$

$$\therefore \text{ Inner diameter of the ram} = \frac{200}{1{\cdot}25} = 160 \text{ mm.}$$

Packing for Hydraulic rams:

The success of applying fluid power to any application depends largely on the ability of the sealing devices to prevent both internal and external leakages in the system. Losses of fluid from hydraulic machinery caused by leakage can be very costly not only in terms of the cost of the fluid but also in production.

Modern fluid power systems depend more and more on the development of sealing materials that will operate with special fluids and at conditions of high pressures and temperatures. The manufacturers of sealing devices have kept pace with the needs of all industry through research and development programmes. The recommendations of manufacturers in sealing devices are always important in any application

Each application for a seal must be treated as a specific problem because of slight variations in minor conditions of operation. The following are the important factors to be considered *while selecting a seal*·

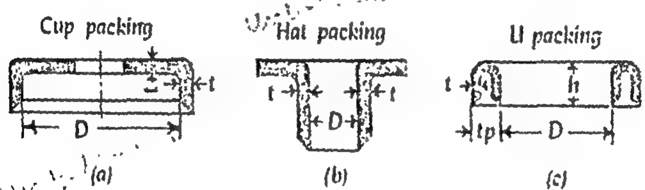
1. Surface speed in combination with size of the member to be sealed
2. Pressure in the system in contact with a sealing device
3. Temperature at the point of seal
4. Compatibility between sealing devices
5. Machine operating conditions such as fast reversals, intermittent operation, etc

The designer who specifies the type of sealing device for any given application should study all of the problems very carefully consulting the suppliers at every stage so that the proper final selection is made

Sealing materials may be classed into three general categories: leather, fabricated rubber and homogeneous

Leather is the oldest material used for sealing devices and is still popular for many applications Leather seals have low frictional properties and relatively high tensile strength which resists extrusion into clearance spaces Thus leather is used for applications where higher system operating pressures are used. Leather does not score metal surfaces. Leather sealing devices are manufactured in "U", cup and hat or flange shapes as shown in fig 18-4. The U leather packing has been used in the press considered here The water leaks past the ram as far as the packing, and entering its interior, presses one side against the recess in the cylinder and the other against the ram. The greater the pressure of the water the greater is the tendency to leak but in the U leather packing the force with which the leather is pressed against the ram and

against the recess is proportional to the pressure of the water. This is one of the great merit of the U leather packing. *This is called a pressure actuated seal that depends on the hydraulic pressure in the system to act against the lip to create contact pressure for sealing.* In order for the lip to move freely, the groove space should be slightly greater than the seal. This type of seal is more effective when the groove is properly proportioned to allow lip flexing with little solid compression. When used at very high operating pressures over 30 kg/sq cm, it is advantageous to use a fabricated back up washer to prevent extrusion of material into the clearance space. The U seal is considered a balanced type of seal. The U leather packing is made from a disc or flat ring of leather, which is moulded between two cast iron blocks. The leather is softened in hot water and placed between the blocks which are then pressed together in a hydraulic press or by bolts and nuts, bolts passing through the blocks. The leather is kept in the mould for about twenty four hours, when it is removed, and after it is dried, it is trimmed to the form as shown in fig. 18-4(*c*).



Leather Packings
Fig. 18-4

An objection to the U leather packing is that the ram must be removed before the packing can be renewed and this, in many cases, is very troublesome. In such cases the ordinary stuffing box is generally used.

The flange seal is a pressure actuated seal that effects sealing through the hydraulic pressure. These types of seals are generally applied in the lower pressure ranges. The flange seal is considered an unbalanced seal.

Cup seals are primarily used for sealing cylinder pistons. This seal is also pressure actuated, with the sealing accomplished by the contact pressure outward against the cylinder bore. It is essential that these seals be clamped in place and the method of

Modern fluid power systems depend more and more on the development of sealing materials that will operate with special fluids and at conditions of high pressures and temperatures. The manufacturers of sealing devices have kept pace with the needs of all industry through research and development programmes. The recommendations of manufacturers in sealing devices are always important in any application.

Each application for a seal must be treated as a specific problem because of slight variations in minor conditions of operation. The following are the important factors to be considered while selecting a seal.

1. Surface speed in combination with size of the member to be sealed
2. Pressure in the system in contact with a sealing device
3. Temperature at the point of seal
4. Compatibility between sealing devices
5. Machine operating conditions such as fast reversals, intermittent operation, etc.

The designer who specifies the type of sealing device for any given application should study all of the problems very carefully consulting the suppliers at every stage so that the proper final selection is made

Sealing materials may be classed into three general categories: leather, fabricated rubber and homogeneous

Leather is the oldest material used for sealing devices and is still popular for many applications. Leather seals have low frictional properties and relatively high tensile strength which resists extrusion into clearance spaces Thus leather is used for applications where higher system operating pressures are used Leather does not score metal surfaces. Leather sealing devices are manufactured in "U", cup and hat or flange shapes as shown in fig 18-4 The U leather packing has been used in the press considered here. The water leaks past the ram as far as the packing, and entering its interior, presses one side against the recess in the cylinder and the other against the ram. The greater the pressure of the water the greater is the tendency to leak but in the U leather packing the force with which the leather is pressed against the ram and

μ = Coefficient of friction

p = fluid pressure

h = height of the U packing and

d = diameter of the ram.

On substitution of values we get, the frictional resistance as $\pi \times 20 \times \frac{4}{2} \times 160 \times 0{\cdot}1 = 2{,}000$ kg.

In the above the coefficient of friction is taken as 0·1. The another way to consider the friction at the packing is to consider the friction factor whose value ranges from 1·03 to 1·07 and to multiply the fluid pressure on the gland by this friction factor.

Total load on the gland studs = 34600 + 2000 = 36,600 kg. Load on each stud, assuming that all the studs share the load equally, will be $\frac{36{,}600}{4} = 9{,}150$ kg.

Assuming the permissible stress intensity for the stud material to be 900 kg/sq cm, the core area of the stud will be $\frac{9150}{900} = 10{\cdot}17$ sq cm. We assume coarse threads. From table we adopt M45 studs.

Outer diameter of the ram = 20 cm

Outer diameter of the gland = 26 cm

Diameter of the stud = 4·5 cm.

We adopt 32 cm as the pitch circle diameter of the circular flange of the gland. In order to determine the thickness of the circular flange of the gland, we consider the flange to be a cantilever fixed to the outer diameter of the gland and loaded by a load of 4 studs, which is 36,600 kg. The length of the cantilever ram = $\frac{1}{2}(32 - 26) = 3$ cm.

Bending moment = 36600 × 3 = 109,800 kg cm.

Modulus of section $= \frac{109800}{900} = 122$ cm³.

If t be the thickness of the flange, then

$\frac{1}{6} t^2 \times \pi \times 26 = 122$

$$\text{or} \quad t = \sqrt{\frac{122 \times 6}{26 \times \pi}} = 3 \text{ cm.}$$

clamping is important. It is important to note that the cup lip be no higher than is necessary to follow the side play. This type of seal is considered unbalanced seal and its operating pressure depends on the materials and conditions under which it operates.

Another type of seal known as *O* rings have been shown in fig 3-5 on page 135, where they form static seals between cylinder tube and its covers and dynamic seals between piston head and cylinder and piston rod and cover.

We adopt U packing, the dimensions of the packing being as under:

$t = 6$ mm

$l_p = 30$ mm

and $h = 40$ mm

Gland:

It will be of cast iron. The thickness of the gland is given by the empirical rule $t = \sqrt{\frac{d}{2.5}}$ to $1.25\sqrt{\frac{d}{2.5}}$

where d is the diameter of the plunger in cm. Hence the thickness of the gland varies from $t = \sqrt{\frac{20}{2.5}}$ to $1.25\sqrt{\frac{20}{2.5}}$

i.e. 2·82 cm to 3·52 cm. The thickness of the gland should be equal to l_p of the U leather packing.

We adopt the thickness of the gland as 3 cm and hence the outer diameter of the gland will be $20+2 \times 3 = 26$ cm. Outer diameter of the gland will be equal to diameter of the ram plus twice l_p of the U packing which amounts to $20+2 \times 3 = 26$ cm. Let us consider the load on the gland studs. We adopt 4 studs.

The load on these studs will consist of

(i) Due to fluid pressure and

(ii) Due to friction at the U packing

Fluid pressure: $\frac{\pi}{4} \times (26^2 - 20^2) \times 160 = 34{,}600$ kg

In order to determine the friction resistance for U packing we assume that half the height is effective. Friction resistance for U packing will be $\pi\, d\, p\, h\, \frac{\mu}{2}$ where

μ = Coefficient of friction

p = fluid pressure

h = height of the U packing and

d = diameter of the ram.

On substitution of values we get, the frictional resistance as $\pi \times 20 \times \frac{4}{2} \times 160 \times 0{\cdot}1 = 2{,}000$ kg.

In the above the coefficient of friction is taken as 0·1. The another way to consider the friction at the packing is to consider the friction factor whose value ranges from 1·03 to 1·07 and to multiply the fluid pressure on the gland by this friction factor.

Total load on the gland studs = 34600 + 2000 = 36,600 kg. Load on each stud, assuming that all the studs share the load equally, will be $\frac{36{,}600}{4} = 9{,}150$ kg.

Assuming the permissible stress intensity for the stud material to be 900 kg/sq cm, the core area of the stud will be $\frac{9150}{900} = 10{\cdot}17$ sq cm. We assume coarse threads. From table we adopt M45 studs.

Outer diameter of the ram = 20 cm

Outer diameter of the gland = 26 cm

Diameter of the stud = 4·5 cm.

We adopt 32 cm as the pitch circle diameter of the circular flange of the gland. In order to determine the thickness of the circular flange of the gland, we consider the flange to be a cantilever fixed to the outer diameter of the gland and loaded by a load of 4 studs, which is 36,600 kg. The length of the cantilever ram = $\frac{1}{2}(32 - 26) = 3$ cm.

Bending moment = 36600 × 3 = 109,800 kg cm.

Modulus of section = $\frac{109800}{900} = 122$ cm³.

If t be the thickness of the flange, then

$\frac{1}{6} t^2 \times \pi \times 26 = 122$

$$\text{or} \quad t = \sqrt{\frac{122 \times 6}{26 \times \pi}} = 3 \text{ cm.}$$

We adopt 5 cm as the thickness of the flange. It has been observed in practice that the thickness of the gland flange is usually from $1\frac{1}{4}$ to $1\frac{1}{2}$ diameter of the stud. Outer diameter of the flange will be $32 + 10$ (more than twice the diameter of the stud) $= 42$ cm.

Cylinder:

We design the cylinder as a thick cylinder. The cylinder will be in contact with the ram only in the vicinity of the gland, so that elsewhere there will be an annular water space between the ram and cylinder. Assuming that this space is to be 3 cm wide the internal diameter of the cylinder will be $20 + 2 \times 3 = 26$ cm. We adopt the permissible stress intensity for the cylinder as 1,000 kg/sq cm. If t be the thickness of the cylinder, then, according to Lamè's equation we get, with usual notations,

$$t = \frac{D}{2}\left[\sqrt{\frac{f+p}{f-p}} - 1\right]$$

$$= \frac{26}{2}\left[\sqrt{\frac{1000+160}{1000-160}} - 1\right] = 2\text{·}21 \text{ cm.}$$

Since the cylinder will be rough, both inside and outside, the nominal wall thickness will be 2·5 cm so that the outside diameter becomes $26 + 2 \times 2\text{·}5 = 31$ cm

The length of the cylinder must be sufficient to contain the ram and make provision for the gland and packing. Fig 18-3 shows the other end of the cylinder, where the arrangement for admitting high pressure water is shown. The arrangement for draining the water from the cylinder is also shown in the figure

Pillars:

The function of the pillars is to support the top plate when the material is not being pressed. It also guides the bottom plate, which is welded to the ram. The bottom plate can also be kept free with the ram. The constrain will be maintained We adopt 4 pillars. When the material is being pressed, the pillars will be in direct tension.

$$\text{Load per pillar} = \frac{48000}{4} = 12{,}000 \text{ kg.}$$

μ = Coefficient of friction

p = fluid pressure

h = height of the U packing and

d = diameter of the ram.

On substitution of values we get, the frictional resistance as $\pi \times 20 \times \frac{4}{2} \times 160 \times 0{\cdot}1 = 2{,}000$ kg.

In the above the coefficient of friction is taken as 0·1. The another way to consider the friction at the packing is to consider the friction factor whose value ranges from 1·03 to 1·07 and to multiply the fluid pressure on the gland by this friction factor.

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Bending moment = $36600 \times 3 = 109{,}800$ kg cm.

Modulus of section = $\frac{109800}{900} = 122\ \text{cm}^3$.

If t be the thickness of the flange, then

$\frac{1}{6} t^2 \times \pi \times 26 = 122$

$$\text{or} \quad t = \sqrt{\frac{122 \times 6}{26 \times \pi}} = 3 \text{ cm}.$$

We adopt 5 cm as the thickness of the flange. It has been observed in practice that the thickness of the gland flange is usually from $1\frac{1}{4}$ to $1\frac{1}{2}$ diameter of the stud. Outer diameter of the flange will be 32 + 10 (more than twice the diameter of the stud) = 42 cm.

Cylinder:

We design the cylinder as a thick cylinder. The cylinder will be in contact with the ram only in the vicinity of the gland, so that elesewhere there will be an annular water space between the ram and cylinder. Assuming that this space is to be 3 cm wide the internal diameter of the cylinder will be 20 + 2 × 3 = 26 cm. We adopt the permissible stress intensity for the cylinder as 1,000 kg/sq cm. If t be the thickness of the cylinder, then, according to Lamè's equation we get, with usual notations,

$$t = \frac{D}{2}\left[\sqrt{\frac{f+p}{f-p}} - 1\right]$$

$$= \frac{26}{2}\left[\sqrt{\frac{1000+160}{1000-160}} - 1\right] = 2{\cdot}21 \text{ cm}$$

Since the cylinder will be rough, both inside and outside, the nominal wall thickness will be 2·5 cm so that the outside diameter becomes 26 + 2 × 2·5 = 31 cm.

The length of the cylinder must be sufficient to contain the ram and make provision for the gland and packing. Fig. 18-3 shows the other end of the cylinder, where the arrangement for admitting high pressure water is shown. The arrangement for draining the water from the cylinder is also shown in the figure.

Pillars:

The function of the pillars is to support the top plate when the material is not being pressed. It also guides the bottom plate, which is welded to the ram. The bottom plate can also be kept free with the ram. The constrain will be maintained. We adopt 4 pillars. When the material is being pressed,the pillars will be in direct tension.

$$\text{Load per pillar} = \frac{48000}{4} = 12{,}000 \text{ kg.}$$

If d_c cm be the core diameter of the threaded portion of the pillar then,

$$\frac{\pi}{4} d_c^2 \times 900 = 12000$$

or $$d_c = \sqrt{\frac{12000 \times 4}{900 \times \pi}} = 4{\cdot}12 \text{ cm.}$$

As the press must be stable and rigid enough, we adopt 60 mm as the diameter of the pillar and a collar of diameter 70 mm is provided on the pillar to take its seat on the flange of the cylinder as shown in Fig. 18-3. The length of the threaded portion of the pillar at the cylinder end may be taken as 120 mm as cast steel acts as a nut and mild steel as the threaded portion. At the top portion of the pillar mild steel nuts can be used and hence the length of the threaded portion can be reduced.

Cylinder flange:

We have to press the largest material of the size 60 cm × 60 cm × 60 cm. The diameter of the pillar is 6 cm. Hence the centre to centre distance between pillars (consecutive) 75 to 80 cm. Let us adopt 75 cm as the distance between the pillar centres.

Let us consider the flange as a cantilever connected to the cylinder at the outer diameter. The arm of the cantilever will be $\frac{75 - 31}{2} = 22$ cm.

Maximum bending moment on the flange where it joins the cylinder is $22 \times 48000 = 11{,}00{,}000$ kg cm.

If t cm be the thickness of the flange, then by equating the moment of resistance to applied bending moment, we get

$$\tfrac{1}{6} \times t^2 \times \pi \times 31 \times 1000 = 1100000$$

or $$t = \sqrt{\frac{1100000 \times 6}{31 \times 1000 \times \pi}} = 8{\cdot}05 \text{ cm.}$$

We adopt 9 cm as the minimum thickness of the flange where it joins the cylinder. The top circular flange can be modified to reduce the weight of the cylinder.

Lower moveable table:

The table can be considered as a square plate loaded uniformly and supported at the centre on a circle of diameter equal to the diameter of the ram, which is 20 cm. It is not easy to obtain the

We adopt 3 cm as the thickness of the flange. It has been observed in practice that the thickness of the gland flange is usually from $1\frac{1}{4}$ to $1\frac{1}{2}$ diameter of the stud. Outer diameter of the flange will be $32 + 10$ (more than twice the diameter of the stud) $= 42$ cm.

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Since the cylinder will be rough, both inside and outside, the nominal wall thickness will be 2·5 cm so that the outside diameter becomes $26 + 2 \times 2{\cdot}5 = 31$ cm

The length of the cylinder must be sufficient to contain the ram and make provision for the gland and packing Fig 18-3 shows the other end of the cylinder, where the arrangement for admitting high pressure water is shown. The arrangement for draining the water from the cylinder is also shown in the figure.

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$$\text{Load per pillar} = \frac{48000}{4} = 12{,}000 \text{ kg.}$$

$$p = \frac{48000}{75 \times 75} = 8{\cdot}5 \text{ kg/sq cm.}$$

The thickness in cm of the plate is given by

$$t = 0{\cdot}92 \times 75 \sqrt{\frac{8{\cdot}5}{1000}} = 6{\cdot}4 \text{ cm.}$$

We adopt 65 mm as the thickness of the upper table.

Thus we have attempted to fix the main dimensions of the hydraulic press from strength view point. These dimensions will be modified in the design office on the drawing table to suit other requirements such as manufacture, assembly, etc. The limitation of space prevents the consideration of all such factors at this stage.

The water for working the hydraulic press will be supplied by an accumulator which is a container in which fluid is stored under pressure as a source of fluid power. There are two general types of accumulators; the hydropneumatic and the mechanical designs. In the hydropneumatic design the compressed gas is used to apply force to the stored liquid. Mechanical designs use a weighted member or spring which applies force to the stored liquid.

The flow of fluid to the cylinder of the press from the accumulator and from the cylinder to the waste is controlled by a hand operated valve.

Project Exercises:

1. *Design a hydraulic press for a repair shop or a small factory. A capacity of 30 tonnes is to be produced by hydraulic ram and hand pump with a pressure gauge. Press is to have a welded structural frame. Press bed is to be about 30 cm by 100 cm and is to be provided with rails for rolling out from under the ram for loading and unloading. A mean is to be provided for adjusting the height of the upper crosshead. The design is to be arranged for maximum utility and economy of construction.*

2. *Design a pipe bender for field use. A straight section of pipe is placed across two pivoted concave supports. Mid-way between a 90° concave bending a shoe is pressed into the pipe until the pipe assumes the curvature of the shoe. The necessary force is supplied by a hydraulic cylinder and hand pump. Shoes and a simple but rugged frame are required that can handle all diameters of pipe between 15 mm to 50 mm.*

3. *A hydraulic testing machine has a maximum capacity of 100 tonnes. The piston diameter is 25 cm. Calculate the thickness of the*

simple formula for maximum stress in such a plate from the standard books. We may be able to obtain the result from the research work. *Hence we shall make the nearest approach to the problem if we neglect the corners of the lower table and assume that the plate is circular of diameter 75 cm.*

According to Grashof the thickness of the circular plate subjected to uniformly distributed pressure intensity p and supported at the centre of circle of diameter d is given by

$$t = a\sqrt{\frac{p}{f}\left[2 \log_e \frac{2a}{d} + \frac{1}{4}\right]}$$

where a is the radius of the circular plate and f is the permissible stress intensity for the material of the plate. First of all let us calculate the intensity of uniformly distributed pressure on the plate, which is due to the pressing of the material. The load of 48,000 kg is supported on the circular area of diameter 75 cm.

$$\text{Hence } p = \frac{48000}{\frac{\pi}{4} \times 75^2} = 11{\cdot}1 \text{ kg/sq cm.}$$

On substitution of values we get

$$t = 37{\cdot}5\sqrt{\frac{11{\cdot}1}{1000}\left[2 \log_e \frac{75}{20} + 0{\cdot}25\right]}$$
$$= 11{\cdot}5 \text{ cm}$$

A 120 mm thick slab if available or nearest higher thickness is adopted and welded to the plunger or ram

Upper table or Bridge piece:

The strength of the upper table or bridge piece can be investigated by treating it as a flat square plate simply supported at four corners and having a central uniformly distributed load spread over a square area of the side equal to the side of the lower movable table. (The theory of such a plate is given on page 220 of second edition of the book "Theory of Plates and Shells" by S. Timoshenko and S. Woinowsky Krieger) The maximum stress will be at the edges For a material having a value of Poisson's ratio as 0·3, the thickness of the plate is obtained as $t = 0{\cdot}92\, c\sqrt{\frac{p}{f}}$ where c is the side of the square and p is the intensity of pressure.

$$p = \frac{48000}{75 \times 75} = 8{\cdot}5 \text{ kg/sq cm.}$$

The thickness in cm of the plate is given by

$$t = 0{\cdot}92 \times 75 \sqrt{\frac{8{\cdot}5}{1000}} = 6{\cdot}4 \text{ cm.}$$

We adopt 65 mm as the thickness of the upper table.

Thus we have attempted to fix the main dimensions of the hydraulic press from strength view point. These dimensions will be modified in the design office on the drawing table to suit other requirements such as manufacture, assembly, etc. The limitation of space prevents the consideration of all such factors at this stage.

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3. A hydraulic testing machine has a maximum capacity of 100 tonnes. The piston diameter is 25 cm. Calculate the thickness of the

cylinder. Also design the necessary gland, gland bolts and hydraulic seal for the cylinder. Give a neat sketch of the cylinder gland in position.

4. Fig. 3-7 shows a line diagram of a certain power circuit. Motor drives the pump, and maintains a pressure of 40 kg/sq cm in the spherical pressure tank A of diameter 50 cm.

(a) Find the thickness of the spherical tank if f_t = 500 kg/sq cm and efficiency of the joints is 90%. Give a free hand sketch of the joint.

(b) Design the hydraulic work cylinder B completely i.e. its thickness, glands, packings, end covers, piston and piston rod and give a dimensioned sketch of the cylinder assembly. Friction losses at stuffing box and gland = 10% of F, which is 4,000 kg.

Stroke of the piston = 300 mm. Length of the piston rod = 400 mm.
U packing proportions:

$$t = 0{\cdot}3 \times d^{0{\cdot}2};\ t_p = 4\,t \text{ and } h = 1{\cdot}2t_p \text{ to } 1{\cdot}8\,t_p$$

where h, t and t_p are shown in fig. 18-4(c), while d is the diameter of the piston rod in mm.

Assume your own values for the stresses.

18-2. Design of a piston for I.C. engines:

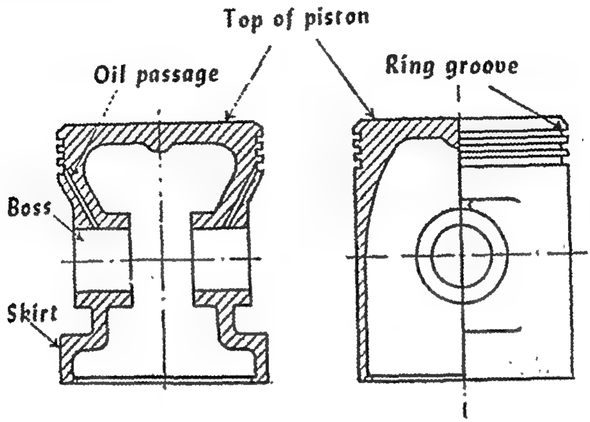
The piston is a disc which reciprocates within a cylinder and is either moved by or moves the fluid which enters the cylinder. The piston of an I.C engine receives the impulse from the expanding gas and transmits the energy through the connecting rod to the crank.

The following points are to be considered in the design of pistons for I.C. engines:

(i) Strength to resist the gas and inertia forces
(ii) Dispersion of the heat of combustion
(iii) Gas and oil sealing of the cylinder
(iv) Bearing area sufficient to prevent undue wear
(v) Minimum weight
(vi) Noiseless operation
(vii) Resistance to mechanical and thermal distortion
(viii) Adequate support for piston pin.

Trunk pistons are used for I.C engines. The main constituents of trunk piston are: (i) head to withstand the pressure of the gas, (ii) skirt to act as a bearing for the side thrust, (iii) gudgeon

pin or wrist pin to connect the piston to the connecting rod and (iv) piston rings to prevent the leakage of the gas past piston. Fig. 18-5 shows the piston for an I.C. engine.



Internal combustion engine trunk piston

FIG. 18-5

Piston Materials:

Materials used in pistons are, for the most parts, either aluminium alloy or some form of cast iron. Various alloys of cast iron are used including the special form *Meehanite.* A few engines use malleable cast iron. Pin carrier inserts are generally cast iron but sometimes are made of heat treated steel forgings. A few large assembled pistons have a separate crown made of either cast steel or a steel forging.

The coefficient of expansion, the increase in size per degree of temperature increase of aluminium is approximately twice that of cast iron. This fact must be taken into account when determining minimum piston clearance.

The heat conductivity, the rate of heat flow, of aluminium is approximately three times that of cast iron. The result is that an aluminium piston has less variation in temperature from top to bottom.

The density of cast iron is three times as much as aluminium. This does not mean that an aluminium piston weighs only a third as much as a cast iron piston because strength and heat transfer problems dictate that the metal sections of an aluminium pistons be made proportionately thicker.

cylinder. Also design the necessary gland, gland bolts and hydraulic seal for the cylinder. Give a neat sketch of the cylinder gland in position.

4. Fig. 3-7 shows a line diagram of a certain power circuit. Motor drives the pump, and maintains a pressure of 40 kg/sq cm in the spherical pressure tank A of diameter 50 cm.

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Stroke of the piston = 300 mm. Length of the piston rod = 400 mm. U packing proportions :

$$t = 0{\cdot}3 \times d^{0{\cdot}5};\ t_p = 4\,t \text{ and } h = 1{\cdot}2t_p \text{ to } 1{\cdot}8\,t_p$$

where h, t and t_p are shown in fig. 18-4(c), while d is the diameter of the piston rod in mm.

Assume your own values for the stresses.

18-2. Design of a piston for I.C. engines:

The piston is a disc which reciprocates within a cylinder and is either moved by or moves the fluid which enters the cylinder. The piston of an I.C engine receives the impulse from the expanding gas and transmits the energy through the connecting rod to the crank.

The following points are to be considered in the design of pistons for I.C. engines.

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(vi) Noiseless operation
(vii) Resistance to mechanical and thermal distortion
(viii) Adequate support for piston pin.

Trunk pistons are used for I.C. engines. The main constituents of trunk piston are: (i) head to withstand the pressure of the gas, (ii) skirt to act as a bearing for the side thrust, (iii) gudgeon

Even though aluminium is weaker than cast iron, formula (ii) is applicable for both the materials as aluminium pistons are ribbed under the heads, the number of ribs being from 4 to 6 having thickness ranging from $\frac{1}{2}t$ to $\frac{1}{3}t$.

The piston may absorb some portion of heat in the burning gases and if not cooled in some manner the piston would melt. In small engines the heat is passed from piston to the cylinder through direct contact and through rings, which assist the piston. On large pistons much of this heat is carried away by oil which is introduced into the cavity under the top of the piston. Hence when we consider the design of piston we should consider this point. The amount of heat absorbed by the piston varies considerably in many engines, depending upon their designs. It varies from 5 to 20 per cent of the heat supplied to the engine in form of fuel.

As the piston has to transmit the absorbed heat to the cylinder walls, the considerable temperature difference exists between the inner and outer surfaces of the piston crown. As a result thermal stresses are induced which are supplemented to the mechanical stresses.

To determine the thermal stresses let us assume that the piston crown to be a flat wall and the temperature changes through the crown thickness are of a linear nature. The temperature difference δT between the outer and inner surfaces is given by the equation

$$\delta T = \frac{q \times t}{\lambda} \quad \text{...(iii)}$$

where q = quantity of heat passing through per sq metre of the crown surface per hour

t = thickness of the piston crown

λ = heat conduction coefficient.

The value of the heat conduction coefficient is 50 kcal/metre hour °C for steel and cast iron and 175 kcal/metre hour °C for aluminium.

The value of q can be obtained if we know the specific fuel consumption, the brake horse power of the engine, calorific value of the fuel, portion of the heat transmitted to the crown and area of the piston.

$$q = \frac{K \times C \times W \times \text{B.H.P.}}{A} \quad \text{...(iv)}$$

Strength of aluminium decreases faster than that of cast iron when temperature is increased. In pistons this is compensated for use by thick sections. Actual design of piston section is dependent upon size, method of cooling employed and other factors. In contrast cast iron pistons are made thin with several ribs to provide (a) the required strength of the crown and lower structure, and (b) additional surface for oil cooling.

Wear of aluminium pistons in many instances may be greater than for corresponding cast-iron pistons. However, this is compensated by the protection against serious scoring furnished by aluminium.

The top of the piston may be treated as a flat plate fixed on the cylindrical portion of the piston crown and subjected to a uniformly distributed load of the maximum intensity of gas pressure p. The thickness of the piston top is given by

$$t = D\sqrt{\frac{3}{16} \times \frac{p}{f}} \quad \ldots\ldots(i)$$

where p is the maximum combustion pressure, D is the cylinder diameter and f is the permissible stress in tension

The maximum combustion pressure may rise upto 80 kg/sq cm. The average value may be taken from 40 to 50 kg/sq cm.

For ribbed crowns or crowns of complex shape the section modulus is determined by graphical and other methods and the formula (i) will have to be modified accordingly In calculating the thickness of the piston crown by equation (i) the following values of the bending stresses are permissible

Material	f kg/sq cm
Cast iron	350 to 400
Steel	600 to 1,000
Aluminium Alloy	500 to 900

In fact there is much doubt about the validity of application of the flat plate theory in design of pistons, as the results differ in practice. Therefore, we adopt the empirical formula, recommended by Held and Favary for calculation of the thickness of the piston head.

Another important design consideration is how much combustion chamber volume is to be accommodated in the top of the piston. The exact amount depends on the arrangement of the valve gear. If the inlet and exhaust valves open and close at angles near top dead centre, the inlet valve may strike the piston or the exhaust valve be struck by piston because of overtaking. When the cavity is in the form of spherical segment, its radius is such that shallow depression is made except in cases where $\frac{L}{D}$ ratio is more than 1·5. For engines of $\frac{L}{D}$ ratio upto 1·5, the cup in the top of the piston head may be drawn with a radius equal to $0{\cdot}7D$.

Automobile and air craft engine pistons have usually three compression rings; sometimes four. Stationary compression ignition engines have five to seven rings. These rings are placed at the head of the piston where the leakage starts. For the better heat transfer, it is advisable to use many narrow rings than using few wide shallow rings.

As a rule the radial thickness t_r of the piston ring can be given in terms of the bore of the cylinder D as

$$t_r = (0{\cdot}029 \text{ to } 0{\cdot}033)\, D \qquad \text{(vii)}$$

and the ring thickness

$$b = (0{\cdot}6 \text{ to } 1)\, t_r \qquad \text{(viii)}$$

The radial thickness of the ring can be calculated by considering the radial pressure between the cylinder wall and ring. The maximum bending moment occurs in the ring at *a section opposite to the ring joint* on closing the gap. From bending stress consideration in the ring we get the radial thickness as under:

$$\text{Radial thickness, } t_r = D\sqrt{\frac{3\,p_w}{f_t}} \qquad \text{(ix)}$$

where p_w is the radial wall pressure and f_t the permissible stress for the piston ring.

The value of p_w lies between 0·4 to 0·7 kg/sq cm and the value of f_t between 1,000 to 1,500 kg/sq cm for cast iron.

When the piston ring is slipped on the piston the stress in the section rises to $\dfrac{1{\cdot}6 \times E \times t_r}{D - t_r + \frac{l}{\pi}}$ kg/sq cm where l is the ring gap when

where K is the constant representing the part of the heat absorbed by the piston, C is the higher calorific value of the fuel, W is the *specific fuel consumption and A is the piston head area*

The relative compression and tension of the layers on the outer and inner sides of the flat top piston crown is given by

$$f_c = f_t = \alpha \frac{qt}{2}\left[\frac{E}{1-m}\right] \qquad \text{(v)}$$

where α is the coefficient of linear expansion, E is the modulus of elasticity of the crown material and m is Poisson's ratio.

In making calculations by the *above described* method, the following temperature stresses are allowed:

Cast iron 1,500 to 2,000 kg/sq cm
Steel 2,000 to 4,000 kg/sq cm

The highest temperature of the crown of the cast iron piston may be taken as 400°C and that for aluminium alloy pistons as 250°C.

According to another design procedure the thickness of the piston head for the heat flow is given by

$$t_h = \frac{qD^2}{16C_1\theta} \qquad \text{(vi)}$$

where θ is the temperature difference between the centre and edge of the piston head and C_1 is the heat conduction factor in kcal/cm *hour °C, its value being 0·134 for cast iron and 0·47 for aluminium.*

In the above equation t_h denotes the thickness of the piston head for heat flow considerations.

The temperature difference θ°C may be taken 220° for cast iron and 75° for aluminium The piston designed for heat transfer must be designed to prevent the distortion of the skirt. A stiffening rib at the centre line of the boss extending around the skirt distributes the side thrust and prevents distortion

The thickness of the piston head in terms of the diameter D of engine cylinder *as found in practice for various engines is given below.*

Thickness of piston head

Type of engine	Piston material	Four stroke	Two stroke
Compression ignition	Cast iron	$0.11D - 0.15D$	$0.16D - 0.18D$
,,	Aluminium	$0.13D - 0.16D$	$0.17D - 0.20D$
Spark ignition	Cast iron	$0.12D - 0.14D$	$0.20D - 0.23D$

Reduction in diameter	Materials	
	Cast iran	Aluminium
Crown	(0·005 to 0·007)D	0·01D
Skirt	(0·001 to 0·0013)D	(0·0018 to 0·0025)D

The side clearances of the piston rings are as follows:

Upper grooves ....0·15 to 0·20 mm

Lower grooves ....0·08 to 0·12 mm

The maximum *side pressure* occurs at a crank angle of about 20° to 25°. The normal pressure varies from 0·08 to 0·1 of maximum gas load the lower value being for the higher values of the ratio of the length of connecting rod to crank.

Piston pin or wrist pin should be designed for the maximum combustion pressure or inertial force of the piston whichever is larger. The centre of the wrist pin should be from 0·02D to 0·04D above the centre of the skirt to offset the turning effect of the friction. The piston pin is usually hollow to reduce its weight and is often tapered on the inside, the smallest inside diameter being at the centre of the pin. The pin should be hardened and ground and should turn in phosphor bronze bushing. The material for the pin may be carbon steel or steel alloy. The allowable bending stress for these materials are as under:

Carbon steel	900 to 1,200 kg/sq cm
Alloy steel	1,500 to 2,300 kg/sq cm

The bearing area should be about equally divided between the bearing in the connecting rod and in the piston. The length of the pin in the connecting rod bearing will be about 0·45 of the piston diameter, allowing for end clearance of the pin, etc. The outside diameter of the pin varies from a value less than the length in the connecting rod bearing to a diameter one third larger. Thus, the piston area will be about three to four times larger than the projected area of the bearing in the connecting rod, making the maximum specific piston pin bearing load three to four times the maximum combustion pressure or piston inertia force intensity.

The allowable bearing pressure in a sliding friction bearing is 120 to 200 kg/sq cm for babbitt lined shell and 200 to 250 kg/sq cm for bronze shell. If needle type rolling bearings are used the value

the ring is being slipped on the piston and E is the modulus of elasticity of cast iron which is 800,000 kg/sq cm. The value of this stress is limited to 1,800 kg/sq cm.

The minimum axial thickness of the piston ring is given by $\frac{D}{10\,n}$ where D is the diameter of the engine cylinder and n is the number of piston rings.

The thickness of piston wall t_1 under rings is to be taken equal to the thickness of the head and decreases towards the end of the piston down to $0{\cdot}25\,t_1$ to $0{\cdot}35\,t_1$ in order to make the piston lighter.

Piston rings are made of cast iron. The distance of the first ring to the edge of the piston crown is (0·15 to 0·30) D for low speed and (0·10 to 0·18) D for high speed engines. The width of lands between the ring grooves is about ¾th the axial thickness of the ring, top land being wider than the ring and has upper edges bevelled. The wider the top land, lower will be temperature of the top ring.

In Diesel engines, additional snap ring is placed near the open end to retain the oil; but in gas engine trunk pistons, two or three oil grooves are cut near the open ends for the same purpose.

Special oil rings have been designed to prevent the oil leakage past piston into combustion chamber. Such rings have slotted openings which in conjunction with small holes in the grooves of the oil rings drain the oil from the piston into the crank case.

The piston skirt acts as a bearing for side thrust. The length of piston below the ring section should be such that the bearing pressure due to side thrust is limited to 3 kg/sq cm of projected area. To reduce the weight of reciprocating parts for high speed engines, the bearing pressure upto 7 kg/sq cm is permitted with a slightly greater wear. The maximum thrust will be during the expansion stroke. The side thrust can be calculated provided the crank length, the connecting rod length and the pressure variation on the piston head for various positions of the piston are known. In calculating the projected area, the space occupied by rings must be omitted. The length of piston is taken from $1{\cdot}25D$ to $1{\cdot}75D$.

To ensure the necessary piston to liner clearance in a hot engine the diameters of the piston crown and skirt should be made smaller by a value δD as given in the following table:

Values of δD in terms of D, the bore of the cylinder.

Radial thickness of the ring $= 20\sqrt{\frac{3 \times 0\cdot4}{1000}} = 0\cdot7$ cm.

Axial thickness of the ring may be taken as 0·6 cm.

Let us take the distance of the first ring from the edge of the crown as $0\cdot1D$, i.e. $0\cdot1 \times 20 = 2$ cm.

Width of piston land between rings $= \frac{3}{4} \times$ axial thickness of the ring $= \frac{3}{4} \times 0\cdot6 = 0\cdot45$ cm; we adopt 0·5 cm.

Length of the piston is adopted as $1\cdot5D = 1\cdot5 \times 20 = 30$ cm.

Length of piston skirt $= 30 - 2 - 4 \times 0\cdot5 - 5 \times 0\cdot6 = 23$ cm.

The centre of the piston pin above the centre of the skirt equals $0\cdot02D = 0\cdot02 \times 20 = 0\cdot4$ cm. Therefore, the distance from the bottom of the piston to the axis of the gudgeon pin is equal to $\frac{23}{2} + 0\cdot4 = 11\cdot9$ cm, say 20 cm.

Thickness of piston wall below the ring $= 2\cdot8$ cm.

Thickness of piston wall at open end $= 1\cdot2$ cm.

Bearing area provided by skirt $= 23 \times 20 = 460$ sq cm.

Maximum gas load $= \frac{\pi}{4} \times 20^2 \times 40 = 12{,}560$ kg.

Assuming that the maximum side pressure is 0·1 of the gas load,

side thrust $= 12560 \times 0\cdot1 = 1{,}256$ kg.

Bearing pressure between the side walls and piston $= \frac{1256}{460}$

$= 2\cdot74$ kg/sq cm,

which is within limits.

Maximum load on gudgeon pin $= 12{,}560$ kg.

Let the length of the pin in the connecting rod be $= 0\cdot45D$
$= 0\cdot45 \times 20 = 9$ cm.

Let us assume the permissible bearing stress as 200 kg/sq cm.

This value is suitable for babbitt lined shell as well as for bronze shell.

We assume the material of the pin as carbon steel for which the permissible stress may range from 900 to 1,200 kg/sq cm.

If d cm be the outside diameter of the pin, then

$d \times 9 \times 200 = 12560$

or $$d = \frac{12560}{9 \times 200} = 7 \text{ cm.}$$

of permissible bearing pressure may be taken as 300 to 600 kg/sq cm.

The bearing pressure intensity for cast iron boss is limited between 250 to 450 kg/sq cm and for aluminium alloy piston it is limited between 250 to 350 kg/sq cm.

After calculating the dimensions of the piston pin from bearing consideration, it should be checked for flexural stresses. It is to be considered a simple beam uniformly loaded for a length of the pin in the connecting rod bearing, with supports at the centres of the bosses at both ends. The bosses are at least $1\frac{1}{2}$ times the outer diameter of the pin. The pin is subjected to double shear at the boss. The allowable shear stress intensity is limited to 500 kg/sq cm. The wrist pin is fixed in the piston by a set screw, which enters one of the boss or the upper end of the connecting rod may be clamped to the pin so that the pin turns in the wrist pin boss or it may float both in piston and rod. In the last arrangement, the pin is retained by circlips or soft plugs at the ends of the piston pin to prevent the pin from contacting and scoring the cylinder wall.

Example:

1. Design a cast iron piston for a single acting internal combustion engine having 20 cm as the cylinder bore. The maximum explosion pressure may be taken as 40 kg/sq cm

We adopt the material of the piston as cast iron.

The thickness of the piston head is calculated from flat plate theory assuming that it is a circular plate fixed at the edges and loaded transversely with uniformly distributed load

$$t = D\sqrt{\frac{3}{16} \times \frac{p}{f}}.$$

The permissible stress for cast iron piston is taken as 385 kg/sq cm.

$$t = 20\sqrt{\frac{3}{16} \times \frac{40}{385}} = 2\cdot8 \text{ cm}.$$

The top of the piston head may be cupped with a radius of $0{\cdot}7D = 0{\cdot}7 \times 20 = 14$ cm. This space will provide a combustion chamber.

We adopt 5 compression rings and 1 oil ring.

Radial thickness of the ring $= D\sqrt{\dfrac{3\,p_w}{ft}}$.

We take $p_w = 0{\cdot}4$ kg/sq cm and $ft = 1{,}000$ kg/sq cm.

2. Design a trunk type piston for a single cylinder, four stroke cycle engine developing 8 B.H.P. at 600 r.p.m. Draw to scale a fully dimensioned working drawing of the piston with the piston rings, scraper rings and the piston pin in position. Show clearly the arrangement for fitting the gudgeon pin.

Diameter of piston is 12 cm and the maximum explosion pressure is 40 kg/sq cm. Heat supplied to the engine is 3,200 kcal/B.H.P./hour. About 6% of the heat is conducted through the piston crown. The heat conduction factor for cast iron may be taken as 0·14 kcal/cm hour °C. The temperature difference between the centre of the crown and the edge of the crown may be taken as 250°C.

Assume any data necessary for the design.

3. Design a four stroke gasoline engine developing 120 B.H.P. and operating at 3,600 r.p.m. Assume volumetric efficiency as 70% and compression ratio as 6:1.

Assume all other design parameters and dimensions necessary for estimation of the following:

(i) Cylinder dimensions

(ii) Cylinder cover including bolts

(iii) Piston including piston rings

(iv) Gudgeon pin. *(M. S. University of Baroda, 1969)*

4. Design a trunk type piston for a four stroke single cylinder Diesel engine running at 350 r.p.m. The maximum pressure in the cylinder is 50 kg/sq cm. The average brake mean effective pressure is 7 kg/sq cm. The specific fuel consumption is 0·21 kg/bhp/hour. Diameter and stroke are 90 cm and 45 cm respectively. The connecting rod length is 90 cm. The piston should have 4 piston rings and 1 oil ring. Radial pressure on the rings should be between 0·3 to 0·45 kg/sq cm. Allowable bending stress for the rings is 800 kg/sq cm. Heat conducted through the piston crown is approximately 5% of the total heat produced. The heat conductivity for the piston material is 177 kcal/metre hour °C. The temperature at the centre and at the edge of the piston crown may be assumed to be 400°C and 200°C respectively.

Take the calorific value of the fuel as 10,500 kcal/kg. The gudgeon pin is of the case hardened steel and bearing bushes are of phosphor bronze. The pin is fully floating. Bearing pressure for the pin should not exceed 120 kg/sq cm.

Give a neat sketch. *(M. S. University of Baroda, 1969)*

The bending stress in the pin is determined from the consideration that the pin is a beam uniformly loaded for a distance of 9 cm (length of pin in the connecting rod) and supported at the centres of the bosses. The maximum bending moment will work out to be

$$\frac{P \times D}{8} = \frac{12560 \times 20}{8} = 31{,}400 \text{ kg cm.}$$

If f be the maximum bending stress, then

$$\frac{\pi}{32} \times 7^3 \times f = 31400$$

or $$f = \sqrt[3]{\frac{31400 \times 32}{\pi \times 7^3}} = 935 \text{ kg/sq cm,}$$ which is within limits. The weight of the gudgeon pin can be reduced by providing hollow gudgeon pin.

Let us consider a hollow gudgeon pin of 8·5 cm outside diameter and 7 cm inside diameter

$$\text{Modulus of section} = \frac{\pi}{32}\left[\frac{8{\cdot}5^4 - 7^4}{8{\cdot}5}\right] = 32{\cdot}7 \text{ cm}^3.$$

$$\text{Bending stress} = \frac{31400}{32{\cdot}7} = 960 \text{ kg/sq cm}$$

Thus, by adopting a hollow pin, the stress has increased by $\frac{(960-935)}{935} \times 100 = 2{\cdot}67\%$, while the weight has been reduced by

$$\frac{7^2 - (8{\cdot}5^2 - 7^2)}{7^2} \times 100 = 52{\cdot}5\%$$

Thus, by adopting a hollow gudgeon pin, the piston can be made lighter

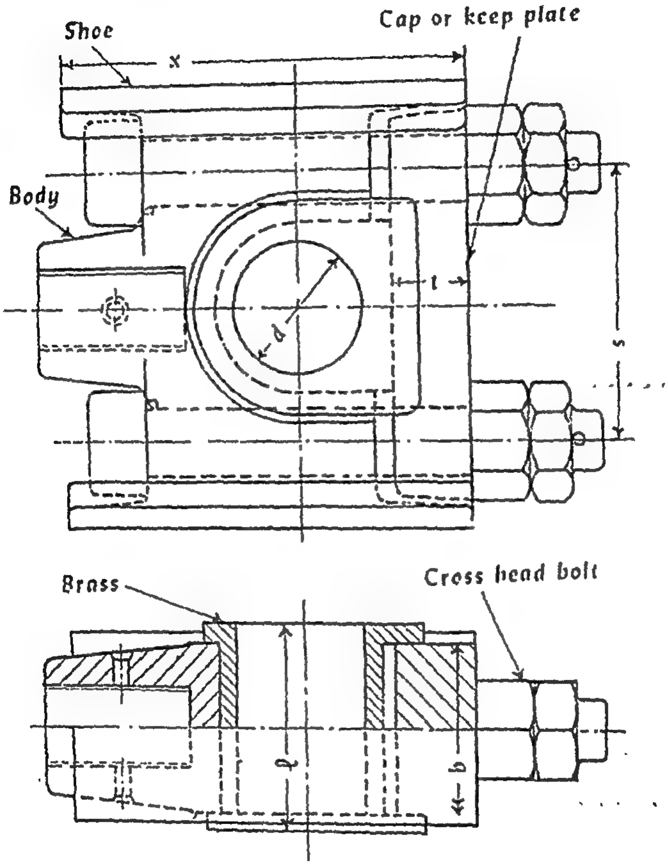
Exercises:

1. A trunk type cast iron piston for an internal combustion engine has a diameter of 10 cm and length of 15 cm. The maximum pressure is 35 kg/sq cm. Maximum permissible tension for cast iron for the design of head thickness is 300 kg/sq cm, and for piston pin material is 450 kg/sq cm. Bearing pressure for piston pin should not exceed 100 kg/sq cm. Design (a) the thickness of the head treating it as a flat plate fixed at the edges and uniformly loaded, and (b) dimensions of the piston pin.

Sketch the cross section of the piston through the piston bosses.

Ans. (a) 1·5 cm (b) Hollow pin of 65 cm outside diameter and 55 mm inside diameter having bearing length in small end as 45 mm.

The proportions of crossheads are largely empirical due to complex stresses and the only parts which permit of calculations are the wrist pin and shoes.



Marine type of crosshead

FIG. 18-7

Wrist pin:

The maximum load T on the wrist pin is given by

$$T = \frac{Pn}{\sqrt{n^2 - 1}} \quad \text{(i)}$$

where P = maximum load on the piston

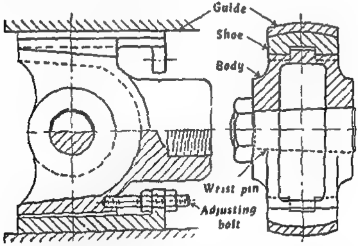
n = the ratio of the lengths of the connecting rod to crank.

5. *Design an aluminium alloy piston with a flat head for an I.C. engine having 100 mm bore. Draw a neat dimensioned sketch of the piston to bring out the details clearly. The maximum gas pressure may be taken as 45 kg/sq cm.*

18-3. Design of a crosshead:

Introduction:

Crossheads are essential parts of all double acting steam engines and several other machines in which reciprocating motion is converted into rotary motion or vice-versa. They are made of cast iron or cast steel. Two distinct forms—the box and the marine types are used in present designs and they are illustrated respectively in figs. 18-6 and 18-7 in which the important parts are given with their names



Box type of crosshead
FIG 18-6

The shoes which guide the crosshead and take all the reaction of the guide should be adjustable for wear. The wearing surface may be left plain, since cast iron on cast iron is quite satisfactory for bearing surfaces where the pressures and speeds are low. The bearing surface may be lined with babbitt metal.

The box type of crosshead is heavier than the marine type. So marine type crossheads are well suited for high speed engines to keep down inertia forces.

should be placed in such a position relative to the wrist pin that the pressure will be distributed uniformly over the bearing surface. The value of allowable bearing pressure varies from 2 to 7 kg/sq cm.

Guide:

The guides for crossheads may be made of cast iron, wrought iron or steel. When the guides are not cast with the frame of the engine, they are generally of rectangular or T section. Fig. 2-41 shows a cast iron guide bar of T section. The maximum load on the guide bar is due to reaction whose magnitude is given as

$$R = \frac{P}{\sqrt{n^2 - 1}} \quad \text{(vi)}$$

If we assume that the maximum load on the guide bar acts at the centre, then maximum bending moment M is given by

$$M = \frac{Ry}{4} \quad \text{(vii)}$$

where y is the distance between the supports of the bar. Equating the bending moment to the moment of resistance of the section of the bar, the dimensions of the section of the bar may be determined. The stress f may be taken at 200 kg/sq cm for cast iron and 400 kg/sq cm for wrought iron or steel.

Crosshead bolt:

When the piston rod and connecting rod are in tension, each bolt is subjected to a tensile load of $\frac{P}{2}$. These bolts are not subjected to a reversal of stresses so higher permissible stress values are adopted. Fine threads are employed.

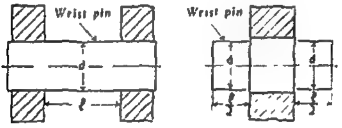
Cap or keep plate:

The cap may be considered as a beam fixed at two ends with a central load P. The span of the beam s equals the distance between bolt centres. The greatest bending moment will occur at the ends and at the centre and both are equal to $\frac{Ps}{8}$. Equating the bending moment to the moment of resistance of the section, the dimensions of the section of the keep plate may be calculated. The permissible stress value for mild steel keep plate is taken to be 450 kg/sq cm.

The maximum reaction R at the guide bar is given by

$$R = \frac{P}{\sqrt{n^2 - 1}} \qquad \text{...(ii)}$$

The wrist pin is designed to give sufficient bearing area and then it is checked for strength as a beam loaded with the thrust or pull in the connecting rod. The load may be taken as equally distributed over the journal length. The journal on the cross-head pin is either supported at both ends as shown in fig 18-8 or there are two separate end journals as shown in fig 18-9. In both the cases, the bending moment is equal to $\frac{Tl}{8}$.



Connections of gudgeon pin in connecting rod

Fig 18-8 Fig 18-9

If f be the permissible bending stress, then

$$\frac{\pi}{32} d^3 f = \frac{Tl}{8} \qquad \text{...(iii)}$$

If p be the bearing pressure per sq cm on journal, then

$$T = p\, d\, l \qquad \text{...(iv)}$$

Eliminating l from equations (iii) and (iv), we get

$$d = \sqrt[4]{\frac{4\, T^2}{\pi\, p f}} = \sqrt[4]{\frac{1{\cdot}273 T^2}{pf}} \qquad \text{...(v)}$$

The safe bearing pressure varies from 70 to 110 kg/sq cm for ordinary lubrication and from 140 to 210 kg/sq cm for forced lubrication. The value of bending stress may be taken as 600 kg/sq cm for wrought iron and 850 kg/sq cm for steel. The length of the pin varies from 1·25d to 2·25d

Shoe:

The shoes should be designed with an area sufficient to support the guide reaction with a specific safe pressure. They

Assuming a bearing pressure on crosshead pin as 85 kg/sq cm, the projected area of the pin required will be $\frac{2690}{85} = 32$ sq cm.

This area will be provided by a pin 4 cm diameter and 8 cm as the bearing length. The total length of the pin will be some what more than double the bearing length of the pin.

Let us determine the diameter of the crosshead bolts.

Load per bolt $= \frac{2580}{2} = 1{,}290$ kg.

The safe stress for the bolts may be taken as 350 to 400 kg/sq cm. We adopt fine threads. Minimum area required $= \frac{1290}{400}$

$= 3{\cdot}25$ sq cm.

From metric table, we adopt $M24 \times 2$, which will provide cross sectional area of 3·84 sq cm and pitch of 2 mm.

The width of keep plate will be equal to width across corners of piston nut. This distance we take as 6 cm. The distance between bolt heads is taken as the sum of the width across corners of piston nut + bolt head diameter + clearance. This dimension is taken as 11 cm.

The thickness of the bearing is taken as $t_1 = 0{\cdot}09d_1 + 0{\cdot}4$ cm where d_1 is the diameter of crosshead pin. On substitution of values, we get

$t_1 = 0{\cdot}09 \times 4 + 0{\cdot}4 \simeq 0{\cdot}8$ cm.

If we take the thickness of each side flange as 0·30 cm, then breadth of the cap $= 8 - 2 \times 0{\cdot}3 = 7{\cdot}4$ cm.

The thickness of the cap above bolt = 25 mm.

The permissible stress for the cap material is taken as 400 kg/sq cm. If t cm be the thickness of the cap, we get

$$\frac{2580 \times 11}{8} = \frac{1}{6} \times 7{\cdot}4 \times t^2 \times 400$$

or $$t = \sqrt{\frac{2580}{400} \times \frac{11}{7{\cdot}4} \times \frac{6}{8}} = 2{\cdot}68 \text{ cm; we adopt } 2{\cdot}8 \text{ cm.}$$

Let us check the crosshead pin for bending.

Maximum bending moment on pin $= \frac{Tl_1}{8}$

$$= \frac{2690 \times (8 - 0{\cdot}6)}{8}$$

Example:

1. Design a suitable crosshead for a single cylinder, non-condensing steam engine to develop 50 i.h p., the steam chest pressure being 7 kg/sq cm gauge. The cylinder is supplied with a D-slide valve giving a mean cut-off at $\frac{5}{8}$ of the stroke. The piston speed is to be 120 metre/min, the revolutions 240 per min and a diagram factor of 0·8 is to be taken.

The ratio of expansion of the engine $= \frac{1}{\frac{5}{8}} = 1{\cdot}6$.

The pressure of steam admission $= 7 + 1 = 8$ kg/sq cm absolute.

Let us take a back pressure as 1·2 kg/sq cm absolute.

Mean effective pressure $p_m = 0{\cdot}8\left\{\frac{8}{1{\cdot}2}(1 + \log_e 1{\cdot}6) - 1{\cdot}2\right\}$

$= 5$ kg/sq cm

$$\text{I.H.P.} = \frac{p_m \times \text{area of piston} \times \text{piston speed in metre/min}}{4500}$$

If D cm be the diameter of piston, then

$$50 = \frac{5 \times \pi \times D^2 \times 120}{4 \times 4500}$$

or $$D = \sqrt{\frac{50 \times 4 \times 4500}{5 \times \pi \times 120}} = 22 \text{ cm}$$

$$\text{Stroke of piston} = \frac{120 \times 100}{2 \times 240} = 25 \text{ cm}$$

Length of the connecting rod is taken as 50 cm

Maximum load on piston $P = \frac{\pi}{4} \times 22^2 \times 6{\cdot}8 = 2{,}580$ kg

$$\text{Maximum thrust } T \text{ in the connecting rod} = \frac{2580 \times 50}{\sqrt{50^2 - 12{\cdot}5^2}}$$

$= 2{,}690$ kg

$$\text{Reaction at the guide bar} = \frac{2580}{\sqrt{4^2 - 1}} = 670 \text{ kg.}$$

We assume the safe bearing pressure for the guide bar as 5 kg/sq cm

$\therefore$ Area of crosshead slipper $= \frac{670}{5} = 136$ sq cm. We adopt $B_0 = 8$ cm and $L_0 = 18$ cm, where B_0 and L_0 are the dimensions of the shoe, providing 144 sq cm shoe area

Angle degrees	0	20	40	60	80	90	100	120	140	160	180
Net steam pressure kg/sq cm	6·5	6·55	6·4	6·4	6·3	6·3	4·8	3·45	1·75	0	-1·7
Inertia force kg/sq cm	-2·54	-2·32	-1·7	-0·85	0·3	0.43	0·77	1·27	1·56	1·67	1·7

The other particulars are as follows:

Stroke 25 cm; length of connecting rod, 63 cm; crosshead guides have cylindrical surface of 18 cm diameter; small end of the connecting rod is forked and the gudgeon pin bearing is incorporated in the crosshead; diameter of piston rod 3·5 cm and it is connected to crosshead by a cotter.

Design and draw working sketches of the crosshead showing the method of adjusting the wear in gudgeon pin bearing.

Design and draw working sketches of the crosshead showing the method of adjusting the wear in gudgeon pin bearing.

6. The following particulars refer to a steam engine: Cylinder diameter, 30 cm; stroke 60 cm; connecting rod 150 cm; steam pressure 7 kg/sq cm; and cut-off at $\frac{5}{8}$ of the stroke. Calculate:

(i) Gudgeon pin dimensions

(ii) Area of crosshead shoe.

Bearing pressure for pin may be taken as 70 kg/sq cm and for shoe as 4 kg/sq cm. The bending stress in the pin is not to exceed 700 kg/sq cm.

(Sardar Patel University, 1968)

18-4. Design of connecting rods:

The types of connecting rod ends for steam engines are shown in fig. 6-7 and 9-16. The solid rod shown in fig. 18-10 is the strongest and most common form used with side crank engines. Connecting rods for the centre crank engines are made with the crank pin end in two or more pieces to facilitate assembly on the crank pin. Suitable arrangements should be made for taking up the wear.

In this article we shall consider in detail the design principles as related to internal combustion engines.

$$= 2{,}500 \text{ kg cm.}$$

The modulus of section of the pin $= \frac{\pi}{32} d_1^3 = \frac{\pi}{32} \times 4^3 = 6{\cdot}28 \text{ cm}^3$.

$\therefore$ Bending stress $= \frac{2500}{6{\cdot}28} = 400$ kg/sq cm; the stress value is well within safe limits.

Exercises:

1. Calculate the area of a crosshead shoe for a 4 cylinder horizontal Diesel engine developing 850 B.H.P running at 160 r.p.m. Diameter of the cylinder is 47 cm and the stroke length 60 cm. The ratio of the length of the connecting rod to crank length is 5 and the permissible bearing stress is limited to 2·5 kg/sq cm. *Ans* 400 sq cm.

2. The maximum thrust in the connecting rod of a steam engine is 4,983 kg. Suggest the suitable dimensions of the gudgeon pin. Assume that the bearing pressure on the pin is limited to 90 kg/sq cm and that $\frac{l}{d}$ ratio is 2. Bending stress in the pin is limited to 600 kg/sq cm.

Ans $d = 75$ mm.

3. The crosshead for a double acting steam engine has a single slide of forged steel which has a width equal to 0·4 of its length. The diameter of the cylinder is 30 cm; stroke 55 cm; and length of connecting rod is 5 times that of the crank. Steam admission pressure is 10 kg/sq cm and back pressure 1·2 kg/sq cm. The safe bearing pressure for the slide is 5 kg/sq cm. Design and sketch the crosshead.

Ans Length 25 cm, width of guide bar 10 cm.

4. The diameter of a steam engine cylinder is 30 cm, stroke 60 cm and connecting rod length 1·5 metres. The steam pressure is 7 kg/sq cm. Safe bearing pressure for guides is 3·5 kg/sq cm. Determine the minimum area of the guide bars assuming that the side thrust is maximum when the crank is at right angles to the line of stroke. If the safe bearing pressure for gudgeon pin is 100 kg/sq cm, determine the dimensions of the gudgeon pin if its length is twice the diameter.

Ans. 290 sq cm; diameter of pin 5·5 cm.

5. In a H.P. cylinder of 20 cm diameter, the net forward steam pressure and inertia force per sq cm of piston area are observed to be given as under:

bearing. This support is fairly narrow, has some clearance for oil, the piston has some freedom in the cylinder, and the piston pin and crankpin may not be exactly parallel. As a result, stress calculations are often based on the assumption that the rod is supported freely in both planes with point contact. Allowance must be made for the rapid rate of application and release of the loads imposed.

In 2-stroke cycle engines, the load is partially released during the operational cycle but never completely. There is always a compressive load on the rod due to (a) compression of the air, (b) the power impulse or, near bottom dead center (c) the inertia force of the piston and rod.

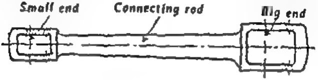
In 4-stroke cycle engines, there is a reversal of loading on the connecting rod from compression to tension each cycle. Compression loads are present during compression and power strokes but during the last part of the exhaust stroke and first part of the intake stroke, the rod is in tension to absorb the inertia forces since there is no gas pressure in the cylinder. In high-speed engines this tensile load may be even greater than the compressive load on the power stroke.

Maximum allowable stress in a 4-stroke cycle rod is less than that in a 2-stroke cycle rod. The 4-stroke cycle stress reversal condition is more severe than a 2-stroke cycle partially relieved compressive load.

A connecting rod is subjected to an additional stress due to bending as the rod is whipped back and forth by the crankpin. The condition is the same as if the rod was swung back and forth rapidly by a force at the big end while supported on a stationary piston pin. Weight of the rod creates a bending stress which is a maximum at the outer limits of swing and is dependent on rod weight, length, shape of cross section, radius of the crank arm and engine speed. This maximum stress is developed when the crank arm and connecting rod are at right angles to each other. At this position on the power stroke, the piston is nearly half way down, the compressive column load is much below the maximum, so that in nearly all cases the combination of compressive stress plus bending stress is much less than the maximum stress of column action alone. As a result, this bending load is seldom found to be serious and is often neglected.

Connecting Rods and Piston Pins:

The connecting rod and piston pin are the connecting links between piston and crankshaft. They transform the power delivered to the reciprocating piston into a rotating torque in the crankshaft.



Connecting rod
FIG 18-10

Connecting Rod Type:

Many large slow-speed engines use connecting rods of the so-called *marine-type construction.* This is particularly true when the bore is over 40 cm. The piston-pin eye is generally a solid integral part of the rod column. The crankpin end consists of two separate parts which bolt together to locate the bearing on the pin and the assembly is then bolted to the foot of the column This design facilitates bearing service on large engines when working through access doors in the sides of the frame.

Most engines have the conventional two-piece connecting rod. The whole rod may be forged in one piece, the bearing cap being cut off, faced and bolted in place for final machining of the big end. In large sizes, the cap is forged separately, the joint machined, and then bolted to the rod for final machining.

The small end of the rod is generally made as a solid eye.

Loads on Connecting Rods:

During the compression and power strokes, the connecting rod functions as a column, and is subject to compressive loads due to downward forces on the piston The resulting piston force is the gas pressure in kg/sq cm times the piston area in sq cm minus the inertia force of the piston and rod.

Stress in the rod can be calculated by conventional column formulas. The rod is supported on two parallel cylindrical bearings and, therefore, is freely supported about the bearings. At 90 degrees, each end of the rod is supported by the length of the

Materials:

The forces of gas pressure in the piston and the inertia forces of the reciprocating masses of the piston and connecting rod cause variable loading of the connecting rod. The connecting rod is one of the most heavily stressed parts of an engine and is therefore made of high grade steels.

Most connecting rods are made of a medium-carbon steel or alloy steel forgings. If heat treated, they are drawn at a suitable temperature to leave the material soft so as to better resist fatigue failure.

The bolts of connecting rod bearings are made of high grade medium carbon alloy steel, heat treated to provide high strength along with good fatigue resistant properties. Many of these bolts are machined to provide high fatigue resistance by having a large portion of the body turned to a smaller diameter. All the body except the ends and the central portion which acts as a dowel is machined to the smaller diameter.

The small end bushes are made of phosphor bronze. The shells of the connecting rod big end are usually made of steel-lined with babbitt or lead bronze.

Calculations of connecting rods:

Calculations of the connecting rod are confined to checking the strength of the shank, small end, big end and big end bolts.

The connecting rod shank is subjected to the force of gas pressure, the force of inertia of the reciprocating masses and the transverse inertia forces of the connecting rod mass.

When the piston is at top dead centre or inner dead centre position the connecting rod is compressed by the resultant of gas force and the inertia force. As the gas force is compressive and the inertia force at top dead centre or inner dead centre position is tensile, the resultant of the two will be less than the gas force. *Hence for higher reliability the connecting rod shank is calculated only for the gas force.*

The cross sectional area of the connecting rod may be determined by principles explained in chapter on "Struts and Columns".

The allowable compressive stresses for the material of the connecting rod are as follows:

Connecting Rod Length

The shorter the connecting rod for a specific stroke, or crank-arm radius, the greater the angular swing of the rod, and the greater the piston side thrust. However, no piston or cylinder wall was ever scored due to piston side thrust alone.

Length of the connecting rod is dependent for the most part on the piston stroke and length of the piston skirt below the piston pin. In high-speed engines, where fairly short pistons are used, the l/r ratio is generally in the neighbourhood of 4, or less. In large low-speed engines, it varies from 4 to over 5, in the case of extremely long pistons.

Connecting Rod Sections

Most connecting rods are made with I-beam sections, while a few engines use round-sectioned rods. Most rods of either shape have a rifle-drilled hole from end-to-end to carry oil for pin lubrication and piston cooling

The I-beam is positioned for greatest support against flexure around the bearings. The round rod section provides uniform strength in all directions. It also provides a simple means of producing a completely-machined rod, and thus provides additional insurance against fatigue failure as compared to an irregular unfinished rod

The duty of the connecting rod is to transmit the load received from the piston pin to the crankpin. To prevent highly localized loads on portions of the bearings, both ends of the rod column section must be flared out and filleted well into the cylindrical sections which support the bearings, so that load is distributed uniformly over the bearing surface. Liberal fillets will provide the required support at the rod eye At the crankpin end, width of the I-beam should increase as it approaches the bearing support. Large fillets are desirable from the web of the I-beam to the bearing support for rigidity along the length of the bearing

Since 4-stroke cycle rods are subjected to tensile as well as compression loads, the bearing caps must be made with deep sections of sufficient rigidity to carry these loads without bending and deforming the bearing shell The cap bolts, or cap screws, must be capable also of withstanding these repeated tensile loads without fatigue. In a 2-stroke cycle rod the bolt and cap loads are practically zero

Acceleration of the crank pin $= r\omega^2$.

$\therefore$ Inertia force per unit length at crank pin $= \frac{\rho}{g} a r\omega^2$.

Inertia force per unit length at a distance x from the gudgeon pin $= \frac{x}{l}\frac{\rho}{g} a r \omega^2$. Thus we see that the load distribution on the connecting rod shank is represented by a triangle as shown in fig. 18-11.

Total inertia force P on the connecting rod $= \int_0^l \frac{x}{l}\frac{\rho}{g} a r \omega^2 dx$.

On integration, we have

$$P = \frac{\rho}{g} a r\omega^2 \frac{l}{2} \qquad \text{(i)}$$

This load acts at a distance $\frac{2}{3}l$ from the gudgeon pin.

$\therefore$ Reaction at gudgeon pin will be $\frac{P}{3}$ and at crank pin $\frac{2P}{3}$.

Bending moment M at any section at a distance x from the gudgeon pin $= \frac{P}{3}(x - \frac{x^3}{l^2})$. This bending moment will be maximum for that value of x, which gives

$$\frac{dM}{dx} = 0.$$

$$\therefore \quad \frac{dM}{dx} = \frac{P}{3}\left(l - \frac{3x^2}{l^2}\right) = 0$$

or $\qquad x = \frac{l}{\sqrt{3}}$.

Substituting in bending moment equation, we get

$$M_{max} = \frac{2\sqrt{3}}{27} Pl \qquad \text{(ii)}$$

Maximum bending stress $= \frac{M_{max}}{Z}$.

The maximum bending stress in the connecting rod is given by

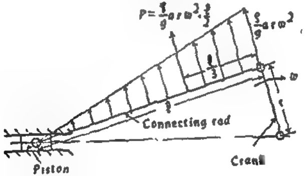
$$f = \frac{N^2 r \rho a l^2}{14400 Z} \text{ kg/sq cm} \qquad \text{(iii)}$$

where N is the crank speed in *r.p.m.*; Z the section modulus in cm^3, r the crank length in metre, ρ density in kg/cm^3, l the length of the rod in cm and a the area of cross section of the rod in sq cm.

Carbon Steel 800 to 1,200 kg/sq cm
Alloy Steel 1,200 to 1,800 kg/sq cm

Bending of connecting rod shank by the transverse forces of inertia of the connecting rod mass is taken into account only when checking the dimensions of the connecting rods of high speed engines.

Now we consider in detail the analysis of transverse force analysis. In high speed engines the connecting rod of the engine has a lateral oscillation in addition to its longitudinal motion. As a result inertia forces are set up in the rod which tend to bend it in the plane of oscillation. This is due to the fact that one end of the rod is fastened to the crosshead or piston having a motion of pure translation, while the other end is fastened to the crank pin which is rotating about the fixed axis. *Such an action is known as the whipping action due to the oscillation of the rod.* If the rod were to be made cylindrical, it will bend very easily. In order to strengthen the rod in the plane of oscillation, it is made rectangular or of *I* section with the greatest depth in the plane of oscillation.



Inertia loading on a connecting rod in its plane of oscillation
Fig. 18-11

Let us consider the stresses in the connecting rod due to the whipping action. The maximum stresses due to bending will occur when the crank and connecting rod are at right angles as shown in fig. 18-11. Let r be the radius of the crank, ω the angular velocity of the crank, a the area of cross section of the connecting rod, l the length of the connecting rod and ρ the density of the material of the rod. We assume for the analysis that the area of cross section of the connecting rod is constant throughout, which is not the case.

majority of rods are tapered so that the crank pin end is 1·1 to 1·15 times the depth computed for the centre section. The outer ends of solid rods and cap of marine type and split end rods are designed as beams simply supported at ends and loaded at the centre by a concentrated force P which is equal to the inertia force of reciprocating masses. If this force is insignificant as is the case with low speed engines, the force should be taken as a arbitrary force of piston seizure which may be taken as (10 to 15) $\frac{\pi}{4}D^2$ kg where D is the bore of the cylinder in cm. Afterwards they should be checked for stiffness and the deflection at the centre should not exceed 0·03 mm. (Please refer to article 9-10.)

The width of the connecting rod big end must be smaller than the cylinder bore diameter D if the piston connecting rod group is to be removed upward through the cylinder liner.

In four stroke cycle engine, the big end bolts are designed for the inertia forces of the reciprocating masses of the piston and connecting rod plus the rotating mass of the connecting rod. If F be the inertia force on which the bolts are to be designed then

$$d_b = \sqrt{\frac{F \times 4}{\pi n f}} \quad \text{(v)}$$

where n = number of connecting rod bolts

d_b = core diameter of a bolt

f = allowable tensile stress intensity.

The permissible values of allowable tensile stress intensities are as under:

Carbon Steel	500 to 600 kg/sq cm
Alloy Steel	900 to 1,000 kg/sq cm

In two stroke cycle engines and low speed four stroke cycle engines these bolts are checked for the force of piston seizure which may be taken as (10 to 15) $\frac{\pi}{4}D^2$ kg where D is the engine bore in cm.

The required tightness of the joint between the halves of the connecting rod end during engine operation should be ensured by tightening up the connecting rod to 1·35 to 1·5 times the value of F, the inertia force.

The flexural or bending stresses in the engine connecting rods are usually as follows:

Low speed engines	75 to 100 kg/sq cm
High speed engines	150 to 200 kg/sq cm.

The above results are obtained on the assumption that the rod is of uniform section but the taper in the rod will not change the stresses appreciably The maximum values of the stresses due to gas loads and due to whipping action do not occur at the same instant in case of internal combustion engines, so we cannot get the maximum values by adding them It is unnecessary to investigate the maximum tensile stress because the maximum stresses due to gas load occur near about at dead centres when the transverse bending stresses are insignificant *For steam engines, we should check the section for maximum stresses as the maximum values of both stresses are likely to occur at the same time, because the pressure of steam remains constant upto the point of cut-off.*

It should also be remembered that as the thrust along the connecting rod acts along the friction axis, which does not lie along the geometrical axis of the connecting rod, flexural or bending stresses are induced in each normal section of the connecting rod These stresses vary from section to section

If we consider the inertia of the reciprocating masses of the engine, their effect is to reduce the maximum gas load along the connecting rod during the first part of the stroke and increase it during the latter half Hence while proportioning the cross sectional dimensions of the rod we have neglected the inertia effect due to reciprocating masses.

Inertia force, I, due to reciprocating masses equals

$$\frac{W}{g} r\omega^2 \left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \qquad (v)$$

where W is the weight of reciprocating masses and θ the angle turned by the crank from the inner dead centre position

The thrust along the connecting rod at any instant will be gas load minus the inertia force.

The connecting rod may be circular, rectangular or I sectioned. The ratio of width to depth of a rectangular section should be 0·5 to 0·7. I section rods have generally a width 0·6 times the depth and a flange and web thickness 0·20 to 0·25 times the depth The

majority of rods are tapered so that the crank pin end is 1·1 to 1·15 times the depth computed for the centre section. The outer ends of solid rods and cap of marine type and split end rods are designed as beams simply supported at ends and loaded at the centre by a concentrated force P which is equal to the inertia force of reciprocating masses. If this force is insignificant as is the case with low speed engines, the force should be taken as a arbitrary force of piston seizure which may be taken as (10 to 15) $\frac{\pi}{4} D^2$ kg where D is the bore of the cylinder in cm. Afterwards they should be checked for stiffness and the deflection at the centre should not exceed 0·03 mm. (Please refer to article 9-10.)

The width of the connecting rod big end must be smaller than the cylinder bore diameter D if the piston connecting rod group is to be removed upward through the cylinder liner.

In four stroke cycle engine, the big end bolts are designed for the inertia forces of the reciprocating masses of the piston and connecting rod plus the rotating mass of the connecting rod. If F be the inertia force on which the bolts are to be designed then

$$d_b = \sqrt{\frac{F \times 4}{\pi n f}} \qquad \text{(v)}$$

where n = number of connecting rod bolts

d_b = core diameter of a bolt

f = allowable tensile stress intensity.

The permissible values of allowable tensile stress intensities are as under:

Carbon Steel	500 to 600 kg/sq cm
Alloy Steel	900 to 1,000 kg/sq cm

In two stroke cycle engines and low speed four stroke cycle engines these bolts are checked for the force of piston seizure which may be taken as (10 to 15) $\frac{\pi}{4} D^2$ kg where D is the engine bore in cm.

The required tightness of the joint between the halves of the connecting rod end during engine operation should be ensured by tightening up the connecting rod to 1·35 to 1·5 times the value of F, the inertia force.

In more accurate calculations the maximum tensile stress in the connecting rod bolts is determined by the methods explained in art. 5-13.

$$f = \frac{1}{a_{min}}\left[P_i + \frac{F}{1 + \frac{\lambda_1}{\lambda_2}}\right] \text{kg/sq cm.} \quad \text{(vi)}$$

where

a_{min} = core area of the bolt

P_i = initial tightening load in the bolt

$\lambda_1 = \frac{E_1 \times a_1}{l_1}$ = rigidity of the clamped parts of the connecting rod end

l_1 = length of the parts before tightening

a_1 = cross section of the parts being clamped

$\lambda_2 = \frac{E_2 \times a_2}{l_2}$ = rigidity of the bolt

l_2 = length of the bolt between nuts

a_2 = mean cross section of the bolt.

The maximum allowable tensile stress in the above equation is given as under

Carbon Steel 800 to 1,200 kg/sq cm
Alloy Steel 1,300 to 2,000 kg/sq cm

To prevent forcing out the oil film and destroying the anti-friction lining of the bearing the load on the crank pin should not exceed 90 to 120 kg/sq cm of projected area for slow speed Diesel engines and 200 to 220 kg/sq cm of projected area for high speed engines

Another application of a machine element where transverse loading is to be considered is *the coupling rod of a locomotive*, which undergoes considerable bending stresses when running at a high speed on account of transveres inertia loading At its highest position the downward acceleration is maximum and is equal to $r\omega^2$. The acceleration of each particle on the coupling rod is the same. Similarly when the crank is vertical in lowest position, the upward acceleration is maximum In this phase the gravity and inertia force are in the same sense and hence the flexural stresses are maximum. The coupling rod is considered as a beam, simply supported at each end and loaded with a uniformly distributed load (due to inertia) of $\frac{\rho}{g} a r\omega^2$ per unit length. The

maximum bending stress will occur at the centre. When modulus of section is known, the stresses can be calculated. If the permissible stresses are known, the suitable section can be suggested.

Examples:

1. *The following section has been suggested for I section connecting rod of a petrol engine running at 2,000 r.p.m. The depth of the section is 4 cm, the width of the flange 2·5 cm and the thickness of the flange and the web 0·6 cm. If the stroke of the piston be 15 cm, determine the maximum stress in the connecting rod due to inertia of the connecting rod if the length of the connecting rod be 30 cm. The density of the material is 7·8 kg/dm³.*

$$\text{Area of cross section} = 2 \times 2{\cdot}5 \times 0{\cdot}6 + 2{\cdot}8 \times 0{\cdot}6 = 4{\cdot}68 \text{ sq cm.}$$

$$\text{Modulus of section} = \frac{\frac{1}{12}[2{\cdot}5 \times 4^3 - 1{\cdot}9 \times 2{\cdot}8^3]}{2} = 4{\cdot}92 \text{ cm}^3.$$

The maximum bending stress in the connecting rod due to whipping action is given by

$$f = \frac{N^2\, r\, \rho a l^2}{14400 Z} \text{ kg/sq cm},$$

where N is the crank speed in r.p.m., Z the section modulus in cm³, r the crank length in metre, ρ density in kg/cm³, l the length of the rod in cm and a the area of cross section of the rod in sq cm.

$$f = (2000)^2 \times \frac{7{\cdot}5}{100} \times \frac{7{\cdot}8 \times 4{\cdot}68 \times 30^2}{14400 \times 4{\cdot}92} \times \frac{1}{1000}$$

$$= 139 \text{ kg/sq cm.}$$

2. *The connecting rod of a slow speed Diesel engine is 3 metre long and is made of St 55. Determine the suitable dimensions for a circular section of a rod. The bore and stroke of the cylinder are 90 and 120 cm respectively. The maximum combustion pressure is 48 kg/sq cm.*

Determine, also, the whipping stresses if the engine runs at 150 r.p.m.

$$\text{Maximum gas load} = \frac{\pi}{4} \times 90^2 \times 48 = 305{,}000 \text{ kg.}$$

As the engine is slow speed, the inertia effect of reciprocating masses is to be neglected.

Let us determine the cross section of the connecting rod by using Euler's formula assuming that it has hinged ends. We take factor of safety 20.

In more accurate calculations the maximum tensile stress in the connecting rod bolts is determined by the methods explained in art. 5-13.

$$f = \frac{1}{a_{min}}\left[P_i + \frac{F}{1 + \frac{k_1}{k_2}}\right] \text{kg/sq cm.} \qquad \text{(vi)}$$

where

a_{min} = core area of the bolt

P_i = initial tightening load in the bolt

$k_1 = \frac{E_1 \times a_1}{l_1}$ = rigidity of the clamped parts of the connecting rod end

l_1 = length of the parts before tightening

a_1 = cross section of the parts being clamped

$k_2 = \frac{E_2 \times a_2}{l_2}$ = rigidity of the bolt

l_2 = length of the bolt between nuts

a_2 = mean cross section of the bolt.

The maximum allowable tensile stress in the above equation is given as under:

Carbon Steel 800 to 1,200 kg/sq cm
Alloy Steel 1,300 to 2,000 kg/sq cm

To prevent forcing out the oil film and destroying the anti-friction lining of the bearing the load on the crank pin should not exceed 90 to 120 kg/sq cm of projected area for slow speed Diesel engines and 200 to 220 kg/sq cm of projected area for high speed engines.

Another application of a machine element where transverse loading is to be considered is *the coupling rod of a locomotive*, which undergoes considerable bending stresses when running at a high speed on account of transveres inertia loading At its highest position the downward acceleration is maximum and is equal to $r\omega^2$. *The acceleration of each particle on the coupling rod is the same.* Similarly when the crank is vertical in lowest position, the upward acceleration is maximum In this phase the gravity and inertia force are in the same sense and hence the flexural stresses are maximum. The coupling rod is considered as a beam, simply supported at each end and loaded with a uniformly distributed load (due to inertia) of $\frac{\rho}{g} a r\omega^2$ per unit length. The

We neglect the inertia of the reciprocating masses.

$$T = \frac{1720 \times 5}{\sqrt{5^2 - 1}} = 1{,}760 \text{ kg.}$$

The stroke is 25 cm. Hence the crank length is $\frac{25}{2} = 12{\cdot}5$ cm and as the value of n is 5, the length of the connecting rod is $5 \times 12{\cdot}5$ cm. For a circular section the radius of gyration is $\frac{d}{4}$.

We determine the diameter of the cross section of the connecting rod by using Rankine's formula for hinged ends. If d cm be the diameter of the connecting rod, then

$$1760 = \frac{400 \times \frac{\pi d^2}{4}}{1 + \frac{1}{7500} \times \left(\frac{62{\cdot}5 \times 4}{d}\right)^2}$$

Solving the above equation we get d^2 as $9{\cdot}18$ cm² hence we adopt d as $3{\cdot}2$ cm. Total area of the section of the jaws is usually from $1{\cdot}25$ to $1{\cdot}8$ times the area of the section of the adjacent part of the rod, and in the case of solid forked end, the total area of the section through the eyes is usually from $1{\cdot}4$ to $1{\cdot}9$ times the area of the section of the rod. Hence the various proportions of the forked end of the connecting rod as shown in Fig. 18-12 may be as under.

$B = d$ to $1{\cdot}2d$ average value $1{\cdot}1d$ and $C = 0{\cdot}5d$ when $B = 1{\cdot}1d$.

Thickness of eye $E = C$ to $1{\cdot}35C$; average value $1{\cdot}2C = 0{\cdot}6d$ when $C = 0{\cdot}5d$

Height of each fork $F = 0{\cdot}5d$ when $E = 0{\cdot}6d$.

We have determined the diameter of the connecting rod shank as 32 mm, hence the numerical values of the various proportions are as:

If d cm be the diameter of the connecting rod in cm, then by employing Euler's formula, we have

$$305000 \times 20 = \frac{\pi^2 \times 2{\cdot}1 \times 10^6 \times \frac{\pi}{64} d^4}{300^2}$$

$$\text{or} \quad d = \sqrt[4]{\frac{305000 \times 20 \times 300^2 \times 64}{\pi^3 \times 2{\cdot}1 \times 10^6}}$$

$= 27{\cdot}1$ cm, we adopt 28 cm

$$\text{Slenderness ratio } \frac{l}{k} = \frac{300 \times 4}{28} = 42{\cdot}8.$$

As the slenderness ratio is less than 105, Euler's formula is not applicable and hence we check the design by Tetmajer's formula. According to Tetmajer, buckling load on mild steel column with hinged ends is given by

$$P = A\,(3100 - 11{\cdot}4\lambda) \text{ kg}$$

where A = area of cross section in sq cm and

λ = the slenderness ratio.

$$P = \frac{\pi}{4} \times 28^2\,[3100 - 11{\cdot}4 \times 42{\cdot}8]$$

$$= 1{,}600{,}000 \text{ kg.}$$

$$\text{Factor of safety} = \frac{1600000}{305000} = 5{\cdot}25.$$

Thus, the column is strong to withstand the buckling action. In fact we can reduce the diameter of the column because according to ten Bosch the factor of safety of 3·5 to 5 is sufficient for Tetmajer's formula.

Let us check the design for whipping stress

$$\text{Whipping stress} = \frac{N^2\, r\, \rho\, a\, l^2}{14400\, Z} \text{ kg/sq cm}$$

where N is the crank speed in r.p.m., r the crank radius in metre, ρ the density in kg/cubic cm, a area of cross section in sq cm, l the length of the connecting rod in cm and z the section modulus in cm^3.

On substitution of values in the above formula, we get

$$f = \frac{150^2 \times 0{\cdot}6 \times \frac{\pi}{4} \times 28^2 \times 0{\cdot}0078 \times 300^2}{14400 \times \frac{\pi}{32} \times 28^3}$$

$= 190$ kg/sq cm.

We neglect the inertia of the reciprocating masses.

$$T = \frac{1720 \times 5}{\sqrt{5^2 - 1}} = 1{,}760 \text{ kg.}$$

The stroke is 25 cm. Hence the crank length is $\frac{25}{2} = 12{\cdot}5$ cm and as the value of n is 5, the length of the connecting rod is $5 \times 12{\cdot}5$ cm. For a circular section the radius of gyration is $\frac{d}{4}$.

We determine the diameter of the cross section of the connecting rod by using Rankine's formula for hinged ends. If d cm be the diameter of the connecting rod, then

$$1760 = \frac{400 \times \frac{\pi d^2}{4}}{1 + \frac{1}{7500} \times \left(\frac{62{\cdot}5 \times 4}{d}\right)^2}$$

Solving the above equation we get d^2 as $9{\cdot}18$ cm² hence we adopt d as $3{\cdot}2$ cm. Total area of the section of the jaws is usually from 1·25 to 1·8 times the area of the section of the adjacent part of the rod, and in the case of solid forked end, the total area of the section through the eyes is usually from 1·4 to 1·9 times the area of the section of the rod. Hence the various proportions of the forked end of the connecting rod as shown in Fig. 18-12 may be as under.

$B = d$ to $1{\cdot}2d$ average value $1{\cdot}1d$ and $C = 0{\cdot}5d$ when $B = 1{\cdot}1d$.

Thickness of eye $E = C$ to $1{\cdot}35C$; average value $1{\cdot}2C = 0{\cdot}6d$ when $C = 0{\cdot}5d$

Height of each fork $F = 0{\cdot}5d$ when $E = 0{\cdot}6d$.

We have determined the diameter of the connecting rod shank as 32 mm; hence the numerical values of the various proportions are as under:

$B = 36$ mm; $C = 16$ mm; $E = 20$ mm and $F = 16$ mm.

The gudgeon pin or wrist pin is designed to give sufficient bearing area and then it is checked for strength as a beam loaded with the thrust or pull in the connecting rod. The load may be taken as equally distributed over the journal length. The journal on the cross head pin is supported as both ends as shown in Fig. 18-8. According to equation (v) of art 18-3, the diameter of the gudgeon pin d' is given by

$d' = \sqrt[4]{\dfrac{1{\cdot}273T^2}{pf}}$ where T is the thrust in the connecting rod, p is the permissible bearing pressure intensity and f is the permissible flexural stress intensity,

On substitution of values we get

$$d' = \sqrt[4]{1{\cdot}273 \times \frac{1760}{400} \times \frac{1760}{100}}$$

$= 3{\cdot}14$ cm; we adopt 32 mm.

We assume that there are two cap bolts. Load on each bolt $= \dfrac{1760}{2} = 880$ kg

Core area of the bolt $= \dfrac{880}{400} = 2{\cdot}2$ sq cm. We adopt fine threads $M20 \times 1{\cdot}5$ or we can adopt M 20 bolt of uniform strength the latter will be preferable as the connecting rod is subjected to shock loading

5. The high speed Diesel engine has the following particulars: Bore of the cylinder 9 cm, stroke 14 cm, speed 1,500 r p m., compression ratio 16, maximum pressure 45 kg/sq cm upto 7% of the stroke, weight of reciprocating parts 3 kg and length of connecting rod 35 cm.

Design a suitable connecting rod for the given duty.

The shanks of the connecting rods are made with an I section in medium and high speed engines. The cross section being equal, I section shanks possess a higher section modulus in connecting rod swinging plane than do round shanks.

The connecting rod shank is subjected to the force of gas pressure and the force of inertia of the reciprocating masses, in addition to transverse inertia stresses. The gas force is maximum at inner dead centre position when the transverse inertia forces are zero and the maximum bending moment caused by the transverse inertia forces is originated *when the connecting rod is positioned at 90° to the crank for the section located at a distance of 0·577 l from the centre of the connecting rod small end.*

For higher reliability the connecting rod shank is calculated only for the gas force, neglecting the inertia force.

Gas pressure at inner dead centre $= \dfrac{\pi}{4} \times 9^2 \times 45$

$= 2{,}880$ kg.

The maximum value of thrust along the connecting rod is obtained by resolving this force when the fuel injection ceases i.e. at 7% of the stroke. The value of the thrust at this instant will be more than 2,880 kg, the excess will be of the order of 3 to 4%. In order to reduce the arithmetical and trignometrical calculation work we adopt another procedure. The material of the connecting rod is carbon steel for which the minimum allowable stress is 800 kg/sq cm. We design the connecting rod shank for a load of 2,880 kg reducing the value of the minimum allowable stress to 700 kg/sq cm. Thus we have indirectly considered the effect of increase in thrust in the rod due to obliquity of the connecting rod. We adopt I section having proportions as shown in Fig. 10-4(*b*). If t cm be the thickness of the web as well as flange, then area of the section will be $11\,t^2$ sq cm and radius of gyration will be $\sqrt{3{\cdot}18}\,t$ cm.

According to Rankine's formula, we have

$$2880 = \frac{11t^2 \times 700}{1 + \left[\frac{1}{7500} \times \frac{35^2}{3{\cdot}18t^2}\right]}$$

Solving the above equation we get $t = 0{\cdot}85$ cm.

We adopt section as web and flange of thickness 8·5 mm, flange width as 3·4 cm and the depth of section as 4·3 cm. This will be the section at the middle of the rod.

Big end bearing: The ratio of the length to diameter for big end varies from 1·25 to 1·5 and the permissible bearing pressure intensity varies from 50 to 100 kg/sq cm. We adopt $\frac{l}{d}$ ratio to be 1·3 and permissible bearing pressure intensity as 70 kg/sq cm.

We get $2880 = d \times 1{\cdot}3d \times 70$

or $$d = \sqrt{\frac{2880}{1{\cdot}3 \times 70}} = 5{\cdot}62 \text{ cm.}$$

We adopt d as 6 cm and length of the pin in the bearing as 7·5 cm.

Small end: The ratio of length to diameter for small end is usually about 1·5 to 2 and the bearing pressure intensity varies from 100 to 150 kg/sq cm. We adopt the length of the pin in small end as 5·5 cm and diameter of the pin as 3·5 cm. The $\frac{l}{d}$

$d' = \sqrt[4]{\dfrac{1{\cdot}273T^2}{pf}}$ where T is the thrust in the connecting rod, p is the permissible bearing pressure intensity and f is the permissible flexural stress intensity,

On substitution of values we get

$$d' = \sqrt[4]{1{\cdot}273 \times \frac{1760}{400} \times \frac{1760}{100}}$$

$= 3{\cdot}14$ cm; we adopt 32 mm.

We assume that there are two cap bolts. Load on each bolt $= \dfrac{1760}{2} = 880$ kg

Core area of the bolt $= \dfrac{880}{400} = 2{\cdot}2$ sq cm We adopt fine threads $M20 \times 1\ 5$ or we can adopt M 20 bolt of uniform strength the latter will be preferable as the connecting rod is subjected to shock loading

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Design a suitable connecting rod for the given duty.

The shanks of the connecting rods are made with an I section in medium and high speed engines The cross section being equal, I section shanks possess a higher section modulus in connecting rod swinging plane than do round shanks.

The connecting rod shank is subjected to the force of gas pressure and the force of inertia of the reciprocating masses, in addition to transverse inertia stresses. The gas force is maximum at inner dead centre position when the transverse inertia forces are zero and the maximum bending moment caused by the transverse inertia forces is originated *when the connecting rod is positioned at 90° to the crank for the section located at a distance of 0·577 l from the centre of the connecting rod small end.*

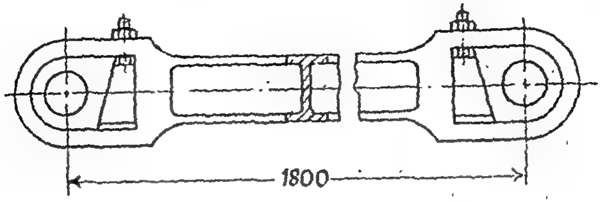
For higher reliability the connecting rod shank is calculated only for the gas-force, neglecting the inertia force.

$$\text{Gas pressure at inner dead centre} = \frac{\pi}{4} \times 9^2 \times 45$$

$$= 2{,}880 \text{ kg.}$$

180 cm. Materials used are brass for bearings and forged steel for other parts.

Assume your own data for stresses and factor of safety.



Locomotive coupling rod
FIG. 18-13

3. *A locomotive coupling rod 2·4 metre long between centres is of uniform section 9 cm × 4 cm. Find the maximum stress in the rod. The crank radius is 30 cm and it makes 200 r.p.m. Diameter of the cylinder is 45 cm and the steam pressure is 10 kg/sq cm. Assume that the thrust in the rod is as half of the load on the piston and the density of the material of the rod as 7·8 gm/cu cm.*

4. *Design and draw the connecting rod of a petrol engine having the following specifications:*

Piston diameter 120 mm, weight of reciprocating parts 2·5 kg, length of rod centre to centre 380 mm; stroke 150 mm, normal speed 2,000 r.p.m. and overspeed 2,800 r.p.m., compression ratio 5·5 and probable maximum explosion pressure 30 kg/sq cm.

5. *A forged steel connecting rod is to be designed for a high speed Diesel Engine operating at 1,800 r.p.m. The maximum gas load can be taken as 5,000 kg. The length of the connecting rod is 240 mm and crank radius 60 mm. The maximum stresses are to be limited to 1,200 kg/sq cm. The gudgeon pin diameter is 45 mm and crank pin diameter is 75 mm.*

The section of the connecting rod is I section having flange width = 4·5t and height 6t where t is the thickness of the flange and web. Determine the section of the rod and nominal diameter of bolts presuming the gas load to occur at top dead centre.

Bolt material: Alloy steel En. 24 having a permissible stress value of 800 kg/sq cm.

(Bombay University, 1965)

ratio will be $\frac{5 \cdot 5}{3 \cdot 5} = 1 \cdot 57$ and the bearing pressure intensity as $\frac{2880}{3 \cdot 5 \times 5 \cdot 5} = 150$ kg/sq cm.

Cap bolts:

The connecting rod small end has one piece construction; the big end is provided with split steel backed babbitt lined shells. We assume two bolts of alloy steel for which the allowable tensile stress is 900 to 1,000 kg/sq cm In four stroke cycle engine the bolts are subjected to inertia of reciprocating masses. We assume that the engine is likely to over speed upto 2,000 r.p.m.

$$\text{Inertia force} = \frac{3 \times 7}{981}\left[\frac{2000 \times 2}{60}\right]^2 \left(1 + \frac{1}{5}\right)$$

$$= 1{,}130 \text{ kg.}$$

$$\text{Core area of the bolt} = \frac{1130}{2 \times 900} = 0 \cdot 63 \text{ sq cm.}$$

We adopt *M*12 having 0·843 sq cm of core area.

The whipping stresses can be checked in a manner similar to example 1 on page 750.

The illustrative examples showing the principles of connecting rod design are from pages 235 to 236 and pages 421 to 423.

Exercises:

1. The critical section of a connecting rod for a Diesel Engine is at a [illegible] th distance from the gudgeon pin It is an I section having the following proportions:

Width of the flange 4 cm; thickness of the flange 4 mm; depth of the section 5 cm and thickness of the web 1 cm

The diameter and stroke of the piston are 12 cm and 22 cm respectively and the length of the connecting rod is 45 cm. The maximum gas pressure in the cylinder = 47 kg/sq cm. The speed of the engine is 1,200 r.p.m. Check whether the factor of safety is sufficient or not.

2. Design a shunting locomotive coupling rod of the type as shown in fig. 18-13.

The maximum static draw bar pull or push is 4,000 kg. Maximum speed of locomotive 80 km/hour, crank length 30 cm and length of the rod

Allowable stress for cap 800 *kg/sq cm*
Allowable stress in bolts 400 *kg/sq cm.*

(Bombay Univerity, 1965)

11. A locomotive coupling rod has a rectangular section of 8 cm × 4 cm. It is 1·05 metre long. The cranks connected by the rod have a radius of 25 cm and the speed 300 r.p.m. The maximum power developed is 1,200 H.P. and is applied to the rear axle. Determine,

(i) the stresses in the rod due to useful effort
(ii) the stresses due to inertia of the rod
(iii) the maximum resultant stress.

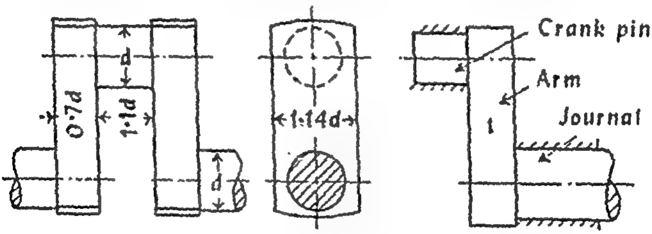
(Gujarat University, 1966)

18-5. Design of crankshafts:

The design of overhung crank is explained in articles 12-9 and 12-10. The proportions for the forged crankshaft are shown in fig. 18-14. Here, we shall consider illustrative examples to explain the main principles involved in the design.

Examples:

1. Fig. 18-15 shows the overhung crank of an engine having the cylinder diameter 25 cm and stroke 40 cm. The connecting rod is 5 cranks



Centre crank
Fig. 18-14

Overhung crank
Fig. 18-15

long. The maximum explosion pressure in the cylinder is 17 kg/sq cm, and engine runs at 200 r.p.m. Assuming suitable stresses for the material, design the overhung crank. Also, calculate the maximum stress in the crank arm when the crank is at 30° to the i.d.c. position and the gas pressure at this instant is 8 kg/sq cm. Take the modulus of section for torsion for rectangular section as 0·269 bt².

6. Design the big end of a marine type of connecting rod for a vertical steam engine for the following data:

Load along the rod 4,500 kg, and the length of the connecting rod measures 100 cm between crank pin and gudgeon pin centres.

Choose the suitable value for the $\frac{l}{d}$ ratio for the crank pin Also select the suitable materials for the connecting rod and cap bolts

7. Explain why in the design of the cross section of the connecting rod I sectioon is preferred. Why proportions of the section are adopted as 5t for the depth 4t as the width and t as the thickness of the flange as well as of the web?

8. Check the suitability of the I section 13 mm × 12 mm × 3 mm for designing the connecting rod in the case of single cylinder motor cycle engine of 40 mm bore by 55 mm stroke where in the maximum pressure produced is 36 kg/sq cm and in which the connecting rod is four times the crank. Draw a neat sketch of the connecting rod

9. State the various forces which affect the design of the connecting rod, and explain their influence choosing the suitable materials for various parts and adopting suitable stress values Design a connecting rod of I section for a Diesel Engine running at 1,500 r p.m. Bore of the engine is 12 cm and stroke is 15 cm Length of the connecting rod is 35 cm Maximum pressure is 50 kg/sq cm which is constant upto 5% of the stroke. Weight of reciprocating parts is 0 15 kg/sq cm of piston area. Law of expansion is $PV^{1.25}$ = constant

Also check the design for transverse inertia stresses

10. Explain the design features of a connecting rod

The following particulars relate to a four stroke cycle Diesel vertical engine:

Cylinder bore 7·5 cm; stroke 10 5 cm; length of connecting rod 21 cm, maximum pressure which is constant upto 6% of the stroke 45 kg/sq cm; speed 2,000 r.p.m.; weight of reciprocating parts 0·04 kg/sq cm of piston area and law of expansion $PV^{1.3}$ = constant.

Design and prepare a working sketch of a nickel steel connecting rod with cap and big end bolts. Assume additional data if necessary.

Safe stress for nickel steel 750 kg/sq cm

Safe bearing stress for gudgeon pin 75 kg/sq cm

Thickness of crank arm cm	5	6	7	7·5
Eccentricity cm	$\frac{10\cdot5+5}{2}=7\cdot75$	$\frac{10\cdot5+6}{2}=8\cdot25$	$\frac{10\cdot5+7}{2}=8\cdot75$	$\frac{10\cdot5+7\cdot5}{2}=9$
Bending moment kg cm	66,800	69,000	73,000	75,000
Bending stress kg/sq cm	$\frac{16000}{b}$	$\frac{11500}{b}$	$\frac{8960}{b}$	$\frac{7970}{b}$
Direct stress kg/sq cm	$\frac{1670}{b}$	$\frac{1390}{b}$	$\frac{1195}{b}$	$\frac{1150}{b}$
Resultant stress kg/sq cm	$\frac{17670}{b}$	$\frac{12890}{b}$	$\frac{10055}{b}$	$\frac{9020}{b}$
Width of the crank cm	20·8	15·5	11·8	10·5
Area of cross section sq cm	108	93	82·7	79

We adopt the section for the crank arm as 7 cm × 12 cm. We can also adopt 7·5 cm × 10·5 cm.

Resolving T into tangential and radial components T_t and T_r respectively, we have

$$T_t = T\sin\ (\alpha + \theta) = 3960\ \sin (35° - 42') = 2{,}305\ \text{kg.}$$
$$T_r = T\cos\ (\alpha + \theta) = 3960\ \cos (35° - 42') = 3{,}220\ \text{kg.}$$

The critical section of the crank, which is at the hub, is subjected to bending and torsion due to tangential component of the magnitude 2,305 kg and bending and direct compression due to radial component of the magnitude 3,220 kg.

$$\text{Direct compressive stress} = \frac{3220}{7 \times 12} = 38\cdot4\ \text{kg/sq cm.}$$

Modulus of section for bending due to tangential component will be $\frac{1}{6} \times 12^2 \times 7 = 168\ \text{cm}^3$, while for bending due to radial component will be $\frac{1}{6} \times 7^2 \times 12 = 98\ \text{cm}^3$.

Assuming the lever arm of the crank web extends upto the axis of the shaft, maximum bending stress due to tangential component will be $\dfrac{2305 \times 20}{168} = 275$ kg/sq cm.

Load on piston $= \frac{\pi}{4} \times 25^2 \times 17 = 8{,}350$ kg.

We assume the permissible bending stress for the pin to be 100 kg/sq cm and bearing stress 840 kg/sq cm. According to article 12·10, the length to diameter ratio for the crank pin is given as $\sqrt{\frac{0.2 \times f_b}{p}} = \sqrt{\frac{0.2 \times 840}{100}} = 1.30.$

The projected bearing area $= 1.3d^2$

$\therefore\quad 8350 = 1.3d^2 \times 100$

or $\quad d = \sqrt{\frac{8350}{1.3 \times 100}} = 8$ cm.

Length of the crank pin $= 8 \times 1.3 = 10.4$ cm, we adopt 10·5 cm, thus providing $8 \times 10.5 = 84$ sq cm projected area.

We assume the crank arm thickness to be $0.6d = 0.6 \times 8 = 4.8$ cm; we adopt 5 cm. Let us determine the dimensions of the crank web when the crank is at its inner dead centre position As explained in art. 12-9, in this position the web section is subjected to direct compressive stress accompanied by bending stresses. The eccentricity of the load is equal to half the sum of the length of the crankpin and the thickness of web

Let b be the width of the crank web.

Here we shall take four values of the crank arm thicknesses and determine the corresponding values of the width of the web and adopt those dimensions which will give least area of cross section thus providing lighter crank We assume permissible stress as 850 kg/sq cm The calculations are shown in tabular form as on page 762.

When the crank has turned through an angle $\theta = 30°$ from inner dead centre, the inclination, of the connecting rod, α, to the line of stroke is given as

$\alpha = \sin^{-1}\frac{\sin\theta}{n}$ where n is the ratio of the length of connecting rod to crank

$\therefore\quad \alpha = \sin^{-1}\frac{\sin 30°}{5} = 5° - 42'.$

The axial thrust T in the connecting rod $= \dfrac{\frac{\pi}{4} \times 25^2 \times 8}{\cos(5° - 42')}$
$= 3{,}960$ kg.

Tangential component of this thrust equals
$9800 \sin(24° - 18' + 5° - 54') = 4{,}930$ kg.

Radial component of maximum thrust equals
$9800 \cos(24° - 18' + 5° - 54') = 8{,}460$ kg.

Let us calculate the maximum thrust in the connecting rod of the compressor. The crank for the compressor is inclined at an angle of $(90° - 24° - 18') = 65° - 42'$ to the compressor cylinder centre line. The connecting rod for the compressor is inclined at an angle $\sin^{-1} \dfrac{\sin(65° - 42')}{4} = 12° - 7'$ to the cylinder centre line.

Maximum thrust in the compressor connecting rod equals
$\dfrac{780}{\cos(12° - 7')} = 800$ kg.

Tangential component of this thrust equals
$800 \sin(65° - 42' + 12° - 7') = 782$ kg.

Radial component of maximum thrust is equal to
$800 \cos(65° - 42' + 12° - 7) = 156$ kg.

$\dfrac{l}{d}$ ratio for crank pin lies between 0·7 to 1·2. We adopt it as 0·84. Maximum thrust in the connecting rod is 9,800 kg. Projected area at the crank pin is $0{\cdot}84d^2$. The bearing pressure for the crank pin lies between 105 to 120 kg/sq cm. We assume it to be 120 kg/sq cm.

$\therefore \quad 0{\cdot}84d^2 \times 120 = 9800$

or $$d = \sqrt{\frac{9800}{0{\cdot}84 \times 120}} = 10 \text{ cm.}$$ The length of the pin will be 8·5 cm.

The thickness of the web = 0·64 times the crank pin diameter.
$= 0{\cdot}64 \times 10 = 6{\cdot}4$ cm; we adopt 6·5 cm.

The length of the main bea[illegible] 0·6 t[illegible] the cylinder bore. We take the length o[illegible] bea[illegible]

As two crank[illegible] [illegible]h of the pin will be 8·5 [illegible]

Bending stress due to radial component $= \dfrac{3220 \times 17.5}{98 \times 2}$
$= 288$ kg/sq cm

The total stress f at the upper left corner of the crank is given by

$f = 275 + 288 + 38.4 = 601.4$ kg/sq cm.

The twisting moment on the section $= 2305 \times 8.75$
$= 20,200$ kg cm.

Torsional modulus of section $= 0.269 \times 12 \times 7^2 = 159$ cm³.

Maximum torsional shear stress at the middle of the long side $= \dfrac{20200}{159} = 128$ kg/sq cm

Torsional shear stress is zero at all the corners of the section.

Maximum stress $= 601.4$ kg/sq cm at the upper left corner of the crank

2. Connecting rods of a Diesel engine and a compressor are arranged to run on the same crank pin The maximum torque is developed when the crank has turned through 24° — 18' from the inner dead centre position of the engine. (Refer fig 18-16) The centre lines of the engine and the compressor are at right angles to each other with the compressor cylinder in the horizontal position

The following particulars refer to this combination unit

Diameter of the engine cylinder 175 mm, diameter of compressor cylinder 190 mm, stroke 200 mm; length of connecting rod 400 mm and speed in r p m 1,000

Maximum gas loads on the pistons at the maximum torque position are·

Engine piston 9,750 kg and compressor piston 780 kg

Suggest the suitable crankshaft for the combination unit.

The ratio of the length of connecting rod to crank is $\dfrac{16}{4} = 4$.

The inclination φ of the connecting rod of the engine to the cylinder centre line is given by

$$\varphi = \sin^{-1} \frac{\sin (24° - 18')}{4} = 5° - 54'.$$

Maximum thrust in the engine connecting rod $= \dfrac{9750}{\cos (5° - 54')}$
$= 9,800$ kg.

As the reaction at bearing No. 1 is greater, it is considered in further calculations.

Resultant reaction at $1 = \sqrt{3290^2 + 5038^2} = 5{,}940$ kg.

Let us consider the bending moment at the engine crank pin.

B.M. in plane of crank $= 5038 \times 17 = 86{,}000$ kg cm.

B.M. in perpendicular plane $= 3290 \times 17 = 56{,}000$ kg cm.

$\therefore$ Resultant B.M. $= \sqrt{86000^2 + 56000^2} = 102{,}000$ kg cm.

Torque produced due to 3,290 kg component of reaction equals $3290 \times 10 = 32{,}900$ kg cm.

Equivalent torque according to Guest formula

$$= \sqrt{102000^2 + 32900^2} = 107{,}500 \text{ kg cm.}$$

The permissible value of shear stress for nickel steel lies between 600 to 750 kg/sq cm. Let us adopt the value of shear stress as 700 kg/sq cm.

If d be the diameter of the crank pin, then

$$\frac{\pi}{16} d^3 \times 700 = 107500$$

or
$$d = \sqrt[3]{\frac{107500}{700} \times \frac{16}{\pi}} = 9{\cdot}2 \text{ cm.}$$

We have adopted 10 cm. Therefore the design is safe. The width of the web of the crank arm is taken as 13 cm.

Note: The stress calculation for the web is left as an exercise for the student. Also refer example 2 on page 65.

Exercises:

1. *The crank pin of a forged side crank is acted upon by a force of 2,500 kg at right angles to the crank. The length of the crank is 15 cm. The distance of the plane of rotation of the centre of the crank pin from the centre of the adjacent bearing is 20 cm.*

Safe bearing pressure for pin, 70 kg/ ... afe ... ressure for the crankshaft journal, 15 kg/sq cm; perm ... stress intensities for crank material are 700 and 60 ... and give two views of the same. Mark importa ...

2. *A double ac ... engine ... and is directly connected t ... or thro ... the engine is 15 cm ... ng rod occurs at 60° from i ... whe 1,800 kg. A flywhe ... 180 k is situated centrally b ... beari*

Reactions in the plane of tangential component:

Bearing 1	Bearing 2
Engine:	
$R_1 = \dfrac{4930 \times 26}{43} = 2{,}980$ kg. $\leftarrow$	$R_2 = 4930 - 2980 = 1{,}950$ kg
Compressor:	
$R_1 = \dfrac{782 \times 17}{43} = 310$ kg. $\leftarrow$	$R_2 = 782 - 210 = 472$ kg.
Resultant $R_1 = 2980 + 310 = 3{,}290$ kg.	$R_2 = 1950 + 472 = 2{,}422$ kg.

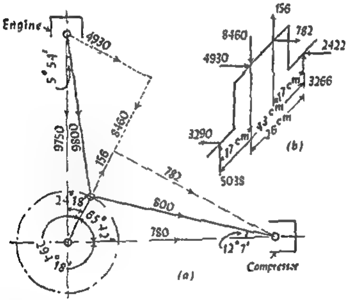


Fig. 18-16

Reactions in the plane of radial component:

Engine:	
$R_1 = \dfrac{8460 \times 26}{43} = 5{,}100$ kg. $\uparrow$	$R_2 = 8460 - 5100 = 3{,}360$ kg.
Compressor:	
$R_1 = \dfrac{156 \times 17}{43} = 62$ kg.	$R_2 = 156 - 62 = 94$ kg.
Resultant:	
$R_1 = 5100 - 62 = 5{,}038$ kg.	$R_2 = 3360 - 94 = 3{,}266$ kg.

As the reaction at bearing No. 1 is greater, it is considered in further calculations.

Resultant reaction at 1 $= \sqrt{3290^2 + 5038^2} = 5{,}940$ kg.

Let us consider the bending moment at the engine crank pin.

B.M. in plane of crank $= 5038 \times 17 = 86{,}000$ kg cm.

B.M. in perpendicular plane $= 3290 \times 17 = 56{,}000$ kg cm.

$\therefore$ Resultant B.M. $= \sqrt{86000^2 + 56000^2} = 102{,}000$ kg cm.

Torque produced due to 3,290 kg component of reaction equals $3290 \times 10 = 32{,}900$ kg cm.

Equivalent torque according to Guest formula

$$= \sqrt{102000^2 + 32900^2} = 107{,}500 \text{ kg cm.}$$

The permissible value of shear stress for nickel steel lies between 600 to 750 kg/sq cm. Let us adopt the value of shear stress as 700 kg/sq cm.

If d be the diameter of the crank pin, then

$$\frac{\pi}{16} d^3 \times 700 = 107500$$

or $$d = \sqrt[3]{\frac{107500}{700} \times \frac{16}{\pi}} = 9{\cdot}2 \text{ cm.}$$

We have adopted 10 cm. Therefore the design is safe. The width of the web of the crank arm is taken as 13 cm.

Note: The stress calculation for the web is left as an exercise for the student. Also refer example 2 on page 65.

Exercises:

1. The crank pin of a forged side crank is acted upon by a force of 2,500 kg at right angles to the crank. The length of the crank is 15 cm. The distance of the plane of rotation of the centre of the crank pin from the centre of the adjacent bearing is 20 cm.

Safe bearing pressure for pin, 70 kg/sq cm; safe bearing pressure for the crankshaft journal, 15 kg/sq cm; permissible tensile and shear stress intensities for crank material are 700 and 600 kg/sq cm. Design the crank and give two views of the same. Mark important dimensions on your sketch.

2. A double acting steam engine developes 25 h.p. at 450 r.p.m. and is directly connected to a generator through a flexible coupling. The stroke of the engine is 15 cm and connecting rod 70 cm long. The maximum torque occurs at 60° from i.d.c. position when the thrust of the connecting rod is 1,800 kg. A flywheel weighing 180 kg forms one half of the coupling and is situated centrally between main bearings of the engine and the generator.

The main bearings of the crankshaft are situated at 15 and 16 cm from the centre line of the cylinder. The eccentric is situated outside the main bearing and requires a space of 5 cm.

Design the crankshaft if the permissible shear stress is 300 kg/sq cm. Balance weights may be omitted. Assume suitable bearing pressures. Draw a dimensioned sketch and indicate lubricating arrangements.

3. Design an overhung crank for a 200 mm bore and 350 mm stroke steam engine. Steam pressure at inlet is 12 kg/sq cm gauge. For maximum torque the crank makes an angle of 77° with the top dead centre and in this position the steam pressure is 10 kg/sq cm. Allowable tensile stress in the shaft 800 kg/sq cm. Safe tensile stress in the crank pin 630 kg/sq cm. Maximum stress allowed in the web is 850 kg/sq cm. The length of the connecting rod is 4·5 times the crank radius.

Indicate an efficient lubrication system for the above crank in the sketch.
(University of Rajasthan, 1969)

18-6. Design of a spring-loaded Hartnell Governor:

Introduction: The function of a governor is to regulate the mean speed of a machine or prime mover, or to keep to mean speed within certain limits, the limits of variation depends on the nature of the work which the machine or prime mover has to do. The limits of variation of mean speed also depends on the sensitiveness of the governor.

A well known form of a spring-loaded governor, designed by Mr. Wilson Hartnell of Leeds is shown in fig. 12-18. Two bell crank levers *L* are mounted on pins *I*, carried by a frame *A*, which is attached to a rotating spindle *S*. Each lever carried a ball *B* at the end of one arm and a roller *R* at the end of the other. The centrifugal forces of the balls cause the rollers *R* to press against the collar *C* on the sleeve *E* The upward pressure of the rollers on the collar of the sleeve is balanced by the downward thrust of the helical spring *H* which is in compression. The angle of bell crank levers is 90° but in practice it may be greater. Let us consider the important points in the design of main parts of the governor.

(a) Spring design:

In order to design the spring, the maximum spring force and the stiffness of the spring should be calculated. The relation between the dimensions of the governor, equilibrium speed and

As the reaction at bearing No. 1 is greater, it is considered in further calculations.

Resultant reaction at $1 = \sqrt{3290^2 + 5038^2} = 5{,}940$ kg.

Let us consider the bending moment at the engine crank pin.

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Equivalent torque according to Guest formula

$$= \sqrt{102000^2 + 32900^2} = 107{,}500 \text{ kg cm.}$$

The permissible value of shear stress for nickel steel lies between 600 to 750 kg/sq cm. Let us adopt the value of shear stress as 700 kg/sq cm.

If d be the diameter of the crank pin, then

$$\frac{\pi}{16} d^3 \times 700 = 107500$$

or

$$d = \sqrt[3]{\frac{107500}{700} \times \frac{16}{\pi}} = 9{\cdot}2 \text{ cm.}$$

We have adopted 10 cm. Therefore the design is safe. The width of the web of the crank arm is taken as 13 cm.

Note: The stress calculation for the web is left as an exercise for the student. Also refer example 2 on page 65.

Exercises:

1. *The crank pin of a forged side crank is acted upon by a force of 2,500 kg at right angles to the crank. The length of the crank is 15 cm. The distance of the plane of rotation of the centre of the crank pin from the centre of the adjacent bearing is 20 cm.*

Safe bearing pressure for pin, 70 kg/sq cm; safe bearing pressure for the crankshaft journal, 15 kg/sq cm; permissible tensile and shear stress intensities for crank material are 700 and 600 kg/sq cm. Design the crank and give two views of the same. Mark important dimensions on your sketch.

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The main bearings of the crankshaft are situated at 15 and 16 cm from the centre line of the cylinder. The eccentric is situated outside the main bearing and requires a space of 5 cm.

Design the crankshaft if the permissible shear stress is 300 kg/sq cm. Balance weights may be omitted. Assume suitable bearing pressures. Draw a dimensioned sketch and indicate lubricating arrangements.

3. *Design an overhung crank for a 200 mm bore and 350 mm stroke steam engine. Steam pressure at inlet is 12 kg/sq cm gauge. For maximum torque the crank makes an angle of 77° with the top dead centre and in this position the steam pressure is 10 kg/sq cm Allowable tensile stress in the shaft 800 kg/sq cm. Safe tensile stress in the crank pin 630 kg/sq cm. Maximum stress allowed in the web is 850 kg/sq cm. The length of the connecting rod is 4·5 times the crank radius.*

Indicate an efficient lubrication system for the above crank in the sketch.
(University of Rajasthan, 1969)

18-6. Design of a spring-loaded Hartnell Governor:

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(a) Spring design:

In order to design the spring, the maximum spring force and the stiffness of the spring should be calculated. The relation between the dimensions of the governor, equilibrium speed and

the spring load may be calculated by considering the equilibrium of the bell crank lever.

Let W be the weight of each ball, S the spring force exerted on the sleeve, k the stiffness of the spring, ω the speed of rotation, r the radius of rotation, a and b the vertical and horizontal arms of the bell crank levers and F the centrifugal force on the ball.

By taking moments about the fulcrum of the bell crank lever, neglecting the effect of pull of gravity on the governor balls and arms, we get

$$F.a = \frac{S}{2}\,b \text{ or } S = 2F\frac{a}{b} \quad \text{...(i)}$$

Let the suffixes 1 and 2 denote the values at maximum and minimum radii respectively. Then, at maximum radius

$$S_1 = 2\,F_1\frac{a}{b} \quad \text{...(ii)}$$

At minimum radius

$$S_2 = 2\,F_2\frac{a}{b} \quad \text{...(iii)}$$

$$\therefore \quad S_1 - S_2 = 2\,\frac{a}{b}\,(F_1 - F_2).$$

The lift of the sleeve $h = \frac{b}{a}\,(r_1 - r_2)$.

$$\therefore \quad k = \frac{(S_1 - S_2)}{h} = 2\left(\frac{a}{b}\right)^2 \frac{F_1 - F_2}{r_1 - r_2} \quad \text{...(iv)}$$

Thus, the maximum spring force S_1 coming on the spring is known, from which the diameter of the spring wire can be calculated and from the stiffness number, the number of active turns can be calculated.

We assume the suitable value for the spring index C and determine A.M. Wahl correction factor K, given by

$$K = \frac{4C - 1}{4C - 4} + \frac{0{\cdot}615}{C},$$ to account for stress concentration.

Higher the spring index, less will be the number of turns and lower will be the stress concentration factor. From the value of the permissible stress, the value of stress to be used in torque formula is determined. We determine the value of the maximum torque on the spring wire and determine d_w, diameter of the wire, from the equation,

$S_1 \times$ mean radius of the coil $= \frac{\pi}{16} d_w^3 f_s$ ....(v)

We select the diameter of the wire from SWG table or IS: 1137-1959 and calculate the new value of the spring index. The number of active turns n can be calculated from,

$$k = \frac{G\, d_w}{8C^3 n} \quad \text{(vi)}$$

where G is the modulus of rigidity.

Free length of the spring $= (n+1)\, d_w +$ compression $+$ gap clearance ....(vii)

Generally the spring index is taken to be 8

(b) Design of a spindle:

It is subjected to a direct tensile load which is equal in magnitude to the maximum spring force S_1. *It is also subjected to a torque during starting and during load changes*. This torque induces torsional shear stresses in the spindle. As the magnitude of the torque is indeterminate, we take the low value of the tensile stress in the spindle and design for the axial spring load S_1.

(c) Design of a cast steel body:

It is used as the casing for the spring and supports for the fulcrum of bell crank levers. The inner diameter of the casing is slightly larger than the outside diameter of the coil. The thickness of the body is fixed from minimum requirement for casting, which may be taken to be 6 mm. The body is subjected to direct tensile stresses from the maximum spring load S_1 and to centrifugal stresses due to rotation of the body. If we check the body for the stresses induced, the stresses will be within safe limits.

(d) Design of a bell crank lever:

(i) *Roller end*: The maximum load on the roller is $\frac{S_1}{2}$.

The relation between the width and the diameter of the roller is:

width of roller $= \frac{1}{2}$ diameter of the roller $+ 4$ mm

The bearing stress per unit width of roller must not exceed 90 kg/cm width.

The initial trial value for the roller diameter may be taken as twice the pin diameter. The pin diameter must be sufficient to limit the shear and bending stresses. The roller is free to rotate

on the pin fixed in the sides of the fork and the details of the fork end can be completed. The lever arm can be designed from the bending consideration adopting a rectangular section. The thickness of the lever is kept constant as we go from roller end to fulcrum end and the height of the section is varied.

(ii) *Design of the fulcrum pin:* The maximum load on the fulcrum pin is determined, and assuming $\frac{l}{d}$ ratio as 1·2 and bearing pressure as 70 kg/sq cm, the dimensions of the pin are calculated. The pin is checked for bending and shear. Finally, the dimensions for the boss of the lever are finalised.

(iii) *Design of the governor balls:* The governor balls are made of cast iron and may be spherical or cylindrical. The radius of the spherical ball can be calculated by the equation $W = \frac{4}{3}\pi r^3 \times$ density.

(iv) *Design of the end of the ball arm:* The ball is screwed into the lever end, which is subjected to a bending moment which is equal to the product of the maximum centrifugal force F_1 and the radius of the spherical ball. By equating the bending moment to the moment of resistance, we shall get the diameter at the bottom of the threaded part. We adopt metric threads and fix the dimensions for the screwed end of the ball arm.

Example:

1. *The pivots of the bell crank levers of a spring loaded governor of Hartnell type are fixed at 95 mm radius from the spindle axis. The length of the ball arm of each lever is 150 mm; the length of the sleeve arm is 75 mm and two arms are at right angles. The weight of each ball is 2 kg. The equilibrium speed in the lowest position of the governor is 300 r.p.m. when the radius of rotation of the ball path is 82 mm. The speed is to be limited to 6% more than the lowest equilibrium speed. The lift of the sleeve for, the operating speed range, is 16 mm. Design the suitable spring and bell crank lever for the governor. Choose your own values for the stresses.*

The lift of the sleeve is 16 mm. The change in radius during the operating range of speed equals

$$\frac{\text{lift} \times \text{length of the ball arm}}{\text{length of the roller arm}} = 16 \times \frac{150}{75} = 32 \text{ mm.}$$

The minimum radius of the ball path = 82 mm. The maximum radius of the ball path = 82 + 32 = 114 mm.

$S_1 \times$ mean radius of the coil $= \frac{\pi}{16} d_w^3 f_s$ ....(v)

We select the diameter of the wire from SWG table or IS: 1137-1959 and calculate the new value of the spring index. The number of active turns n can be calculated from,

$k = \frac{G\, d_w}{8C^3 n}$ ....(vi)

where G is the modulus of rigidity.

Free length of the spring $= (n+1)\, d_w +$ compression $+$ gap clearance ....(vii)

Generally the spring index is taken to be 8.

(b) Design of a spindle:

It is subjected to a direct tensile load which is equal in magnitude to the maximum spring force S_1. *It is also subjected to a torque during starting and during load changes* This torque induces torsional shear stresses in the spindle. As the magnitude of the torque is indeterminate, we take the low value of the tensile stress in the spindle and design for the axial spring load S_1.

(c) Design of a cast steel body:

It is used as the casing for the spring and supports for the fulcrum of bell crank levers. The inner diameter of the casing is slightly larger than the outside diameter of the coil The thickness of the body is fixed from minimum requirement for casting, which may be taken to be 6 mm The body is subjected to direct tensile stresses from the maximum spring load S_1 and to centrifugal stresses due to rotation of the body. If we check the body for the stresses induced, the stresses will be within safe limits.

(d) Design of a bell crank lever:

(i) *Roller end:* The maximum load on the roller is $\frac{S_1}{2}$.

The relation between the width and the diameter of the roller is.

width of roller $= \frac{1}{2}$ diameter of the roller $+ 4$ mm.

The bearing stress per unit width of roller must not exceed 90 kg/cm width.

The initial trial value for the roller diameter may be taken as twice the pin diameter. The pin diameter must be sufficient to limit the shear and bending stresses. The roller is free to rotate

Free length of the spring $= 7{\cdot}15 \times 0{\cdot}8229 + 2{\cdot}84 + 1{\cdot}6$ + clearance

We adopt the spring of 11 cm free height.

The outside diameter of spring coil is $7 + 0{\cdot}8229 = 7{\cdot}8229$ cm. The inside diameter of the cast steel body is taken as 9 cm and the thickness of the body as 5 mm. The tensile stresses will be induced in the body due to a spring load of 103·2 kg and due to centrifugal stresses. These stresses will be within safe limits.

The diameter of the spindle is taken as 25 mm.

The maximum load on the roller $= \dfrac{103{\cdot}2}{2} = 51{\cdot}6$ kg. The roller is free to rotate on a pin fixed in the sides of the fork. Let d be the diameter of the pin of the roller and width of the roller is adopted *same as the diameter*. Assuming bearing pressure of 70 kg/sq cm, we see that 1 cm diameter pin will suffice. The width of the roller is also 1 cm, which gives 51·6 kg/cm bearing pressure, which is within safe limits. The diameter of the roller will be 2 cm. The thickness of the eye will be 6 mm. The pin is in double shear and the value of the shear stress will be 32·8 kg/sq cm.

The load on the fulcrum pin $= \sqrt{51{\cdot}6^2 + 25{\cdot}8^2} = 57{\cdot}7$ kg. Assuming the same proportions and stresses as the roller pin, the diameter of the pin will be 12 mm. The boss diameter will be 24 mm. The section of the lever at the boss will be 1 cm × 2·5 cm for a permissible stress of 700 kg/sq cm.

We adopt spherical balls of cast iron, having density as 7·25 gm/cu cm. If r cm be the radius of the ball, then

$$2270 = \tfrac{4}{3}\pi r^3 \times 7{\cdot}25$$

or $$r = \sqrt[3]{\frac{2270 \times 3}{4\pi \times 7{\cdot}25}} = 4{\cdot}27 \text{ cm.}$$

Maximum bending moment on the screwed portion of the lever equals $25{\cdot}8 \times 4{\cdot}27 = 1{,}100$ kg cm.

We adopt *M* 16 threads, having pitch 2 mm, the permissible stress value in the material shall not exceed 700 kg/sq cm.

Note: The student should refer the example 2 on page 498, in which detailed calculations for the bell crank lever for Hartnell governor are shown.

Exercises:

1. In a Hartnell type spring controlled governor ball arms are 130 mm long and sleeve arms are 80 mm long. The ball and sleeve arms are

The centrifugal force at minimum radius equals

$$\frac{2}{981} \times 8\cdot2 \left(\frac{300 \times 2\pi}{60}\right)^2 = 16\cdot5 \text{ kg.}$$

Spring force $S_2 = 2 \times 16\cdot5 \times \frac{150}{75} = 66$ kg.

The centrifugal force at maximum radius equals

$$16\cdot5 \left(\frac{114}{82}\right)(1\cdot06)^2 = 25\cdot8 \text{ kg.}$$

Spring force $S_1 = 2 \times 25\cdot8 \left(\frac{150}{75}\right) = 103\cdot2$ kg.

Stiffness of the spring $k = \frac{103\cdot2 - 66}{1\cdot6} = 23\cdot3$ kg/cm.

Initial compression of the spring $= \frac{66}{23\cdot3} = 2\cdot84$ cm.

We assume the maximum stress for spring material to be 4,000 kg/sq cm.

We adopt the spring index (provisionally) 8.

Stress concentration factor

$$K = \frac{4C-1}{4C-4} + \frac{0\cdot615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0\cdot615}{8} = 1\cdot175.$$

Stress to be adopted for pure torque formula is

$$\frac{4000}{1\cdot175} = 3{,}400 \text{ kg/sq cm}$$

Maximum torque on the spring wire $= 103\cdot2 \times 4d_w = 413d_w$.

$$\therefore \quad 413d_w = \frac{\pi}{16} d_w^3 \times 3400$$

or $$d_w = \sqrt{\frac{413 \times 16}{\pi \times 3400}} = 0\cdot785 \text{ cm.}$$

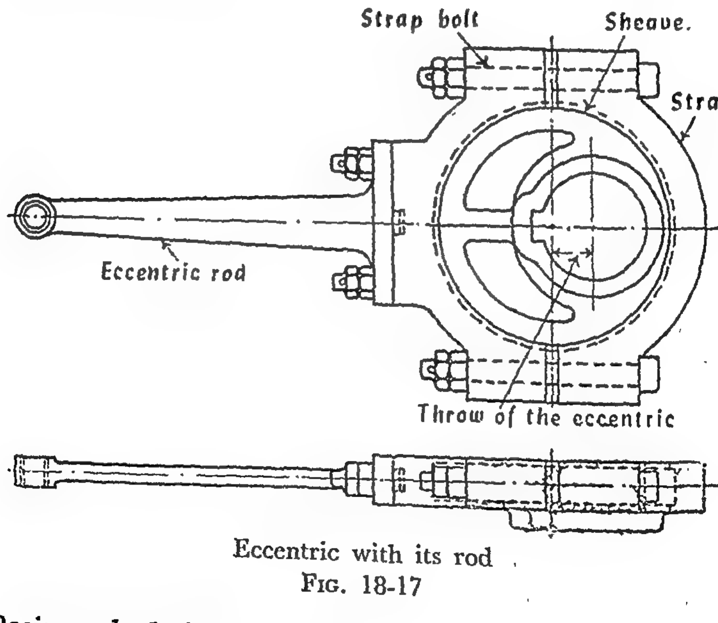
From S.W.G. table, we adopt 8·229 mm diameter, the description number being 0. The mean diameter of spring coil is taken as 70 mm. The spring index will be $\frac{70}{8\cdot229} = 8\cdot5$. If n be the number of active turns, then

$$k = \frac{G\,d_w}{8C^3n}.$$

On substitution of values, we get

$$n = \frac{Gd_w}{8C^3k} = \frac{0\cdot84 \times 10^6 \times 0\cdot8229}{8 \times 8\cdot5^3 \times 23\cdot3} = 6\cdot15 \text{ turns.}$$

The main parts of the eccentric are sheave, straps, eccentric rod and strap bolts (fig. 18-17). The eccentric sheave is made of cast iron and it is made in one piece if the eccentric can be put on over the end of the shaft. Generally, the sheave is made in two parts, which are connected by bolts, studs and cotters. The smaller part of the sheave is made solid and is frequently made of wrought iron or steel. The larger part of the sheave is made with a boss and rim, connected by arms. The sheave is secured to the shaft by a key and sometimes one or more set screws in addition. Eccentric straps are made of cast iron, cast steel or bronze and are bolted by means of mild steel bolts. Flanges are provided to keep straps on place on sheave. Eccentric rods are made of mild steel. When eccentric straps are made of cast iron or brass no liner is necessary, but when made of other materials a brass or white metal liner is essential.



Eccentric with its rod
FIG. 18-17

Design calculations:

The proportions for various parts are based entirely on empirical rules. The most important dimensions are D, the diameter of the sheave, B, the breadth of the sheave and d, the diameter of the strap bolts. The diameter of the sheave should

at right angles to each other. The pivots of the levers are at a distance of 100 mm from the axis of the spindle of the governor. Each ball weighs 4 kg. The sleeve begins to lift at 300 r.p.m. and is to lift 10 mm for 5% increase in speed. The sleeve arm of each lever presses against the governor sleeve, the movement of which is controlled by a spring. The ball centres are vertically above the pivots at the lowest position of the sleeve.

Design a suitable spring, bell crank lever, and spring housing containing the pivots for the bell crank levers.

Make a dimensioned sketch of the governor.

Select your own materials and suitable design stresess for these materials.
(University of Bombay, 1965)

2. A centrifugal governor for a Diesel Engine operates at a rated speed of 750 r.p.m. and is to be designed so as to limit the permanant speed rise to 4% of the rated speed when load is thrown off

The governor weights weigh 300 gm each and can be mounted on bell crank levers. The fulcrum of the bell crank can be spaced at a radius of 6·5 cm The weights are connected to each other by two tension springs

The spindle diameter is 3·5 cm and sleeve movement required is 1 cm Determine the spring specifications and bell crank dimensions and sketch your lay out of the governor.

Use spring steel wire having $G = 8 \times 10^5$ kg/sq cm and safe shear stress 4,000 kg/sq cm.

(University of Bombay, 1965)

18-7. Design of an eccentric:

Introduction: When a radius of a crank motion is so small that the crank pin would not fall outside the crankshaft, the crank is converted into eccentric and the large end of the connecting rod is converted into eccentric strap An eccentric contains both the main shaft and the crank arm. Obviously, a function of an eccentric will be to impart a rather short reciprocating motion from the rotation of a comparatively large shaft. They are used for driving valves, mechanical stokers, small pump plungers, shaking screens, etc. The peculiar characteristic of an eccentric is such that with its help rotary motion can be converted into reciprocating motion, but due to excessive friction between the sheave and the strap reciprocating motion of the slider cannot impart rotary motion to the shaft.

The design equation for the sheave will be

$$F = D\ B\ p \quad \text{(iv)}$$

where p is the permissible bearing pressure intensity between the sheave and the strap, D is the diameter of the sheave and B is the bearing thickness of the sheave.

The strap is designed as a beam simply supported at the strap bolts axes. If L be the distance between the strap bolt axes,

then maximum bending moment on strap $= \frac{FL}{4}$ ....(v)

If t be the thickness of the strap and f the permissible stress intensity for the strap mateiral, then

$$\frac{FL}{4} = \frac{1}{6}\ Bt^2 f$$

or $$t = \sqrt{\frac{1 \cdot 5\ FL}{Bf}} \quad \text{(vi)}$$

The value of f for cast iron is 140 kg/sq cm and from 280 to 560 kg/sq cm for malleable cast iron and gun metal.

The deflection at the centre of the strap should not exceed 0·025 mm.

Deflection at the centre $= \frac{FL^3}{48EI}$ ....(vii)

where E is the modulus of elasticity for the strap material.

Each strap bolt is designed for a load $\frac{F}{2}$. The bolts connecting the eccentric strap and rod should be of the same diameter as those connecting the two halves of the strap.

Eccentric rods are usually very long compared to their transverse dimensions. They are designed as columns hinged at ends according to Euler's formula. The length of eccentric rod is taken as 20 times the throw of the eccentric. The eccentric rods may be circular or rectangular in section. In many cases the centre of the eccentric is not in line with the valve rod, which is always the case of one eccentric at least in Stephenson reversing gear. In such cases the eccentric rod will be subjected to direct axial stresses together with bending stresses. The resultant stresses can be calculated by the methods explained in article 2-16.

The bearing at the small end of the eccentric rod should be designed for a bearing pressure which lies between 420 to 700 kg/sq cm. If the end of the rod be forked, the length of each

be small so as to reduce the velocity of the rubbing surface; as a result the friction will be minimised. The breadth of the sheave should be large to give a low value of the bearing pressure.

$D = 2$ (throw of the eccentric + shaft radius + E)....(i)

where E is thickness of the sheave at the thinnest part, which may be made $\frac{d_1}{6} + 6$ mm to $\frac{d_1}{5} + 8$ mm where d_1 is the diameter of the shaft. If the eccentric is mounted on lay shaft, the value of E will be $\frac{d_1}{3}$.

For a steam engine valve, the throw of the eccentric equals the sum of steam lap and the maximum port opening.

The thickness of the sheave is calculated from bearing considerations. The projected area of the bearing equals DB. The value of the bearing pressure should be low, because the lubrication is not efficient as the diameter of the bearing is large and the length small. For a cast iron strap, the bearing pressure lies between 4.5 to 5·5 kg/sq cm and for lined strap the bearing pressure may be from 7 to 8·5 kg/sq cm

The load on the sheave can be calculated from the duty of the eccentric In case of *D*-slide valve, the force required to work the valve will be the load on the sheave. This force depends on the net steam pressure on the valve, the coefficient of friction between the surfaces of contact and the inertia of the valve mechanism The coefficient of friction between the surfaces of contact may be taken as 0·2. If l be the length and b the breadth of the slide valve, μ the coefficient of friction and p_1 the effective pressure of steam on back of *D*-slide valve, then

$P = \mu\, l\, b\, p_1$ ....(ii)

where P = force required to move the valve against friction

In case of piston valves, the frictional forces are small while the inertia forces may be very large

Inertia force I = mass × acceleration ... . (iii)

The motion of the valve is taken as simple harmonic so the inertia forces will be maximum at the beginning or end of the stroke. While calculating the maximum value of the resultant force F, the proper direction of the inertia force should be taken

If d_1 be the diameter of the solid shaft, then

$$\frac{\pi}{16} d_1^3 \times 560 = 20200$$

or $$d_1 = \sqrt[3]{\frac{20200}{560} \times \frac{16}{\pi}} = 5{\cdot}68 \text{ cm; we adopt 6 cm.}$$

The eccentric sheave is made in two parts, the thickness of the sheave at the thinnest part being $\frac{d_1}{3} = \frac{6}{3} = 2$ cm.

The minimum diameter of the cast iron sheave will be $2\,[2 + 7{\cdot}5 + 3] = 25$ cm.

We adopt brass strap. The permissible value for the bearing stress will be adopted as 8 kg/sq cm.

Width of the sheave will be $= \frac{1034}{8 \times 25} = 5{\cdot}5$ cm.

The other dimensions we get by proportions and are as follows:

Thickness of strap $= 4{\cdot}5$ cm.

Diameter of strap bolts = M27.

Thickness of palm on end of the eccentric rod $= 2{\cdot}5$ cm.

Diameter of circular eccentric rod at strap end $= 5$ cm.

Length of the eccentric rod $= 150$ cm.

By adopting 55 kg/sq cm as the bearing pressure for the small end of the eccentric rod, the length of each fork will be equal to the diameter of the pin, which will be 35 mm.

Note: For the complete design of eccentric rod, please refer illustrative example 1 on page 419. Also, refer illustrative example 3 on page 193 and exercise 5 on page 49.

Exercises:

1. *An eccentric is used to give S.H.M. to a vertical follower system weighing 2 kg. The eccentric runs at 600 r.p.m. and the travel of the follower is 5 cm. The follower is always kept in contact with the eccentric by means of a spring which has a margin of 3 kg i.e. total load on the eccentric is never less than 3 kg. Design a suitable spring. Safe stress 4,200 kg/sq cm.* $G = 0{\cdot}8 \times 10^6$ *kg/sq cm.*

Determine the maximum torque on the eccentric shaft and the size of the shaft if the shear stress is not to exceed 420 kg/sq cm. Neglect bending.

Which of the following materials will you use for (a) the eccentric and (b) the follower face?

fork is equal to the diameter of the pin. When the bearing is in one piece, it is usually 1·4 diameters long.

The following are some of the proportions for eccentric straps and rod:

Thickness of strap	cast iron	$= 0{\cdot}7\,B$ to $0{\cdot}9\,B$
" " "	steel	$= 0{\cdot}5\,B$ to $0{\cdot}6\,B$
" " "	brass	$= 0{\cdot}6\,B$ to $0{\cdot}8\,B$
Diameter of strap bolts, d		$= 0{\cdot}4\,B$ to $0{\cdot}5\,B$

Thickness of palm on end of eccentric rod $= 0{\cdot}45\,B$

Diameter of eccentric rod at strap end $= 0{\cdot}9\,B$ to B

Breadth of rectangular eccentric rod at strap end $= 1{\cdot}3B$ to $1{\cdot}5B$

Thickness " " " " " " " $= 0{\cdot}9B$ to B.

Example:

1. *The eccentric to drive the water pump which pumps water against a head of 120 metre is mounted on the middle of the shaft which is 60 cm long between centres. The bore of the pump is 10 cm and stroke 15 cm. (a) Determine the axial force along the eccentric rod assuming the friction at the gland is 10% of the load on the piston. (b) Determine the torque on the driving shaft assuming the efficiency of the drive to be 60%. (c) Calculate the diameter of the shaft if the permissible shear stress in the shaft is not to exceed 560 kg/sq cm. (d) Suggest the suitable dimensions for the sheave, strap bolts, strap and eccentric rod for the strap end. Choose your own materials for the parts and values of stresses.*

120 meter head = 12 kg/sq cm.

Load on the piston $= \dfrac{\pi}{4} \times 10^2 \times 12 = 940$ kg

As the friction at the gland is 10% of the load on piston, the axial force along the eccentric rod will be $1{\cdot}1 \times 940 = 1{,}034$ kg.

The throw of the eccentric is 7·5 cm

The torque on the shaft $= \dfrac{7{\cdot}5}{2} \times 1034 = 7{,}770$ kg cm.

The efficiency of the drive is 60%, therefore the maximum torque on the shaft will be $\dfrac{7770}{0{\cdot}6} = 12{,}900$ kg cm

Maximum bending moment on the shaft $= \dfrac{1034 \times 60}{4}$

$= 15{,}500$ kg cm.

Equivalent twisting moment $= \sqrt{15500^2 + 12900^2}$

$= 20{,}200$ kg cm.

By equating two areas, we get

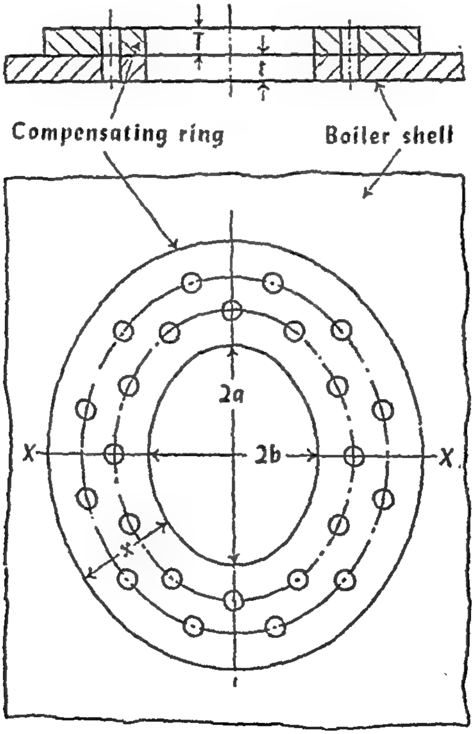
$$2(x-d)T = 2bt$$

or $$x = \frac{bt}{T} + d. \quad \text{(i)}$$

If the thickness of the shell plate be the same as that of the compensating ring.

$$x = b + d. \quad \text{(ii)}$$

The effect of cutting a manhole in the end or flat plate of a boiler is to transfer the load on the manhole to the edge of the hole. In order to strengthen the edge of the hole, the plate is flanged inwards as a result the edge of the hole is stiffened.



Compensating ring for a manhole

FIG. 18-18

Exercise:

1. *Design and sketch a man hole with its compensating ring for a boiler 1·8 metre in diameter working under a pressure of 14 kg/sq cm.*

Cast iron, mild steel, hardened steel, phosphor bronze, hardened nickel steel and nylon.

2. How will you calculate the load coming on the eccentric sheave and strap bolts in case of D slide valve engine? Why the eccentric sheave is made in two parts, if the eccentric were to be mounted between bearings? Are these two parts equal? If not, why? How will you determine the thickness of the strap and width of the strap? State your justifications in using the formula What will be the load in the strap bolts when the eccentric rod is subjected to a compressive load? What are the suitable materials for a sheave of an eccentric? How will you determine the cross sectional dimensions of the eccentric rod? How will you fix the length of the eccentric rod?

3. The pressure of steam in a steam chest is 10 kg/sq cm by gauge and the effective area of the valve face is 700 sq cm. Design a valve rod, eccentric sheave and strap for operating the slide valve. State your assumptions clearly

4. Design and sketch an eccentric from the following data·

Net area 'a' of valve face=775 sq cm, steam pressure 'p' 10·5 kg/sq cm, coefficient of friction 0 2, diameter of crankshaft 17·8 cm; [illegible]

safe stress for rod 280 kg/sq cm Design the valve rod, eccentric sheave, strap and strap bolts.

18-8. Compensating ring for a manhole (Fig 18-18)

In cutting a manhole in the shell plate of a boiler, since the plate is in tension, the sectional area cut through must be compensated for. The area cut through is measured by the length of the minor axis multiplied by the thickness of the plate. The cut area is compensated by providing a compensating ring whose area is same as the cut area in the direction of the minor axis in the shell plate.

Let $2b$ be length of the minor axis, t the thickness of the shell plate, x the width of the compensating ring, d the diameter of the rivet hole and T the thickness of the compensating ring

The area cut through $= 2bt$

The area compensated by the ring $= 2\,(x - d)\,T.$

provided by the dead weights which are directly mounted on the valve.

Dead weight safety valve is susceptible to vibratory and other disturbing influences and it is not used on locomotive boilers and marine boilers, where the inevitable jolting and rolling would produce many variations in the intended blow off pressures. Dead weight safety valve is used on Lancashire boilers as an independent auxiliary safety valve, which is usually set to blow off at a pressure slightly higher than the pressure for which the spring loaded safety valves are required to blow. Thus the spring loaded safety valves are the first to take the control of any over pressure. If the spring loaded safety valve is incapable of coping with the evaporation, the dead weight safety valves come into operation.

Sometimes two types of valves are used in combination and mounted on a common type; in this arrangement also the dead weight safety valve is usually set to blow off at a pressure in excess of its companion spring loaded valve.

The fundamental requirement of valve arrangement is that *the thrust of the spindle should be imparted to the valve member at or point below the level of the seat contacting faces.*

In any safety valve there must be a downward force opposing the upward force due to fluid pressure; in addition there must be an additional downward force sufficient to produce a *clamping pressure* between the seating faces of sufficient intensity to prevent leakage across these faces. In order to maintain fluid tightness in a metal to metal joint a clamping pressure of *one and a half to twice* the pressure of the fluid being controlled is generally required. The lower pressure may be adopted in the case of accurately lapped surfaces, which are absolutely essential for satisfactory performance of a safety valve.

Most boilers safety valves are set to blow off at a pressure 5% in excess of the normal working pressure of the boiler. For a valve diameter less than 3 cm bore it shall be increased to 10% excess. This requirement of excess pressure should be referred from Indian Boiler Regulations.

To ensure fluid tightness at the seat contacting faces, the radial width of the seating face should be equal to 0·01 of the effective diameter of the seating; this value should be doubled for values of seating diameter less than 3 cm bore. In actual practice

The efficiency of the longitudinal joint may be assumed to be 80%. The allowable stress in the boiler shell plate is 700 kg/sq cm. The thickness of the compensating ring plate may be assumed to be the same as boiler shell plate. The ring is to be flanged inwards to a total depth of 7·5 cm.

The oval hole in a boiler shell is 40 cm × 50 cm.

The oval hole in the compensating ring is 30 cm × 40 cm.

Assume shearing stress in rivets to be 0 837 × tensile stress in the compensating ring.

2. Design a man hole cover for a Lancashire boiler 180 cm diameter and having a working pressure of 15 kg/sq cm by gauge. The cover has two bridge bars and studs. Prepare a dimensioned sketch. Choose suitable stresses.

18-9. Design of safety valves for boilers:
Introduction:

The function of a safety valve is to prevent an undue rise in pressure in any vessel. Such a valve should be entirely automatic in its action and should operate independently of any human agency. When a safety valve opens it discharges the fluid; after relieving the pressure it should close down again automatically and should remain closed until such time as it is again required to perform its appointed function

Safety valves may be broadly classified into four basic types:

(i) Lever loaded safety valve (Fig. 12-13 and Fig. 18-19)
(ii) Spring balance safety valve (Fig. 18-21)
(iii) Direct spring loaded safety valve (Fig 18-20)
(iv) Dead weight safety valve

In lever loaded safety valve the controlling force for the valve is provided by the dead weight, which is placed at the end of a lever and its effect is transferred at the valve through lever action In spring balance type of safety valves the controlling force is provided by the spring which is attached at the end of a lever and its effect is transferred at the valve through lever action. *The use of spring balance type of safety valves is abandoned long back.*

In direct spring loaded type of safety valve the controlling force is provided by the spring directly and *the lever action is absent.* Similarly in dead weight type of safety valve, the controlling force is

p = Highest pressure to which any safety valve is to be set to lift in kg/sq cm absolute.

The value of the constant is given by the following table:

For boilers having an evaporative capacity of less than 1,[illegible] kg of water per hour

Types of Valve	Spring loaded Valves	Weight loaded valves
Ordinary lift	4	4·8
High lift	8	9·6
Full lift	16	16

For boilers having an evaporative capacity of more than 1,[illegible] kg per hour the value of the constant may be taken as under:

Ordinary lift	4·8
High lift	9·6
Full lift	20

When two valves are loaded by a single spring as in case of Ramsbottom safety valve, the areas calculated as above shall be increased by 50%.

If the valves have to pass superheated steam, the area shall be increased as under

$$A_{sup} = A_{sat}\sqrt{1+\frac{2\cdot 7\theta}{1000}} \quad \text{(iii)}$$

where θ is the degree of superheat in °C.

The third method of determining area of flow through a safety valve is to determine the velocity of escaping steam considering the flow through a safety valve as a flow through a *convergent nozzle*. When the velocity of the escaping steam is known, by knowing the evaporation we can determine the volume of steam to be passed, through safety valve, per second. By flow equation we can find out the minimum aggregate area through the valve.

When a waste steam pipe is fitted in order to discharge steam away from the boiler house, sufficient cross sectional area of the pipe should be provided otherwise severe back pr[illegible] will be experienced on the back of the valve and the free l[illegible] will be affected. Cross sectional area not less than combined area of the safety valve req[illegible] or lift valves and not less than double th[illegible] full

the radial width of the seating face will be more in order to provide against premature failure due to wire drawing and also as a precaution against distortion. It depends on the material of construction of the surfaces of the valve seating.

It is reasonable to suggest that the unit pressure between safety valve seating may be rated much higher than that of a screw down stop valve because there is none of the screwing action which usually accompanies the closing down of a stop valve.

Minimum Valve Area:

According to Indian Boiler Regulations the essential requirement of any safety valve is that its area should be sufficient to discharge the steam as quickly as it is generated with a rise in pressure not more than 10% of the safety valve blow off pressure. The rise in pressure is called the accumulation.

The minimum total area of a safety valve can be given

$$A = \frac{H \times \text{constant}}{p} \text{ sq cm} \quad \ldots \quad (i)$$

where H is the heating surface of the boiler in sq metre and p is the absolute pressure of steam in kg/sq cm. The value of the constant varies from 5·5 to 7·5 depending upon the kind of boiler. For water tube boilers, oil fired or coal fired, with forced draught installation its value is 6·25. The above formula is based on the heating surface.

The other formula based on evaporation of steam per hour is given as under:

$$A = \frac{E}{p \times \text{constant}} \text{ sq cm} \quad \ldots \quad (ii)$$

where A = aggregate area in sq cm of the orifices through the seating of the valve for ordinary and higher lift valves and for full lift safety valves the net area through the seats after deducting the area of guides or other obstructions when the valves are fully lifted.

E = Total peak load evaporation in kg/hour (including evaporation from water walls, steaming economiser and other heating surfaces in direct communication with the boiler) for which the boiler is specified. In no case, however, shall the evaporation as calculated be based on less than 30 kg/hour/sq metre of heating surface (exclusive of superheater and non steaming economics).

7. When the pins in a lever loaded safety valve are of different material than the material of the lever or the bushes that fit into the pin holes of the lever?
8. When the boiler inspector reduces the working pressure of the boiler, he also reduces the evaporation of the boiler for the same size valve. Give reasons. If the original evaporation is desired, the valve requires to be changed. Why?
9. What is the normal value of the leverage in a lever loaded safety valve? What are the advantages and disadvantages of providing a leverage of say 10:1?
10. What is the material of the lever? How will you make it inexpensive?
11. As the lever is subjected to bending, the section that provides beam of uniform strength would be appropriate; however such sections are not used in practice. Why?
12. What can be the possible reason of valve chatter and how to get rid of it in a new design?
13. Why is it desirable that the seatings of all the valves should knife edged as far as practicable?
14. Explain how you will determine the diameter of the valve?
15. Explain the terms: Ordinary lift, high lift and full lift; which type of valve should we design and why?
16. What is the distance between the fulcrum and valve axis and why should it be kept minimum?
17. How will you design the fulcrum of the lever? What types of fulcrums are used in the design of lever loaded safety valves?
18. Why do we prefer ball contact in place of a thrust pin?
19. How will you fix the size of the mounting bolts?
20. How will you fix the thickness of the casting? How will you determine the thickness of the flange?

Now let us consider the design procedure for such a valve, which is simplest of all the valves. An objection to these valves is that they readily lend themselves to surreptitious over loading by ant attendants. *The weight should be in one piece and the safety s shall be mounted so that the axis of the valve is vertical.*

In ordinary lift safety valves, which are commonly employed in simpler and cheaper valves, the lift is much less than $\frac{D}{24}$. In order to increase the lift, and thus to obtain a greater area of discharge with a corresponding reduction in the size of the valve required to pass a given volume, many artifices have been devised principally directed at utilising the kinetic energy of this high velocity escaping steam by permitting it to impinge on to a suitably shaped projection on the valve member.

Classification of Valves:

An ordinary safety valve is a valve which lifts automatically at least $\frac{D}{24}$ where D is the diameter of the valve seating. A high lift valve is a valve which lifts automatically at least $\frac{D}{12}$. A full lift safety valve is a valve which lifts automatically a distance giving a discharge area round the edge of the valve seating equal to area through the valve orifice when the valve is fully lifted after deducting the area of guides or obstructions.

Design of Lever loaded safety Valves: (Fig. 18-19)

The working principle of a lever loaded safety valve is explained in art. 12-6 and a schematic diagram of such a valve is given by Fig. 12-13. Fig. 18-19 shows some of the various forms of the arrangements for lever loaded safety valves.

When we want to design a lever loaded safety valve the number of questions may arise to the designer when he carries out the preliminary design. Here we list some of these questions

1. Explain how the lever loaded safety valve works.
2. What is clamping pressure and how much it should be?
3. What is the fundamental law governing valve design?
4. How will you determine the load to be placed at the end of a lever?
5. Can you adopt a lever loaded safety valve on a locomotive, a steam road roller, a stationary steam engine boiler? Give reasons?
6. How will you determine the load on the fulcrum pin and the thrust pin? Why are these pins normally of the same size?

The centre of gravity of each cheese weight may be taken to coincide with the geometrical centre of the casting proper. The spindle is designed from compressive stress consideration. Let us consider the design of the pins. Fulcrum pin and the pin securing the spindle to the lever will be subjected to shearing and bending forces. The fulcrum pin will be subjected to an upwardly directed shearing force, and the thrust pin will be subjected to a downwardly directed shearing force. These pins are in double shear. In order to provide more wearing surfaces the calculated diameters are increased by 5 to 6 mm. The following values of permissible shear stresses are taken:

450 kg/sq cm for rolled phosphor bronze or manganese bronze
300 kg/sq cm for rolled brass bars
550 kg/sq cm for monel metals.

A fulcrum is to be designed as a tension member. We may have a linked fulcrum in place of a rigid fulcrum bolt or a ball thrust arrangement in which a thrust pin is abandoned in favour of a ball contact constrained to abut against the underside of the lever, two artifices employed either singly or in combination. Reduction of friction to a minimum is essential in any safety valve if the maximum lift and therefore discharge is to be secured. In this connection knife edge contacts at all swivelling points are advocated. The fulcrum bolt or stud, usually, at its upper extremity is in the form of a fork, and hence it should be designed as a knuckle joint. For gun metal the permissible tensile stress is taken as 350 kg/sq cm and for mild steel 700 kg/cm.

The thickness of the branch of the fork will be determined from forging consideration. It is also determined from the necessity of providing sufficient bearing area for the pin. Too little area will result in too concentrated distribution of load over the bearing area and spalling or grooving of the surface of the pin or holes will ensue.

Thus after deciding the thickness from forging consideration, the pin is checked for bearing consideration. The value of bearing pressure is taken as 150 to 200 kg/sq cm for gun metal or phosphor bronze.

The shank and screwed mechanism of the fulcrum bolt or stud be in pure tension and the diameter at the bottom of the thread culated and nominal diameter increased by 5 to 6 mm to

The axial force due to the set or blowing off pressure acting on the underside of the valve member is determined as under:

$$P' = \frac{\pi}{4} D^2 p + 1{\cdot}5p\,(\pi D W) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

Steam force + clamping force

where D is inside diameter of the seating and W is the width of the contact face which may be taken as $0{\cdot}02D$.

In order to determine the force to be applied at the end of a lever and the load in the fulcrum we employ moment equation. In taking moment equation we neglect the frictional resistance of the fulcrum pin and thrust pin and other rubbing parts, will be ignored but it should be remebered that the freedom of movement at these points is most essential. Pins of non-ferrous materials should be adopted but where steel pins are preferred it is important that lever should be bushed with bronze. In preliminary calculations we neglect the weight of the lever but allowance should be made for it. *The omission of this factor in low operating pressure will be dangerous.*

The fulcrum distance is mainly governed by the external diameter of the valve seating or valve member and should be kept as small as possible in order to employ the lightest possible cheese weight. Knowing the overall length of the lever its cross sectional dimensions may be assumed and its weight can be calculated. The combined weight of the valve member and spindle is in the designer's hand. The length of the lever is measured from the centre *of the fulcrum pin.* Cheese weights fall into one of the two kinds

Sliding weight type in which case the lever passes through the cheese weight or they may be pendulously mounted on the lever The latter expedient is not altogether a desirable one in as much that it does not permit of any subsequent adjustment of centres. Moreover it is not immune from unauthorised interference its removal being a simple matter by any one possessed of requisite strength

According to regulations the weights shall be attached to the lever in such a way that they can not be moved inadvertently

Pitching screw fixes the cheese weight to the lever There is, however, one pronounced disadvantage in this arrangement, the screw is liable to become rust bound in course of time making any subsequent adjustment a most difficult operation.

Materials:
Body and weight cast iron
Fulcrum and lever M.S.

Design of a spring loaded safety valve:

The maximum compressive force which may be imparted to the lower extremity of the spindle is due to the spring thrust and is obtained as under:

$$P = \frac{\pi}{4} D^2 p + 1{\cdot}5p\,(\pi D W) \quad \ldots\ldots\ldots\ldots \quad \text{(v)}$$

Steam force + clamping force

where D is the inside diameter of the seating and W is the width of the contact face which may be taken as $0{\cdot}02D$.

The initial compression to give the desired load is $\frac{1}{4}$ of the diameter of the valve seating, but the Admirality specify double this amount. According to *BS* 759, for ordinary lift safety valves the total compression or extension of the springs required to load the valves to the set pressure shall not be less than one quarter of the valve diameter. This value is not exceeded by the designer as the stress induced in the material of the spring is directly proportional to the compression or extension necessitating increased dimensions.

The proportion of unloaded length to external diameter of the spring shall not exceed 4:1. If the free length of compression springs is very much in excess of four times the outside diameter, the spring will lack in lateral stiffness and will tend to become bowed. This tendency may cause tilting of valve member and undue flexing of the spindle whose thrust should be always co-axial with the valve member if the freedom of movement of the parts is to be assured.

The maximum shear stress as determined by the torsional formula shall not exceed 7,000 kg/sq cm for compression springs and 5,600 kg/sq cm for extension springs. The commonly adopted torsional formulas are given below:

(a) *Round section wire:*

$$\text{Shear stress} = \frac{16PR}{\pi d_w^3} \times K \quad \ldots\ldots\ldots\ldots \quad \text{(vi)}$$

where p = maximum load at set pressure
R = mean radius of the coil

d_o = diameter of round wire

K = Wahl stress concentration factor.

(b) *Square section wire:*

$$\text{Shear stress} = \frac{4 \cdot 8\ PR}{d^3} \times K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

where d = side of a square section

(c) *Rectangular section wire.*

$$\text{Shear stress} = \frac{(3b + 1 \cdot 8h)\ PR}{b^2h^2} \times K \quad \ldots\ldots\ldots\ldots \text{(viii)}$$

where b = radial width of cross section and

h = axial depth of cross section

The values of stress concentration factors are given in art 8-2.

For small valves and moderately low pressures springs of round section wire are usually employed, square section being reserved for the larger sizes of valves or those intended for high pressure while springs of rectangular section are specifically intended for valves having high lift characteristics in order to provide for more than the usual amount of subsequent deflection

The number of active coils are obtained by the stiffness consideration. By active coils is meant the actual number of coils which take part in the resistance of the applied loads and not the dead coils or portions of coils, at each end of the spring. Having determined the number of active coils, the overall length of the spring may now be determined having due regard to the minimum allowable space between the coils, working deflection, initial compression and allowing for number of dead coils

The valve chest is usually made of cast iron for a pressure of 10 atg saturated Wherever there is any doubt regarding the choice of the material, cast steel should be adopted.

The spring casing may be of cast iron and it will be subjected to a direct tensile force resulting from the spring thrust and transmitted by the compression screw and top plate to this member. If we base the design only on the tensile force imparted, the wall thickness will be relatively thin Hence we adopt that thickness which gives us sound castings.

The bolts and studs securing the spring casing to the chest are also subjected to tension.

For any given type of spring loaded safety valve, the force analysis should be made for each component and then the dimensions of various components should be decided after selecting the proper material. Space considerations prevent us from the detailed design considerations of all these components.

Examples:

1. The spring loaded safety valve for a boiler is required to blow off at a pressure of 11 kg/sq cm. The diameter of the valve is 6 cm and the maximum lift of the valve is 1 cm.

Design the suitable compression spring for the safety valve assuming the spring index of 6 and providing initial compression of 3 cm. Maximum shear stress is limited to 4,200 kg/sq cm.

According to equation (v) of this article the axial load can be written as under:

$$\text{Axial load} = \frac{\pi}{4} \times 6^2 \times 11 + 1{\cdot}5 \times 11\ (\pi \times 6 \times 0{\cdot}02 \times 6)$$
$$= 350 \text{ kg.}$$

As the initial compression is 3 cm, the stiffness of the spring will be $\frac{350}{3} = 116{\cdot}6$ kg/cm. Maximum spring load $= 4 \times 116{\cdot}6 = 467$ kg.

$$\text{Stress concentration factor} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0{\cdot}615}{6}$$
$$= 1{\cdot}27$$

Allowable stress in shear for torsional formula $= \frac{4200}{1{\cdot}27} = 3{,}300$ kg/sq cm.

If d cm be the diameter of the circular wire of the spring which is in compression, then

$$467 \times 3d = \frac{\pi}{16}\, d^3 \times 3300$$

or
$$d = \sqrt{\frac{467 \times 3 \times 16}{\pi \times 3300}} = 1{\cdot}47 \text{ cm; we adopt 15 mm wire.}$$

If n be the number of active coils, then

$$n = \frac{0{\cdot}84 \times 10^6 \times 1{\cdot}5}{8 \times 6^3 \times 116{\cdot}6} = 6{\cdot}3 \text{ turns.}$$

$$\text{Mean diameter of the coil} = 6 \times 1{\cdot}5$$
$$= 9 \text{ cm}$$

d_w = diameter of round wire

K = Wahl stress concentration factor.

(b) *Square section wire:*

$$\text{Shear stress} = \frac{4{\cdot}8\ PR}{d^3} \times K \qquad \text{(vii)}$$

where d = side of a square section

(c) *Rectangular section wire.*

$$\text{Shear stress} = \frac{(3b + 1{\cdot}8h)\ PR}{b^2h^2} \times K \qquad \text{(viii)}$$

where b = radial width of cross section and

h = axial depth of cross section

The values of stress concentration factors are given in art 8-2.

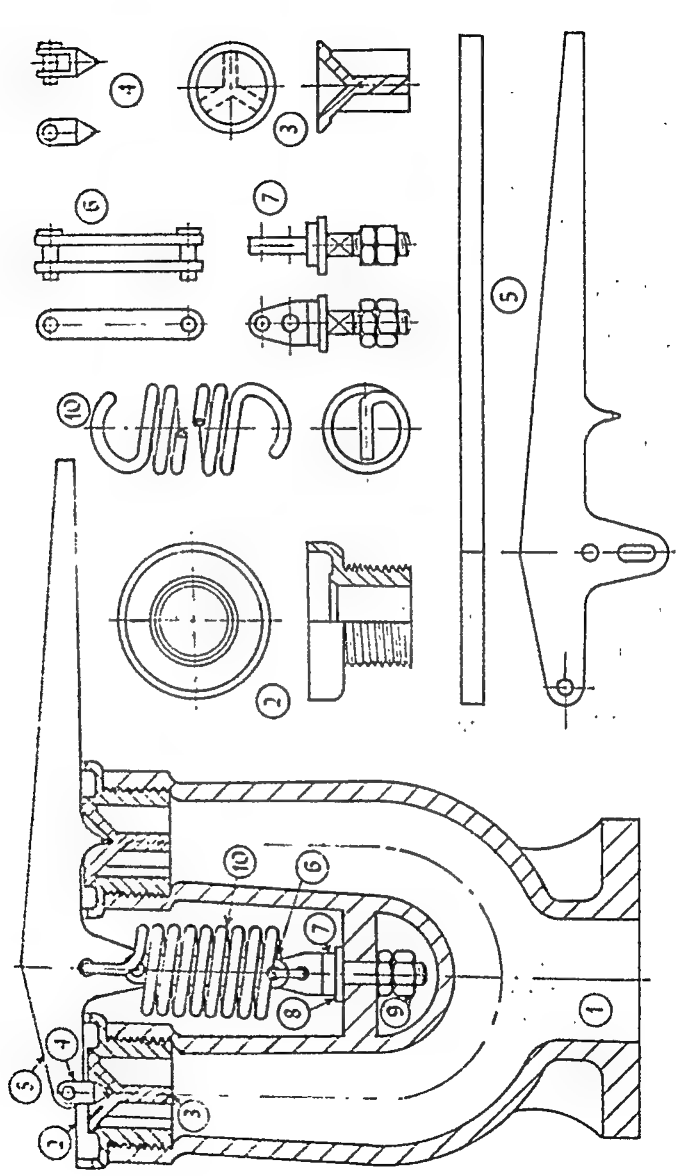
For small valves and moderately low pressures springs of round section wire are usually employed, square section being reserved for the larger sizes of valves or those intended for high pressure while springs of rectangular section are specifically intended for valves having high lift characteristics in order to provide for more than the usual amount of subsequent deflection

The number of active coils are obtained by the stiffness consideration By active coils is meant the actual number of coils which take part in the resistance of the applied loads and not the dead coils or portions of coils, at each end of the spring. Having determined the number of active coils, the overall length of the spring may now be determined having due regard to the minimum allowable space between the coils, working deflection, initial compression and allowing for number of dead coils

The valve chest is usually made of cast iron for a pressure of 10 atg saturated Wherever there is any doubt regarding the choice of the material, cast steel should be adopted

The spring casing may be of cast iron and it will be subjected to a direct tensile force resulting from the spring thrust and transmitted by the compression screw and top plate to this member If we base the design only on the tensile force imparted, the wall thickness will be relatively thin Hence we adopt that thickness which gives us sound castings

The bolts and studs securing the spring casing to the chest are also subjected to tension.



Ramsbottom spring loaded safety valve

Fig. 18-20

Free length $= 6{\cdot}3 \times 1{\cdot}5 + 3 + 1 + 1$

$= 14{\cdot}45$ cm. say 15 cm

Thus the proportion of unloaded length to external diameter of the wire $\frac{15}{10{\cdot}5}$ does not exceed 4

The space between the coils when the valve head is lifted the required distance, it shall not be less than 1·5 mm for full lift safety valves and not less than half its amount for ordinary and high lift safety valves.

2. Design a spring loaded safety valve of the Ramsbottom type for a boiler working at a pressure of 10 kg/sq cm gauge The diameter of the valve is 50 mm The set pressure is to be 5% more than the working pressure and the valves are to be lifted through 5 mm, when the pressure rises by 10% of the set pressure

Fig. 18-20 shows the sectional elevation of a Ramsbottom type safety valve fitted on boilers In this design two valves are loaded by a single spring, which is in tension. This valve discharges directly to atmosphere The valve members are in no way connected to the loading lever. In the event of spring breaking the valves are prevented from being blown away by the safety links

The following table gives the list of various parts of this type of valve.

Reference No	Description	Material	Number required
1	Casting	Cast Iron	1
2	Valve seat	Gun metal	2
3	Valve	Gun metal	2
4.	Fulcrum bracket	Mild steel	1
5	Lever	Mild Steel	1
6	Safety links and pins	Mild Steel	2 each
7	Eye bolt	Mild Steel	1
8	Washer	Mild Steel	1
9	Nuts	Mild Steel	2
10	Spring	Spring Steel	1

From the above list, the student should make the force analysis for each part. After element recognisation, the various dimensions can be fixed for individual parts

3. *A dead weight safety valve is to be designed for a Lancashire boiler for a working pressure of 10 kg/sq cm gauge. Over all height from the boiler seating is to be not greater than 90 cm. Grate area is 4 sq metre and 60 kg of coal are burnt per square metre of grate area per hour generating 10 kg of steam per kg of coal. The valve is to be designed on the assumption that it will pass 25% more steam than the boiler will generate and that the speed of the steam through the valve is not to exceeed 40 metre/sec. Assuming that the weight of the valve and the casing without weights is 135 kg, determine the diameter of the valve, the lift of the valve and the additional weights required.*

Steam generated per hour $= 4 \times 60 \times 10 = 2{,}400$ kg/hour. As the dead weight safety valve under consideration should be capable of allowing 25% more steam, the quantity of steam to be discharged $= 2400 \times 1{\cdot}25 = 3{,}000$ kg/hour. According to I.B.R., assuming *full lift valve*, the minimum area for a saturated steam is given as

$$A = \frac{\text{Evaporation in kg/hour}}{\text{Constant} \times \text{absolute steam pressure (ata)}}$$

$$= \frac{3000}{16 \times 11}$$

$$= 17 \text{ sq cm.}$$

Let us calculate the area from first principles. The specific volume of dry saturated steam at 11 kg/sq cm absolute is 0·181 cu metre. Assuming the dryness fraction of steam to be 0·98, the amount of steam to pass per second is equal to $\dfrac{3000 \times 0{\cdot}98 \times 0{\cdot}181}{3600}$ $= 0{\cdot}148$ cu metre/second. If A sq cm be the area of flow through the valve, then $\dfrac{A}{10000} \times 40 = 0{\cdot}148$

or $$A = \frac{0{\cdot}148 \times 10000}{40}$$

$$= 3{\cdot}7 \text{ sq cm.}$$

We adopt the diameter of the valve as 7 cm, and the lift of the valve as 18 mm.

The maximum force required to be imparted to the valve number $= \dfrac{\pi}{4} \times 10 \times 7^2 + \pi \times 7 \times 1{\cdot}5 \times 10 \times 0{\cdot}02 \times 7$

$$= 431 \text{ kg.}$$

Area of flow through each valve $= \frac{\pi}{4} \times 5^2$

$= 20$ sq cm.

Total area of flow will be $2 \times 20 = 40$ sq cm.

Diameter of the main pipe, from the boiler, which connects two branches $= 7 \cdot 5$ cm.

The main body is made of cast iron upto a pressure of 10 kg/sq cm gauge and temperature below 250°C. We determine the thickness of the casting by employing thin cylinder formula We adopt 200 kg/sq cm as the permissible stress As we can verify that the thickness obtained from strength view point is less, we adopt the thickness which will give sound casting. A liberal thickness is suggested in the region of the neck to cater for the effects of vibration in addition to that of meeting pressure requirements. For the same reason the inlet flange should be made larger than that appropriate to the pipe size denoted by the bore.

Set pressure $= 1 \cdot 05 \times 10 = 10 \cdot 5$ kg/sq cm

Lift of the valve $= 5$ mm when the pressure rises to $1 \cdot 1 \times 10 \cdot 5 = 11 \cdot 6$ kg/sq cm

Total steam load on the valve when it begins to lift $= 20 \times 10 \cdot 5 = 210$ kg.

Total steam load when the valve is lifted by 5 mm $= 20 \times 11 \cdot 6 = 232$ kg.

Increase in spring load $= (232 - 210)2 = 44$ kg

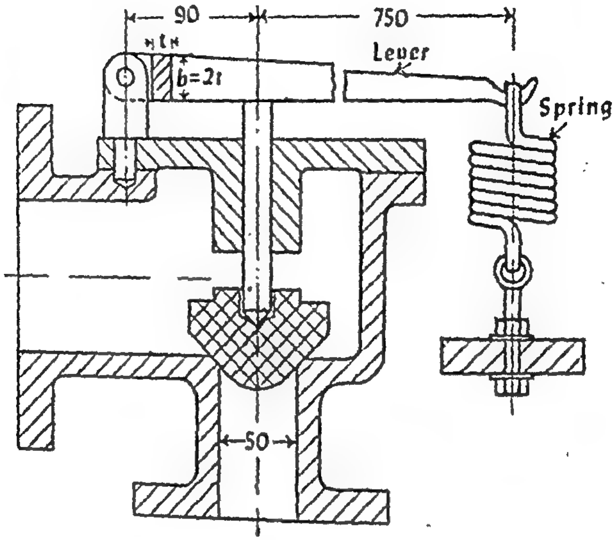
Stiffness of the spring $= \frac{44}{0 \cdot 5} = 88$ kg/cm

From fig. 18-20 it can be seen that the thrust is imparted to the *valve member* at a point below the level of the seat contacting faces. By this arrangement valve chatter is eliminated

When the maximum load in the spring is known, by the torsional formula we can find out the diameter of the spring wire From the stiffness consideration, we can determine the number of active turns of the spring which is subjected to an axial extensional loading

The calculations for various components such as safety link, pins, lever, eye bolt and fulcrum brackets can be carried out according to the principles explained in the text earlier.

Valve diameter 50 mm; maximum pressure when the valve begins to blow off — 7 kg/sq cm gauge; maximum lift when pressure rises from 7·0 to 7·7 kg/sq cm by gauge is 7 mm.



Spring loaded lever safety valve

FIG. 18-21

Assume: Allowable stress for spring 4,200 kg/sq cm; ratio of mean diameter of coil to the diameter of the wire — 12:1; G = 7 × 10^5 kg/sq cm; maximum allowable stress in the lever, fulcrum and fulcrum pin 350 kg/sq cm; bearing pressure for pin 140 kg/sq cm; safe stress in the valve body 140 kg/sq cm.

18-9. Design of a steam stop valve:

Fig. 18-22 shows a steam stop valve of the screw lift type i.e. the valve is attached to the spindle so that the valve rises or falls with the spindle. The spindle has an external screw. The steam is admitted through the vertical passage to the underside of the valve; this arrangement facilitates the adjustment of the cover gland and packing even though the line is live.

The diameter of the stop valve is given by,

$$d = \sqrt{\frac{Q}{V \times 0{\cdot}7854}} \quad \text{.......... (i)}$$

Thus total dead weight required will be 431 kg, which includes the weight of the plate, weight carrier, valve member and other appurtenances attached there to. Hence additional dead weight required $= 431 - 135 = 296$ kg.

Note: Refer the following pages for additional examples pertaining to design of safety valves

(i) Example 4 on page 193 for the design of a fulcrum
(ii) Example on page 348 for the design of a spring for a spring loaded safety valve.
(iii) Art. 12-6 for the design of lever loaded safety valve; pages 491-493.

Exercises:

1. Design the elements for a direct spring loaded 5 cm diameter cast iron pressure relief valve to operate at 15 kg/sq cm gauge. The valve should have steel side columns, a steel cross head, and a closed coiled helical spring. Also specify general metal thickness for the valve body and flange thickness. State all assumptions made

2. Design and draw a neat dimensioned sketch of a spring loaded safety valve of 'Ramsbottom type' for the following duty:

Blow off pressure 100 psi (7 kg/sq cm), pressure at which the valve blows off freely is 105 psi (7·35 kg/sq cm), diameter of each valve 1½ in (38 mm); lift of the valve when it is blowing off freely ⅛ in (3.2 mm) Materials used; casing-cast steel, spring—Ni-Cr steel, valve gun-metal, all other parts mild steel

Safe shear stress in the spring steel 80,000 psi (5,600 kg/sq cm)
(Bombay University, 1951)

3. Design a spring loaded safety valve from the following particulars

Diameter of valve 2 in (5 cm); blow off pressure 150 psi (10·5 kg/sq cm gauge); the valve to blow-off freely at 158 psig (11 kg/sq cm gauge); lift of the valve ¼ in (6 mm)

The spring may either be in tension or in compression The material used are:

Cast iron for casing, phosphor bronze for valve and seat; nickel steel for spring; mild steel for all other parts.
(Gujarat University, 1955)

4. Design the spring, the lever, the fulcrum pillar, the pin and the thickness of the valve body of the spring balanced safety valve, as shown in fig. 18-21 for the following duty:

the valve, which will be the product of the area of the valve and the steam pressure intensity. The spindle is operated by a hand-wheel and the screw. When the valve is tightly screwed down, the pressure between the valve and its seat will be far in excess of that due to steam load. There are many unknown factors, that enter while calculating the actual force acting on the spindle. The maximum load on the spindle we take to be twice the steam load.

(ii) The spindle is subjected to twisting moment when it is being screwed down hard on its seat. This stress is temporary and will be relaxed when the turning effort at the hand-wheel is removed.

(iii) The spindle is made of rust-proof material such as phosphor-bronze, gun-metal, etc. It should be designed as a short compression member. If the slenderness ratio exceeds 60, it should be designed as a column.

(iv) The thickness of the valve body should be calculated by considering it to be the thick cylinder. The commonly adopted materials for the body are cast iron, cast steel or gun-metal. As I.B.R. require the body to be hydraulically tested, it should be designed for hydraulic test pressure. The thickness of the flange should be taken 3 mm more than the thickness of the body. The size of the bolts to connect the valve to the boiler or the steam pipe should be at least M 18.

(v) The valve seat is made of gun-metal. The valve seat may be either a push fit in the body or screwed in the body or fixed with a set screw. The thickness of the seat is calculated by Lamé's formula and width of the seat from bearing considerations.

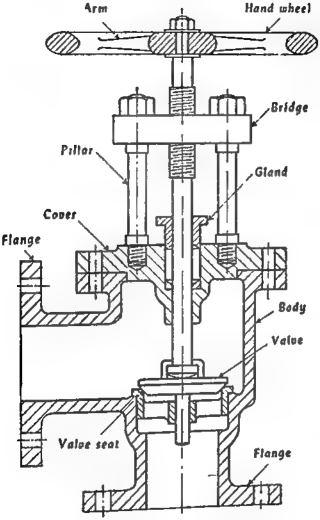
(vi) The thickness of the valve is calculated by considering it to be a plate supported at the edges and loaded at the centre. The thickness t of the valve may be calculated by the equation

$$t = r\sqrt{\frac{p}{f}} \qquad \text{....(ii)}$$

where r is the radius of the valve, p the pressure of the steam and f the permissible stress intensity. The valve is provided with ribs or feathers to guide the valve in the valve seat. The maximum lift of the valve should not exceed one-fourth the diameter of the valve.

(vii) The cover bolts must be sufficiently in number to resist the steam load safely, and to ensure a leak-proof joint. The thick-

where Q is the volume of steam in cu metre/sec and V the velocity of steam in metre/sec.



Boiler stop valve
Fig 18-22

The following salient points should be noted in the design of the stop valve:

(1) The spindle, when screwed down, will be subjected to a minimum compressive force equal to the steam load acting on

2. A 75 mm screw down stop valve having a flat faced phosphor bronze seating is to work against a steam pressure of 10 kg/sq cm gauge. Cover joint with resilient gasket is continuous to the inside of cover bolts and pillar ends are continuous through both flanges.

Design (i) cover plate (ii) bolts for cover plate (iii) pillars supporting the main spindle nut bridge (iv) main spindle and nut (v) valve disc and (vi) valve seatings.

Assume your own stresses. Prepare a complete drawing of the stop valve designed.

18-10. Design of tangent cams: (I.C. Engines)

The requirements for valve cams are some what conflicting in that for high volumetric efficiency, the valves should be opened and closed quickly and held wide open for the longer period while to reduce inertia forces and hence to keep down the spring force they should be opened and closed gradually. *In addition the cam contours should be cheap to manufacture.*

Common cams in use:

(i) Tangent cams with roller followers
(ii) Convex flank cams with flat faced followers in sliding contact
(iii) Concave flank cams with roller followers
(iv) Generated cams.

All cams in common have

(i) a base circle on which the follower rides during the time valve is closed,
(ii) an opening flank so shaped as to open the valve in desired way,
(iii) a cam nose on which the follower rides during the time when the valve is wide open and it may include the period of dwell and
(iv) a closing flank which allows the valve to close properly.

The tangent cams are simpler to manufacture and are therefore used more often than the circular cams.

Tangent cams (Fig. 18-23):

These cams are cheaper to manufacture but require a stiff valve spring to ensure contact of follower at all points. The tangent cams have straight line flanks tangent to base circle and nose circle.

ness of the cover must be sufficient to act as nuts for two mild steel pillars. The compressive load in the spindle is transmitted by bridge to the mild steel pillars, which are subjected to tensile load when the valve is closed. The weakest section will be at the *bottom of the threads*. In order to distribute strain, the pillars are designed as bolts of uniform strength (Please refer pages from 203 to 206.) The pillars are provided with bearing collars at the top and bottom. *In order to facilitate the fitting operations*, two flats are machined on lower collars to take a spanner (See illustrative example 5 on page 194)

(viii) The function of the bridge is to provide a fixed *nut for* the screw of the spindle, when opening and closing the valve. The bridge is subjected to a load to which the spindle is subjected. The height of the bridge is calculated by considering the bearing stresses on the threads. The bearing area per thread will be the annular area. After the number of threads have been fixed from bearing considerations, they are checked for the induced shear stresses. In order to calculate the width of the bridge, it is in the condition of a centrally-loaded beam supported at each end by the pillars and subject to a load equal to compression in the spindle While calculating the modulus of section, due allowance should be made for the central hole in the bridge.

(ix) The spindle is not rigidly attached to the valve, but is allowed a slight play and the valve is free to rotate in order to adjust itself to its seat. To facilitate the insertion of the spindle from the side, the top of the valve is given a horse-shoe shape. *The spindle is retained in the groove by a split pin*

(x) The handwheel is made of cast iron and is provided with four or six arms of elliptical section The torque on handwheel *to overcome frictional torque* at screw and nut is calculated from the principles explained in article 11-4 The arms are subjected to bending. Generally, the handwheel has a square hole to receive the squared portion of the spindle, thus eliminating the use of the key. For the wheel, a force of 30 kg can be applied by a person for intermittent work.

Exercises:

1. Design and draw a suitable stop valve for a boiler supplying 5,000 kg of steam per hour at 17 kg/sq cm Calculation in respect of spindle, bridge and bridge supports must be clearly shown. The body is to be made of cast steel 13 mm thick and body flange 22 mm thick.

(8) If the maximum retardation exceeds the acceleration due to gravity, the spring is necessary to keep the follower in contact with the cam face.

When the angle of crankshaft travel from initial opening to final closing of the valve is quite large or when it is desired to keep the valve in its fully open position for a longer period, a period of dwell is built in. This period of dwell is an arc of a circle concentric with the base circle and having a radius equal to the radius of the base circle plus the total lift.

The period of dwell is arbitrary with the designer but it is limited by the capacity of the valve spring and to a lesser extent by the allowable stress at the line of contact between cam and roller follower.

Let us denote the angle of dwell by 2β. The following modifications should be made in the conclusions stated earlier.

(3) Radius of nose circle $= R_n = R_b - \dfrac{\text{lift} \times \cos(\alpha - \beta)}{[1 - \cos(\alpha - \beta)]}$

(4) $h = \dfrac{R_b - R_n}{\cos(\alpha - \beta)}$

(5) $\theta_{max} = \tan^{-1} \dfrac{h \sin(\alpha - \beta)}{R_f + R_b}$

The maximum retardation will not be at the end of the stroke. It will be somewhat earlier.

The retardation of the follower should not exceed the following values:

Low speed engines	40 to 60	metre/sec^2
Medium speed engines	100 to 120	,,
Light high speed engines	350 to 450	,,

The roller diameter in I.C. engine practice is usually 0·5 to 0·75 times the minimum cam diameter. The width of roller is from 0·3 to 0·5 times its diameter in I.C. engine practice, and in other cases it may be made $\frac{1}{4} \times$ (roller diameter) $+ 5$ mm. If the load on the cam is known, the width should be calculated on the basis of 90 kg/cm width. The roller pins should have $\frac{1}{3}$ to $\frac{1}{2}$ the diameters of the rollers which turn on them. The width of the hub is from 0·6 to 0·8 times the minimum cam diameter.

The usual materials for cam are cast iron for low speed and case hardened and ground machinery steel for high speed cams. Manytimes cams are forged integral with the cam shaft.

Design data:

(a) angle of action, (b) lift of the valve, (c) diameter of cam shaft, (d) diameter of roller $= 2\ R_f$, (e) speed in r.p.m. of cam shaft, N and (f) no dwell.

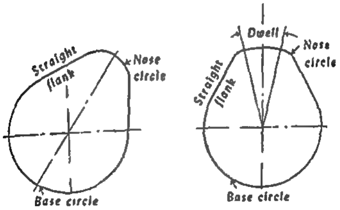
Conclusions:

(1) Diameter of base circle $= 2R_b =$ diameter of cam shaft $+$ 2·5 mm.

(2) Determine half the angle of action; call it α

(3) Radius of nose circle $R_n = R_b - \dfrac{\text{lift} \times \cos\alpha}{(1 - \cos\alpha)}$

(4) Distance between base circle centre and nose circle centre $= h = \dfrac{(R_b - R_n)}{\cos\alpha}$



Tangent cams

FIG. 18-23

(5) Angle turned by cam shaft when contact is on straight flank $= \theta_{max} = \tan^{-1} \dfrac{h \sin \alpha}{R_b + R_f}$.

(6) Angle turned by camshaft when contact is on nose circle $\varphi_{max} = \alpha - \theta_{max}$.

(7) Maximum retardation $= \left(\dfrac{2\pi N}{60}\right)^2 h \left[1 + \dfrac{h}{R_n + R_f}\right]$

This value will be at the end of the stroke.

Exercise:

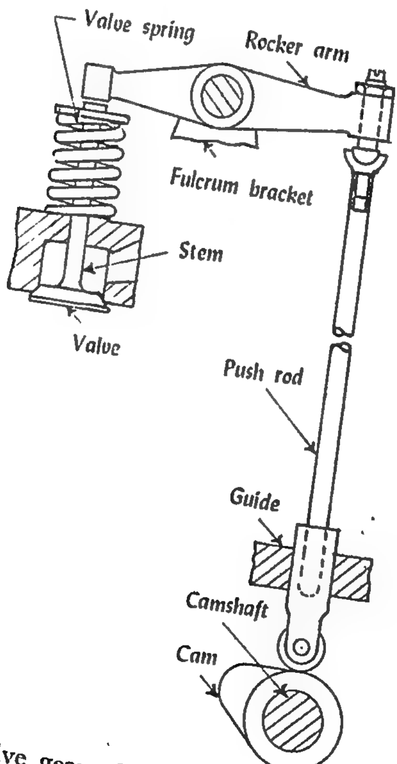
1. Determine the dimensions for tangent cam to operate the exhaust valve of an engine running at 500 r.p.m. The valve opens 33° before outer dead centre and closes 1° after inner dead centre. The lift of the valve is 25 mm. The radius of the base circle may be taken as 50 mm. The diameter of roller follower is 25 mm.

Ans. $R_n = 13{\cdot}2$ mm; $h = 61{\cdot}8$ mm.

18-11. Design of a valve gear for I. C. Engines: (Fig. 18-24)

Introduction:

The valve gear of a four stroke cycle engine consists of the following components:



Valve gear of an I. C. engine
FIG. 18-24

(i) Inlet and exhaust valves
(ii) Valve levers or rocker arms

Example:

1. Determine the dimensions for tangent cam to operate air inlet valve on an inline engine (a) for zero dwell (b) for a period of dwell equal to 20° of camshaft travel.

Inlet valve opens 15° before top dead centre and closes 65° after bottom dead centre. Maximum lift is 1 cm, diameter of cam shaft 3·15 cm, diameter of roller follower 2 cm, and the speed of the engine 2,000 r.p.m.

With no dwell:

The speed of the cam shaft $= \frac{2000}{2} = 1{,}000$ r.p.m.

Radius of the base circle $= \frac{3{\cdot}15 + 0{\cdot}25}{2} = 1{\cdot}7$ cm.

Lift $= 1$ cm.

The angle of action of cam shaft $= \frac{15 + 180 + 65}{2} = 130°$.

$$\alpha = \frac{130}{2} = 65°.$$

Radius of nose circle $= 1{\cdot}7 - \frac{1 \times \cos 65°}{(1 - \cos 65°)} = 0{\cdot}968$ cm

$$h = \frac{(1{\cdot}7 - 0{\cdot}968)}{\cos 65°} = 1{\cdot}732 \text{ cm}$$

Maximum retardation

$$= \left(\frac{2\pi \times 1000}{60}\right)^2 \times 1{\cdot}732\left[1 + \frac{1{\cdot}732}{1 + 0{\cdot}98}\right] = 35{,}160 \text{ cm/sec}^2.$$

With dwell of 20°:

The dwell period is 20° of cam shaft, therefore $\beta = 10°$

$\therefore \quad (\alpha - \beta) = 65 - 10 = 55°$.

Radius of the nose circle $= 1{\cdot}7 - \frac{1 \cos 55°}{(1 - \cos 55°)} = 0{\cdot}358$ cm

$$h = \frac{1{\cdot}7 - 0{\cdot}358}{\cos 55°} = 2{\cdot}34 \text{ cm}$$

Retardation at the end of stroke equals

$$= \left(\frac{2\pi \times 1000}{60}\right)^2 \times 2{\cdot}34\left[1 + \frac{2{\cdot}34}{1 + 0{\cdot}358}\right] = 69{,}960 \text{ cm/sec}^2.$$

When there is dwell, the maximum retardation will be not at the end of the stroke It is somewhat earlier. For this example the exact mathematical calculations show that the maximum retardation is 94,080 cm/sec².

The allowable bending stress is:

Carbon Steel	500 to 600 kg/sq cm
Alloy Steel	1,000 to 1,200 kg/sq cm.

The thickness of the valve disc at the edges is given as 0·75 to 0·85 times the thickness of the valve disc calculated earlier.

The surface of the valve disc seat is always made tapered with the cone generatrix inclined at $\alpha = 30°$ to $45°$. If we assume that the mean velocity of gas flow through the valve is equal to that through the opening of the valve when it is lifted to a maximum, then according to the law of continuity of flow, the maximum lift of the valve is given by

$$h_{max} = \frac{d}{4\cos\alpha} \quad \text{.........................(ii)}$$

The mean velocity of gas flow through the valve at maximum lift is obtained by the continuity of gas flow

$$v_m = V_m \frac{A}{a} \quad \text{.........................(iii)}$$

where v_m = mean velocity of gas flow,
V_m = mean velocity of piston
A = piston area and
a = area through the valve opening when the valve is lifted to the maximum.

The allowable mean velocities of the gas flow in metre/sec are given as under:

	Inlet valves	Exhaust Valves
Low speed engines	35 — 40	40 — 50
Medium speed engines	35 — 45	50 — 60
High speed engines	80 — 90	90 — 100

Design of a spring:

Valve springs must be sufficiently stiff to prevent jumping of the tappet roller from the cam when the valve moves with a negative acceleration. Otherwise knocking will develop in the valve operating gear, causing premature wear of its parts and loud noise. When the valve does not operate its spring must have a certain preload to provide the resilience required for tight seating of the valve which otherwise tends to open under its own weight and due to vacuum during the suction stroke.

(iii) Tappets or push rods
(iv) Cams
(v) Cam shafts
(vi) Transmission device from the crank shaft to the cam shaft.

Materials:

The valves of low speed engines are often of a composite construction with cast iron valve discs and steel stems. High speed engines always have one-piece pressed or forged valves. The valve is forced down on its seat in the cage by one or two springs in low speed engines and two or three in medium and high speed engines. The springs are installed between retainers, one of which is secured to the valve stem and the other rests on the valve cage or the cylinder head. The end of the valve stem is provided with a cap for adjusting the clearance between the lever roller and the cam.

The inlet and exhaust valves are subjected to dynamic loads occasioned by the inertia forces of the masses of the valve operating gear. In addition, they are acted upon by the high pressure hot gases which create mechanical and thermal stresses The exhaust valves have to withstand the severest operating conditions They are made from heat resistant austenitic steels, which possess high heat resisting properties Valve cages are made of high grade pearlitic cast iron; the valve stem bushes and seats are made of bronze.

Valve springs of light duty engines are made from manganese steel and those of heavy duty engines from chrome vanadium steel Cams may be made from cast iron, heat treated steel and case hardened carbon steel. Cam shafts are made of either steel or alloy steel.

Valve design:

We assume that the valve disc is a circular plate freely supported around the periphery and subjected to uniformly distributed load p. Its thickness is given by equation

$$t = 0{\cdot}5d\sqrt{\frac{p}{f}} \quad \ldots\ldots(i)$$

where d is the inside diameter of the valve and f is the permissible bending stress.

should be designed on bearing considerations and checked for bending and shear stress considerations.

The other arrangement of a rocker arm is shown in fig. 12-31 on page 523.

Design of a push red:

It is introduced between the rocker arm (the lever) and the cam roller in order to enable the cam shaft to be located at a low level. Bright drawn steel tube of 0·4% carbon is usually employed for push rods. They are designed as struts by employing either Euler's formula or Rankine-Gordan formula. The aim is to make the rod as light as possible so that it would seem reasonable to choose a ratio of length to diameter which would reduce buckling to negligible proportions. If this is done, however, the wall thickness may be so low as to make accidental damage probable.

The following table gives the value of working stresses for various values of length to diameter ratio:

Ratio $\frac{\text{length}}{\text{diameter}}$	10	20	30	40	50	60	70	80	90	100
Permissible stress kg/sq cm	1,000	700	500	350	280	200	150	120	90	75

If the push rods are short, they may be safely made of centreless ground stock. In all cases, great care should be taken in manufacture to ensure absolute straightness.

The ends of the push rod will depend on the general design of the gear. If the push rods are guided then the ends may consist simply of flat ended plugs pressed in and brazed or welded. If ball and socket joints are used, then the rods will not as a rule be guided and sockets must be sufficiently deep to prevent the rod from jumping out in the event of a valve sticking.

In cases where some of the spring work is provided at the plunger it may be necessary to attach the rods positively at one or both ends. This may be done either by providing a forked end and pivot pin or by using a restrained ball end.

Bearing pressures on ball ends may be 60 to 100 kg/sq cm depending on lubrication conditions. Pins in forked ends may have the bearing pressure of the same order. A shear stress of 100 to

The maximum force of the spring can be determined when the diagram of inertia forces of the valve operating gear are known. *In order to determine the maximum force of the spring, we assume that the spring is linear and the spring is given initial compression.*

$$\text{Minimum spring force} = (0{\cdot}5 \text{ to } 0{\cdot}8)\ \frac{\pi}{4}\ d^2 \text{ kg} \qquad (iv)$$

where d is the inside diameter of the valve disc in cm.

While determining the maximum inertia force of the valve operating gear masses referred to the valve, certain mathematical calculations are to be carried out to consider the effect of mass of lever, push rod, etc.

The allowable twisting stress for the spring material may be taken as 2,500 to 3,500 kg/sq cm.

If the natural frequency of vibration of a spring is equal or is in a simple ratio to the frequency of disturbing force, there will be synchronous vibrations and the centre coils will begin to surge back and forth, changing the force exerted by the spring. For this reason it is desirable to use two concentric springs *having different natural frequencies*, since the tendency of one spring to surge would be damped out by the other spring. To avoid interlocking one spring should be wound right hand and the other left hand.

Design of a lever or a rocker arm:

Valve levers are made of steel and more or less it resembles to a beam of constant strength when the valve opens, it is subjected on the side of the valve to the following forces

(i) Force of gas pressure

(ii) Inertia of the mass of the valve operating gear

(iii) Spring force.

Assuming that the lever is clamped at the fulcrum, the bending moment can be obtained and when we know the permissible stress intensity, the cross sectional dimensions of the lever can be obtained. The permissible values for flexural stress for cast steel lie between 500 to 600 kg/sq cm and those for forged steel 700 to 800 kg/sq cm.

The force acting on the other end of the lever, which can be determined by taking moment about the lever fulcrum, is transmitted to the push rod and further to the cam. The lever fulcrum,

Modulus of elasticity will be taken as $2{\cdot}1 \times 10^6$ kg/sq cm.

According to Euler's formula, we get

$$2500 = \frac{\pi^2 \times 2{\cdot}1 \times 10^6 \times \pi \times 0{\cdot}344 D^4}{40^2 \times 64}$$

or $$D = \sqrt[4]{\frac{2500 \times 40^2 \times 64}{\pi^3 \times 2{\cdot}1 \times 10^6 \times 0{\cdot}344}}$$

$= 1{\cdot}85$ cm; we adopt 20 mm outside diameter tube having 20 SWG thickness.

This push rod can withstand a design load of

$$\frac{\pi^2 \times 2{\cdot}1 \times 10^6 \times \pi}{40^2 \times 64} [2^4 - 1{\cdot}828^4]$$
$$= 2{,}860 \text{ kg.}$$

Thus the design is safe. The push rod will be a 40 cm long tube of 20 mm outside diameter tube of 20 SWG thickness.

Resultant load on the fulcrum pin will be equal to

$$\sqrt{500^2 + 500^2 + 2 \times 500 \times 500 \times \cos(180° - 160°)} = 950 \text{ kg.}$$

Let us assume that the cross section be rectangular having thickness equal to three eighth of the depth.

Maximum bending moment $= 500 \times 30$
$= 15{,}000$ kg cm.

If d cm be the depth of the section, then

$$\tfrac{1}{6} \times \tfrac{3}{8} d \times d^2 \times 500 = 15000$$

or $$d = \sqrt[3]{\frac{15000 \times 6 \times 8}{1500}} = 7{\cdot}8 \text{ cm;}$$

We adopt 8 cm.

Thickness of the section will be $\dfrac{8 \times 3}{8} = 3$ cm.

Minimum bearing area required for the pin $= \dfrac{950}{170}$
$= 5{\cdot}6$ sq cm.

Diameter of the fulcrum pin $= \dfrac{5{\cdot}6}{3} = 1{\cdot}9$ cm; we adopt 2 cm diameter pin.

2. *Design the tappet, rocker arm and its bearings, the spring and roller for an engine from the following data: Diameter of the valve 8 mm; the lift of the valve 25 mm; the weight of associated parts with the valve 0·4 kg; the angle of action of cam shaft 110°; r.p.m. of the crankshaft 1,500 r.p.m.*

140 kg/sq cm may be allowed on solid pins and 140 to 200 kg/sq cm in hollow pins.

Design of a cam shaft:

The cam shaft is subjected to bending as well as torsion. Hence it should be designed according to maximum shear stress theory. The allowable stress for carbon steel is limited to 500 kg/sq cm. The deflection of the cam shaft should not exceed 0·05 to 0·1 cm.

Note: (i) Refer articles from 8-2 to 8-5 and illustrative example 9 on page 350 for the design of springs
(ii) Refer article 12-7 and pages 502 to 505 for the design of rocker arm for a Diesel Engine
(iii) Refer article 10-7 for the design of push rods

Examples:

1. For operating the exhaust valve of a petrol engine the maximum load required on the valve is 500 kg. This load is exerted by a rocker arm which is actuated through a ball and socket joint from a push rod. The rocker arm oscillates around a pin whose centre line is 30 cm away from the valve axis. The two arms of the rocker are equal and make an included angle of 160°. Design

(a) Push rod if it is 40 cm long and is made of bright steel having an ultimate strength of 3,150 kg/sq cm assuming a factor of safety 5

(b) Rocker arm with the fulcrum, if the safe tensile stress is 500 kg/sq cm and its bearing pressure is 170 kg/sq cm

The push rod is subjected to a maximum compressive load of 500 kg The length of the push rod is normally such that it is to be designed as a long hinged ended column. We employ Euler's formula. Normally standard steel tubes are employed as push rods The thicknesses of these tubes are in accordance with standard wire gauges. In order to simplify the numerical calculations we assume that the inner diameter of the tubular rod is 0·9 of the outer diameter

Second moment of area of cross section

$$I = \frac{\pi}{64}\,[D^4 - (0{\cdot}9D)^4]$$

$$= \frac{\pi}{64} \times 0{\cdot}344D^4.$$

As the factor of safety is 5, the design load will be 500 × 5 = 2,500 kg.

We adopt the material of the rocker arm as cast steel with ultimate strength as 8,000 kg/sq cm. Since the lever is under a varying stress of the same kind, but with some shock a factor of safety of 10 will be chosen. So the permissible stress will be

$$\frac{8000}{10} = 800 \text{ kg/sq cm.}$$

We assume the proportions of I section as under:

Let t be the thickness of the flange as well as web, $6t$ the depth of the I section and $2{\cdot}5t$ as the width of the flange.

Modulus of section for bending about principal axis

$$= \frac{\frac{1}{12}t^4\,[2{\cdot}5 \times 6^3 - 1{\cdot}5 \times 4^3]}{3t}$$

$$= 12{\cdot}33\ t^3.$$

We assume that the arm of the bending moment on the lever extends from the point of application of load to the centre of the point of application of load to the centre of the pivot of the rocker. The section that we determine will be in the region of the central boss whose size is fixed by other considerations. This assumption is commonly made and results in a slightly stronger section. The section through the boss is usually amply strong to resist bending.

$$\therefore \quad 225 \times 18 = 12{\cdot}33\ t^3 \times 800$$

or
$$t = \sqrt[3]{\frac{225}{800} \times \frac{18}{12{\cdot}33}} = 0{\cdot}75 \text{ cm.}$$

We adopt thickness of the flange as well as that of the web as 8 mm; the depth of the section as 48 mm and the width of the section as 20 mm.

Fulcrum for the rocker arm:

The reaction at the fulcrum =

$$\sqrt{225^2 + 225^2 + 2 \times 225 \cos (180 - 135)^\circ} = 418 \text{ kg.}$$

We adopt a bearing pressure of 50 kg/sq cm of projected area. We have determined the width of the I section of the rocker arm as 2 cm. The bearing length of the pin in the rocker arm should be at least this dimension. Let the ratio of bearing length of the pin to its diameter be 1·2.

$$\therefore \quad 1{\cdot}2d^2 \times 50 = 418$$

or
$$d = \sqrt{\frac{418}{1{\cdot}2 \times 50}} = 2{\cdot}64 \text{ cm.}$$

From the probable indicator diagram it has been observed that the greatest back pressure when the exhaust valve opens is 4 kg/sq cm and the greatest suction pressure is 0.2 kg/sq cm below atmospheric.

The rocker arm is to be of I cross section and effective length of each arm may be taken as 180 mm, the angle between the two arms being 135°.

The motion of the valve may be assumed simple harmonic, without dwell in the fully opened position.

Choose your own materials and suitable values for the stresses.

Rocker arm: (Refer fig. 12-20 on page 503.)

Refer the article on pages 502 and 503, and illustrative example 1 on page 503.

The load on the rocker arm consists of

(a) the pressure of the gas on the valve when it opens
(b) the force required to accelerate the valve which will be maximum when the valve just opens as the motion of the valve is harmonic
(c) the initial pressure of the spring necessary to hold the valve on its seat against suction on the induction stroke

Maximum gas load on the valve $= \frac{\pi}{4} \times 8^2 \times 4$

$= 200$ kg.

Gravity effect of 0·4 kg can be neglected.

The speed of the cam shaft is $\frac{1500}{2} = 750$ r.p.m. The half the angle of action of the cam is $\frac{110}{2} = 55°$

During this period the valve moves a distance of 25 mm with simple harmonic motion

∴ Force required to accelerate the valve at the beginning of the stroke (lift) $= \frac{0{\cdot}4}{981} \times \frac{2.5}{2} \times \frac{180}{55} \times \left[\frac{2\pi \times 750}{60}\right]^2 = 10.4$ kg.

The force on the valve tending to draw it into the cylinder on the suction stroke $= \frac{\pi}{4} \times 8^2 \times 0.2 = 10$ kg.

The total maximum force on the rocker arm $= 200 + 10 + 10{\cdot}4 = 220{\cdot}4$ kg; say 225 kg.

As the two arms of the rocker arm are equal, the force at the cam end will also be 225 kg.

this circular end will be $2 \times 15 = 30$ mm. The height of this end will be also 30 mm.

Spring: The maximum load on the spring will be when the valve will be fully lifted. From the observed existing design we adopt a spring of stiffness 9 kg/cm and the spring index as 8.

Total load on the spring required will be $10 + 9 \times 2{\cdot}5 = 32{\cdot}5$ kg. We design the spring for 35 kg load. We adopt spring steel for which permissible torsional shear stress value is 4,200 kg/sq cm.

$$\text{Stress concentration factor} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0{\cdot}615}{8}$$
$$= 1{\cdot}2.$$

If dw be the diameter of the wire, then

$$35 \times 4dw = \frac{\pi}{16} dw^3 \times \frac{4200}{1{\cdot}2}$$

or
$$dw = \sqrt{\frac{35 \times 4 \times 16 \times 1{\cdot}2}{\pi \times 4200}} = 0{\cdot}47 \text{ cm.}$$

We adopt 5 mm wire having mean diameter of coil as 40 mm.

If n be the number of active coils, then

$$n = \frac{G\,dw}{8C^3 k}$$ where G is the modulus of rigidity and C is the spring index.

On substitution of numerical values we get

$$n = \frac{0{\cdot}84 \times 10^6 \times 0{\cdot}5}{8 \times 512 \times 9} = 11{\cdot}5.$$

Since the spring is to be used in compression, $1\frac{1}{2}$ more turns must be used to allow for the bearings at the ends. Therefore 13 coils of 5 mm diameter wire having 40 mm as the mean coil diameter are required for the spring.

Free length of the spring can be obtained as under:

Solid height $= 13 \times 0{\cdot}5 = 6{\cdot}5$ cm.

$$\text{Maximum compression} = \frac{35}{9} = 4 \text{ cm.}$$

$\therefore$ Free length we take as 12 cm so as to allow for adjustment.

Exercises:

1. *The exhaust valve of a four stroke cycle Diesel Engine running at 1,500 r.p.m. has a diameter of 3 cm and a lift of 0·6 cm. The valve opens when the pressure in the engine cylinder is 6 kg/sq cm by gauge.*

We adopt diameter as 2·8 cm and bearing length in the rocker arm as 3·2 cm which will provide a bearing area of 9 sq cm.

We use a brass bush pressed into the boss as a bearing so that the renewal will be simple when wear occurs. The thickness of the boss will be 3 mm. Hence internal diameter of the boss will be $2{\cdot}8 + 2 \times 0{\cdot}3 = 3{\cdot}4$ cm. The external diameter of the boss will be 5·5 cm (twice the diameter of the pin). Lubrication arrangement should be provided at the bearing surfaces to minimise the wear.

Roller end:

The roller end of the rocker arm should be forked and the roller is carried by a pin, which should be free to revolve in the eyes. It is necessary to provide a cut away portion to clear the nose of the cam. We find the dimensions of the pin by bearing consideration, assuming $\frac{l}{d}$ ratio as 1·2 and a bearing pressure intensity of 70 kg/sq cm.

$\therefore \quad 225 = 70 \times 1{\cdot}2\, d^2$

or $\quad d = \sqrt{\dfrac{225}{70 \times 1{\cdot}2}} = 1{\cdot}67$ cm; we adopt 16 mm diameter pin with a bearing length of 20 mm.

This value of bearing length is the minimum distance between the eyes, each eye must be at least 10 mm in each case. 3 mm thick phosphor bronze bush must be provided in the eye to account for wear. This provision need not be made in the roller as wear will also have occured in the profile and hence instead of providing for wear, we replace the roller.

The outer diameter of the roller should be as large as is reasonably possible and should be at least 2 × diameter of the pin = $2 \times 1{\cdot}6 = 3{\cdot}2$ cm.

Since clearance is necessary between the roller and the inside of the fork there will be some bending on the pin so this should be checked by the method used for the forked pin joint.

Tappet:

It will be adjustable. It will be a threaded rod provided with a lock nut. Coarse threads are to be used. We adopt M 15 and it will safely carry the load. The end of the rocker arm to carry this stud will be circular and the diameter of

EXAMPLES XVIII

1. Design a steam chest cover with the following particulars:

diameter of cylinder 30 cm	stroke of engine 45 cm
stroke of valve 10 cm	width of steam port 2·5 cm
width of exhaust 6·5 cm	valve rod diameter 2·5 cm
steam pressure 10 kg/sq cm.	

Assuming the cover to be rectangular and made of cast iron, draw a dimensioned drawing of the cover.

Permissible stress in studs 300 kg/sq cm.

2. A locomotive coupling rod is of I cross section. The maximum thrust in the rod is 10 tonnes. The rod is 2·5 metre between centres. Neglecting friction at the pins, if the maximum intensity of stress in the rod equals 10 kg/sq mm, determine the thickness of the web and flanges, assuming depth as 100 mm.

The rocker arm has two equal arms of 8 cm length. The push rod is 30 cm long. The angle between the two arms of the rocker arm is 180°. The valve is to remain open 30° before outer dead centre and 24° after inner dead centre. Simple harmonic motion without duell in the fully lifted position may be assumed. Taking the weight of the valve, the push rod and the parts attached to it as 1 kg, design the following:

(i) *Rocker arm and the fulcrum pin*

(ii) *Push rod*

Choose your own materials and suitable values for the stresses.

2. *Sketch a rocker and push rod arrangement for operating the overhead valves of an I.C. engine. Make a dimensioned sketch of such a rocker given that the each of the arms of the rocker arm is 6 cm long. The maximum load on push rod is 55 kg. Maximum stress in the cast steel is limited to 400 kg/sq cm.*

3. *An overhead exhaust valve of an I.C. engine opens at 30° before bottom dead centre and closes 10° after top dead centre. The maximum net force exerted by the push rod is 120 kg. The length of the push rod is 450 mm. Suggest the suitable cross sectional dimensions for the tubular push rod assuming that the end fixity coefficient is 2.*

Take the factor of safety as 3.

(*Rajasthan University*, 1969)

4. *The following particulars refer to the valve spring of an automobile petrol engine:*

Length of the spring when the valve is open	= 40 mm
Length of the spring when the valve is closed	= 48 mm
Spring load with valve open	= 40 kg
Spring load with valve closed	= 25 kg
Maximum inner diameter of the spring	= 26 mm

Calculate:

(i) *size of the wire required for a maximum operating stress of 50 kg/sq mm*

(ii) *number of active coils required*

(iii) *spring length when completely closed, assuming square and ground ends* $G = 0.83 \times 10^6$ *kg/sq cm*

(iv) *pitch of the spring.*

(*Rajasthan University*, 1969)

7. A spring controlled governor of Hartnell type has ball arms 125 mm long, sleeve arms 75 mm long, the pivots of the bell crank in which the ball and sleeve arms are at right angles to each other are at a distance of 100 mm from the axis of the governor spindle and the weight of each ball is 3·53. The sleeve begins to lift at 270 r.p.m. and is to lift 13 mm for an increase of speed of 6%. The sleeve arm of each lever presses against the sleeve, the movement of which is controlled by a spring. The ball centres are vertically above the pivot at the lowest position of the sleeve.

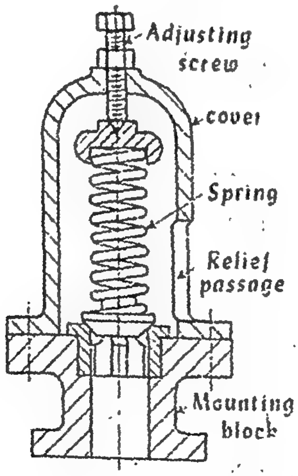
Design a suitable spring, spring housing containing the pivot for the bell crank and the spindle supporting the housing. Make dimensioned sketches of the free spring and the housing. Select your own materials and suitable design stresses for these materials.

8. Design and draw an eccentric complete with straps, disc, bolts, key and eccentric rod from the following data:

Diameter of crank shaft 10 cm—eccentricity of eccentric 4 cm; valve load on eccentric—180 kg; length of eccentric rod 120 cm; r.p.m. of crank shaft—100.

The straps and disc are made of cast iron while the eccentric rod bolts, studs and key are made of mild steel.

9. A spring loaded relief valve, fig. 18-25 is to be designed for the following particulars:



Cylinder relief valve

FIG. 18-25

Blow off pressure 14 kg/sq cm gauge; lift of the valve when the pressure rises from 14 to 15 kg/sq cm gauge 8 mm; diameter of the valve 50 mm.

Use the following materials:

(i) Valve body of C.I. safe stress 150 kg/sq cm in tension.

3. The following data refer to a *four stroke cycle, single cylinder horizontal* Diesel engine:

suction pressure	0 95 kg/sq cm	cylinder diameter	20 cm
ratio of compression	15	stroke	25 cm
engine speed	750 r.p m.		

equivalent weight of reciprocating parts 0 2 kg/sq cm of piston area

ratio of $\dfrac{\text{connecting rod length}}{\text{crank length}}$ 4.

Design and prepare a working sketch of Nickel steel connecting rod of I section, choosing suitable values for the permissible stresses for the materials.

4. Design and prepare fully dimensioned drawings of the connecting rod of a *steam engine* to the following data *Length of the rod* 84 cm; diameter of the crank pin 16 cm, diameter of the crosshead pin 9 5 cm and the maximum load on the rod 16,000 kg. The rod is to be made hollow by boring a central hole of 28 mm dia throughout the length Calculation should be shown for:

(i) the external diameter of the rod at the centre

(ii) length of the crosshead pin

(iii) length of the crank pin

(iv) diameter of the big end bolt

(v) width and thickness of the cap.

5. A twin cylinder V-engine has cylinder centre lines at right angles The stroke of each piston is 115 cm and length of connecting rod is 230 mm Both the connecting rods drive on to the same crank pin. The span of main bearing is 280 mm. There are two flywheels each requiring an extension of 100 mm The maximum torque occurs when the crank has turned through 30° from the inner dead centre of the left hand cylinder. The net forces (inclusive of inertia forces) on the pistons at this instant are as follows: Left hand cylinder 6,230 kg (on expansion stroke), right hand cylinder 266 kg (on exhaust stroke)

Design and draw a working drawing of the crankshaft Show also the method of lubrication of crank pin bearings.

Material *Ni*-steel forging — f_s 550 kg/sq cm
Bearing pressures — crank pin 100 to 125 kg/sq cm
" " — main bearings 35 to 55 kg/sq cm

6. A single cylinder *Diesel* engine having a cylinder diameter of 15 cm and a stroke 20 cm develops 20 h.p. at 600 r p m The maximum torque occurs when the crank shaft has turned through 25° from the inner dead centre position during the expansion stroke The net gas force at that instant is 40 kg/sq cm The crankshaft is *supported in three bearings* . *The main bearings* are 35 cm apart and the outer bearing is at [illegible] cm from the main bearing. *A flywheel weighing* 300 kg and a driving pulley of 60 cm diameter are placed between the main bearing and the outer bearing

Design and prepare a dimensioned drawing of the crankshaft. Calculate the stresses in the webs. The permissible shear stress in crankshaft is 520 kg/sq cm.

the ball path is 17·8 cm. For proper speed regulation the spring force must be 59 kg in limiting position. When the radius of rotation of the ball path changes from 12·7 cm to 17·8 cm, the spring is compressed by the amount 4·5 cm. Assuming the spring index to be 8, design the spring which is to be used for the spring loaded governor.

Maximum permissible stress = 4,200 kg/sq cm

Modulus of rigidity $G = 0{\cdot}84 \times 10^6$ kg/sq cm

The size of the spring wire should be selected from SWG table.

(University of Bombay, 1970)

14. (i) Make a sketch of a disc cam with a radial roller follower, label the pitch surface and the working surface and indicate the pitch point and the pressure angle for a given phase.

(ii) What is generally regarded as limiting value for the pressure angle? Why should this angle be limited?

(iii) Explain why roller follower is to be preferred to a direct contact follower.

(iv) Explain how the size of the base circle affects the pressure angle.

(v) Discuss the factors in detail that affect the size of the cam.

(University of Bombay, 1970)

15. Design four stroke petrol engine developing 200 BHP at 3,000 r.p.m. Assume volumetric efficiency as 75% and compression ratio of 6·5 : 1. Assume all other design parameters and proportions necessary for the design.

(a) Cylinder dimensions

(b) Cylinder cover including bolts

(c) Piston including piston rings and gudgeon pin, and

(d) Suction valve.

Draw neat sketches of the parts designed.

(M. S. University of Baroda, 1970)

(ii) Valve, valve seat, compression screw and spring cage of gun metal, safe stresses, $f_s = f_c = 200$ kg/sq cm and $f_t = 350$ kg/sq cm.

(iii) Spring *Ni-Cr* steel, safe stress in shear 5,600 kg/sq cm

(iv) Bolts or studs of M S , $f_t = 350$ kg/sq cm, $f_c = 550$ kg/sq cm, $f_s = 300$ kg/sq cm.

Also, assume the spring index as 6 and take $G = 8{\cdot}4 \times 10^5$ kg/sq cm.

Sketch dimensioned sectional elevation and the plan of the valve of your design

10. The cast iron cylinder of a low speed steam engine is 20 cm in diameter and the steam pressure is 9 atg The length of the piston rod is 75 cm and the length of the connecting rod is 120 cm.

Determine

(a) the thickness of the cylinder wall if the allowable stress is 120 kg/sq cm

(b) the diameter of the piston rod,

(c) the dimensions of the connecting rod which has got a rectangular cross section, and

(d) the dimensions of the overhung crank pin if the bending stress is to be 700 kg/sq cm and bearing pressure 45 kg/sq cm

Sketch the ends of the piston rod so that it may be suitably fastened

Also suggest the suitable number of studs to connect the cylinder cover with the cylinder flange; also suggest the suitable thickness for the cylindrical flange, cylinder cover and size of the studs Also sketch the cylinder cover for this engine

11. Design the crank shaft for a single throw pump to have a stroke of 40 cm and connecting rod equals 5 times the crank The pressure in the cylinder is [illegible] effort is 4 tonnes The ratio [illegible] Bearing [illegible] al is to be made same as the crank pin *diameter The web thickness should not exceed* 1 5 cm and the flanges at the junction of the shaft and web 15 mm Also calculate the principal stress in the crank pin and the maximum shear stress in the web.

12. Calculate the main dimensions of the following parts of 6 cylinder Diesel engine developing 60 H P running at 1 500 r.p.m Mention clearly the values of design parameters Assume the stresses in each case Draw the neat sketches of the parts designed

(a) Cylinder (diameter, stroke, wall thickness, cylinder flanges and studs)

(b) Crank case

(c) Piston including rings and pins, and

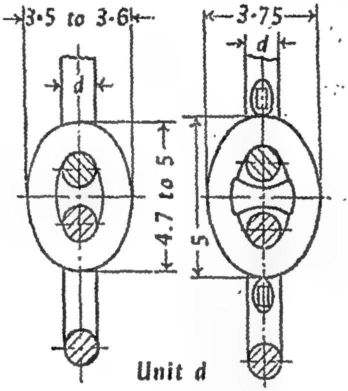
(d) Suction and exhaust valves

(M. S. University of Baroda, 1969)

13 A force analysis of the spring loaded governor suggests that a spring force of 19 kg is required to permit the proper speed when the radius of rotation of the ball path is 12 7 cm. Assume that in limiting position the radius of rotation of

safe for coil links. However, the addition of stud results in a decrease of the ultimate strength of the chain.

With respect to external forces the links of welded chains are statically determinate and with respect to internal stresses they are statically indeterminate. It is, therefore, extremely difficult to find the actual stresses, which can be determined only approximately.



Coil chain *Stud link chain*

Chains

Fig. 19-1

When tensile load is applied to a chain, each link is subjected to bending stresses in addition to direct tensile stresses. The bending is greatest at the extremities of the longer diameter of the link.

As a rule chains are checked for tension by taking some what reduced safe stress to account for the statically indeterminate feature of the links with respect to the stresses and additional bending stresses when the chain runs over pulleys and drums.

The permissible load P in kg on the chain of the coil types is given by

$P = 940\ d^2$ .... (i)

where d is the diameter of the link in cm.

The chains used for crane are subjected to shock loading so the working load for the same diameter of the link is reduced and is given by the equation

$P = 500d^2$ .... (ii)

The diameter of the drum on which the chain is being wound depends on the speed of hoisting, the load to be raised and the life

CHAPTER **XIX**

DESIGN OF MISCELLANEOUS MACHINE PARTS—II BRAKES AND CLUTCHES

(A) HOISTING EQUIPMENTS

19·1. Introduction:

The component parts and units of hoisting equipment include the following items

(i) Flexible hoisting appliances such as chains and ropes
(ii) Pulleys, sprockets and drums
(iii) Load handling attachments such as hooks
(iv) Brakes
(v) Drives
(vi) Transmission components, such as axles and shafts, bearings, clutches, etc
(vii) Rails and travelling wheels
(viii) Machine structures such as frames
(ix) Control devices.

Due to shortage of space it may not be possible for us to consider all the parts in detail; however in this chapter we shall consider the design of flexible hoisting appliances, pulleys or sheaves, drums, crane hooks, brakes and clutches In earlier chapters we have considered the design principles of transmission components such as axles, shaft, bearings and gears.

19·2. Design of hoisting chains and drums:

Two types of chains are found in engineering practice Coil chains are used on hoists, cranes and dredges, while stud link chains are found in marine practice in connection with anchors and moorings. Both these types of chains are shown in fig. 19-1. The chains are forged out of round iron bars of best quality. The experiments carried out by G. A. Goodenough and L. E. Moore reveal that for the same size of the link, a stud link within elastic limit will carry 20 per cent greater load than that which is

safe for coil links. However, the addition of stud results in a decrease of the ultimate strength of the chain.

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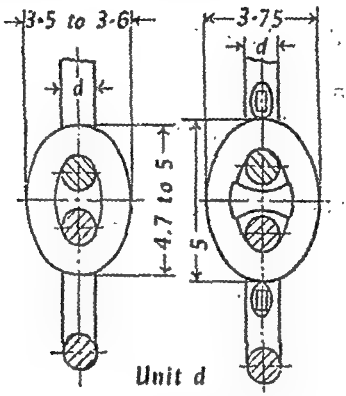

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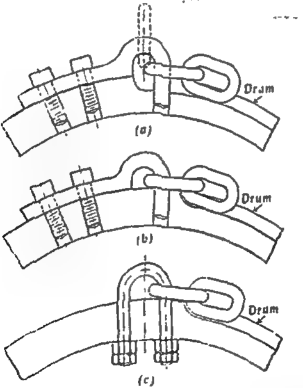
The chains used for crane are subjected to shock loading so the working load for the same diameter of the link is reduced and is given by the equation

$$P = 500 d^2 \qquad \text{(ii)}$$

The diameter of the drum on which the chain is being wound depends on the speed of hoisting, the load to be raised and the life

of the chain. The drum diameter should be more than twenty diameters of the link and preferably it should be thirty diameters. Machined helical grooves are provided on the drum for the chain. The length of the drum should be such that the required length of the chain may be wound upon the drum in one layer It is considered good design practice to have one or two coils on the chain remaining on the drum when the load is in its lowest position, thus reducing the stress coming upon anchor. Fig. 19-2 shows the various methods of connecting the chain to the drum, while fig. 19-3 shows the common forms of helical groove for the drum

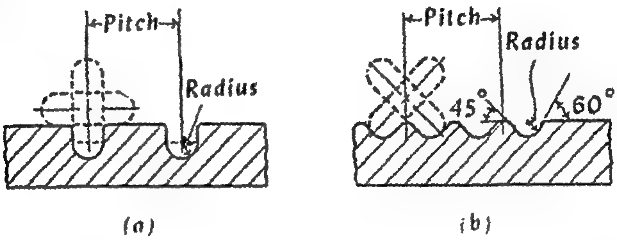


Connections of chains to drum
Fig. 19-2

A good design for the drum has shaft stationary while the drum rotates with which the driving gear is rigidly connected The drum hub is provided with bronze bushes Manytimes the shaft is cast into the drum and the whole combination rotates as a single unit.

The exact stress analysis for a hoisting drum is very complicated and hence the following approximate method may be used for arriving at or checking the thickness of the metal below the groove:

(i) The minimum thickness t of the metal is decided from casting considerations and considerations for machining the groove.



Groove for chains on drum
FIG. 19-3

(ii) The bending stresses are calculated by treating the drum as a hollow cylindrical beam loaded at the centre and supported at the ends. The value should not exceed 210 kg/sq cm for cast iron and 350 kg/sq cm for cast steel.

(iii) The tangential crushing stress, f, due to tension in the coils of rope about the drum is calculated. The rope tension varies from coil to coil, and since maximum values are sought, we consider the coil supporting the load.

$$f = \frac{F}{t\,p} \quad \text{(iii)}$$

F = maximum rope tension
t = thickness of the metal below the groove
p = pitch of the groove.

The value of f should not exceed 1,050 kg/sq cm for ordinary cast iron, 1,200 kg/sq cm for the best cast iron or 840 to 1,150 kg/sq cm for cast steel.

(iv) Shearing stresses due to torsional moment transmitted should be calculated. As a rule this stress is very small and is usually not considered.

(v) The stresses calculated in (ii) and (iii) are combined. Drums, thus, designed have sufficient strength and in general the weight is not excessive.

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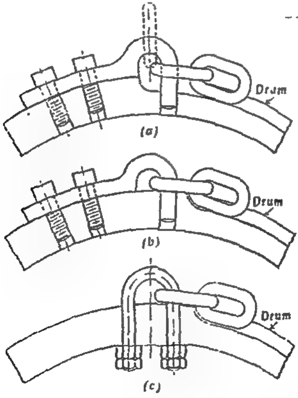


Connections of chains to drum
FIG. 19-2

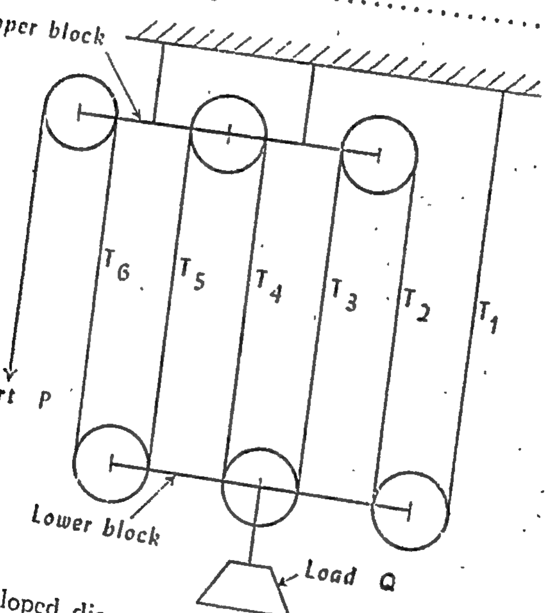
A good design for the drum has shaft stationary while the drum rotates with which the driving gear is rigidly connected. The drum hub is provided with bronze bushes Manytimes the shaft is cast into the drum and the whole combination rotates as a single unit.

From equilibrium of forces on the lower block, we have

$$Q = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$$
$$= T_1 (1 + C + C^2 + C^3 + C^4 + C^5)$$
$$= T_1 \left(\frac{C^6 - 1}{C - 1}\right) = P \frac{(C^6 - 1)}{C^6 (C - 1)}$$

or $$P = Q \cdot \frac{C^6 (C - 1)}{(C^6 - 1)} \quad \text{(iii)}$$

Without friction $$P_o = \frac{Q}{6} \quad \text{(iv)}$$



Developed diagram for a hoisting tackle

FIG. 19-4

Hence efficiency of the hoist is given by

$$\eta = \frac{P_o}{P} = \frac{C^6 - 1}{6\, C^6 (C - 1)} \quad \text{(v)}$$

In general, when the block and tackle has n sheaves and n supporting the load Q, we shall have

$$P = Q\, C^n \left[\frac{C - 1}{C^n - 1}\right] \quad \text{(vi)}$$
$$= \frac{C^n - 1}{nC^n (C - 1)} \quad \text{(vii)}$$

19·3. Design of a hoisting rope:

Vegetable fibre ropes made from manila or cotton fibres are used for temporary hoists and slings. In hoisting operations, ropes are wound upon drums and sheaves are used for changing the direction of the rope.

It is necessary to know the relation existing between the effort and the resistance applied to the ends of the rope running over a sheave. The rigidity of the rope and the friction of the sheave pin increase the resistance that the effort applied to the running side must overcome.

The relation between the effort P and resistance Q is given by the equation

$$P = CQ \quad \text{.................................} \quad (i)$$

where C is a constant greater than unity and depends on the size of the rope, the relative size of the sheave and the pin and the coefficient of friction. For a manila rope an average value of C is 1·14 and the value of C for wire rope varies from 1·09 to 1·04 for rope diameters ranging from 9 mm to 25 mm. The larger values are taken for smaller diameters.

The efficiency of a mechanism is defined as the ratio of the useful work to the total work put in. So it is evident that the efficiency of the guide sheave is given as

$$\eta = \frac{Q}{P} = \frac{1}{C} \quad \text{.................................} \quad (ii)$$

Analysis of a hoisting tackle:

The common hoisting block and tackle consists of two pulley blocks one above the other. Each block has a series of sheaves mounted side by side on the same axle pin. The principle explained earlier can be used to analyse the hoisting tackle. The arrangement shown in fig. 19·4 is used. Beginning with the end of the rope fastened to the upper block, let successive tensions in the parts of the rope supporting the load Q be denoted by T_1, T_2, T_3, etc.

Let us consider the case when the load is being raised.

From the previous analysis,

$$T_2 = CT_1;\; T_3 = CT_2 = C^2T_1;\; T_4 = CT_3 = C^3T_1;$$
$$T_5 = CT_4 = C^4T_1;\; T_6 = CT_5 = C^5T_1;\; P = CT_6 = C^6T_1$$

While raising the load the rope P is the maximum loaded one while lowering the load the rope T_1 is the maximum loaded one. The capacity of a hoist depends on the most heavily loaded rope.

While lowering the load, the general relation between P and Q is given by

$$P = Q\frac{(C-1)}{C(C^n-1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{viii})$$

$$T_1 = PC^n = Q\frac{C^n(C-1)}{C(C^n-1)}$$

$$= Q\frac{C^{n-1}(C-1)}{(C^n-1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{ix})$$

The ultimate strength S of the manila rope is given by

$$S = 500\ d^2\ \text{kg} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{x})$$

where d is the diameter of the rope in cm

The ultimate strength for the cotton rope is given by

$$S = 357\ d^2\ \text{kg} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(\text{xi})$$

The factor of safety ranges from 30 to 35

The size of the sheave, over which the rope runs, has a considerable influence on the life of the rope. To ensure a reasonable rope life, for slow speed haulage upto 15 metre/min the minimum sheave diameter may be as low as 8 to 10 times the rope diameter. For high speeds upto 180 metre/min, the sheave diameter may be as high as 40 to 50 times the rope diameter.

The poor mechanical properties of hemp ropes make them suitable only for hand operated hoisting machinery

The sheaves used for hoisting purposes vary considerably in design. The sheaves are provided with arms of cross shaped section. The arms are under direct compression. These are of cast iron so there will be residual stresses in the rim and arms. For crane work, the sheaves are usually constructed with a central web in place of arms and openings are provided in the web to reduce the weight of the sheave

19.4. Design of wire ropes:

The wire ropes are used for hoisting and haulage and for static loading such as guys and supporting wire for stacks, masts, etc. The wire rope consists of cold drawn steel wires wrapped into strands and twisted around a hemp centre or core saturated with

The effort is 136 kg. Let us adopt a factor of safety to be 30.

Working load in the rope will be $\frac{500d^2}{30} = 136$.

$$\therefore \qquad d = \sqrt{\frac{136 \times 30}{500}} = 2{\cdot}87 \text{ cm; we adopt 3 cm.}$$

Note: The wire rope would be more suitable for the service.

2. *A hand operated winch has a drum 30 cm in diameter. Suggest the suitable wire rope for a load of 600 kg. Suggest the suitable thickness for the drum radius of the groove and the pitch of grooves.*

Let us adopt 6 × 19 rope having ultimate strength $5100d^2$ kg. The maximum rope diameter d based upon the minimum sheave diameter from the table will be $\frac{30}{30} = 1$ cm. We adopt 1 cm rope. The diameter of the wire will be $0{\cdot}063d = 0{\cdot}063$ cm.

Bending load in the rope

$$= \frac{0{\cdot}84 \times 10^6}{30} \times 0{\cdot}063 \times 0{\cdot}38 \times 1^2 = 670 \text{ kg.}$$

Service load = 600 kg.

Total load in the rope = 600 + 670 = 1,270 kg.

Breaking load = $5100d^2 = 5100\ (1)^2 = 5{,}100$ kg.

The factor of safety $= \frac{5100}{1270} = 4{\cdot}02$.

As the value of the factor of safety is satisfactory we adopt 1 cm diameter 6 × 19 rope.

The thickness of the drum below the helical groove is adopted as 1 cm. The radius of the groove will be $0{\cdot}5d + 1{\cdot}5$ mm = 6·5 mm. The pitch of the groove will be $d + 1{\cdot}5$ mm. Therefore the pitch of the groove will be 12 mm.

3. *A hand operated wire rope hoist is to raise a load of 200 kg. The force on the operating lever is limited to 20 kg and the rope is being wound on a drum of 25 cm diameter. The effective length of lever is 35 cm. Determine the number of ropes leading to the hook block the efficiency of the hoist and the factor of safety of the hoist, assuming that it is reefed with 15 mm — 6 × 37 wire rope.*

The maximum effort $P = \frac{20 \times 35}{12{\cdot}5} = 56$ kg.

Let us adopt a hoisting block with 5 sheaves.

$$\frac{f}{\frac{1}{2}d_w} = \frac{E}{\frac{1}{2}D} \text{ or } f = E.\frac{d_w}{D} \qquad \text{(i)}$$

where E is Young's modulus.

Since the twisted rope is flexible, the stress will be less than the value given by equation (i). The modulus of elasticity for the rope is taken as 0.84×10^6 kg/sq cm.

The following table gives the necessary information for the commonly adopted wire ropes. In the following table, d is the diameter of rope in cm.

Rope	Nature	Breaking load in kg	Diameter of wire in cm	Area of wires in rope sq cm	Diameter of sheave in cm	
					Minimum	Advisable
6×7	Coarse	$4800\ d^2$	$0.106\ d$	$0.38\ d^2$	$42\ d$	$72\ d$
6×19	Flexible	$5100\ d^2$	$0.063\ d$	$0.38\ d^2$	$30\ d$	$45\ d$
6×37	Extra flexible	$4800\ d^2$	$0.045\ d$	$0.38\ d^2$	$18\ d$	$27\ d$
8×19	Flexible	$4400\ d^2$	$0.050\ d$	$0.35\ d^2$	$21\ d$	$31\ d$

The factor of safety varies from 8 to 12 for elevator service, from 3 to 5 for mine hoists, derrick service and hand operated cranes and from 4 to 6 for power actuated cranes.

Spliced ropes have 75% of the breaking strength of the unspliced rope and spliced ropes are not used for important service.

Examples:

1. *A block and tackle, having two sheaves at the top block and two at the hook with the rope anchored at the top block, is reefed with a manila rope. Suggest the suitable diameter for the rope if the maximum load to be raised is 400 kg. Also calculate the efficiency of the hoist.*

For manila rope $C = 1.14$

$$\eta = \frac{C^n - 1}{nC^n\,(C-1)} = \frac{1.14^4 - 1}{4 \times 1.14^4\,(1.14-1)} = \frac{0.69}{4 \times 0.14 \times 1.69}$$

$$= 0.735 \text{ i.e. } 73.5\%.$$

$$P = Q\,.\,C^n\left[\frac{C-1}{C^n-1}\right] = \frac{400 \times 1.14^4}{1.14^4 - 1}\left[1.14 - 1\right] = 136 \text{ kg.}$$

load on the drum at the rated capacity of the hoist is 40 kg. Assuming 100% overload capacity and allowable shear stress of 420 kg/sq cm, calculate the dimensions for the shaft, key, drum and the hand lever. The length of the lever is 25 cm and is made of steel for which the bending stress may be taken as 1,050 kg/sq cm.

Ans. *Diameter of shaft 25 mm; thickness of drum 1 cm; 6 × 37, 1 cm wire rope; the pitch of groove 11 mm; the circular part of the lever 15 mm dia 15 cm long.*

19-5. Stresses in curved beams:

We come across many applications of curved beam principles in engineering, the most common being the frames of machines such as punches, presses, planners, etc. The common example of curved beam is the crane hook.

In case of straight beams, the neutral axis of the section coincides with the gravity axis, which is not true for the sections of the curved beam. In case of straight beam, the stress in any fibre is proportional to the distance of the fibre from the gravity axis.

In case of curved beams, the neutral axis is shifted towards the centre of curvature of the section and is nearer to the centre of curvature than the gravity axis.

If the section of curved beam of area A is subjected to a bending moment M, the general equation for the stress at any fibre at a distance y from the neutral axis is given as

$$f = \frac{M}{Ae}\left(\frac{y}{r_n + y}\right) \qquad \text{(i)}$$

where e = the distance from the gravity axis to the neutral axis

r_n = the radius of curvature of the neutral axis.

The maximum stresses will occur at the extreme fibres.

The general expression for r_n is given as

$$r_n = \int \frac{dA}{\frac{dA}{r}} \qquad \text{(ii)}$$

For convenience the values of r_n for various sections are given on page 835.

If we consider a beam of rectangular cross section, the depth of which is twice the inner radius and it is subjected to a bending moment M, the shift of the neutral axis will be 0·0897d where d

The effort required to raise a load of 200 kg is given as
$P = \frac{C^5\,(C-1)}{C^5-1} \times 200.$

For wire rope the value of C is 1·076.

$\therefore \quad P = \frac{200 \times 1{\cdot}076^5\,(1{\cdot}076-1)}{(1{\cdot}076)^5-1} = 49{\cdot}3 \text{ kg}$

Thus the arrangement will be suitable.

The efficiency of the hoist $= \frac{(C^5-1)}{5\,C^5\,(C-1)}$

$= \frac{(1{\cdot}076)^5-1}{5 \times 1{\cdot}076^5\,(1{\cdot}076-1)} = 0{\cdot}81$ i.e. 81%.

Breaking strength of the rope $= 4800 \times d^2 = 4800 \times 1{\cdot}5^2$
$= 10{,}400$ kg.

Diameter of the wire $= 0{\cdot}045 \times d = 0{\cdot}045 \times 1{\cdot}5 = 0{\cdot}0675$ cm.

Area of the rope $= 0{\cdot}38 \times 1{\cdot}5^2 = 0{\cdot}85$ sq cm

Bending load $= \frac{0{\cdot}84 \times 10^6 \times 0{\cdot}0675}{25} \times 0{\cdot}85 = 1{,}930$ kg.

Service load $= 56$ kg.

Total load $= 1930 + 56 = 1{,}986$ kg

Factor of safety $= \frac{10400}{1986} = 5{\cdot}32.$

Exercises:

1. A block and tackle has six sheaves on each block and is reefed with 2·5 cm diameter manila rope for which the working load is given as $P = 30d^2$ kg. Determine the capacity of block and tackle and its efficiency. Ans. *1 tonne; 47·2 %.*

2. A high speed mine hoist has a lift capacity of 2,000 kg and the hoist drum speed of 500 r.p.m. The hoist drum is 90 cm in diameter The hoist lift is 600 metre and the hoist attains a full speed in 30 seconds. Select a suitable wire rope for the service.

3. Design a hand operated winch to lift a load of 400 kg through a maximum height of 3 metre. The winch is fixed to the vertical stanchion and the rope leads vertically upwards from the drum.

Assume suitable materials for the various parts and safe working stresses in them. Draw an assembly drawing of the winch.

4. The hand operated wire rope hoist shaft has bearings 30 cm apart. The drum of 20 cm diameter is keyed to the shaft. The tangential

were to be a straight beam, the flexure stress in extreme fibres will be $\frac{6M}{bd^2}$. Thus we see that because of the curvature the stress at the inner fibre is about 52% greater than the stress in a straight beam and the stress at the outer fibres is only 73% of the stress in a straight beam. Thus we see that *the effect of curvature is to result in non-linear distribution of stresses. The curvature changes the force flow lines so that there is stress concentration towards the concave side of a beam.*

Curved beams subjected to bending moment alone are of rare occurance, but examples of curved beams subjected to bending and direct stress are numerous. Hooks, rings or other links, frames of machines, *C* clamps and tools of various sorts are common examples. The stress at any point on a cross section of such a member is the algebraic sum of the direct stress and the stress due to bending moments as in straight beams; however the flexural stress should be calculated according to curved beam theory.

It is natural that appearance of the large stress concentration at the inside of the curve has set designers busy making cross sections *of which the centre of gravity is shifted towards the inside so that* by the straight beam theory the stress in outer fibre should be greater than the inside fibre stress which effect is to be neutralised by the curved beam stress concentration as a result approximately equal stresses at the extreme outer and inner fibres are achieved. Such sections are trapezium, triangle, *T* section and non symmetrical I sections.

We have three symmetrical sections: circular, rectangular and symmetrical I. The stress concentration is more for circular sections and less for symmetrical I section.

If holes must be put in a curved beam, they should be located on the neutral axis to decrease the effect of stress concentration.

As it is not possible to get the direct solution for the required dimensions of a curved beam, the best procedure is to assume the dimensions for the section and then to calculate the stresses.

Example:

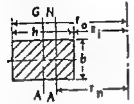
1. A punch press, used for stamping sheet metal, has a punching capacity of 7,500 kg. The shape of the dangerous section is as shown in fig. 19-6. Calculate the maximum stress in the section of the frame.

Area of the section $= 30 \times 10 + 20 \times 10$

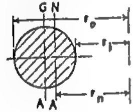
$= 500$ sq cm.

VALUES OF r_n FOR CURVED BEAM

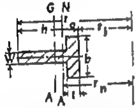
Shape of section | Radius of neutral surface r_n



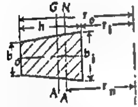
$$r_n = \frac{h}{\log_e \frac{r_o}{r_i}}$$



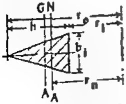
$$r_n = \frac{(\sqrt{r_o}+\sqrt{r_i})^2}{4}$$



$$r_n = \frac{(b-W)\,t + Wh}{(b-W)\log_e \frac{(r_i+t)}{r_i} + W\log_e \frac{r_o}{r_i}}$$



$$r_n = \frac{\left(\frac{b_i+b_o}{2}\right) h \,\cancel{(r_o-r_i)}}{\left(\frac{b_i r_o - b_o r_i}{h}\right)\log_e \frac{r_o}{r_i} - (b_i - b_o)}$$



$$r_n = \frac{\frac{b_i(r_o-r_i)}{2}}{\frac{b_i\, r_o}{(r_o-r_i)}\log_e \frac{r_o}{r_i} - b_i}$$

is the depth of the beam. The stress at the concave inner surface will be $\frac{9 \cdot 14M}{bd^2}$ where b is the breadth of the section and the stress at the convex outer surface will be $\frac{4 \cdot 38M}{bd^2}$. If the beam

and compressive stress is 1,000 kg/sq cm, is the C clamp properly designed from the stand point of strength? If the design is not proper, what changes could be made to improve the design? Assume that the analysis is to be made after the screw has been tightened.

Ans. The design will not be satisfactory. Suggestions to improve the design:

(i) *Non-symmetrical sections (T or I) should be used.*

(ii) *Radius of curvature should be increased.*

(iii) *Diameter (25 mm) of the circular section should be increased.*

2. *It is necessary to bend a link as shown in fig. 19-7 in order to prevent interference with another part of the machine. The link is to support a load of 1,250 kg with a design factor of 2·5 on the yield strength of 25 kg/sq mm. Determine the thickness of the rectangular cross section of the link if the depth is to be 5 cm, neglecting stress concentration due to curvature and then find the increase in stress in this section when curvature is taken into account.*

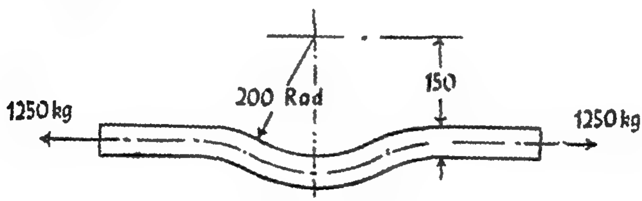


FIG. 19-7

19-6. Design of a crane hook:

The hook is a curved bar subjected to direct and bending stresses. In this article we adopt the principles of design for a crane hook as laid down in the paper "Design of crane hooks and other components of lifting gear" by H. J. Gough, H. L. Cox and D. G. Sopwith, published as proceedings of the Institution of Mechanical Engineers, in 1934.

In this paper, it is shown that the most suitable section for the body of the crane hook will be of triangular form with the proportions shown in fig. 19-8. The bed diameter c and the width of the throat must be sufficient to accommodate the necessary slings but should not be in excess of requirements. The bed diameter denoted by c is given by

The centre of gravity of the section is at a distance x from the top of the flange, where x is given by

$$x = \frac{300 \times 5 + 200 \times 20}{500} = 11 \text{ cm.}$$

The eccentricity of the load $= 75 + 20 + 11 \doteq 106$ cm.

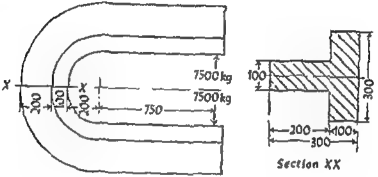


Fig 19-6

The bending moment on the section $= 106 \times 7500 =$ 795,000 kg cm. The radius of curvature of the neutral surface

$$r_n = \frac{500}{30 \log_e \frac{30}{20} + 10 \log_e \frac{50}{30}} = 28.75 \text{ cm}$$

The radius of curvature of the gravity surface $= 31$ cm

The dangerous section is subjected to direct tensile load of 7,500 kg and the bending moment of 795,000 kg cm. The maximum stress will be tensile and will be along the inner fibre.

$$\text{Direct stress} = \frac{7500}{500} = 15 \text{ kg/sq cm}$$

$$\text{Maximum bending stress} = \frac{My_i}{Aer_i} = \frac{795000 \times 8.75}{500\ (31 - 28.75) \times 20}$$

$$= 308 \text{ kg/sq cm.}$$

Combined stress on the inner fibre $= 308 + 15 = 323$ kg/sq cm.

Exercises:

1. *In a small C clamp of circular section of 25 mm, a force of 500 kg is exerted between the C clamp and the screw. The eccentricity of the load is 8 cm and the inner radius of curvature is 20 mm. If the maximum allowable shear stress is 500 kg/sq cm and the maximum allowable tensile*

The maximum allowable bearing pressure for the trunion bearing in the housing should not exceed 500 kg/sq cm of projected area.

The block is designed as a beam freely supported at its ends and loaded at the centre due to the given load. The length of the beam depends upon the number of sheaves to be carried in a hoisting tackle. We have assumed the severe conditions for the block in calculating the bending moment. Such conditions are not likely to be encountered in practice.

The width of the block at the centre will be 3 to 5 mm more than the flange diameter of the protecting cover for the trunion bearing. While calculating the modulus of section, the weakening effect of the hole for the shank of the hook should be considered.

The pins for the block and the sheave spindle are stationary in the side plates as there is no necessity for the movement of the hook about the horizontal axis of the block. The pin should be an inteference fit in the side plates. The diameter of the pin should be calculated from shear considerations allowing a low value for the shear stress. The thickness of the side plate is obtained from bearing considerations. The ends of the pins are reduced and screwed to take standard nuts. The generous fillets are provided to minimise stress concentrations. The minimum width of the side plate is calculated from tensile stress considerations allowing for the weakening effect of holes for the pin. Each side plate carries half the load on the hook.

When the load on the crane is large, double hooks are provided. The load will be uniformly distributed between the hooks as a result the stress distribution will be more favourable than simple hooks. The approximate value of the maximum stress is given by the equation

$$f = \frac{M}{Z\left(1 - \frac{c_1}{r}\right)} - \frac{M}{Ar} + \frac{P}{A} \quad \text{(iii)}$$

where M = bending moment acting on the section, Z, modulus of section, c_1, the distance of the c.g. of the section from intrados, r, radius of curvature of the gravity axis, A, area of the section and P, load on the hook.

For the section shown in fig. 19-9,

$$P = \frac{Q}{2}\sin\alpha;\ M = \frac{Q}{2}.l;\ r = \frac{l}{\sin\alpha} \doteq 1{\cdot}5l;\ A = 0{\cdot}78\ bh;$$

$$Z = 0{\cdot}1bh^2; \qquad e_1 = 0{\cdot}5h \quad \text{(iv)}$$

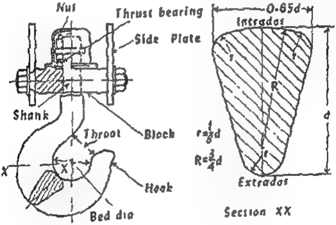
$e = \mu \sqrt{P}$ ....(i)

where P is the load in tonnes and μ a constant varying from 3·8 to 7·6. For economy of material, it should be kept as low as possible.

The throat of the hook is taken 0 75 of the bed diameter. The dimension d for the triangular form is given as

$d = 3{\cdot}2 \sqrt{P} + \frac{e}{10}$ cm ....(ii)

This will be the value of d at the horizontal and vertical centre lines of the hook At a plane midway between these sections, the value of d is taken 8 per cent more. With these proportions, the maximum tensile stress at the intrados will be 1,800 kg/sq cm As the body curves to join the shank, the section may be reduced, provided the specified stress limit is not exceeded.



Crane hook
Fig 19-8

In the said paper the working tensile stress for the shank is 450 kg/sq cm. The hook load is carried by a thrust bearing through the nut screwed on to the end of the shank The nut is locked to the shank by suitable arrangement. The suitable ball thrust bearing must be selected for the hook The allowable load on each ball is given as 210 d^2 kg, where d is the diameter of ball in cm. The ball bearing should be self adjusting and should be protected [illegible]

The provision should be made in design [illegible] lubricate the ball bearing and shank of the hook.

The maximum allowable bearing pressure for the trunion bearing in the housing should not exceed 500 kg/sq cm of projected area.

The block is designed as a beam freely supported at its ends and loaded at the centre due to the given load. The length of the beam depends upon the number of sheaves to be carried in a hoisting tackle. We have assumed the severe conditions for the block in calculating the bending moment. Such conditions are not likely to be encountered in practice.

The width of the block at the centre will be 3 to 5 mm more than the flange diameter of the protecting cover for the trunion bearing. While calculating the modulus of section, the weakening effect of the hole for the shank of the hook should be considered.

The pins for the block and the sheave spindle are stationary in the side plates as there is no necessity for the movement of the hook about the horizontal axis of the block. The pin should be an inteference fit in the side plates. The diameter of the pin should be calculated from shear considerations allowing a low value for the shear stress. The thickness of the side plate is obtained from bearing considerations. The ends of the pins are reduced and screwed to take standard nuts. The generous fillets are provided to minimise stress concentrations. The minimum width of the side plate is calculated from tensile stress considerations allowing for the weakening effect of holes for the pin. Each side plate carries half the load on the hook.

When the load on the crane is large, double hooks are provided. The load will be uniformly distributed between the hooks as a result the stress distribution will be more favourable than simple hooks. The approximate value of the maximum stress is given by the equation

$$f = \frac{M}{Z\left(1-\frac{c_1}{r}\right)} - \frac{M}{Ar} + \frac{P}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

where M = bending moment acting on the section, Z, modulus of section, c_1, the distance of the c.g. of the section from intrados, r, radius of curvature of the gravity axis, A, area of the section and P, load on the hook.

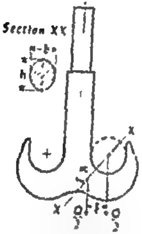
For the section shown in fig. 19-9,

$$P = \frac{Q}{2}\sin\alpha;\ M = \frac{Q}{2}.l;\ r = \frac{l}{\sin\alpha} \doteq 1{\cdot}5l;\ A = 0{\cdot}78\ bh;$$

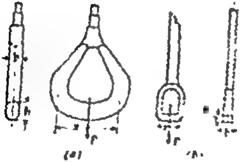
$$Z = 0{\cdot}1bh^2; \qquad c_1 = 0{\cdot}5h \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

For extra ordinary heavy loads the open hooks are not employed but loops are used as shown in fig. 19-10(a) or eyelets as shown in fig. 19-10(b).

Load-loops	Eyelets
$M = \dfrac{Px}{8}$	$M = \dfrac{Px}{8}$
$f = \dfrac{M}{Z}$	$f = \dfrac{M}{Z}$



Double Hook
Fig. 19-9



Loop and Eyelet
Fig. 19-10

Example:

1. Design a crane hook for a 3 tonne crane. The hook is to be of swivelling type and of (a) trapezoidal section [illegible]. Choose the suitable values for the stresses.

We assume that the centre of curvature lies along the load line.

The bed diameter is obtained by the equation

$c = \mu\sqrt{P}$ tonnes.

We take the value of μ as 5.

$C = 5\ \sqrt{3} = 8{\cdot}68$ cm; we adopt 9 cm.

The dimension of the throat $= 9 \times \frac{3}{4} = 6{\cdot}75$ cm; we adopt 7 cm.

The proportion d for the triangular section will be

$$d = 3{\cdot}2\sqrt{P} + \frac{c}{10} = 3{\cdot}2\sqrt{3} + \frac{9}{10} = 6{\cdot}5 \text{ cm.}$$

The height of the section will be 6·5 cm while the base of the triangle will be 4·5 cm.

If d_1 cm be the diameter at the bottom of the thread, then

$$\frac{\pi}{4}\, d_1^2 \times f = 3000$$

We assume 500 kg/sq cm as the permissible stress value, then area at the bottom of the thread will be 6 sq cm. From table of metric threads we adopt 33 mm diameter. The minimum diameter of the plain portion of the shank will be 33 mm and increases to 40 mm at the junction with the body of the hook. The height of the threaded portion will be 33 mm. The diameter of the hole in the block will be 37 mm.

(a) Let us calculate the stresses in the triangular section.

Area of the section, $A = \frac{1}{2} \times 4{\cdot}5 \times 6{\cdot}5 = 14{\cdot}6$ sq cm.

Direct tensile stress across the section $= \dfrac{3000}{14{\cdot}6} = 205$ kg/sq cm.

Radius of the intrados $= \frac{9}{2} = 4{\cdot}5$ cm.

Radius of the extrados $= 4{\cdot}5 + 6{\cdot}5 = 11$ cm.

Radius of centre of gravity $= 4{\cdot}5 + \dfrac{6{\cdot}5}{3} = 6{\cdot}66$ cm.

Eccentricity of the load $= 6{\cdot}66$ cm.

Bending moment acting on the section $= 3000 \times 6{\cdot}66 = 19{,}980$ kg cm.

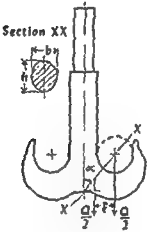
Radius of curvature of neutral axis

$$= \frac{\frac{1}{2} \times 4{\cdot}5 \times 6{\cdot}5}{\dfrac{4{\cdot}5 \times 11}{6{\cdot}5} \log_e \dfrac{11}{4{\cdot}5} - 4{\cdot}5}$$

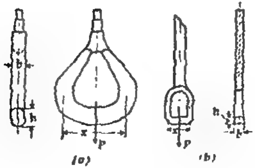
$$= 6{\cdot}35 \text{ cm.}$$

For extra ordinary heavy loads the open hooks are not employed but loops are used as shown in fig. 19-10(*a*) or eyelets as shown in fig. 19-10(*b*).

Load-loops	Eyelets
$M = \dfrac{Px}{8}$	$M = \dfrac{Px}{8}$
$f = \dfrac{M}{Z}$	$f = \dfrac{M}{Z}$



Double hook
FIG. 19-9



Loop and Eyelet
FIG. 19-10

Example:

1. Design a crane hook for a 3 tonne crane. The hook is to be of swivelling type and of (a) triangular section (b) circular section. Choose the suitable values for the stresses.

Exercises:

1. Fig. 19-11 shows a crane hook and its bearings for a 2·5 tonne crane. The hook is to be of swivelling type and of trapezoidal section having a bed diameter of 75 mm. Design and draw a detailed drawing of the hook and its supports.

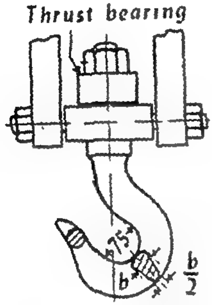


FIG. 19-11

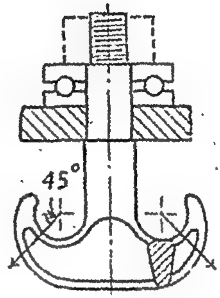


FIG. 19-12

2. A special hook is shown in fig. 19-12. The wire sling ends are inclined at 45° as shown. The load handled is 1·5 tonnes with wire rope size of 15 mm diameter. Design and prepare a dimensioned sketch. Safe tensile stress in the hook is 1,200 kg/sq cm.

3. Fig. 19-13 shows the crane hook with the supporting block. Calculate the dimensions of the hook, the supporting block and the thrust block. The hook is to be designed for a maximum load of 20 tonnes. All parts except the thrust block are made of forged steel and the thrust block of phosphor bronze.

Make neat dimensioned sketches (elevation and end view) of the hook and the supporting block designed by you. Use the following data for your design:

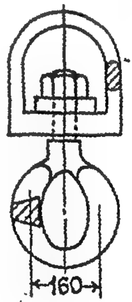


FIG. 19-13

Ultimate stresses for forged steel:

$f_t = 55$ *kg/sq mm;* $f_c = 65$ *kg/sq mm;* $f_s = 40$ *kg/sq mm.*

Allowable bearing pressure for thrust block 150 to 350 kg/sq cm.

4. Explain why non symmetrical sections are employed for the design of crane hooks or frames of machines.

5. Design a snatch block for a crane hook for a maximum load of 5 tonnes. Choose your own materials and values for permissible stresses. Give three views of the block designed by you.

Maximum bending stress $= \dfrac{M y_i}{A e r_i}$

$$= \frac{19980\ (6{\cdot}35 - 4{\cdot}5)}{14{\cdot}6\ (6{\cdot}66 - 6{\cdot}35) \times 4{\cdot}5}$$

$$= 1{,}810 \text{ kg/sq cm.}$$

The maximum stress in the hook will be at intrados and is equal to $205 + 1810 = 2{,}015$ kg/sq cm.

(b) *Let us consider the hook of circular section*

Let d cm be the diameter of the section of the hook.

Radius of curvature of the gravity axis be $2d$

Eccentricity of load will be $2d$.

The radius of curvature of the intrados will be $1{\cdot}5d$ while that of extrados will be $2{\cdot}5d$.

Area of cross section $= \dfrac{\pi}{4} d^2$

$\therefore$ Direct tensile stress $= \dfrac{3000}{\frac{\pi}{4} d^2} = \dfrac{3820}{d^2}$ kg/sq cm.

Bending moment across the section $= 3000 \times 2d = 6\,000d$ kg cm.

Radius of curvature of the neutral axis

$$= \frac{(\sqrt{2{\cdot}5d} + \sqrt{1{\cdot}5d})^2}{4}$$

$$= 1{\cdot}95d.$$

$e = 2d - 1{\cdot}95d = 0{\cdot}05d.$

Maximum bending stress $= \dfrac{M y_i}{A e r_i}$

$$= \frac{6000d \times (1{\cdot}95d - 1{\cdot}5d)}{0{\cdot}785d^2 \times 0{\cdot}05d \times 1{\cdot}5d} = \frac{45700}{d^2} \text{ kg/sq cm.}$$

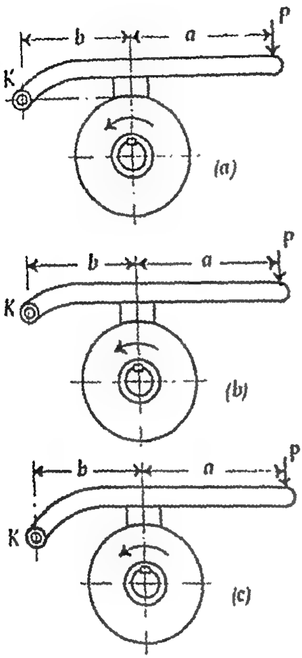
Resultant stress $= \dfrac{3820}{d^2} + \dfrac{45700}{d^2} = \dfrac{49520}{d^2}$ kg/sq cm

The permissible stress for the body of the hook is adopted as 1,000 kg/sq cm.

$\therefore \quad 1000 = \dfrac{49520}{d^2}$

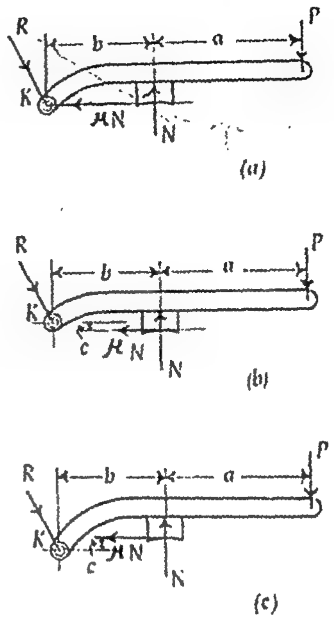
or $\quad d = \sqrt{\dfrac{49520}{1000}} = 7$ cm.

When the brake is applied by operating a single lever, the operating lever with a friction block can be considered as a free body in equilibrium under the action of the following forces: (Fig. 19-15)



Simple block brake

FIG. 19-14



Brake lever with a block as a free body diagram

FIG. 19-15

(i) Normal reaction N between the brake drum and the block

(ii) Frictional force μN between the brake drum and the block

(iii) Applied force P or the brake actuating force

(iv) Pin reaction R.

Resultant, of normal reaction and the frictional force, which is known in magnitude and direction is called the *braking force*. It is inclined at an angle $\lambda = \tan^{-1}\mu$ where μ is the coefficient of friction between the surfaces in contact. The operating force, whose magnitude can be calculated from the braking torque required, by taking moment about the fulcrum of the lever, is known as the *brake actuating force*, whose direction can be assumed. *The braking force,*

(B) BRAKES

19-7. Introduction:

The primary function of the brake is to control the motion of a machine or a machine element. Brakes may be used to slow down, stop or hold a load or to release a load and control its speed. In performing its function brake is frequently required to convert large amount of kinetic energy or potential energy or both into internal energy of the brake components and dissipation in the form of heat. This heat may be carried away by circulating water or it may be transferred to surrounding air.

Brakes are generally mounted on rotary machine elements such as wheels, drums, gears, sheaves, sprockets, etc. where the rotary motion of machine elements is to be controlled. However there are certain machine parts such as work tables, which have translational motions controlled by means of brakes.

19-8. Types of brakes:

Brakes may be classified according to the means for transforming energy by braking elements.

- Brakes
  - Mechanical
    - Block
      - Single
      - Double
    - Band brake
    - Block and band brake
      - Simple
      - Differential
      - Back stop
    - External and internal shoe
  - Electric
    - Generator
    - Eddy current
  - Hydraulic
    - Pump or Hydro-dynamic
    - Agitator

We shall consider in brief the design procedure for block brakes and band brakes. *The students should refer pages 433 to 451 of the book entitled "Theory of Machines Vol. I edition I" written by the authors for general mechanics of block and band brakes, before considering the design procedure.*

19-9. Design procedure for block brakes:

The simplest form of brake consists of a block pressed against the rim of a brake wheel as shown in fig. 19-14.

coefficient of friction that is too low may result in the inadvertent design of a brake that will grab.

The performance of a brake depends on the effects of the interaction of the brake lining and brake drum materials during brake actuation. The torque produced by the brake lining and the drum and the resistance to wear of the brake lining depends upon the contact pressure between the shoe and drum and upon the temperature of the line-drum interface reached during brake actuation.

	Coefficient of friction μ	Allowable pressure kg/sq cm	Maximum temperature C°
Asbestoes in rubber compound on metal	0·3 to 0·4	5 to 7	90°
Asbestoes in resin binder on metal	0·3 to 0·4 dry 0·1 oil	5 to 7 40	200° to 250°
Powdered metal on metal	0·2 to 0·4 dry 0·05 to 0·08 oil	28	550°

The value of contact pressure is frequently calculated on the basis of the projected area of the block or shoe.

Single block brake is not used much because the normal reaction N exerts a heavy pressure on the shaft bearings and produces bending on the shaft, which is very objectionable in large brakes. In order to reduce the bending loads on the shaft, double block brakes are employed. Two blocks placed on opposite sides of a wheel, as shown in fig. 19-16 and fig. 19-17, may be controlled by levers and rods so that they are pulled together against the wheel to form a brake.

In the above analysis we have assumed that the normal pressure is uniform between the block and the drum and this is true when the angle of contact between the drum and block θ, is less than 60°. When the arc of contact is large, the unit pressure normal to the surface of contact is less at the ends than at the centre and we assume that the wear in the direction of applied force is uniform. In such cases we employ the equivalent coefficient of friction, which is given by

the brake actuating force and the brake arm reaction force are coplaner and keep the brake lever in equilibrium.

The following procedure for the design of a block brake is suggested·

(i) The braking torque is fixed from the required duty.

(ii) The value of coefficient of friction for the combination of materials chosen for the block and the drum is decided upon.

(iii) The diameter of the brake drum is decided upon.

(iv) The braking force is calculated by the formula

$$F_b = \frac{\text{Torque}}{\text{Radius of brake drum}} \times \sin\lambda$$

(v) The direction of the brake operating force P is fixed from the axis of the link actuating the brake. If this link is not existing then we assume the direction

(vi) From the force triangle for the brake lever we calculate the magnitude of the brake actuating force and the magnitude and direction of the brake reaction force R. When the forces are known, by choosing the suitable materials for the brake lever arms and the fulcrum pin, the dimensions can be specified.

(vii) The value of the brake arm reaction force enables the design of pivot pins and pivot pin bearings.

When the pivot point of the brake arm lever lies on the line of the tangential braking force as shown in fig. 19-14(*a*), the same operating force is necessary for clockwise or anticlockwise rotation of the brake drum. *If the moment of the tangential braking force* at the drum about the pivot point aids in actuating the brake, the brake is said to be self energising. *The percentage of total brake* effort that results from self energising action depends on (a) the location of the brake arm pivot point, (*b*) *the coefficient of* friction and (c) the direction of rotation of the brake drum In order to prevent the brake arm from grabbing, *the moment of* friction force about the brake arm pivot point should be less than the total required braking effort.

When determining the length of the moment arm of the friction force for a self energizing brake, care should be exercised *in evaluating the coefficient* of friction. Choosing a value of

Cast iron on cast iron	0·15
Steel on cast iron	0·15
Asbestocs band on CI or steel	0·35 to 0·37
Rolled band on CI or steel	0·42
Wood on CI	0·30
Wood on steel	0·25
Leather on CI	0·20
Leather on steel	0·20
Bronze on CI	0·17
Bronze on steel	0·16
Bronze on bronze	0·18
Steel on laminated fabric	0·15
Steel on fibre	0·17

A new development are sintered metal linings which do not contain organic matter and, therefore, their coefficients of friction are only slightly altered in heating. They possess a relatively high resistance to wear. The coefficient of friction may range from 0·6 to 0·76 and the bearing pressure intensity may be upto 8 kg/sq cm.

Brake levers:

Brake levers are manufactured from die forged or cast steel. The safe bending stress of levers is taken from 400 to 800 kg/sq cm, depending on the brake size. The permissible values take into account the braking shocks. Steel cast levers are more expensive but they possess greater rigidity and have less *lost motions* in pivots.

Pull rods:

They are made *adjustable* and is of round bar steel having permissible tensile stress of 300 to 500 kg/sq cm.

Energy considerations in brakes:

Heat generated during the application of a brake must be dissipated by heat transfer or the brake will overheat and perhaps burn out the lining. When we design the brake it is necessary to determine the amount of energy to be transformed and also the time of the braking cycle in order to estimate the rise of temperature of brake. The temperature rise of the brake depends upon (i) the mass of the parts (ii) the ratio of the braking time of the rest time and (iii) on the heat dissipation capacity of the brake. The maximum temperature of the brake is limited to prevent deterioration of the materials forming the friction surfaces. The temperatures are

$$\mu_e = \frac{4\,\mu \sin \theta/2}{\theta + \sin\theta}$$

where θ = angle of contact in radians

μ = actual coefficient of friction.

We give below some of the points concerning the design of block brakes or shoe brakes.

Brake wheels:

They are made of cast iron and should be dynamically balanced. The width of the wheel should exceed that of the shoe by 5 mm to 10 mm Brake wheels should always be finned for better heat dissipation and provided with holes between the fins for more rapid air circulation and to dissipate the heat more effectively into the atmosphere. If the brake is mounted on a *flexible coupling* it is installed in the half which faces the driving mechanism.

Brake shoe:

Generally for a mechanical drive the shoes are made of cast iron, provided with special brake linings, which may be secured with *rivets* or with counter sunk screws

Brake linings:

It should satisfy the following requirements·

(i) It should have a high coefficient of friction.

(ii) It should retain braking capacity at temperatures upto 300°C

(iii) It should have effective resistance to wear at the highest speeds, unit pressure and temperature.

(iv) Easily yield to treatment

(v) Low cost.

To-day every where use is made of rolled band which is manufactured on rolling machines and it is manufactured from cheap non-textile asbestoes and rubber with an addition *of sulphur* for subsequent vulcanisation. These bands are manufactured upto [illegible] mm thick and upto 100 mm wide. It possesses a high and stable coefficient of friction which ranges between 0·42 and 0·53 and can withstand temperatures upto 220°C.

Experiments recommend the following average coefficients of friction for various unlubricated materials:

On substitution of values, we get

$$\mu_e = 0{\cdot}30 \frac{4 \times \sin 45^\circ}{\frac{\pi}{2} + \sin\frac{\pi}{2}} = 0{\cdot}33.$$

If P be the normal reaction between the block and brake drum the tangential braking force F is given by

$$\frac{F}{P} = \mu_e = 0{\cdot}33.$$

$$\therefore \quad P = \frac{F}{0{\cdot}33} = 3F.$$

Let us consider the forces acting on the left hand yoke whose pivot is at the left. Let S be the axial force along the screw. If we take moment about the pivot, we get

$$24\ S + 2{\cdot}5\ F_L - 12\ P_L = 0$$

We have $P_L = 3F_L$.

$$\therefore \quad 24\ S + 2{\cdot}5\ F_L - 36\ F_L = 0.$$

$$\therefore \quad F_L = \frac{24\ S}{36 - 2{\cdot}5} = 0{\cdot}715\ S.$$

If we consider the forces acting on the right hand yoke and if we take moment about the right hand pivot, we get

$$24\ S - 2{\cdot}5\ F_R - 12\ P_R = 0$$

or

$$24\ S - 2{\cdot}5\ F_R - 36\ F_R = 0.$$

$$\therefore \quad F_R = \frac{24\ S}{38{\cdot}5} = 0{\cdot}625\ S.$$

Braking torque $= (F_R + F_L)\ 9$ kg cm

$= (0{\cdot}715 + 0{\cdot}625)\ S \times 9 = 12{\cdot}06\ S$ kg cm.

$$\therefore \quad 12{\cdot}06 \times S = 2000$$

or $$S = \frac{2000}{12{\cdot}06} = 166 \text{ kg}.$$

Torque required to overcome friction at the thread surfaces (two friction surfaces)

$$T = 2 \times S \times \frac{d_m}{2} \tan(\alpha + \phi)$$

where S = force along the axis of the screw = 166 kg

d_m = mean diameter of the screw = 2 cm

α = thread angle $= \tan^{-1}\dfrac{6}{\pi \times 2} = 43^\circ - 42'$

ϕ = friction angle $= \tan^{-1} 0{\cdot}15 = 8^\circ - 32'$.

limited by the properties of the material. The maximum temperature should not exceed the following values:

Leather, fibre and wood facing	70°C
Asbestos	100°C
Automotive asbestos block lining	250°C

Since the temperature rise of the brake is very difficult to predict we make use of the design coefficients as *kg metre of energy absorbed per sq cm of surface per minute.* This design coefficient is expressed as the product pV where p is the bearing pressure in kg/sq cm and V equals the rubbing velocity in metre/minute.

H.P. absorbed per sq cm of projected area

$$= \frac{pV}{4500}$$

Examples:

1. The arrangement of a transmission brake is shown in fig. 19-16. The arms are pivoted as shown and when force is applied at the end of the hand lever, the screw will rotate. The left and right hand threads working in nuts on the ends of the arm will move the arms together and thus apply the brake.

The force on the hand lever is applied 40 cm from the axis of the screw. The brake drum is 18 cm in diameter and the angle subtended by each block is 90°. The screw has six square threads with a mean diameter of 2 cm and a lead of 6 cm. Assuming a coefficient of friction for the braking surface equal to 0·30 and for the threads equal to 0·15, determine the force on the hand lever required to set the brake when the torque on drum is 2,000 kg cm.

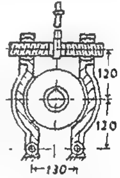


Fig. 19-16

The equivalent coefficient of friction between the block and the brake drum is given by

$$\mu_e = \mu \frac{4 \sin\theta}{2\theta + \sin 2\theta}$$

where μ = coefficient of friction between the block and brake and θ = semi-block angle measured in radians.

$\therefore \quad 12k = 166 + 1{\cdot}2k$

or $\quad k = \frac{166}{10{\cdot}8} = 15{\cdot}4$ kg/cm.

Maximum spring force will occur when the brake lining is new and the brake is released.

Maximum force in the spring when the brake is released

$$= 166 + 1{\cdot}2k + \frac{2 \times 3 \times 2}{10} \times k$$

$$= 166 + 2{\cdot}4k = 166 + 2{\cdot}4 \times 15{\cdot}4 = 203 \text{ kg.}$$

Let us assume the spring index to be 8, and the permissible stress to be 6,000 kg/sq cm.

$$\text{Wahl correction factor} = \frac{4C-1}{4C-4} + \frac{0{\cdot}615}{C}$$

$$= \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0{\cdot}615}{8} = 1{\cdot}176.$$

If d cm be the diameter of spring wire, we have

$$203 \times 4d = \frac{\pi}{16} \times d^3 \times \frac{6000}{1{\cdot}176}.$$

or $$d = \sqrt{\frac{203 \times 4 \times 16 \times 1{\cdot}176}{6000 \times \pi}} = 0{\cdot}8863 \text{ cm.}$$

From SWG table we adopt 3/0 having 0·9449 cm as the diameter.

Mean diameter of the coil = 7·5 cm.

Actual spring index $C = \frac{7{\cdot}5}{0{\cdot}9449} = 7{\cdot}95.$

Stiffness $k = \frac{Gd}{8C^3 n}$

where G is the modulus of rigidity and n is the number of active turns of the coil. Assuming $G = 0{\cdot}84 \times 10^6$ kg/sq cm, we get

$$12{\cdot}6 = \frac{0{\cdot}84 \times 10^6 \times 0{\cdot}9449}{8 \times 7{\cdot}95^3 \times n}$$

or

$$n = \frac{0{\cdot}84 \times 10^6 \times 0{\cdot}9449}{8 \times 7{\cdot}95^3 \times 12{\cdot}6} = 13 \text{ turns.}$$

3. *A double block brake with wooden shoes on a cast iron drum ($\mu = 0{\cdot}3$) is arranged as shown in fig. 19-18. Determine the operating force required to absorb 35 metric horse power with a drum speed of 300 r.p.m. clockwise.*

On substitution of values, we get

$$\text{torque} = 2 \times 166 \times \tfrac{3}{4} \tan (43° - 42' + 8° - 32')$$
$$= 430 \text{ kg cm.}$$

If Q be the force applied at the end of a hand lever 40 cm long, then $Q \times 40 = 430$

or $$Q = \frac{430}{40} = 10{\cdot}75 \text{ kg.}$$

The size of the hand lever can be fixed from bending considerations and the length of the nut can be obtained from bearing stress considerations for the thread The pivot pin is designed from shear considerations The load on each pivot pin is obtained by considering the equilibirum of each yoke. The forces on two pivot pins will be different.

2 *The double block brake shown in fig. 19-17 must provide a braking torque of 2,000 kg cm. The drum diameter is 18 cm and each block subtends an angle of 90°. Assume a coefficient of friction of 0 3 for the brake lining. Brake shoes should clear the drum by about 3 mm when brakes are released and 3 mm lining wear should be allowed Design the coil compression spring used to actuate this brake. Use ordinary spring wire. The spring force variation should not exceed about 10% between replacements of the brake lining.*

Specifications of the brake for this problem are the same as those of problem solved earlier. The main difference between the two problems is in the method of setting the brake. Here the brake is spring set

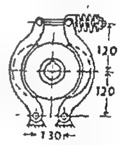


FIG. 19-17

From the analysis of problem (1) we see that the minimum spring force required to set the brake is 166 kg. This force will be exerted by the spring when the lining is worn out. If k be the stiffness of the spring in kg/cm unit, the force in the spring will be

$166 + \frac{2 \times 3 \times 2}{10} \times k = (166 + 1{\cdot}2k)$ kg. The spring force variation should not exceed about 10% between replacement of the brake lining.

$$\therefore \quad 166 + 1{\cdot}2k - 166 = \frac{1}{10}(166 + 1{\cdot}2k).$$

$$3{\cdot}205F_L \times 300 + Q\,(300-65) = 6Q \times 600 + F_L\,(250-65).$$

$$\therefore \quad F_L = \frac{3235}{776{\cdot}5}\;Q = 4{\cdot}17\;Q \text{ kg.}$$

Similarly considering the equilibrium of forces acting on the right hand yoke, we get

$$3{\cdot}205\;F_R \times 300 = 6Q \times 600 - F_R\;(250-65).$$

$$\therefore \quad F_R = \frac{3600\;Q}{1146{\cdot}5} = 3{\cdot}14\;Q.$$

$$\text{Braking torque} = 25\;(3{\cdot}14\,Q + 4{\cdot}17\,Q).$$

$$\therefore \quad 8100 = 25 \times 7{\cdot}31\,Q$$

or $$Q = \frac{8100}{25 \times 7{\cdot}31} = 43{\cdot}2 \text{ kg.}$$

4. *A 1,500 kg automobile moving on a level ground at 108 km/hour is to be stopped in a distance of 100 metre, the tyre diameter being 80 cm. All frictional energy except for the brake is to be neglected.*

(*a*) *What total average braking torque must be applied?*

(*b*) *What must be the minimum coefficient of friction between the tyres and the road in order for the wheels not to skid if it is assumed that weight is equally distributed among the four wheels?*

(*c*) *If the friction energy is momentarily stored in 22 kg cast iron brake drums, what is the average temperature rise of the drum?*

$$\text{Velocity of the vehicle} = \frac{108 \times 1000}{60 \times 60} = 30 \text{ m/sec.}$$

$$\text{K.E. of the vehicle} = \frac{1500}{2} \times \frac{1}{9{\cdot}81} \times 30^2$$

$$= 68,600 \text{ kg metre.}$$

If we assume the uniform frictional resistance to motion, the tangential braking force $= \dfrac{68600}{100} = 686$ kg.

$$\text{Braking torque} = 686 \times 40 = 27{,}440 \text{ kg cm.}$$

$$\text{Coefficient of friction} = \frac{686}{1500} = 0{\cdot}454.$$

Heat to be absorbed for the cast iron brake wheel will be equal to $\dfrac{68600}{426{\cdot}7} = 161$ kcal.

If $0{\cdot}12$ be the specific heat of cast iron, then the temperature rise will be $\dfrac{161}{22 \times 0{\cdot}12} = 61°$C.

$$\text{Braking torque} = \frac{71620 \times \text{metric h p.}}{\text{speed}} \text{ kg cm}$$

$$= \frac{71620 \times 35}{300} = 8{,}100 \text{ kg cm.}$$

The equivalent coefficient of friction between the block and brake drum is given by

$$\mu_e = \mu \frac{4 \sin\theta}{2\theta + \sin 2\theta}$$

$$\mu = 0{\cdot}3 \text{ and } \theta = 25°.$$

$$\therefore \quad \mu_e = \frac{0{\cdot}3 \times 4 \sin 25°}{\frac{50}{180} \times \pi + \sin 50°} = 0{\cdot}312.$$

Let Q be the operating force required to set the brake. From principles of statics it can be shown that the force at the other end will be horizontal force of magnitude $\frac{Q \times 60}{10} = 6Q$ kg.

Free body diagram for the left hand yoke is shown in fig. 19-19.

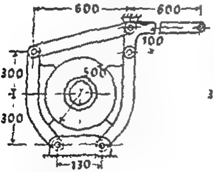


Fig 19-18

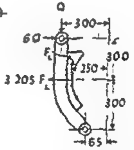


Fig 19-19

Let F be the tangential braking force and P be the radial force on the drum. We have $F = \mu_e P$.

$$\therefore \quad P = \frac{F}{\mu_e} = \frac{F}{0{\cdot}312} = 3{\cdot}205F$$

Let us consider the equilibrium of forces acting on the left hand yoke. When we take moment about the fulcrum of the left hand yoke, we get (refer fig. 19-19.)

3. *The double block brake is required to bring a brake drum to rest within 5 seconds. The diameter of the brake drum is 60 cm and it is rotating at 300 r.p.m. The brake has to absorb 1,000 kg metre of energy each time when it is applied. The bearing pressure between the block and the drum is limited to 5 kg/sq cm. Determine the brake actuating force required at the end of the brake lever 100 cm long. The distance between two fulcrums is 46 cm and the axis of the drum shaft is 35 cm from the line joining two fulcrums.*

Design the size of the shoe, the size of the pivot pin for the shoe and the cross sectional dimensions for the critical section of the lever. Take $\mu = 0{\cdot}3$.

4. *Assume that the operation is intermittent; that cast iron shoes run dry on a steel brake sheave which has a diameter of 50 cm and runs at 90 r.p.m., and that the area of the heat dissipating surface is 50% larger than the braking surface. Determine the braking area for absorbing 7,000 kg metre in 40 seconds.*

5. *Determine the total energy which must be absorbed to slow down to 120 metre per minute a mine hoist cage descending at the rate of 450 metre per minute in 15 sec. The weight of the cage with the load is 2,000 kg and the hoist drum diameter is 180 cm and the brake sheave diameter is 140 cm. The rotative speed of the brake sheave is one third that of the driving engine, and the speed of the hoist drum is one tenth that of the engine. The weight of the hoist drum is 7,500 kg.*

Determine the normal and tangential forces which must be applied to the brake sheave.

Assuming that the brake has wooden blocks on cast iron drum and that the area of the radiating surface consists of the inner surface of the brake drum rim, find the minimum brake drum width required.

6. *Determine the capacity and the main dimensions of a double block brake for the following conditions:*

The brake sheave is mounted on the drum shaft, the hoist with its load weighs 3,000 kg and moves downward with a velocity of 60 metre/minute; the hoist must be stopped in a distance of 3 metre; the kinetic energy of the drum may be neglected.

19-10. Band brakes: Introduction:

Band brakes are not as commonly used as shoe or block brakes, but in some installations they have advantages over shoe brakes

Design notes:

1. Specific heat of steel may be taken as 0·11 and that for aluminium 0·22

For brake calculations specific heat of metals is expressed in $\frac{\text{kg metre}}{\text{kg °C}}$, which can be obtained by multiplying specific heat of metal by 426·7, mechanical equivalent of heat.

2. If the automobile were going down hill, the loss of potential energy would be added to loss of kinetic energy; if up hill, gravity would help slowing down the car.

3. In automobile brakes a desirable maximum instantaneous loading is 0·35 h.p. per sq cm of brake rubbing surface on the drum.

4. The reasonable value of retardation for automobiles is 5 metre/sec²

5. The effectiveness of the brake may greatly decrease shortly after it begins to act continuously. This phenomenon is called fade, which is due to decrease in coefficient of friction at elevated temperatures which are induced during braking. This can be remedied either by more effective dissipation of heat or by design of the braking system, which requires that the ratio of frictional moment to applied moment shows a minimum variation when plotted against the coefficient of friction. This ratio is known as the mechanical advantage.

Exercises:

1. The block brake shown similar to one shown in fig. 19-14(b) balances a torque of 4,000 kg cm. On the drum shaft, the diameter of the drum is 60 cm. The pivot is 5 cm below the horizontal at the point of contact of the block to the highest point of the drum. Assuming a coefficient of friction of 0·35 and safe allowable stresses for the material of the brake lever and the pin in bending, shear and bearing as 600 kg/sq cm, 450 kg/sq cm and 900 kg/sq cm, design the brake lever and pin and show the dimensions on a neat drawing of the brake. Also design the pivot pin. a = 55 cm and b = 25 cm.

2. The double block brake similar to one shown in fig. 19-17 must provide 120 kg metre braking torque. The drum diameter is 35 cm and each block subtends an angle of 120°. The coefficient of friction may be assumed to be 0·3 for the brake lining. Brake shoe should clear the drum by 3 mm when brakes are released and 3 mm lining wear should be allowed.

Design the suitable compression spring required to actuate this brake. Assume that 13 cm diametral space for the spring will be reasonable. Use ordinary spring wire. The spring force variation should not exceed 10% between replacements of brake lining. Calculate the width of the shoe. The distance between the fulcrums may be taken as 8 cm. The length of each lever is 45 cm and the distance of the axis of the drum from the line joining two fulcrums is 21 cm.

$$T_1 = \frac{Fe^{\mu\theta}}{e^{\mu\theta}-1} \text{ and } T_2 = \frac{F}{e^{\mu\theta}-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

wherever possible, it is advantageous to make the tight side the fixed end and the slack side the operating end, where the force required to actuate the brake is at a minimum.

19-11. Design procedure for Band Brakes:

We suggest the following brief procedure for design:

(i) The tight end of the band is fixed rigidly and the slack end is made adjustable.

(ii) To ensure a tight contact with the drum, the width of the band should not exceed 150 mm for drum diameter greater than 1,000 mm and 100 mm for diameter of drum less than 1,000 mm.

(iii) Thickness of bands are normally in the range of 3 to 10 mm. The band thickness may be taken as 0·005 D where D is the diameter of the brake drum.

(iv) The band is commonly made of steel to which the lining is riveted. The bands are checked for tensile failure from the maximum tension T_1 with due regard for the two rivet holes for weakening the cross section. The permissible value of safe tensile stress ranges from 500 to 800 kg/sq cm.

(v) The diameter of the rivets may be taken from 8 to 13 mm, the number of rivets is not below 4. The rivets should be checked for shear and crushing. For possible jerks during brake opration the value of permissible shear stress is limited to 300 to 400 kg/sq cm and the safe crushing stress is double the permissible shear stress intensity.

(vi) The transmission ratio of the lever is usually assumed within 3 to 6 and sometimes the higher limit reaches 10.

(vii) The band tension should be applied as far as possible at an angle of 90° to the pivot of the brake lever.

If this can not be achieved the end of the straight lever should be bent as shown in fig. 19-20. The design of levers have been considered in more detail in chapter XII.

Band brakes are well adopted for manual operation, and at the same time requires very little space beyond the outer diameter of the drum. As with shoe brake ample size hardened pins, equipped with lubrication fittings should be used in the linkage.

The band brake consists of a flexible steel band lined with friction material, the latter usually having an arc of embrace of the order of 270°. Tightening or slackening of the band on to or away from the rotating member may be carried out by many different mechanical arrangement of which space does not permit detailed descriptions or appraisals.

Band brakes are, in general, positive in action and can be prone to such troubles as grabbing These conditions arise from mechanical causes which can be avoided at the design stage Heavy loads can be applied to the band brake anchorages and any weakness at these points may lead to flexing and distortion. Band brakes are more effective with one direction of rotation than with the other direction of rotation In some cases advantage can be taken of this feature by designing the linkage in such a way that the brake will release itself when the drum turns in one direction and apply itself when the drum rotation reverses Such an arrangement is called *back stop* Such types of brakes are desirable in hoisting machineries to automatically prevent the load from lowering when the power is cut-off

Bank brakes, which are effective for one direction of rotation may be either of *simple* type or *differential* type. In simple band brake, one end of the band is attached to the fulcrum of the lever, while the differential band brake has two ends of the band attached to two points on opposite sides of the fulcrum, such that the band pulls give differential moment about the fulcrum of the lever.

When the band is tightened on to the rotating member the friction between them provides the tangential braking force F Owing to the direction of rotation of the rotating member the tension in the band varies between the ends and these tight and slack tensions are denoted by T_1 and T_2 respectively. The following relations can be derived

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{.......(i)}$$

$$T_1 - T_2 = F \quad \text{.......(ii)}$$

(viii) The value of permissible bearing pressure intensity between the brake and the band varies from 4 to 15 kg/sq cm depending upon the material combination and the nature of application of brake.

(ix) In order that the band may withdraw uniformly from the drum, band brakes should have a bent bar made of flat steel and fitted round the outside of the band with adjusting screws spaced at definite intervals to adjust the band departure.

(x) A modification of the band brake is a block and band brake in which the flexible steel band has a number of wooden blocks fixed to the inside surface and the friction of the blocks on the drum provides the braking action. Each block embraces a short arc on the drum.

Examples:

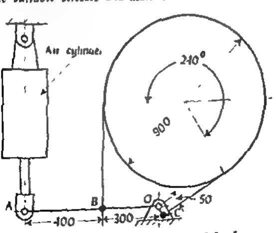
1. An air operated block and band brake is shown in fig. 19-20 and is to have a torque capacity of 1,200 kg metre The brake band consists of a number of asbestoes blocks riveted to a steel band. The coefficient of friction is 0 35 and the pressure of air inside the cylinder is 10 kg/sq cm. Design·

(i) *the diameter of the cylinder and its thickness*

(ii) *the pivot pin diameter and*

(iii) *the brake lever*

Assume suitable stresses and materials.



Air operated block and band brake

Fig. 19-20

The angle of lap is 240°. We assume that there are 14 blocks, each of which subtends an angle of 15° at the drum centre. This leaves us with a clearance of 30° between 14 blocks. The tension ratio is given by

$$\frac{T_1}{T_2} = \left[\frac{1+\mu \tan \theta}{1-\mu \tan \theta}\right]^n$$

where μ is the coefficient of friction, θ is half the angle subtended by each block at the centre and n is the number of blocks.

On substitution of values we get

$$\frac{T_1}{T_2} = \left[\frac{1+0{\cdot}35 \ \tan 7° - 30'}{1-0{\cdot}35 \ \tan 7° - 30'}\right]^{14} = 3{\cdot}61.$$

The arrangement is the differential one. The brake actuating force is determined by the difference in the moments caused by the tension of the band ends relative to the pivot axis of the braking lever.

We assume the direction of rotation of the brake. In order to have the maximum braking torque for the given operating force, the tight end of the band should be connected to a point nearer to the fulcrum. This arrangement will result in lighter and cheaper construction. Therefore the direction of rotation must be anticlockwise.

$$(T_1 - T_2) \times 0{\cdot}45 = 1200$$

Thus we get $T_1 = 3{,}690$ kg and $T_2 = 1{,}020$ kg.

The differential moment about the fulcrum $= 1020 \times 30 - 3690 \times 5 = 12{,}150$ kg cm.

$$\text{Operating force} = \frac{12150}{70} = 173 \text{ kg.}$$

If d cm be the inner diameter of the air cylinder, then

$$\frac{\pi}{4} D^2 \times 10 = 173$$

or $$D = \sqrt{\frac{173}{10} \times \frac{4}{\pi}} = 4{\cdot}7 \text{ cm; we adopt 6 cm diameter.}$$

We adopt 4 mm thick wall for the air cylinder. It should be noted that the stresses in the wall of the cylinder vary from zero to maximum and back to zero each time the brake is applied and released.

The reaction at the pivot can be determined either graphically by means of a force polygon or analytically. The reaction

at the pivot is the resultant of the brake actuating force of 173 kg, the tight side tension 3,690 kg and slack side tension 1,020 kg.

It can be seen that the reaction at the pivot will be 4,200 kg

Assuming a bearing pressure of 200 kg/sq cm and $\frac{l}{d}$ ratio as 1·25, we can see that 4·5 cm diameter pin will be suitable. The bearing length of the pin will be adopted as 5 cm. The shear stress in the pin will be $\dfrac{4200}{2 \times \frac{\pi}{4} \times 4{\cdot}5^2} = 133$ kg/sq cm, which is the safe value. The diameter of the boss will be 9 cm and 3 mm thick phosphor bronze bush will be inserted. The length of pin in each fork will be 3 cm. Generally the pin is case hardened to give a tough interior core. The pin should be checked in bending as a knuckle pin. Refer page 229. Bending stress will be 525 kg/sq cm, which is within limits

Maximum bending moment on the lever will be at the pivot its value being $3690 \times 5 = 18{,}450$ kg cm We adopt a rectangular section having thickness as $\frac{3}{8}$th of the depth. Assuming permissible flexural stress as 700 kg/sq cm, we get

$$\tfrac{1}{6} \times \tfrac{3}{8} d \times d^2 \times 700 = 18450$$

or $d = \sqrt[3]{\dfrac{18450 \times 16}{700}} = 7{\cdot}5$ cm; we increase to 8 cm and thickness will be 3 cm.

As the minimum length of the pin in the bearing is 5 cm, the thickness of the lever at the boss must be increased to 5 cm. The depth of the lever may be increased to 9 cm to allow for the presence of the hole in the lever

It should be noted that the lever is subjected to combination of complex stresses.

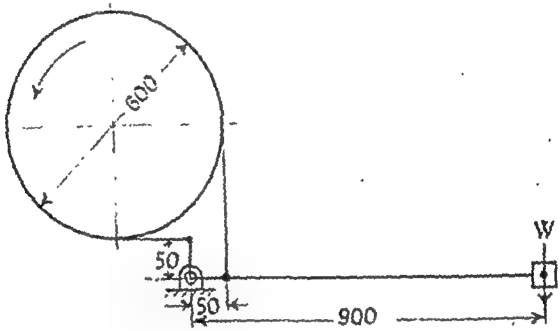
2. *The torque absorbed in the hand brake shown in fig 19·21 is 6,000 kg cm. Design the brake. The coefficient of friction may be taken as 0·27.*

The brake shown in fig. 19-21 may be used for rotation in both directions and hence the value of the brake actuating force will remain the same irrespective of the direction of rotation. This is possible since both the moment of the tension T_1 and T_2 act in the same direction and in the opposite direction to the moment of

the operating force. The moment arms of both the tensions are equal. Therefore, it is possible that T_1 and T_2 may interchange positions. The rivet connections on the two sides must be kept the same.

We have $\dfrac{T_1}{T_2} = e^{\frac{0.27 \times 270 \times 75}{180}} = 3.56$ and $(T_1 - T_2)\,30 = 6000.$

From these two equations we get $T_1 = 278$ kg and $T_2 = 78$ kg.



Band brake equally effective for both the directions of rotation
FIG. 19-21

We adopt four rivets of 8 mm size. There will be two rows of rivets and band will not be weakened by more than two rivet holes. Generally the band thickness may be taken as $0.005D$ where D is the diameter of the brake drum. We adopt band thickness of 3 mm. For a permissible tensile stress of 700 kg/sq cm, 40 mm wide steel band will suffice. Shear stress intensity in the rivet will be $\dfrac{278}{\frac{\pi}{4} \times 0.8^2} \times \dfrac{1}{4} = 139$ kg/sq cm and crushing stress intensity on the rivet will be $\dfrac{278}{4 \times 0.8 \times 0.3} = 290$ kg/sq cm.

These stress values are within limits.

The maximum bearing pressure between the band and the drum $= \dfrac{2 \times T_1}{D \times b} = \dfrac{278 \times 2}{60 \times 4} = 2.31$ kg/sq cm. It decreases gradually towards the slack end of the band to the minimum value of $\dfrac{2.31}{3.56} = 0.65$ kg/sq cm.

at the pivot is the resultant of the brake actuating force of 173 kg, the tight side tension 3,690 kg and slack side tension 1,020 kg.

It can be seen that the reaction at the pivot will be 4,200 kg.

Assuming a bearing pressure of 200 kg/sq cm and $\frac{l}{d}$ ratio as 1·25, we can see that 4·5 cm diameter pin will be suitable. The bearing length of the pin will be adopted as 5 cm. The shear stress in the pin will be $\dfrac{4200}{2 \times \frac{\pi}{4} \times 4{\cdot}5^2} = 133$ kg/sq cm, which is the safe value. The diameter of the boss will be 9 cm and 3 mm thick phosphor bronze bush will be inserted. The length of pin in each fork will be 3 cm. Generally the pin is case hardened to give a tough interior core. The pin should be checked in bending as a knuckle pin. Refer page 229. Bending stress will be 525 kg/sq cm, which is within limits.

Maximum bending moment on the lever will be at the pivot its value being $3690 \times 5 = 18{,}450$ kg cm. We adopt a rectangular section having thickness as $\frac{3}{8}$th of the depth. Assuming permissible flexural stress as 700 kg/sq cm, we get

$$\tfrac{1}{6} \times \tfrac{3}{8}\, d \times d^2 \times 700 = 18450$$

or $d = \sqrt[3]{\dfrac{18450 \times 16}{700}} = 7{\cdot}5$ cm; we increase to 8 cm and thickness will be 3 cm.

As the minimum length of the pin in the bearing is 5 cm, the thickness of the lever at the boss must be increased to 5 cm. The depth of the lever may be increased to 9 cm to allow for the presence of the hole in the lever.

It should be noted that the lever is subjected to combination of complex stresses.

2. *The torque absorbed in the hand brake shown in fig. 19-21 is 6,000 kg cm. Design the brake. The coefficient of friction may be taken as 0·27.*

The brake shown in fig. 19-21 may be used for rotation in both directions and hence the value of the brake actuating force will remain the same irrespective of the direction of rotation. This is possible since both the moment of the tension T_1 and T_2 act in the same direction and in the opposite direction to the moment of

opposite sides of the fulcrum of the brake lever at distances of 4 cm and 12 cm. The length of the lever is 90 cm. The angle of lap may be taken as 240° and the coefficient of friction is 0·3.

Choose your own values for the stresses. Give the main design calculations for the lever, the band, the fulcrum pin and the rivets.

(C) CLUTCHES

19-12. Introduction:

A clutch is a device which engages one rotating part to another rotating part in such a way that parts can be readily engaged and disengaged. Brakes and clutches are similar in their basic function in that they are used to control the flow of mechanical power within a machine. They are similar in other respects too: for instance both must be capable of transmitting specified amounts of torque and both are called upon to convert potential or kinetic energy and to dissipate it in the form of heat, transferred from the brake or clutch to the atmosphere.

Clutches can be roughly grouped into two general types, namely positive acting clutches and friction clutches.

The simplest type of positive clutch is the jaw clutch. One segment of the clutch is permanently fastened to a shaft; the other segment is splined or keyed and allowed to slide axially on the other shaft thus permitting it to be engaged or disengaged by sliding. The greatet difference in the various jaw clutches available to-day lies in the jaw design. In order to provide for a longer period of time during shifting of the clutch, the jaws may be ratchet shaped, gear tooth shaped or spiral shaped. Sometimes a great number of jaws are used; in other instances only a minimum number of jaws are used. A large number of small jaws permit rapid engagement or disengagement with very little motion of the mating jaws. These clutches do not slip and hence no heat is generated at the clutch surfaces. They have the disadvantages of high shock loads when engaged while moving; they can not be engaged at high speeds and sometimes can not be engaged at rest unless the jaws are aligned. The jaw clutch has a very important application where synchronised operation is required, as, for example, in power presses and punches.

Friction Clutches:

Friction clutches transmit torque by virtue of a friction force developed, hence can slip under certain conditions. Thus a fric-

The operating force will be $\frac{(278+78)5}{90} = 18$ kg.

It should be noted that as the brake is to operate for both the direction of rotation, the shorter arm is to be designed for maximum bending moment i.e. $278 \times 5 = 1{,}390$ kg cm.

Assuming a rectangular section with width equal to $\frac{3}{8}$th depth, and permissible tensile stress intensity as 1,000 kg/sq cm we get $\frac{1}{6} \times \frac{3}{8} d \times d^2 \times 1000 = 1390$

$$\text{or} \quad d = \sqrt[3]{\frac{1390}{1000} \times 16} = 3 \text{ cm.}$$

Thickness will be equal to 1·2 cm.

Reaction at fulcrum will be *different* for both the directions of rotation, the larger of the two values will be $\sqrt{278^2 + (78-18)^2} = 282$ kg and the other value will be $\sqrt{78^2 + (278-18)^2} = 274$ kg.

Assuming a bearing pressure intensity of 100 kg/sq cm and ratio as 1·25, we get diameter of the pin as 15 mm and the bearing length of the pin as 20 mm.

The dimensions of the lever near the boss should be modified to accomodate the fulcrum pin

Exercises:

1. *Design a simple band brake to absorb 40 horse power at a speed of 200 r.p.m. The diameter of the brake drum is 750 mm and the length of the lever is 750 mm and the leverage ratio is 5 The angle of lap may be taken as 270°, $\mu = 0{\cdot}3$.*

Permissible tensile stress intensity for the band may be taken as 600 kg/sq cm.

2. *Design a simple band brake to be operated by a lever 50 cm long The brake drum is 50 cm diameter and the brake band embraces five eights of the circumference. One end of the band is attached to the fulcrum of the lever, while the other end is attached to the pin on the lever 10 cm from the fulcrum. The coefficient of friction is 0·25 and the brake actuating force is 50 kg. Choose your own values for the stresses.*

3. *State the advantages of the differential band brake over a simple band brake.*

A differential band brake is to be designed to support a load of 40,000 kg around a barrel of 45 cm diameter. The brake is to be mounted on the drum 80 cm diameter. The two ends of the band are attached to pins on

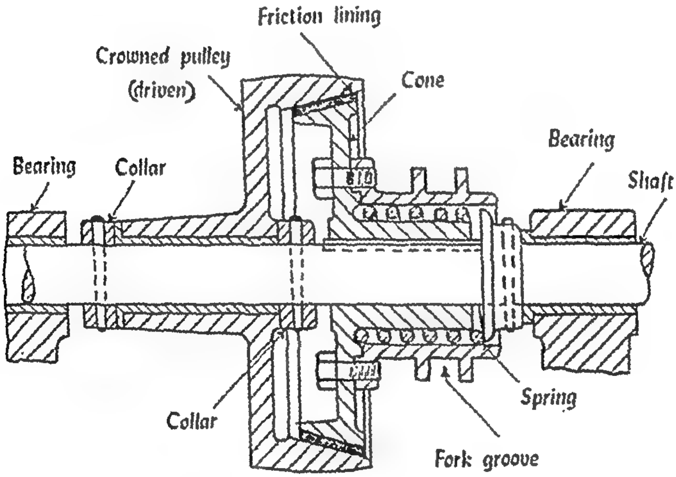
When large torque is to be transmitted multi plate clutches are used in preference to single plate type clutch which possesses only two contact surfaces.

Cone Clutch: (Fig. 19-23)

These clutches are not used now a days in cars and lorries. However they have many other practical applications.

Because of the conical shape of the friction surfaces, a relatively small axial force applied to the inner member or cone provides a large force normal to the friction surfaces. Thus it is seen that a relatively small axial force is needed to engage a cone clutch.

Furthermore, if the cone angle is small enough, the friction surfaces will hold themselves together once the clutch has been engaged.



Cone clutch

Fig. 19-23

The elaborate linkage required with the disc clutch is not needed with the cone clutch, thus giving the cone clutch the advantage of simplicity. However the cone clutch can not be made in multiple units as can the disc clutch, a disadvantage of the conical construction.

Now we consider the design procedure for friction clutches. At this stage the students should refer articles 12 to 16 of chapter

tion clutch can be engaged while the driving member is turning and the driven member is stationary. Because of this the friction clutch is more useful than a positive acting clutch.

Although many friction devices have been used as clutches to-day only the disc clutch, plate clutch and cone clutch are widely used.

Fig. 19-22 shows a typical single plate clutch arrangement and the method of actuating it. The whole of the clutch mechanism is rotating with the input member. The driven shaft passes through the centre of the clutch shell and is splined at the end to take the spinner plate which is situated between the driving and pressure plates. The spinner plate is lined on both the sides with Ferodo friction linings.

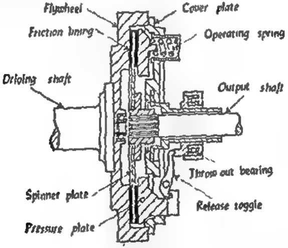


Plate clutch
FIG. 19-22

When the clutch is operative the pressure plate clamps the spinner plate against the driving plates by means of the springs and the drive is then trnsmitted through the pressure and driving plate to the spinner and so as to the transmission shaft.

To disengage the clutch the pressure plate must be withdrawn to release the clutch plate. This is effected by depressing the throw out bearing, while being coupled to the pressure plate by a series of toggles, pulls this plate off against the operating springs

Uniform pressure assumption would be more appropriate where the plates are flexible to permit deflection when wear occurs.

(iv) Consideration may sometimes be given to the question of whether the clutch plate should be operated in a completely dry environment or whether a wet clutch design should be used. While clutches operating dry generally have a high coefficient of friction and hence a high torque transmission capacity in comparision with a wet clutch, the clutch plate surfaces must be kept dry through out the useful life of the clutch. This requirement may lead to difficult sealing problem if the clutch operates in an oily region in a machine.

When the clutch operates in oil, it gives smoother engagement and better dissipation of heat even though the capacity is reduced. Wet clutch design results in longer life and lower operating temperatures for wearing surfaces.

(v) The cone clutch will hold itself in engagement if the tangent of the cone angle is less than the coefficient of friction.

(vi) The following properties are desired for materials of friction surfaces:

(a) A high coefficient of friction retaining a permanent value over a sufficiently wide range of surface velocities, temperature and load.

(b) Adequate mechanical and thermal strength

(c) Little wear and no scoring

(d) High heat conductivity making rapid dispersal of heat from the friction surfaces.

The table on page 871 gives some of the properties which will be of immense value in design of friction clutches:

(vii) The torque that we calculate is called the friction torque and the engagement factor, β, should guarantee the operation of the clutch without slipping under occasional circumstances at intermediate over loads or when the coefficient of friction or the radius of friction are decreased from the design values due to a changed nature of contact between the friction surfaces.

$$T_{friction} = \beta \times \text{design torque}$$
$$= \beta \times \text{motor torque}$$

9 of the book entitled "Theory of Machines Vol. I" written by the authors.

19-13. Design procedure for friction clutches:

(i) The particular attention should be paid to the following items:

(a) The material *forming the contact surfaces* must be selected with great care to ensure that the friction forces are sufficient, engaging pressures are not excessive, the possibility of grabbing is avoided and heat generation will not cause lining deterioration.

(b) The clutch should be as light as possible to minimize the inertia load.

(c) A power transmitting clutch should be so that no external forces are required to maintain contact of the friction surfaces.

(d) The engagement mechanisms should be simple and easy to operate.

(e) Provision must be made for dissipating heat that results from the sliding of surfaces during engagement or disengagement

(f) Design should facilitate repair and provide for wear adjustment.

(g) The clutch should have no projecting parts. If such projecting parts can not be avoided a guard or cover should be provided as an integral part of the design

(ii) In disc clutches the number of pairs of surfaces transmitting power is one less than the sum of the steel and bronze discs and is also an even number if the design is such that no thrust bearings are needed.

(iii) For disc clutches there are two assumptions: (a) uniform wear and (b) uniform pressure. When a clutch is new, it is perhaps true that the pressure may be rather uniform. If the surfaces are relatively rigid, the outer portion where the velocity is high, will wear more than the inner portion. After initial wearing—in it is reasonable to assume that the curve of the profile will maintain its shape or wear thereafter may be considered to be uniform.

engagement and the energy of friction work to be dissipated by the clutch for each engagement.

Engine speed at beginning of clutch engagement 1,900 r.p.m.

Engine torque constant during engagement 800 kg cm.

Automobile weight when loaded 1,400 kg

Automobile wheel diameter 70 cm

Moment of inertia of combined engine rotating parts, flywheel and input side of clutch 0·110 kg metre sec²

Gear reduction at differential 4·1:1

Torque at rear wheels available for accelerating automobile 1,400 kg cm

Coefficient of friction of clutch material 0·3

Design pressure at clutch material 2·5 kg/sq cm.

$$50 \text{ km/hour} = \frac{50 \times 1000}{3600} = 13{\cdot}9 \text{ metre/sec.}$$

$$\text{Angular velocity of wheel} = \frac{13{\cdot}9}{0{\cdot}35} = 39{\cdot}7 \text{ rad/sec.}$$

$$\text{Angular velocity of clutch follower shaft} = 39{\cdot}7 \times 4{\cdot}1 = 163 \text{ rad/sec.}$$

$$\text{Angular velocity of the engine} = \frac{1900 \times 2\pi}{60} = 199 \text{ rad/sec.}$$

Angular acceleration of the engine during the clutch slip period

$$\text{of clutch} = \frac{800 - 4150}{11} = -385 \text{ rad/sec}^2.$$

$$\text{Accelerating force on automobile} = \frac{1400}{35} = 40 \text{ kg.}$$

$$\text{Acceleration of vehicle} = \frac{40 \times 9{\cdot}81}{1400} = 0{\cdot}28 \text{ metre/sec}^2.$$

$$\text{Angular acceleration of clutch output} = \frac{4{\cdot}1 \times 0{\cdot}28}{0{\cdot}35} = 3{\cdot}28 \text{ rad/sec}^2.$$

$$\text{Clutch slip period} = \frac{199 - 163}{3{\cdot}28 - (-385)} = 0{\cdot}0925 \text{ sec.}$$

Angle through which the input side of the clutch rotates during

$$\text{engagement} = 199 \times 0{\cdot}0925 - \frac{385}{2} \times 0{\cdot}0925^2 = 16{\cdot}76 \text{ rad.}$$

TABLE

Material of friction surface		Operating conditions	μ	p kg/sq cm	Operating temperatures C°
Hardened Steel	Hardened Steel	In Oil	0·08	6—8	250
C I	C I or steel	"	0·06	6—8	
C I	"	Dry	0·15	2.5 to 4	"
Bronze	"	In oil	0·05	4	"
Pressed asbestoes	"	Dry	0·3	2 to 3	150 / 150-250
Powder metal	"	"	0·4	3	550
"	"	In oil	0·1	[illegible]	550

where β = 1·25 to 1·5 for metal cutting machine tools
= 1·2 to 1·5 for automobiles
= 2 to 2·5 for tractors
> 1·15 for cranes.

Excessive values of β should be avoided since they cause increased dynamic loads on starting

(viii) As the ratio of $\frac{r_i}{r_o}$ increases the torque transmitting capacity also increases for the same axial force. However when this ratio increases the area of contact of the clutch lining is decreased and the contact pressure is increased Since the rate of wear of materials used for clutch disc facings is generally proportional to contact pressure, the value of $\frac{r_i}{r_o}$ is limited by wear considerations The value of this ratio is never less than 0·5 and usually lies between 0·6 and 0·7.

(ix) $\frac{\text{Width of friction surface}}{\text{Mean radius}}$ may be taken from 0·5 to 0·2

(x) Discs are made of metal

Examples:

1 *Design a single plate automobile clutch to go with a six cylinder engine that has a maximum torque of 2,400 kg cm at 1,900 r.p m The flywheel diameter is large enough so that a 24 cm outside diameter disc clutch can be used. After the clutch dimensions have been decided upon, we have to analyse the characteristics of the drive system, assuming the clutch is engaged when the car is travelling at 50 km/hour after a shift from second to third gear. Determine the number of revolutions of clutch slip during*

engagement and the energy of friction work to be dissipated by the clutch for each engagement.

Engine speed at beginning of clutch engagement 1,900 r.p.m.

Engine torque constant during engagement 800 kg cm.

Automobile weight when loaded 1,400 kg

Automobile wheel diameter 70 cm

Moment of inertia of combined engine rotating parts, flywheel and input side of clutch 0·110 kg metre sec²

Gear reduction at differential 4·1:1

Torque at rear wheels available for accelerating automobile 1,400 kg cm

Coefficient of friction of clutch material 0·3

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$$\text{Angular acceleration of clutch output} = \frac{4{\cdot}1 \times 0{\cdot}28}{0{\cdot}35} = 3{\cdot}28 \text{ rad/sec}^2.$$

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Angle through which the input side of the clutch rotates during

$$\text{engagement} = 199 \times 0{\cdot}0925 - \frac{385}{2} \times 0{\cdot}0925^2 = 16{\cdot}76 \text{ rad.}$$

Angle through which the output side of the clutch rotates during engagement time $= 163 \times 0.0925 + \frac{1}{2} \times 3.28 \times 0.0925^2$

$= 15.1$ rad.

Angle of slip $= 16.76 - 15.1 = 1.66$ rad.

Energy of friction work to be dissipated by the clutch for each engagement $= \frac{4150}{100} \times 1.66 = 69$ kg metre

$$= \frac{69}{426.7} = 0.126 \text{ kcal.}$$

The plate clutch has two friction surfaces.

Torque $= P\mu r_f N$ where

$P = p[\pi\ (r_o^2 - r_i^2)]$

μ = the coefficient of friction between the surfaces in contact

r_f = friction radius, for uniform wear its value is the mean radius of the friction lining

N = number of friction surfaces.

$r_o = \frac{24}{2} = 12$ cm

$\mu = 0.3$ and $p = 2.5$ kg/sq cm

By assuming value of $\frac{r_i}{r_o}$ ranging from 0.5 to 0.7 and solving for normal force P and clutch torque we get the following table

$\frac{r_i}{r_o}$	r_i cm	r_f cm	A sqcm	P kg	N	μ	T kg cm
0.5	6	9	84.83	850	2	0.3	4,580
0.6	7.2	9.6	72.38	722	2	0.3	4,150
0.7	8.4	10.2	57.68	575	2	0.3	3,520

We adopt the ratio $\frac{r_i}{r_o} = 0.6$ for which the margin of clutch torque that is greater than the maximum engine torque is $\frac{4150}{2400} - 1 = 0.73$

This margin is assumed to be sufficient to prevent the clutch from slipping at any time after the engagement period is complete.

When the uniform wear conditions prevail the contact pressure on the clutch facing varies from

$$\frac{P}{2\pi (r_o - r_i)\, r_o} \text{ to } \frac{P}{2\pi (r_o - r_i)\, r_i}.$$

At inner radius $= \dfrac{722}{2\pi \times 4{\cdot}8 \times 7{\cdot}2} = 3{\cdot}33$ kg/sq cm.

At outer radius $= \dfrac{722}{2\pi \times 4{\cdot}8 \times 12} = 2$ kg/sq cm.

2. *A cone clutch of 25 cm mean diameter transmits 50 h.p. at 1,000 r.p.m. The cone angle is 10°, the permissible unit pressure is 4 kg/sq cm. Coefficient of friction surface is 0·3. Find the face width required, the axial force to hold the clutch in engagement while transmitting rated power and shaft diameter for a permissible shear stress of 500 kg/sq cm.*

If the disengaging force is 12% greater than the force to hold the clutch in engagement and the clutch is disengaged by 3 mm of the spring, design the spring.

$$T = \frac{71620 \times 50}{1000} = 3{,}581 \text{ kg cm; say } 3{,}600 \text{ kg cm.}$$

If d cm be the diameter of the solid shaft, then

$$\frac{\pi}{16} d^3 \times 500 = 3600$$

$$d = \sqrt[3]{\frac{3600}{500} \times \frac{16}{\pi}} = 3{\cdot}7 \text{ cm; we adopt 4 cm.}$$

If P be the force normal to the surfaces in contact, then

$$3600 = P \times 0{\cdot}3 \times 12{\cdot}5$$

or $$P = \frac{3600}{0{\cdot}3 \times 12{\cdot}5} = 960 \text{ kg.}$$

If b be the face [illegible] the cone.

Angle through which the output side of the clutch rotates during engagement time $= 163 \times 0.0925 + \frac{1}{2} \times 3.28 \times 0.0925^2$

$= 15.1$ rad.

Angle of slip $= 16.76 - 15.1 = 1.66$ rad.

Energy of friction work to be dissipated by the clutch for each engagement $= \dfrac{4150}{100} \times 1.66 = 69$ kg metre

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Torque $= P\mu r_f N$ where

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μ = the coefficient of friction between the surfaces in contact

r_f = friction radius, for uniform wear its value is the mean radius of the friction lining

N = number of friction surfaces.

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$\mu = 0.3$ and $p = 2.5$ kg/sq cm

By assuming value of $\frac{r_i}{r_o}$ ranging from 0.5 to 0.7 and solving for normal force P and clutch torque we get the following table

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0.6	7.2	9.6	72.38	722	[illegible]	0.3	4,150
0.7	8.4	10.2	57.68	575	2	0.3	3,520

We adopt the ratio $\frac{r_i}{r_o} = 0.6$ for which the margin of clutch torque that is greater than the maximum engine torque is $\dfrac{4150}{2400} - 1 = 0.73$.

This margin is assumed to be sufficient to prevent the clutch from slipping at any time after the engagement period is complete.

When the uniform wear conditions prevail the contact pressure on the clutch facing varies from

The cone pitch angle of the cone is 12·5° and the multiple disc clutch has five steel discs and four bronze discs. Assume uniform wear in both clutches.

Ans. 1·73.

4. *An industrial Diesel Engine develops 35 H.P. at 1,000 r.p.m. Power is to be taken off through a single plate disc clutch from the flywheel end. Design a suitable clutch for the duty. Mean diameter of the clutch is 20 cm and the average pressure is limited to 1·2 kg/sq cm. Coefficient for friction may be taken from 0·4 to 0·5.*

The clutch is normally engaged and the spring used for holding the disc together is made of steel for which safe stress of 4,200 kg/sq cm may be taken. Sketch the clutch showing the arrangement for declutching. Single spring may be used directly or through small levers or a number of springs may be used directly.

5. *A disc clutch transmits 125 H.P. at 600 r.p.m. The number of contact surfaces is 6. The coefficient of friction can be taken as 0·35 and maximum pressure should not exceed 1·5 kg/sq cm. The outer to inner diameter ratio of friction surfaces may be taken as 1·5.*

Determine the main dimensions of the friction surfaces and the necessary axial force at the friction surfaces.

6. *A cast iron cone clutch is to be designed to transmit 30 h.p. at 100 r.p.m. The clutch is operated by a foot lever which has a leverage of 12. The value of coefficient of friction may be taken as 0·3.*

7. *A 60 cm outside diameter plate clutch has a maximum lining pressure of 3·5 kg/sq cm. It is required to transmit 180 h.p. at 400 r.p.m. Design the necessary friction lining and the suitable spring.*

The coefficient of friction may be taken as 0·3.

8. *Design a cone clutch to transmit 200 h.p. at 600 r.p.m. It has a mean radius of 20 cm and the maximum lining pressure is limited to 7 kg/sq cm. The coefficient of friction may be taken as 0·2.*

EXAMPLES XIX

1. A hoisting tackle has two blocks, one upper and the other lower. Each block has two sheaves. Design either the top or bottom block for a lifting load of one tonne. The materials used are: Pulley—cast iron, other parts mild steel and rope manila hemp or cotton.

Stiffness of the spring $= \dfrac{187 - 167}{0 \cdot 3} = 66 \cdot 6$ kg/cm.

Maximum torque on the spring $= \dfrac{187 \times 6 \cdot 5}{2}$

$= 610$ kg cm.

We adopt the spring material having permissible shear stress as 4,500 kg/sq cm. In order to account for the stress concentration we take the stress as 3,600 kg/sq cm in the torque formula

If d_w cm be the size of the wire, then

$$\frac{\pi}{16} d_w^3 \times 3600 = 610$$

or $$d_w = \sqrt[3]{\frac{610}{3600} \times \frac{16}{\pi}} = 0 \cdot 95 \text{ cm}$$

$$k = \frac{G\, d_w}{8 C^3 n} \quad \text{or}$$

n = number of active turns

$$= \frac{0 \cdot 84 \times 10^6 \times 0 \cdot 95}{8 \times \left(\dfrac{6 \cdot 5}{0 \cdot 95}\right)^3 \times 66 \cdot 6} = 4 \cdot 65 \text{ turns.}$$

As the spring will be in compression, the actual turns will be [illegible] turns

Solid height of the spring $= 6 \times 0 \cdot 95 = 5 \cdot 75$ cm

Compression $= \dfrac{187}{66 \cdot 6} = 2 \cdot 88$ cm.

We adopt free height as $5 \cdot 75 + 2 \cdot 88 + 0 \cdot 60 = 9 \cdot 3$ cm

Exercises:

1 What are the items to which the particular attention should be paid when designing friction clutches?

2. Show that a cone clutch will hold itself in engagement [illegible] tangent of the cone pitch angle is less than the coefficient of friction.

3 Compare the horse power capacity of two clutches one a [illegible] disc clutch and the other a cone type of clutch Both clutches [illegible] same speed, both have the same mean diameter and the same [illegible] exerted in both clutches. The coefficient of friction is the same [illegible] clutches.

Design the clutch and the operating mechanism and draw a sectional elevation of the assembly.

Any additional data required may be assumed, the assumptions being clearly stated.

You may estimate the dimensions of the operating link work without calculations. (University of Bombay, 1969)

8. Design a single plate automobile clutch capable of transmitting 90 h.p. at 3,000 r.p.m. The maximum outside diameter of the clutch plate should not be more than 25 cm and the clutch is engaged by means of nine springs. Spring compression required for disengagement is about 2·4 mm and the variation of spring force should be less than 15%. Allowable working shear stress of spring steel is 3,800 kg/sq cm. Modulus of rigidity is $8 \cdot 3 \times 10^5$ kg/sq cm. Take the coefficient of friction for the clutch as 0·25. Pressure on the lining should not exceed 2·5 kg/sq cm.

Design the size of the lining, springs and driven shaft. Draw a simple sketch of the clutch giving an idea of the mechanism for disengagement.

(Gujarat University, 1969)

9. Design a crane hook and a bridge piece of a hook block of 4 tonne capacity. The crane hook is forged from mild steel, having either triangular or trapezoidal section. Evaluate the major dimensions and calculate the maximum stresses involved. Assume any suitable data required.

Sketch two views of the hook block.

(Sardar Patel University, 1970)

The working load for the rope is 30 d^2 kg where d is the diameter of the rope in cm. The bearing pressure for pins is 200 kg/sq cm. The permissible stresses for mild steel are 850 kg/sq cm in tension or compression and 650 kg/sq cm in shear.

2. Design a hook and its support along with a thrust bearing for a 5 tonne load. The hook is to be of swivelling type and of triangular section Choosing your own materials and the probable value of permissible stresses, design and draw to scale at least two views of the arrangement

3. State the relative advantages and disadvantages of cone and disc clutches.

A cone clutch is to be engaged with both parts stationary. Derive the equation for the force required to engage the clutch in terms of the normal force, the friction coefficient, and the cone angle

Is the engaging force found by the equation derived above satisfactory if the the clutch is engaged when one part is rotating and the other part stationary? Explain briefly

4. A cone clutch is required to transmit 40 kg metre of torque at 200 r p.m, and is to hold itself in engagement Design the clutch, stating a satisfactory combination of mean diameter of the cone, cone angle, cone face width, friction surface materials and force required to engage the clutch

5. Design a dry single plate clutch to transmit 15 h p at 1,200 r p m

Number of springs 6

Number of toggle levers 3

Mechanical advantage from pedal to clutch, between 16 and 20

Ratio $\dfrac{\text{mean radius of friction faces}}{\text{radial width of friction faces}}$ 4

Choose the other data so as to have economical design

[illegible]

steel band, brake lever and the pin The length of the lever may be taken as [illegible] cm. Show by a sketch how the band is connected to the lever Assume your own materials and stresses (Gujarat University, 1969)

7. A cone clutch is to be designed to transmit 150 h p at 750 r p m The following is the proposed scheme and the data for this clutch.

Semi cone angle 12°

Radial width of face to be $\frac{1}{4}$ the mean diameter

Normal pressure on contact surfaces to be assumed uniform and not to exceed 1·2 kg/sq cm

Coefficient of friction 0 2

Maximum shear stress in the shaft 600 kg/sq cm

The axial force is proposed to be applied by means of a pneumatic cylinder and a pivoted lever so that the piston need give only 1/3 the value of the axial thrust. The air pressure available for operating the clutch = 5 kg/sq cm above atmosphere.

TABLE

Derived Units

Unit	Name	Symbol
Area	square metre	m^2
Volume	cubic metre	m^3
Frequency	hertz	Hz 1/s
Density (mass)	kilogramme per cubic metre	kg/m^3
Velocity	metre per second	m/s
Angular velocity	radian per second	rad/s
Acceleration	metre per second squared	m/s^2
Angular acceleration	radian per second squared	rad/s^2
Force	newton	N kg. m/sec^2
Pressure (mechanical tension)	newton per square metre	N/m^2
Dynamic viscosity	newton-second per square metre	$N.s/m^2$
Kinematic viscosity	square metre per second	m^2/s
Work, energy, quantity of heat	joule	J N.m
Power	watt	(W J/s)
Entropy	joule per kelvin	J/K
Specific heat	joule per kilogramme kelvin	J/kg. K
Thermal conductivity	watt per metre kelvin	W/(m. K)

Note:

1. A temperature interval can also be expressed in degrees Celsius.

2. Old definition of litre is abrogated. According to resolution 6 of the 12th General Conference of Weights and Measures, *litre* is a special name given to cubic decimetre and the word litre should not be used for expressing results of high precision volume measurements.

3. While writing symbols for plurals, do not add '*s*' to indicate plurality.

4. Units with names of scientists *should not be capitalised* when written in full.

5. Use the symbols *as they are* to avoid confusion.

6. Certain units, although strictly incompatible with SI units, are at least nitially considered acceptable the examples being km/h and rotational speed in ev/min. As usual the unit of a derived quantity is obtained by taking the physical w connecting it with the other basic or fundamental quantities and putting ch of the other basic quantities involved there in equal to unity in this law pressed as a mathematical relation. Some of the important derived units, ich will be of immense value in proportioning parts of a machine are given table on page 881.

Quantity in number of units	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

In compounded units (x e.g. N/m^2) the prefix, where used must always precede the first symbol, i.e. although MN/m^2 is strictly the same as N/mm^2; however the *latter should never be used.*

20-4. Relation between the Units of SI and MKS Systems:

In this text we have made use of MKS system through out. When one wants to use this text with SI system, it is necessary to know the relation between the units of the SI and MKS systems. To facilitate calculation work, below in the table are given the SI units and their equivalents in MKS system:

TABLE

Quantity	SI System	MKS system
Length:		
metre	m	m
centimetre	cm	cm
millimetre	mm	mm
Area:		
square metre	m^2	m^2
square centimetre	cm^2	cm^2
square millimetre	mm^2	mm^2
Force:		
newton	N	0·102 kg
meganewton	$MN = 10^6$ N	$1{\cdot}02 \times 10^5$ kg
Speed:		
metres per second	m/s	m/s
Angular speed:		
radians per second	rad/s	rad/s
Moment of force:		
newton-metre	N.m	0·102 kg. m = 10·2 kg. cm.

TABLE

Derived quantity	Name of unit and symbol in bracket	Physical law connecting the quantity to fundamental quantities or previously obtained derived quantities	Definition of unit	Remarks
Force	Newton(N)	Force = Mass x Acceleration	It is that force which produces unit acceleration i.e. 1 m per sec in unit mass (kg)	
Work energy or quantity of heat	Joule (J)	Work done = Force x Displacement of point of application of force along its direction	It is the work done by unit force (one Newton) over a unit displacement (one metre)	For expressing quantities of heat in Joules, the relationship 1 calorie = 4·1868 Joules will have to be used.
Power	Watt(W)	Power = Work done per sec	It is the power of an agent doing 1 Joule of work/sec	With the use of *kW* (1000 *W*) and *MW* (10^6 watts) there is no place for horse power whether English or French.

20-3. Prefixes for Multiples and Sub-multiples of Units:- When the numerical value of a quantity is inconveniently large or small, the prefixes to the basic units are allowed. Consecutive multiples and sub-multiples are separated by powers of 10 upto quantities 1,000 times or $\frac{1}{1000}$ times the unit quantity and in powers of 10^3 or 10^{-3} in respect of still larger and smaller quantities respectively. However SI system has preference for those prefixes which are separated by factors of 10^3 and 10^{-3}. Thus lengths would be measured generally in kilometres, metres and millimetres only. Table on page 882 gives the certain prefixes for defining multiples of unit quantities as specified by SI system.

Assume that the end journal is a cantilever with a uniformly distributed load.

The permissible flexural stress is limited to 60 MN/sq m and the bearing pressure intensity is limited to 4 MN/sq m.

We assume that the end journal of a hand winch shaft is a cantilever of 0·06 m length and loaded with a uniformly distributed total load of 16,000 N.

$$\text{Maximum bending moment} = \frac{16000 \times 0{\cdot}06}{2}$$
$$= 480 \text{ N.m}$$

$$\text{Modulus of section} = \frac{\pi}{32} \times 0{\cdot}05^3$$
$$= 12{\cdot}25 \times 10^{-6} \text{ m}^3$$

$$\text{Maximum flexural stress} = \frac{480}{12{\cdot}25 \times 10^{-6}}$$
$$= 39{\cdot}1 \text{ MN/sq m.}$$

As the induced stress intensity is less than permissible stress intensity the design is safe.

$$\text{Projected bearing area} = 0{\cdot}003 \text{ sq m.}$$

$$\text{Bearing pressure intensity} = \frac{16000}{0{\cdot}003}$$
$$= 5{\cdot}33 \times 10^6 \text{ N/sq m}$$
$$= 5{\cdot}33 \text{ MN/sq m.}$$

Conclusion:

In spite of considerable margin of bending strength the end journal operates under unfavourable conditions since induced bearing pressure intensity exceeds the allowable value which is 3 MN/sq m.

It means that either the diameter or the length of the end journal has to be increased (with a corresponding increase in the length of the bearing).

The increased diameter is preferred since high ratios $\frac{l}{d}$ have an adverse effect on bearing operation.

3. *A countershaft receives power 20 kW from a motor through a coupling. The speed of the shaft is 30 rad/s. If the diameter of the shaft is 0·07 m determine the value of torsional shear stress induced, due to power transmitted.*

Pressure (stress):		
newtons per square metre	N/m^2	1.02×10^{-5} kg/cm²
meganewtons per square metre	MN/m^2	10.2 kg/cm²
Work, energy:		
joule	J	0.102 kg m.
	(J = 1 N m)	= 10.2 kg. cm.
Power:		
watt	W	0.102 kg m/s
	(W = J/s)	
kilowatt	kW = 10^3 W	kW
Heat.		
joule	J	0.23×10^{-3} kcal
Temperature:		
Kelvin degree	°K	°C + 273·15°

20-5. Illustrative Examples:

Here we give some illustrative examples to make students familiar with SI system.

1. The crank arm of a steam engine has a rectangular cross section of 0.2 m by 0·1 m. In dead centre position of the crank the maximum compressive force acting is 70,000 N. The line of action of the force is parallel to and at a distance of 0.12 m from the principal axis of the section. Determine the maximum tensile stress induced in the crank arm

The cross-sectional area is $0.2 \times 0{\cdot}1 = 0.02$ sq m

Direct compressive stress $= \dfrac{70000}{0.02} = 3{,}500{,}000$ N/sq m

Due to the eccentricity of 0.12 m, a bending moment of the magnitude $70000 \times 0{\cdot}12 = 8400$ N m is acting on the section

Maximum value of the bending stress

$$= \frac{8400}{\frac{1}{6} \times 0{\cdot}2 \times 0{\cdot}1^2} = 25{,}200{,}000 \text{ N/sq m}$$

$\therefore$ Maximum compressive stress $= 25{,}200{,}000 + 3{,}500{,}000$
$= 28{,}700{,}000$ N/sq m.
$= 287$ MN/sq m

Maximum tensile stress $= 25{,}200{,}000 - 3{,}500{,}000$
$= 21{,}700{,}000$ N/sq m.
$= 21{\cdot}7$ MN/sq m

2. Check the end journal of a hand winch shaft which acts on a cast iron bearing with a force of 16,000 N. Length of the end journal is 0.06 m and the diameter is 0·05 m

$f_t = 90\ MN/sq\ m$, $f_s = 75\ MN/sq\ m$ and $f_c = 150\ MN/sq\ m$.

Refer fig. 4-7.

The diameter of the rivets is obtained by the formula

$d = 6\sqrt{t}$ mm $= 6\sqrt{20} = 28{\cdot}8$ mm; we adopt 25·5 mm as the rivet diameter.

We assume that the resistance of a rivet in double shear is 1·75 times that in single shear. Resistance of plate to tearing at

$$\text{outer row} = \frac{(350 - 25{\cdot}5) \times 20}{1000 \times 1000} \times 90 \times 10^6$$

$$= 585{,}000 \text{ N.}$$

$$\text{Shear resistance of rivet} = \frac{1{\cdot}75 \times \frac{\pi}{4} \times 25{\cdot}5^2 \times 75 \times 10^6}{10^6}$$

$$= 67{,}000 \text{ N.}$$

$$\text{Bearing resistance of one rivet} = \frac{20 \times 25{\cdot}5 \times 150 \times 10^6}{1000 \times 1000}$$

$$= 76{,}500 \text{ N.}$$

As the shearing resistance of the rivet is less than the bearing resistance, we use the former in deciding upon the number of rivets. Equating the tensile strength of the plate to the shearing resistance of n rivets, we get

$$585000 = n \times 67000$$

or $$n = \frac{585000}{67000} = 8{\cdot}2.$$

Hence 9 rivets may be used and they may be arranged as shown in fig. 4-7.

The thickness of the butt straps will be $0{\cdot}75t = \frac{3}{4} \times 20 = 15$ mm.

We investigate the strength of the joint at four critical sections AA, BB, CC, or DD, it cannot fracture along BB without shearing one rivet in double shear, along CC, without shearing three rivets in double shear and along DD without shearing six rivets in double shear.

Along AA, joint has a strentgh of

$$\frac{(350 - 25{\cdot}5) \times 20 \times 90 \times 10^6}{1000 \times 1000} = 585{,}000 \text{ N.}$$

If T N.m be the torque acting on the shaft, then

$$20 = \frac{T \times 30}{1000}$$

or $$T = \frac{20000}{30} = 6{,}660 \text{ N.m.}$$

If f_s N/sq m be the maximum shear stress induced due to torque transmitted then,

$$6660 = \frac{\pi}{16} \times 0.07^3 \times f_s$$

$$\therefore \quad f_s = 101 \times 10^6 \text{ N/sq m}$$
$$= 101 \text{ MN/sq m.}$$

4. *A steam boiler has 80 sq m of heating surface and the rate of evaporation is 20 kg/sq m/hour of heating surface The pressure of steam generation is 0·8 MN/sq m. The specific volume of steam is 0·24 cu metre/kg. Determine the diameter and thickness of the steel steam pipe to carry the steam from this boiler with a velocity of steam in pipe at 25 metre/sec. The permissible stress intensity in the pipe material is 40 MN/sq m.*

Amount of steam generated $= 80 \times 20$
$= 1{,}600$ kg/hour

Volume of steam flowing in the pipe

$$= \frac{1600 \times 0.24}{60 \times 60}$$
$$= 0.107 \text{ cu metre/sec}$$

If D metre be the inner diameter of the steam pipe, then

$$\frac{\pi}{4} D^2 \times 25 = 0.107$$

or $$D = \sqrt{\frac{0.107 \times 4}{25 \times \pi}} = 0.0739 \text{ metre, we adopt } 0.075 \text{ m.}$$

If t be the minimum thickness of the pipe, then

$$t = \frac{pD}{2f_t} = \frac{0.8 \times 0.075}{2 \times 40} = 0.00075 \text{ m}$$
$$= 0.75 \text{ mm.}$$

This thickness is too small. We adopt 3 mm thick solid drawn steel tubes.

5 *A mild steel tie bar for a bridge structure 350 mm wide and 20 mm thick are to be connected by a double cover butt joint. Design this joint allowing safe working stresses as follows:*

Load on the fulcrum will be $5670 - 576 = 5{,}184$ N.

Fulcrum of the lever loaded safety valve is subjected to axial tensile load (fig. 12-23).

Minimum cross sectional area required at the bottom of the threads

$$= \frac{5184}{40 \times 10^6}$$
$$= 129{\cdot}6 \times 10^{-6} \text{ sq m} = 129{\cdot}6 \text{ sq mm.}$$

We assume fine threads. We adopt M16 × 1·5 which has 167 sq mm area at the bottom of the threads. The pitch of the threads is 1·5 mm.

7. The big end (common strap end type) of a connecting rod as shown in fig. 6-7 is subjected to a maximum load of 35,000N. The diameter of the circular part of the rod adjacent to the strap end is 60 mm. Determine (a) the width of the strap end, (b) the thickness of the strap at the thinnest part, at the cotter hole and at the crown and (c) the width and thickness of gib and cotter. Safe tensile stress value in the material of the strap is limited to 22·5 MN/sq m. Safe shear stress value in cotter and gib is not to exceed 17·5 MN/sq m.

The width of the strap is generally equal to or slightly greater than the diameter of the adjacent end of the round part of the rod. The width of the strap is taken to be 60 mm. The connecting rod is subjected to a tensile load of 35,000N. As the permissible tensile stress intensity is limited to 22·5 MN/sq m, the minimum cross sectional area to be provided at the thinnest part of the strap will be

$$\frac{35000}{22{\cdot}5 \times 10^6} = 0{\cdot}00155 \text{ sq m.}$$
$$= 1{,}550 \text{ sq/mm.}$$

As the width of the strap is 60 mm, the thickness of the strap at the thinnest part will be taken as 13 mm. The area provided at the thinnest part of the strap is $60 \times 2 \times 13 = 1{,}560$ sq mm, thus inducing 22·5 MN/sq m as the tensile stress intensity. The thickness of the gib and cotter is taken as one-fourth the width of the strap which gives us 15 mm as the thickness of gib and cotter.

If t_2 mm be the thickness of the strap across the cotter holes, by equating the area of the thinnest part of the strap to area of the cross section of the strap at the cotter hole, we get

Along BB joint has a strength of

$$\frac{(350-2\times 25\cdot5)\times 20\times 90\times 10^6}{1000\times 1000}+1\times 67000$$

$= 605{,}000$ N as the fracture along BB cannot take place without shearing one rivet in double shear. Similarly we can determine the strength along CC and DD.

Strength along CC

$$=\frac{(350-3\times 25\cdot5)\times 20\times 90\times 10^6}{1000\times 1000}+3\times 67000$$

$= 693{,}000$ N.

Strength along DD

$$=\frac{(350-3\times 25\cdot5)\times 20\times 90\times 10^6}{1000\times 1000}+6\times 67000$$

$= 894{,}000$ N.

Shearing resistance of all rivet $= 9\times 67000 = 603{,}000$ N.

The lowest strength of the joint is along AA

$$\text{Efficiency of the joint} = \frac{(b-d)}{b}$$

$$=\frac{(350-25\cdot5)}{350}$$

$$= 0\cdot928 \text{ i.e. } 92\cdot8\%$$

Note: It should be noted that if instead of diamond form of joint, had we adopted chain riveting with three rows of three rivets in each, the least strength of the joint would be $\frac{(350-3\times 25\cdot5)\times 20\times 90\times 10^6}{1000\times 1000} = 492{,}000$ N, which gives an efficiency of $\frac{350-3\times 25\cdot5}{350} = 0\cdot782$ i.e. $78\cdot2\%$.

6. *A lever loaded safety valve has a diameter of 70 mm and blow off pressure of 1·5 MN/sq m. The fulcrum of the lever is screwed into the cast iron body of the cover. Suggest the suitable size of threaded part of the fulcrum if permissible tensile stress intensity is limited to 40 MN/sq m. The leverage ratio is 10.*

The valve is required to blow at a pressure of 1·5 MN/sq m.

$$\text{Load on the valve is} = \frac{\pi}{4}\times\frac{70^2\times 1\cdot5\times 10^6}{1000^2}$$

$$= 5{,}760 \text{ N}.$$

As the leverage is 10, the weight at the end of the lever will be

$\frac{5760}{10} = 576$ N.

If d_1 be the diameter of the bolt, then

$$\frac{\pi}{4} d_1^2 \times 63 \times 10^6 = 156000$$

or

$$d_1 = \sqrt{\frac{156000 \times 4}{63 \times 10^6 \times \pi}} = 55 \text{ mm.}$$

We adopt 65 mm.

If t mm be the minimum thickness of the flange, then

$$\frac{\pi \times t \times 300 \times 63 \times 10^6 \times 0{\cdot}3}{1000 \times 1000 \times 2} = 300000$$

or $$t = \frac{300000 \times 2 \times 1000 \times 1000}{\pi \times 300 \times 60 \times 10^6 \times 0{\cdot}3} = 35{\cdot}4 \text{ mm.}$$

We adopt 40 mm. Crushing area provided $65 \times 40 = 2{,}600$ sq mm which will give reasonably low value of crushing stress intensity.

9. *The spring loaded safety valve of a boiler is required to blow off at a pressure of 1 MN/sq m. The diameter of the valve is 60 mm, and the maximum lift of the valve is 15 mm.*

Design the suitable compression spring for the safety valve assuming the spring index to be 6 and providing initial compression of 30 mm.

The maximum shear stress in the material of the wire is limited to 450 MN/sq m. G = 84,000 MN/sq m.

Load on the valve when it just begins to lift

$$= \frac{\pi}{4} \times \frac{60^2}{1000^2} \times 1 \times 10^6 = 2{,}820 \text{ N.}$$

The valve is kept tight on its seat against steam load of 2,820 N providing initial compression of 30 mm. Therefore, the stiffness of the spring $= \frac{2820}{30} = 94$ N/mm.

The lift of the valve is 15 mm. Therefore, the maximum compression of the spring is $30 + 15 = 45$ mm.

Maximum load on the spring when the valve is in fully open position $= 45 \times 94 = 4{,}230$ N.

The assumed spring index is 6. The stress concentration factor as defined by A. M. Wahl, is

$$= \frac{4C - 1}{4C - 4} + \frac{0{\cdot}615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0{\cdot}615}{6} = 1{\cdot}252.$$

$$2580000 = \frac{\frac{\pi}{4} d^2 \times 330}{1 + \frac{1}{17500}\left[\frac{1400 \times 4}{d}\right]^2}$$

The above equation after simplification gives us an equation $d^4 - 9940d^2 - 17800000 = 0$, which is a quadratic in d^2.

After solving, we get $d = 107$ mm; we adopt 120 mm.

11. The compressive load on the nut and screw clamp similar to one shown in fig. 11-7 is 30,000N. Calculate the diameter of the screw, height of the nut and dimensions of the handle if a force of 300 N is required to be applied at the end of a handle to operate the screw. Assume the following:

Safe compressive stress for screw	*= 120 MN/sq m*
Bearing pressure for screw and nut	*= 17·5 MN/sq m*
Coefficient of thread friction	*= 0·14*
Frictional torque of pad B	*= 35 N.m*
Bending stress in handle	*= 100 MN/sq m.*

The screw is subjected to a direct compressive stress and torsional shear stress. In order to find the diameter of the screw, we must consider the principal stress. Let us find the diameter of the screw taking the lower value of stress, say 85 MN/sq m.

If d be the diameter of the screw at the bottom of the thread,

$$\frac{\pi}{4} d^2 \times 85 \times 10^6 = 30{,}000$$

or $$d = \sqrt{\frac{30000}{85 \times 10^6} \times \frac{4}{\pi}} = 0{\cdot}0212 \text{ m, we adopt 25 mm.}$$

We adopt single start square threads having 5 mm pitch.

∴ Mean diameter of the screw

$= 25 + 2{\cdot}5 = 27{\cdot}5$ mm.

Outside diameter of the crew

$= 25 + 5 = 30$ mm.

Bearing area of each thread

$$= \frac{\pi}{4} \times [30^2 - 25^2] = 216 \text{ sq mm.}$$

If n be the number of threads, then

$$\frac{n \times 216 \times 17{\cdot}5 \times 10^6}{1000^2} = 30000$$

or $$n = \frac{30000 \times 1000^2}{216 \times 17{\cdot}5 \times 10^6} = 8 \text{ threads.}$$

Torque $= 4230 \times 3d_w$ where d_w is the diameter of the spring wire.

$$\therefore \quad 4230 \times 3d_w = \frac{\pi}{16} d_w^3 \times \frac{450 \times 10^6}{1 \cdot 252}$$

$$\text{or} \quad d_w = \sqrt{\frac{4230 \times 3 \times 16 \times 1 \cdot 252}{450 \times 10^6 \times \pi}} = 0{\cdot}0134 \text{ m.}$$

$$= 13{\cdot}4 \text{ mm.}$$

From IS: 1137 – 1959, we adopt 14 mm.

Mean diameter of the coil $= 14 \times 6 = 84$ mm.

If n be the number of active turns in the spring then,

$$94 = \frac{84000 \times 14^4 \times 1000^3}{8 \times 84^3 \times 1000^4 \times n}$$

$$\text{or} \quad n = \frac{84000 \times 14^4 \times 1000^3}{8 \times 84^3 \times 1000^4 \times 94}$$

$$= 7{\cdot}4 \text{ turns}$$

Assuming squared and ground ends we have 9 actual turns
Free length of spring = solid length + maximum compression + clearance between adjacent coils (1 mm between adjacent coils) $= 9 \times 14 + 45 + 1 \times 8 = 179$ mm say 180 mm

10 In a certain water works installation the water is pumped against a head of 165 metre. The bore of the reciprocating pump is 0·45 m The unsupported length of the piston rod is 1·4 m. Determine the diameter of the piston rod by using Rankine's formula, taking the factor of safety to be 10. Take f_c *= 330 MN/sq m and Rankine constant* $a = \frac{1}{17500}$

Water is pumped against a head of 165 metre. This head of water is equivalent to $\frac{165 \times 9{\cdot}81}{1000} = 1{\cdot}62$ MN/sq m.

Maximum compressive load on piston rod

$$= \frac{\pi}{4} \times 0{\cdot}45^2 \times 1{\cdot}62 \times 10^6 = 258{,}000 \text{ N}$$

As the factor of safety is 10. the buckling load on the column will be $258000 \times 10 = 2{,}580{,}000$ N.

If d mm be the diameter of the solid rod, by using Rankine's formula, we get

EXAMPLES XX

1. A steel connecting rod is to be subjected to a reversed axial load of 150,000 N. Determine the diameter of the rod, using a factor of safety 1·8. The ultimate tensile strength of the material is 1,000 MN/sq m. Ans. 30 mm

2. A spherical metal vessel 1·2 m diameter is subjected to an internal pressure of 1·5 MN/sq m. If the permissible stress in the metal is 62·5 MN/sq m and the efficiency of the riveted joints is 75%, determine the required thickness of the plate. Ans. 10 mm

3. Design a diamond, double cover butt joint for a tie bar of 25 mm thickness subjected to an axial load of 0·35 MN. Maximum tensile and shear stress intensities are limited to 110 MN/sq m and 85 MN/sq m respectively.

Ans. Use 30 mm diameter 3 rivets on each side, assuming 1 and 2 rivets in rows; 15 cm wide.

4. Fig. 5-15 shows a cap subjected to a load of 20,000 N inclined at an angle of 45°. Determine the size of the screw if the tensile stress intensity in the material is limited to 56 MN/sq m and shear stress intensity in the material to 45 MN/sq m. Ans. 50 mm

5. The big end of the connecting rod as shown in fig. 6-7 is subjected to a maximum load of 70,000 N. Calculate the diameter of the circular part of the rod adjacent to the strap end if the permissible tensile stress is limited to 25MN/sq m. Also determine the width of the strap end and the thickness of the strap at the thinnest part, at the cotter hole and at the crown.

If the shear stress value in the cotter and the gib is not to exceed 15 MN/sq m, suggest the suitable cross sectional dimensions for the gib and cotter.

Ans. Diameter of circular part 60 mm

6. A 15 kw, 1,400 r.p.m. motor drives a centrifugal pump through a single set of 3:1 reduction gear. The load may be considered to be suddenly applied with minor shocks for which the combined shock and fatigue factor may be taken as 1·5. Determine the diameters of the pump and motor shafts if the permissible stress intensity is not to exceed 45 MN/sq m. Ans. 40 mm; 30 mm

7. Design and draw a fully dimensioned scale drawing of a cast iron flange coupling of protected type to transmit 15 kw at 250 r.p.m. The flanges are of cast iron and all other parts are of mild steel.

8. A semi-elliptic automobile spring 1·45 metre long carries a total load of 9,000 N. The spring is composed of 10 leaves two of which are full length, each 60 mm wide. Determine the necessary thickness and the resultant stress to give a deflection of 63 mm.

$E = 0 \cdot 2 \times 10^6$ MN/sq m Ans. 11·8 mm, 232 MN/sq m

9. The piston rod of a reciprocating pump is subjected to a maximum axial compressive load of 290,000 N. The length of the piston rod is 1·4 metre. Assuming the material of the piston rod to be mild steel, determine the diameter of the piston rod, taking it to be freely hinged at the ends. Take the factor of safety to be 10. Ans. 120 mm

10. Design a screw jack to lift a load of 100,000 N having a lift of 0·5 metre. Choose your own materials and the allowable [illegible]es.

From stability point of view, we adopt 10 threads. The height of the nut will be $10 \times 5 = 50$ mm. The outside diameter of the nut is generally twice the outside diameter of the screw. In the present case it will be $2 \times 30 = 60$ mm.

$$\alpha = \text{helix angle} = \tan^{-1} \frac{p}{\pi d_m} = \tan^{-1} \frac{5}{\pi \times 27 \cdot 5} = 3 \cdot 3^\circ.$$

$$\theta = \text{friction angle} = \tan^{-1} \mu = \tan^{-1} 0 \cdot 14 = 8 \cdot 1^\circ.$$

Torque required to overcome friction at nut will be equal to

$$\frac{pd_m}{2} \tan (\alpha + \theta) = 30000 \times \frac{27 \cdot 5}{2 \times 1000} \times \tan 11 \cdot 1^\circ = 82 \text{ N.m.}$$

Torque required to overcome friction at the pad = 35 N.m.
Total torque to be applied at the handle = 82 + 35 = 117 N.m.

$\therefore$ Effective length of the handle $= \frac{117}{300} = 0 \cdot 39$ m.

If d be diameter of the handle, then

$$\frac{\pi}{32} d^3 \times 100 \times 10^6 = 117$$

or
$$d = \sqrt[3]{\frac{117}{10 \times 10^6} \times \frac{32}{\pi}}$$
$$= 0 \cdot 0228 \text{ m} = 22 \cdot 8 \text{ cm; we adopt 23 mm}$$

If f_s be the shear stress induced in the screw, then

$$\frac{\pi}{16} \times \frac{(25)^3}{1000^3} f_s = 117$$

or
$$f_s = \frac{117 \times 16 \times 1000^3}{\pi \times 25^3} = 38 \cdot 3 \text{ MN/sq m.}$$

Direct compressive stress

$$= \frac{30000}{\frac{\pi}{4} \times \left(\frac{25}{1000}\right)^2} = 61 \text{ MN/sq m.}$$

$$\text{Principal stress} = \frac{61 \pm \sqrt{61^2 + 4 \times 38 \cdot 3^2}}{2}$$

$$= 79 \cdot 4 \text{ MN/sq m}$$

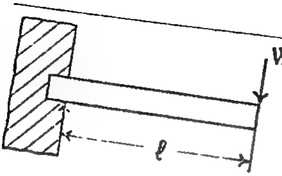
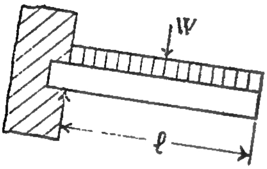
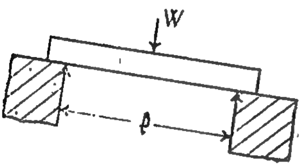
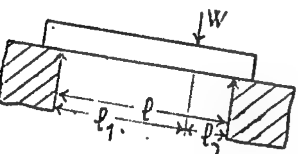
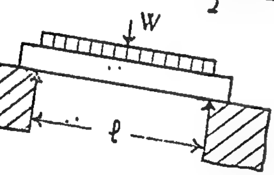
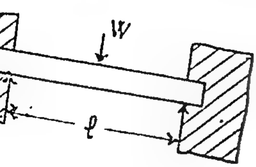
and
$$= -18 \cdot 4 \text{ MN/sq m.}$$

The value of the induced stress is within limits.

Dimensions of the screw: Nominal diameter 30 mm, single start square thread 5 mm, nut of 50 mm height having 60 mm outside diameter. Lever 400 mm of effective length having 23 mm diameter.

APPENDIX

APPENDIX — II Deflection formulas for Machine Parts

Loading	Maximum shearing force	Maximum bending moment	Maximum defl
	W	Wl	$\frac{Wl^3}{3EI}$
	W	$\frac{Wl}{2}$	$\frac{Wl^3}{8EI}$
	$\frac{W}{2}$	$\frac{Wl}{4}$	$\frac{Wl^3}{48EI}$
	$\frac{Wl_1}{l}$ or $\frac{Wl_2}{l}$	$\frac{Wl_1l_2}{(l_1+l_2)}$	$\frac{Wl_2}{3lEI}\left\{\frac{l_1(l_1+2l_2)}{3}\right\}^{\frac{3}{2}}$
	$\frac{W}{2}$	$\frac{Wl}{8}$	$\frac{5Wl^3}{384EI}$
	$\frac{W}{2}$	$\frac{Wl}{8}$	$\frac{Wl^3}{192EI}$
W			

APPENDIX — V

Imperial or Legal Standard Wire Gauge

Descriptive number	Equivalent in Parts of an inch	Equivalent in Millimetres	Sectional area of wire in sq in.	Descriptive number	Equivalent in Parts of an inch	Equivalent in Millimetres	Sectional area of wire in sq. in.
7/0	·500	12·700	·196350	23	·024	·610	·00045239
6/0	·464	11·785	·169093	24	·022	·559	·00038013
5/0	·432	10·973	·146574	25	·020	·508	·00031416
4/0	·400	10·160	·125664	26	·018	·457	·00025447
3/0	·372	9·449	·108687	27	·0164	·4166	·00021124
2/0	·348	8·839	·095115	28	·0148	·3759	·00017203
0	·324	8·229	·082448	29	·0136	·3454	·00014527
1	·300	7·620	·070686	30	·0124	·3150	·00012076
2	·276	7·010	·059828	31	·0116	·2946	·00010568
3	·252	6·401	·049876	32	·0108	·2743	·00009161
4	·232	5·893	·042273	33	·0100	·2540	·00007854
5	·212	5·385	·035299	34	·0092	·2337	·00006640
6	·192	4·877	·028953	35	·0084	·2134	·00005542
7	·176	4·470	·024328	36	·0076	·1930	·00004536
8	·160	4·064	·020106	37	·0068	·1727	·00003632
9	·144	3·658	·016286	38	·0060	·1524	·00002827
10	·128	3·251	·012868	39	·0052	·1321	·00002124
11	·116	2·946	·010568	40	·0048	·1219	·00001810
12	·104	2·642	·008495	41	·0044	·1118	·00001521
13	·092	2·337	·006648	42	·0040	·1016	·00001257
14	·080	2·032	·005027	43	·0036	·0914	·00001018
15	·072	1·829	·004072	44	·0032	·0813	·00000804
16	·064	1·626	·003217	45	·0028	·0711	·00000616
17	·056	1·422	·002463	46	·0024	·0610	·00000452
18	·048	1·219	·001810	47	·0020	·0508	·00000314
19	·040	1·016	·001257	48	·0016	·0406	·00000201
20	·036	·914	·001018	49	·0012	·0305	·00000113
21	·032	·813	·000804	50	·0010	·0254	·00000079
22	·028	·711	·000616				

APPENDIX — III

Metric Coarse Threads

Size	Pitch mm	Stress area sq cm	Size	Pitch mm	Stress area sq cm	Size	Pitch mm	Stress area sq cm
M 1·6	0 35	0 0127	M 5	0 8	0 142	M 20	2 5	2 45
M 1 8	0 35	0 0170	M 6	1 0	0 201	M 22	2 5	3·03
M 2 0	0·40	0 0207	M 7	1 0	0 289	M 24	3 0	3 53
M 2·2	0·45	0 0248	M 8	1·25	0 366	M 27	3 0	4 59
M 2·5	0·45	0 0339	M 10	1 50	0 580	M 30	3 5	5 61
M 3 0	0·50	0 0503	M 12	1 75	0 843	M 33	3 5	6 94
M 3·5	0 60	0 0678	M 14	2·00	1 15	M 36	4 0	8 17
M 4 0	0·70	0 0878	M 16	2 00	1·57	M 39	4 0	9 76
M 4 5	0 75	0 113	M 18	2 50	1·92			

APPENDIX — IV

Metric Fine Threads

Size	Pitch mm	Stress area sq cm	Size	Pitch mm	Stress area sq cm	Size	Pitch mm	Stress area sq cm
M 8×1	1	0 392	M 18×1 5	1 5	2 16	M 30×2	2	6 21
M 10×1 25	1·25	0 616	M 20×1 5	1 5	2 72	M 33×2	2	7 61
M 12×1 25	1·25	0 921	M 22×1 5	1 5	3 33	M 36×3	3	8 63
M 14×1 5	1·5	1 25	M 24×2	2	3 84	M 39×3	3	10 28
M 16×1·5	1·5	1·67	M 27×2	2	4 96			

Nominal diameter d mm	Core diameter d_c mm	Mean diameter d_m mm	Core area a_c sq cm	Pitch p mm
82	71·5	77	40·15	10
85	72·5	79	41·28	12
88	75·5	82	44·77	12
90	77·5	84	47·17	12
92	79·5	86	49·64	12
95	82·5	89	53·46	12
98	85·5	92	57·41	12
100	87·5	94	60·13	12
105	92·5	99	67·20	12
110	97·5	104	74·66	12
115	100	108	78·54	14
120	105	113	86·59	14
125	110	118	95·03	14
130	115	123	103·87	14
135	120	128	113·1	14
140	125	133	122·72	14
145	130	138	132·73	14
150	133	142	138·93	16
155	138	147	149·57	16
160	143	152	160·61	16
165	148	157	172·03	16
170	153	162	183·85	16
175	158	167	196·07	16

APPENDIX — VII

Common sizes of transmission shafts (Dimensions in mm)

25	55	110	220	340	480
30	60	125	240	360	500
35	70	140	260	400	
40	80	160	280	420	
45	90	180	300	440	
50	100	200	320	460	

APPENDIX — VIII

...zes of pulleys for flat and V belts (Dimensions in mm)

...25*, 28*, 32*, 36*, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180,
...4, 250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120,
...400, 1600, 1800, 2000, 2240, 2500, 2800, 3150, 3550, 4000, 4500, 5000

... sizes marked with * are used for only V belt drives.

APPENDIX — VI

Proportions for trapezoidal threads

The trapezoidal threads are commonly adopted for power screws. The angle of thread is 30° *Tr* 60 × 9 means the outside diameter or nominal diameter of the screw is 60 mm and pitch of the thread is 9 mm *Tr* means trapezoidal threads.

Core diameter = Nominal diameter — pitch — clearance.

Mean diameter = Nominal diameter $- \frac{\text{pitch}}{2}$.

Clearance is 0·5 mm for nominal diameter upto 110 mm and 1 mm for nominal diameter more than 110 mm

The table of trapezoidal threads is given below.

Trapezoidal threads

Nominal diameter d mm	Core diameter dc mm	Mean diameter dm mm	Core area ac sq cm	Pitch p mm
10	6·5	8·5	0·33	3
12	8·5	10·5	0·57	3
14	9·5	12	0·71	4
16	11·5	14	1·04	4
18	13·5	16	1·43	4
20	15·5	18	1·89	4
22	16·5	19·5	2·14	5
24	18·5	21·5	2·69	5
26	20·5	23·5	3·30	5
28	22·5	25·5	3·89	5
30	23·5	27	4·34	[illegible]
32	25·5	29	5·11	6
34	27·5	31	5·94	[illegible]
36	29·5	33	6·83	6
38	30·5	34·5	7·91	7
40	32·5	36·5	8·30	7
42	34·5	38·5	9·35	7
44	36·5	40·5	10·46	7
46	37·5	42·0	11·04	8
48	39·5	44	12·25	[illegible]
50	41·5	46	13·53	8
52	43·5	48	14·86	[illegible]
55	45·5	50·5	16·26	9
58	48·5	53·5	18·47	9
60	50·5	55·5	20·03	9
62	52·5	57·5	21·65	9
65	54·5	60	23·33	10
68	57·5	63	25·97	10
70	59·5	65	27·81	10
72	61·5	67	29·71	10
75	64·5	70	32·67	10
78	67·5	73	35·78	10
80	69·5	75	37·94	10

APPENDIX — IX

Width of flat cast iron and mild steel pulleys
(Dimensions in mm)

16, 20, 25, 32, 40, 50, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 315, 355, 400, 450, 560, 630.

APPENDIX — X

Service factor for belt drive

Character of load	Source of power		
	Electric motor	Multi-cylinder engine	High torque drive
Uniform (centrifugal fans and blowers, centrifugal compressors, belt conveyors)	1·1	1·2	1·4
Moderate shock (reciprocating compressors, grinders, bucket conveyors, line shafts)	1·2	1·4	1·6
Heavy shock (propeller fans and blowers)	1·4	1·6	1·8

APPENDIX — XI

Load carrying capacity of V belts

Belt section	Nominal load per belt kg	Minimum sheave diameter mm
A	15	75
B	20	125
C	50	225
D	90	325
E	125	500

APPENDIX — XII

Worm data

Number of threads	Velocity ratio	Number of threads	Velocity ratio
Single	20 and over	Quadruple	6-12
Double	12-36	Sextuple	4-10
Triple	8-12		

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